

Power Allocation in Two-Way Relay Networks with MABC DF Protocol and No Instantaneous Channel State Information

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Abstract In this paper we examine a two way relay network with multiple access broadcast, decode and forward protocol. A new power allocation scheme is proposed which is based on minimizing the outage probability. The outage probability is defined as the event in which the data rate of the different nodes lie outside the achievable rate region. For the first time, a closed-form mathematical expression is derived for the outage probability of such system, in the more general case than previous attempts. The only limiting constraint that is enforced on the problem is that the relay power is less than the power of the end main nodes (terminals). The obtained expression is then considered as the cost function of an optimization problem, in order to be minimized by allocating appropriate power to the nodes. The proposed scheme for power allocation needs no instantaneous channel coefficient estimation, but rather needs only statistical Channel State Information. The correctness of the obtained expression for the outage probability is verified by Monte Carlo simulations and the performance of the proposed scheme for power allocation is analyzed numerically. We have demonstrated that this technique can provide up to 1 dB gain in outage probability for average SNR values.

Keywords Two-way relay network · Multiple access broadcast · Decode and forward · Power allocation · Outage probability

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1 Introduction

1.1 Background of Cooperative Networks

Designers of telecommunication systems have always attempted to design systems transmitting information with higher quality i.e. higher speed and higher reliability. Using multiple antennas in the transmitter and/or receiver has been one of the most effective techniques to achieve this target. In multiple antenna systems, higher speed (or equivalently higher capacity) can be achieved by spatial multiplexing and higher reliability (i.e. lower probability of error or lower probability of outage) is achievable by means of space–time coding schemes [1]. Spatial multiplexing in Multiple-Input Multiple-Output (MIMO) wireless communication is a transmission technique that divides the incoming data into multiple parallel sub-streams and transmits each on a different antenna [2] while space–time coding schemes rely on transmitting multiple, redundant copies of data signals on the antennas.

In practice, it may be impractical to have multiple antennas in one device due to size, cost and/or hardware limitations [3]. The idea of cooperative (or relay) networks is the solution to this problem. In the cooperative networks, different nodes of the network, located in different locations, cooperate with each other and provide a system of distributed antennas and as a result the same benefits as the conventional MIMO wireless systems.

A very vast amount of research in the field of relay networks has been conducted so far and various system models have also been proposed in order to provide a special advantage. One Way Relay Networks (OWRN), as depicted in Fig. 1 are one of the simplest models in which information is flowed in one direction from the source node toward the destination node.

Two-Way Relay Networks (TWRNs) on the contrary are ones in which two distinct terminals exchange information via a single or multiple relay nodes and therefore a bidirectional data flow happens. A OWRN, as depicted in Fig. 2a simply needs four time-slots to handle information bidirectionally (two time-slots for each direction); while in a network with no relays two time-slots are sufficient (Fig. 2b). Therefore the capacity of a bidirectional relay channel is half of the bidirectional channel with no relays.

Recent TWRN protocols complete the information exchange between two source nodes in two or three transmission phases by employing the network coding schemes, designed for multicast [4]. In the next subsection where these protocols are explained, we will elaborate on how network coding schemes can improve the spectral efficiency of a TWRN.

1.2 Protocols of the Cooperative Networks

Several different protocols have been proposed so far which can be served as the basis of the transmission technique in a TWRN. The two most important of these protocols are

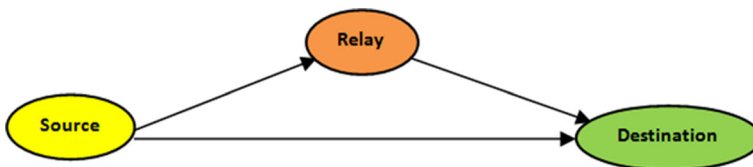


Fig. 1 One-way relay network

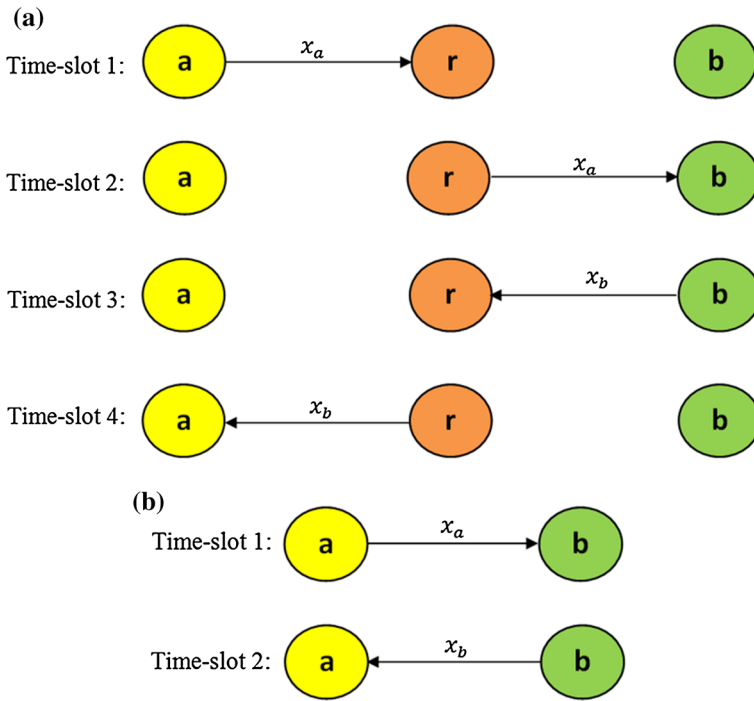


Fig. 2 Steps of information exchange in a **a** TWRN with no use of network coding; **b** direct channel with no relays

called time division broadcast (TDBC) and multiple access broadcast (MABC) which assume three and two time-slots respectively for the transmission of messages [5]. Kim et al. [6] have provided an overview of the different protocols for both the TDBC and the MABC in bidirectional relaying networks along with their corresponding achievable rate region analysis. The TDBC transmission protocol runs in three time-slots (Fig. 3):

- I. Node *a* transmits its message x_a to the relay;
- II. Node *b* transmits its message x_b to the relay;
- III. The relay combines x_a and x_b by means of a network coding scheme and produces $x_r = f(x_a, x_b)$ and then broadcasts (BCs) it (for which this phase is called the BC

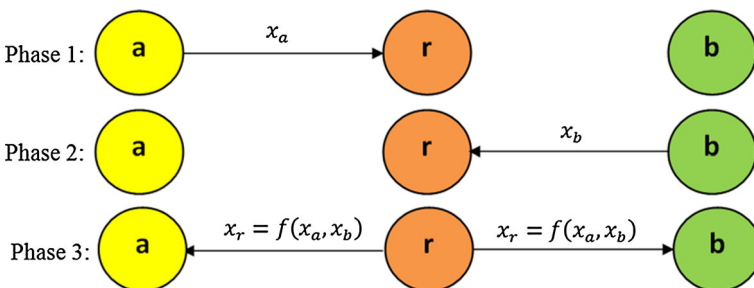


Fig. 3 Steps of information exchange in a TWRN with TDBC protocol

phase). Each of the terminals performs a self-interference cancellation process over the received signal x_r and extracts the intended message (b extracts x_a and a extracts x_b from x_r).

In the MABC transmission protocol on the other hand, messages are exchanged during the two time-slots and therefore it is more spectral efficient than the TDBC, although it is more complex to implement. These two phases are as follows (Fig. 4):

- I. Nodes a and b transmit, synchronously, their messages x_a and x_b to the relay by forming a conventional MAC channel [5] (for which this phase is called the MAC phase). We will give exact details of this phase in part 2.1.
- II. The second phase is exactly the same as the third phase in the TDBC protocol.

In addition to the above mentioned transmission techniques, different relaying schemes have been proposed in the literature [7–11]. The two most important are:

- I. Amplify and Forward (AF): The relay just amplifies the received signal and then retransmits it. Although this scheme needs no computation in the relay, but suffers from the noise amplification in relay.
- II. Decode and Forward (DF): The relay decodes the received messages from terminals and then re-encodes and transmits them. As a result, the noise is not amplified by using this scheme, although a large amount of computation is required at the relay.

1.3 Improving the Performance of Cooperative Networks

One of the most noticeable topics of research in the field of relay networks is the performance improvement. For performance improvement, one should choose a performance criterion and then devise a method to make that network perform better in terms of that criterion. Among various criteria such as minimizing the error probability, outage probability, power consumption or maximizing the energy efficiency, spectral efficiency, etc. we have concentrated, in this paper, on the outage probability. We will clarify the definition of outage probability in part 2.2.

Along with the appropriate criterion, the method for performance improvement is also important. Resource allocation (for instance power allocation) and relay selection (in multiple relay networks) are two examples. Optimal power allocation (OPA) to the active nodes is an effective method to improve the performance of a relay network. Authors in [4, 12–16] have proposed various approaches to determine the optimal power of the nodes in a DF TWRN. The criterion considered in [12, 13, 16] is the achievable sum-rate which is aimed to be maximized. Authors in [14] propose a power optimization scheme in order to minimize the outage probability. In [4, 15], end-to-end BER has been chosen as the

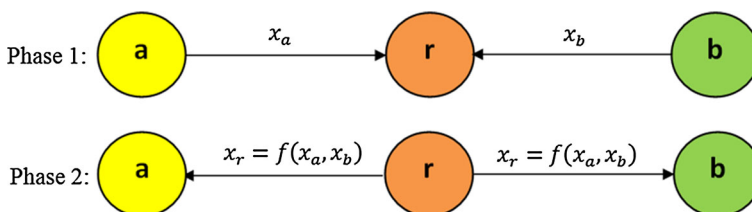


Fig. 4 Steps of information exchange in a TWRN with MABC protocol

performance metric. The power allocation techniques proposed in [12, 13, 15, 16] have been based on the assumption that all the channel coefficients are known prior to determining the power values. In [14] some information such as antenna gains, path loss factor and distance between transmitters and receivers are required to evaluate the power values. It is assumed in [15] that mean channel gains are obtained by channel estimation.

1.4 Main Contributions of This Work

In this paper we propose a novel method for power allocation in a DF TWRN in which neither instantaneous value of the channel coefficients nor the antenna parameters are required. The only assumption we have made about the channels is the Rayleigh flat (not necessarily slow) fading. In other words we have assumed that only statistical Channel State Information (CSI) is available and instantaneous CSI is not required. Authors in [17] have obtained an expression describing the outage probability of a DF TWRN for two special cases: (a) all the nodes transmit with the same power but different rates; (b) all the nodes transmit with the same rate but different power. We have derived, for the first time, a mathematical expression, describing the outage probability of a TWRN in a more general case. In fact we have imposed no restriction on the rates of the nodes. The only constraint we have considered is that the power of the relay is no greater than the power of two end terminals. This expression is then regarded as the cost function and we attempt to minimize it by selecting appropriate values for the power of the nodes.

The remainder of this paper is organized as follows: In Sect. 2 the structure of the network and its performance metric are described. In Sect. 3 the optimization problem is introduced and solved, clarifying the power allocation scheme. Section 4 is devoted to simulation results and numerical analysis. Finally we conclude the paper in Sect. 5.

2 System Model, Notations and the Performance Metric

This section describes the structure of the network and the performance metric used for OPA.

2.1 Network Model

We assume a TWRN consisting of two terminals a and b and one relay. Terminals a and b are supposed to transmit independent messages w_a and w_b respectively to each other. It is assumed that all the nodes use the same codebook. There is no direct link between a and b and the transmission is proceeded only via the relay. All the nodes are considered as half-duplex single antenna transceivers. The physical channels between the terminals and the relay are reciprocal and described by h_a and h_b which are zero-mean independent circularly symmetric complex Gaussian random variables with unit variance. The additive noise in receivers is also considered as a zero-mean circularly symmetric complex Gaussian random variable with variance σ^2 . Noise processes are independent in different receivers. The channel coefficients are not required to be constant during a transmission phase. Furthermore we have made no assumption of instantaneous channel coefficients being known.

We use R_a and R_b to denote the data rate of messages w_a and w_b respectively, i.e., $w_a \in Z_a = \{0, \dots, [2^{R_a}] - 1\}$ and $w_b \in Z_b = \{0, \dots, [2^{R_b}] - 1\}$ and x_a and x_b for corresponding transmitted signals of the two terminals. Moreover the transmission power of the three nodes are represented by P_a , P_b and P_r .

The transmission protocol considered here is MABC DF which assumes two phases for information exchange. In the first phase, a and b transmit their messages to the relay by forming a conventional MAC channel [5]. The relay receives the signal

$$y_r = h_a x_a + h_b x_b + n_r, \tag{1}$$

and then extracts the messages w_a and w_b by Maximum Likelihood (ML) decoding [18]

$$(\bar{w}_a, \bar{w}_b) = \arg \min_{(w_a, w_b) \in Z_a \times Z_b} |y_r - \sqrt{P_a} h_a x_a(w_a) - \sqrt{P_b} h_b x_b(w_b)|. \tag{2}$$

In the second phase the relay exploits a network coding scheme to combine messages and transmit to terminals. It reproduces a new signal x_r , by bit-wise *xor* operation as a network coding scheme ($x_r = x_a \oplus x_b$) and then broadcasts the corresponding signal. Terminals a and b receive the signal x_r and then perform a self-interference cancellation operation (a *xors* x_r with x_a to extract the intended signal x_b and b *xors* x_r with x_b to extract the intended signal x_a). Figure 5 depicts the structure of the network and the steps of information exchange between a and b .

Our attention in this work is only on the outage probability, not error probability; hence we assume that the relay and terminals are able to decode x_a , x_b and x_r correctly without any error.

Although R_a and R_b are not confined to be equal, it is assumed that x_a and x_b are of the same length, since the bitwise *xor* operation requires the signals from a and b to have the same length [19].

Table 1 is included at the end of this part which summarizes all the notations and symbols that have been used in this work.

2.2 Performance Metric

We aim at setting the power values P_a , P_b and P_r in order to minimize the outage probability. For any communication network, if the transmission rates of the nodes exceed out of a specific region, named achievable rate region, an outage will occur. The achievable rate region for the network described in Sect. 2.1 is determined by the constraints [5]

$$R_a \leq \frac{1}{2} \min \left\{ \log_2 \left(1 + P_a \frac{|h_a|^2}{\sigma^2} \right), \log_2 \left(1 + P_r \frac{|h_b|^2}{\sigma^2} \right) \right\}, \tag{3}$$

Fig. 5 Structure of the TWRN with MABC DF protocol and the steps of information exchange. **a** Multiple access phase. **b** Broadcast phase

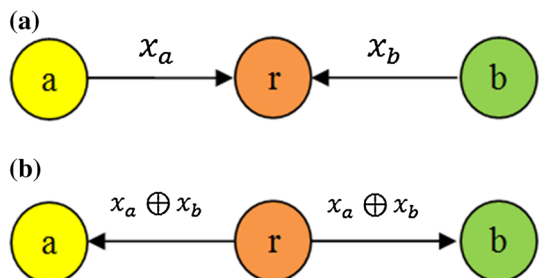


Table 1 Notations and symbols of the article

Symbol	Description	Symbol	Description
$w_i, i \in \{a, b\}$	Message of the terminal i	σ^2	Variance of the noise
$h_i, i \in \{a, b\}$	Channel Coefficient between terminal i and relay	$R_i, i \in \{a, b\}$	Data rate of the message w_i
$x_i, i \in \{a, b\}$	Transmitted signal of the terminal i	$\lfloor n \rfloor$	The first integer number $\leq n$
$P_i, i \in \{a, b, r\}$	Transmission power of the node i	$n_i, i \in \{a, b, r\}$	The AWGN at the receiver of node i
$y_i, i \in \{a, b, r\}$	Received signal at the node i	$\bar{w}_i, i \in \{a, b\}$	Message of terminal i extracted from received signal in relay by ML decoding
$pr\{\cdot\}$	The probability of an random event	\cup	The union symbol in the set theory
\cap	The intersection symbol in the set theory	A'	Complement of a set A
$\&$	Logical AND operation	$ $	Logical OR operation

$$R_b \leq \frac{1}{2} \min \left\{ \log_2 \left(1 + P_b \frac{|h_b|^2}{\sigma^2} \right), \log_2 \left(1 + P_r \frac{|h_a|^2}{\sigma^2} \right) \right\}, \tag{4}$$

$$R_a + R_b \leq \frac{1}{2} \log 2 \left(1 + P_a \frac{|h_a|^2}{\sigma^2}, 1 + P_b \frac{|h_b|^2}{\sigma^2} \right). \tag{5}$$

If at least one of the constraints in (3)–(5) are not satisfied, an outage will occur.

3 Problem Formulation and Solution

We intend to find values P_a, P_b and P_r by solving the problem

$$\begin{aligned} & \text{minimize } P_{outage}(P_a, P_b, P_r) \\ \text{s.t. } & P_a + P_b + P_r = P_{sum}, P_r \leq P_a, P_r \leq P_b \end{aligned} \tag{6}$$

The conditions $P_r \leq P_a$ and $P_r \leq P_b$ have been considered to ensure that the relay is only an assistant node, aiding terminals a and b in their communications and therefore should consume the least amount of power between all the nodes.

3.1 Calculating the Outage Probability

According to Eqs. (3)–(5) we define three probability events A, B and C as

$$A = \left\{ (P_a, P_b, P_r) \mid R_a > \frac{1}{2} \min \left\{ \log_2 \left(1 + P_a \frac{|h_a|^2}{\sigma^2} \right), \log_2 \left(1 + P_r \frac{|h_b|^2}{\sigma^2} \right) \right\} \right\}, \tag{7}$$

$$B = \left\{ (P_a, P_b, P_r) \mid R_b > \frac{1}{2} \min \left\{ \log_2 \left(1 + P_b \frac{|h_b|^2}{\sigma^2} \right), \log_2 \left(1 + P_r \frac{|h_a|^2}{\sigma^2} \right) \right\} \right\}, \tag{8}$$

$$C = \left\{ (P_a, P_b, P_r) \mid R_a + R_b > \frac{1}{2} \log_2 \left(1 + P_a \frac{|h_a|^2}{\sigma^2} + P_b \frac{|h_b|^2}{\sigma^2} \right) \right\}. \tag{9}$$

As mentioned earlier in Sect. 2.2 the outage probability is

$$P_{outage} = pr\{A \cup B \cup C\} = pr\{(A' \cap B' \cap C')'\} = 1 - pr\{A' \cap B' \cap C'\} = 1 - \Gamma, \tag{10}$$

where we have employed De Morgan’s law in the second equality. Now we define two new random variables $X = P_a \frac{|h_a|^2}{\sigma^2}$ and $Y = P_b \frac{|h_b|^2}{\sigma^2}$ and from these $U = P_r \frac{|h_b|^2}{\sigma^2} = \frac{P_r}{P_b} Y$ and $V = P_r \frac{|h_a|^2}{\sigma^2} = \frac{P_r}{P_a} X$. For the sake of simplicity we make use of substitutions $\lambda_1 = \frac{\sigma^2}{P_a}$, $\lambda_2 = \frac{\sigma^2}{P_b}$, $\lambda_3 = \frac{P_r}{P_b}$ and $\lambda_4 = \frac{P_r}{P_a}$. X and Y are exponential random variables with parameters λ_1 and λ_2 respectively, because of the Rayleigh channel assumption. Specifically $f_X(x) = \lambda_1 e^{-\lambda_1 x}$, $x > 0$ and $f_Y(y) = \lambda_2 e^{-\lambda_2 y}$, $y > 0$. It is evident that X and Y are independent random variables; consequently $f_{XY}(x, y) = f_X(x)f_Y(y)$.

As a result of above definitions and assumptions we can write

$$\begin{aligned} \Gamma &= pr \left\{ R_a \leq \frac{1}{2} \min \{ \log_2(1 + X), \log_2(1 + \lambda_3 Y) \}, \right. \\ &\quad R_b \leq \frac{1}{2} \min \{ \log_2(1 + Y), \log_2(1 + \lambda_4 X) \}, \\ &\quad \left. R_a + R_b \leq \frac{1}{2} \log_2(1 + X + Y) \right\}. \end{aligned} \tag{11}$$

Logarithm is a monotonically increasing function. So operators ‘min’ and ‘log’ are able to be exchanged in (11) so that

$$\Gamma = pr\{R_1 \leq \min\{X, \lambda_3 Y\}, R_2 \leq \min\{Y, \lambda_4 X\}, R_3 \leq X + Y\}, \tag{12}$$

where we have used $R_1 = 2^{2R_a} - 1$, $R_2 = 2^{2R_b} - 1$ and $R_3 = 2^{2(R_a+R_b)} - 1$ in (12) for simplicity. By exploiting “Total probability” theorem [20], Eq. (12) can be expanded as

$$\begin{aligned} \Gamma &= pr\{R_1 \leq X, R_2 \leq Y, R_3 \leq X + Y \mid X < \lambda_3 Y, Y < \lambda_4 X\} \cdot pr\{X < \lambda_3 Y, Y < \lambda_4 X\} + \\ &\quad pr\{R_1 \leq X, R_2 \leq \lambda_4 X, R_3 \leq X + Y \mid X < \lambda_3 Y, \lambda_4 X < Y\} \cdot pr\{X < \lambda_3 Y, \lambda_4 X < Y\} + \\ &\quad pr\{R_1 \leq \lambda_3 Y, R_2 \leq Y, R_3 \leq X + Y \mid \lambda_3 Y < X, Y < \lambda_4 X\} \cdot pr\{\lambda_3 Y < X, Y < \lambda_4 X\} + \\ &\quad pr\{R_1 \leq \lambda_3 Y, R_2 \leq \lambda_4 X, R_3 \leq X + Y \mid \lambda_3 Y < X, \lambda_4 X < Y\} \cdot pr\{\lambda_3 Y < X, \lambda_4 X < Y\}, \end{aligned} \tag{13}$$

or equivalently

$$\begin{aligned} \Gamma &= pr\{R_1 \leq X, R_2 \leq Y, X < \lambda_3 Y, Y < \lambda_4 X, R_3 \leq X + Y\} + \\ &\quad pr\{R_1 \leq X, R_2 \leq \lambda_4 X, X < \lambda_3 Y, \lambda_4 X < Y, R_3 \leq X + Y\} + \\ &\quad pr\{R_1 \leq \lambda_3 Y, R_2 \leq Y, \lambda_3 Y < X, Y < \lambda_4 X, R_3 \leq X + Y\} + \\ &\quad pr\{R_1 \leq \lambda_3 Y, R_2 \leq \lambda_4 X, \lambda_3 Y < X, \lambda_4 X < Y, R_3 \leq X + Y\}. \end{aligned} \tag{14}$$

Denoting four terms of (14) with $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 respectively, we attempt to obtain a mathematical solution for them. Specifically

$$\Gamma_i = \iint_{S_i} f_X(x) f_Y(y) dx dy, \quad i = 1, 2, 3, 4, \tag{15}$$

where S_i , the probability region for Γ_i , must be determined. It should be noted that we have assumed, without the loss of generality that $R_a \geq R_b$. Clearly this assumption does not deteriorate the generality of the problem because of the symmetric nature of the network. The probability regions for Γ_i s have been investigated in detail in the appendix.

Based on the probability regions displayed in the appendix, we can perform the integrations in (15). Calculating (15) over different probability regions and then simplifying the terms obtained is a straightforward but long process. We disregard the details and suffice to state the final result.

For the case $P_a \neq P_b$ (or equivalently $\lambda_3 \neq \lambda_4$) Γ will be a four-objective function:

1. If $\left(\frac{R_2}{\lambda_4} \leq \frac{\lambda_3}{1+\lambda_3} R_3 \ \& \ \frac{R_1}{\lambda_3} \leq \frac{1}{1+\lambda_3} R_3 \ \& \ R_1 \leq \frac{R_2}{\lambda_4}\right) \left| \left(\frac{\lambda_3}{1+\lambda_3} R_3 \leq \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4} R_3 \ \& \ \frac{R_1}{\lambda_3} \leq R_3 - \frac{R_2}{\lambda_4}\right)\right.$
then

$$\Gamma = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 R_3} \left[e^{(\lambda_2 - \lambda_1) \left(R_3 - \frac{R_1}{\lambda_3}\right)} - e^{(\lambda_2 - \lambda_1) \frac{R_2}{\lambda_4}} \right] + e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\lambda_1 \left(R_3 - \frac{R_1}{\lambda_3}\right)}; \tag{16}$$

2. If $\left(\frac{R_2}{\lambda_4} \leq \frac{\lambda_3}{1+\lambda_3} R_3 \ \& \ \frac{R_1}{\lambda_3} \leq \frac{1}{1+\lambda_3} R_3 \ \& \ \frac{R_2}{\lambda_4} \leq R_1\right)$ then

$$\Gamma = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 R_3} \left[e^{(\lambda_2 - \lambda_1) \left(R_3 - \frac{R_1}{\lambda_3}\right)} - e^{(\lambda_2 - \lambda_1) R_1} \right] + e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\lambda_1 \left(R_3 - \frac{R_1}{\lambda_3}\right)}; \tag{17}$$

3. If $\left(\frac{R_2}{\lambda_4} \leq \frac{\lambda_3}{1+\lambda_3} R_3 \ \& \ \frac{1}{1+\lambda_3} R_3 \leq \frac{R_1}{\lambda_3}\right) \left| \left(\frac{\lambda_3}{1+\lambda_3} R_3 \leq \frac{R_2}{\lambda_4} \ \& \ \frac{R_2}{\lambda_3 \lambda_4} \leq \frac{R_1}{\lambda_3}\right)\right.$ then

$$\Gamma = e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\lambda_1 R_1}; \tag{18}$$

4. If $\left(\frac{\lambda_3}{1+\lambda_3} R_3 \leq \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4} R_3 \ \& \ R_3 - \frac{R_2}{\lambda_4} \leq \frac{R_1}{\lambda_3} \leq \frac{R_2}{\lambda_3 \lambda_4}\right) \left| \left(\frac{1}{1+\lambda_4} R_3 \leq \frac{R_2}{\lambda_4} \ \& \ R_2 \leq \frac{R_1}{\lambda_3} \leq \frac{R_2}{\lambda_3 \lambda_4}\right)\right.$
then

$$\Gamma = e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\frac{\lambda_1}{\lambda_4} R_2}. \tag{19}$$

For the case $P_a = P_b$ (or equivalently $\lambda_3 = \lambda_4$) Γ will be a two-objective function:

1. If $R_1 \leq \frac{R_2}{\lambda_4}$ then

$$\Gamma = \lambda_1 e^{-\lambda_2 R_3} \left[\left(R_3 - \frac{R_1}{\lambda_3} \right) - \frac{R_2}{\lambda_4} \right] + e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\lambda_1 \left(R_3 - \frac{R_1}{\lambda_3} \right)}; \tag{20}$$

2. If $\frac{R_2}{\lambda_4} \leq R_1$ then

$$\Gamma = \lambda_1 e^{-\lambda_2 R_3} \left[\left(R_3 - \frac{R_1}{\lambda_3} \right) - R_1 \right] + e^{-\frac{\lambda_2}{\lambda_3} R_1} e^{-\lambda_1 \left(R_3 - \frac{R_1}{\lambda_3} \right)}. \tag{21}$$

As a result we have obtained a mathematical solution for $\Gamma = 1 - P_{outage}$ as stated in Eqs. (16)–(21). As it is apparent from these equations, Γ is not dependent on the value of channel coefficients (instantaneous CSI), but rather on statistical CSI ($\lambda_1 = \frac{\sigma^2}{P_a}$, $\lambda_2 = \frac{\sigma^2}{P_b}$) and $\lambda_3 = \frac{P_c}{P_b}$, $\lambda_4 = \frac{P_c}{P_a}$, R_1 and R_2 that are constant parameters. This is why we have claimed that our approach for power allocation doesn't require any instantaneous CSI. In other words we have made use of probabilistic features (probability distribution function, mean and variance value) of channel coefficients and derived an equation for outage probability which is only relevant to the channel parameters.

3.2 Minimizing the Outage Probability

The mathematical expression of outage probability is too complex to examine in terms of convexity. On the other hand we are required to find the optimal value (i.e. global minimum) of the problem (6), while common optimization techniques might result in local minimum points. Analyzing the problem in terms of convexity and proposing an optimization technique can form the basis of another separate work in future. For the present article, we have alternatively utilized a simple and effective technique to solve the optimization problem (6) that is also capable of finding the global minimum. Solving problem (6) is equivalent to maximize $\Gamma(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$. To solve this problem, we use “grid search” technique to find the optimal power values. In fact, the whole domain of the function Γ is investigated to find the optimal solution. Although this technique might seem inefficient and time-consuming, but in the case of our optimization problem and its constraints, it can be useful. Specifically, as we will clarify the details of this technique, the domain which must be investigated is a two-dimensional region, i.e. an 1×1 square in the $\lambda_3 - \lambda_4$ plane and the two other variables, λ_1 and λ_2 , can be calculated from λ_3 and λ_4 .

Details of the steps of this technique is as follows:

1. Let $\lambda_3 = 1$;
2. Let $\lambda_4 = 1$;
3. Calculate λ_1 and λ_2 :
 - (a) $P_r = \frac{P_{sum}}{1 + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}}$;
 - (b) $P_a = \frac{P_c}{\lambda_4}$ and $P_b = \frac{P_c}{\lambda_3}$;
 - (c) $\lambda_1 = \frac{\sigma^2}{P_a}$ and $\lambda_2 = \frac{\sigma^2}{P_b}$;
4. Calculate $\Gamma(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ by Eqs. (16)–(21);
5. Substitute $\lambda_4 = \lambda_4 - \Delta$. if $\lambda_4 > 0$ go to step 3;
6. Substitute $\lambda_3 = \lambda_3 - \Delta$. If $\lambda_3 > 0$ go to step 2;
7. Find the maximum value of Γ and corresponding (P_a, P_b, P_r) .

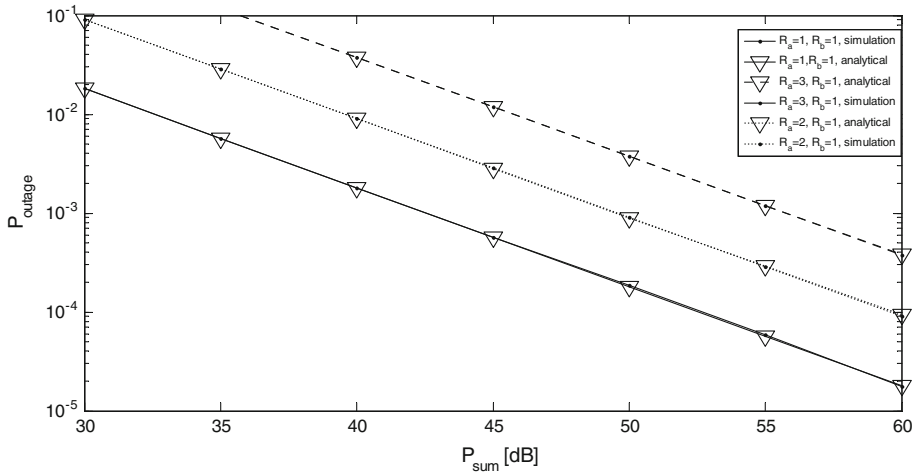


Fig. 6 Outage probability versus $P_{sum} = P_a + P_b + P_r$ under different sum-rate $R_a + R_b$ values and equal power allocation $P_a = P_b = P_r = \frac{P_{sum}}{3}$, by both analytical and simulation schemes

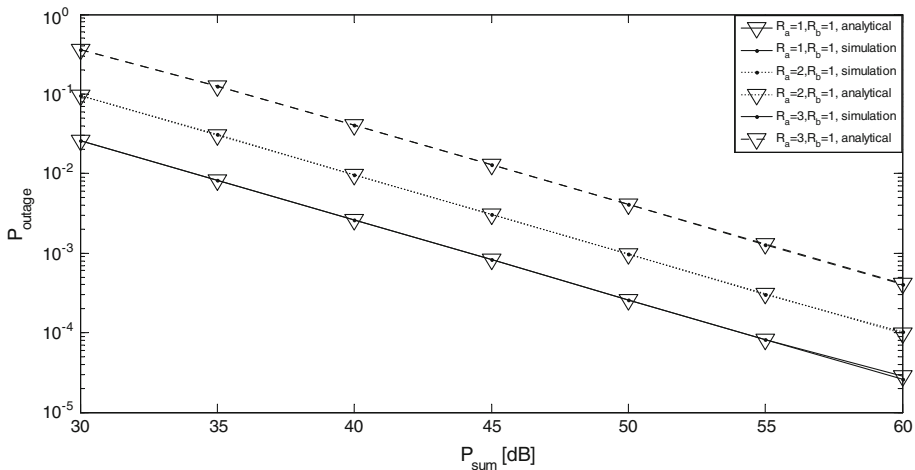


Fig. 7 Outage probability versus $P_{sum} = P_a + P_b + P_r$ under different sum-rate $R_a + R_b$ values and unequal power allocation $P_r = \frac{3}{4}P_b = \frac{1}{2}P_a$; by both analytical and simulation schemes

In this procedure Δ is a step value for changing λ_3 and λ_4 in order to search through the entire feasible set of the optimization problem. It is necessary to select Δ small enough so that the global maximum point will be found in better precision. The proposed grid search technique is able to find the optimum point fast enough owing to the fact that the variation interval of λ_3 and λ_4 is too short (since $\lambda_3, \lambda_4 \in [0, 1]$) and inspection through this limited interval spends little time.

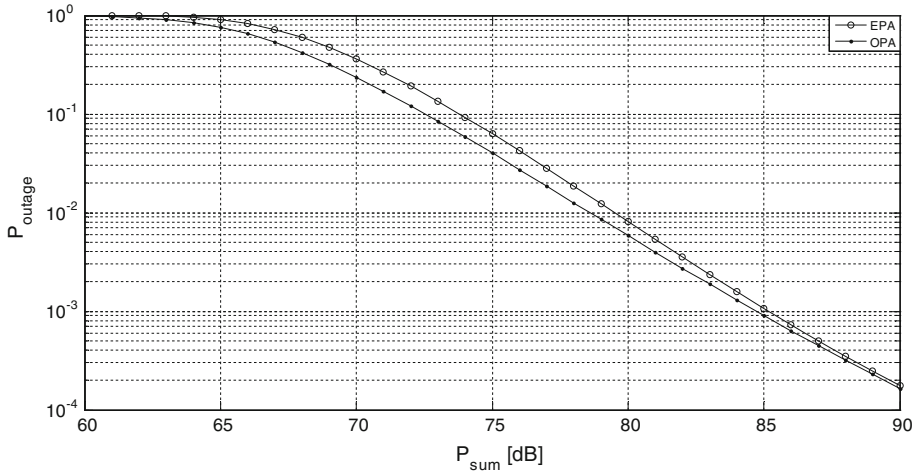


Fig. 8 Comparison of OPA and EPA schemes, $R_a = 7, R_b = 4$ bits/s/Hz

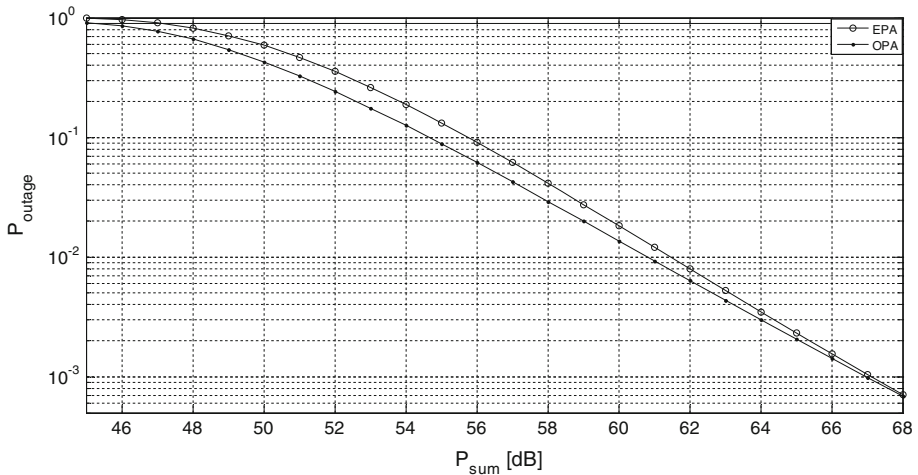


Fig. 9 Comparison of OPA and EPA schemes, $R_a = 4, R_b = 4$ bits/s/Hz

4 Simulation and Numerical Results

In this section we intend to verify Eqs. (16)–(21) by Monte Carlo simulations. Then the proposed scheme for power allocation in a TWRN is examined and numerical results are stated. σ , the variance of the noise, has been set to unity for simulation and analysis of this part. All the figures represent the outage probability P_{outage} versus $SNR = \frac{P_{sum}}{\sigma^2}$ which is equivalent to P_{sum} (since $\sigma = 1$) and specified in dB.

In Figs. 6 and 7, the curves specified by dots, have been plotted by Monte Carlo simulation technique, while in the curves specified by triangles, P_{outage} has been figured out by mathematical Eqs. (16)–(21), derived in part 3.1. As it is indicated by these figures,

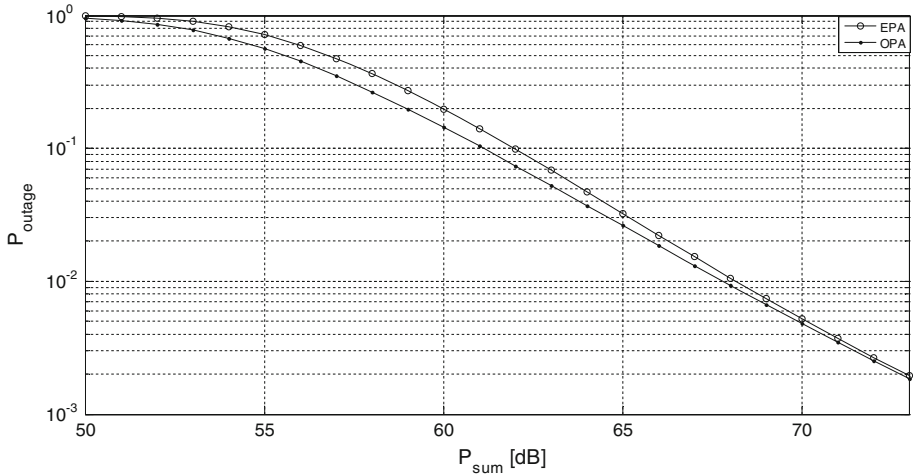


Fig. 10 Comparison of OPA and EPA schemes, $R_a = 6, R_b = 3$ bits/s/Hz

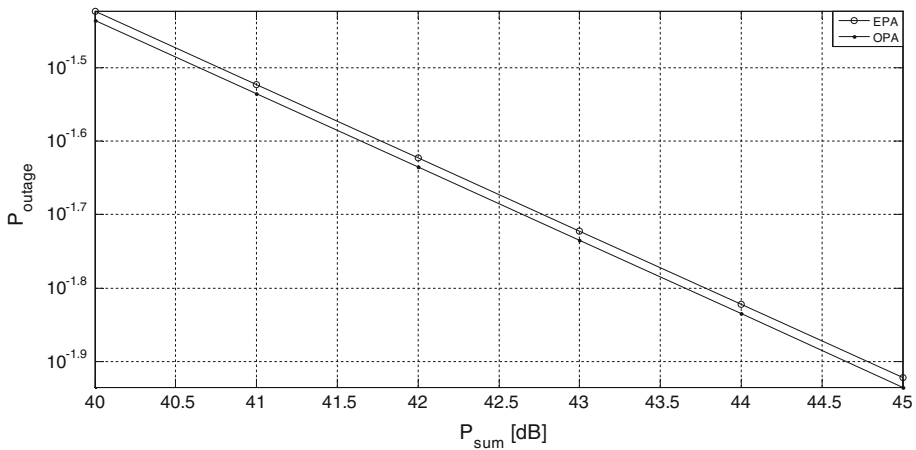


Fig. 11 Comparison of OPA and EPA schemes, $R_a = 3, R_b = 1$ bits/s/Hz

simulation and analytical curves coincide with each other exactly and therefore simulation curves verify the correctness of mathematical analysis.

In Fig. 6 the total power has been divided equally between three nodes but in Fig. 7 different power has been considered for the nodes.

In Figs. 8, 9 and 10 we have demonstrated the superiority of our proposed scheme for power allocation (i.e. OPA) over the equal power allocation (EPA) scheme for various values of data rate. In the curves specified by black dots, power of the nodes has been figured out by the OPA scheme; while in the curves specified by white dots, EPA scheme has been utilized. As it is indicated by these figures, for average SNR values, OPA scheme results in about 1 dB gain over EPA; although as $SNR \rightarrow \infty$ the advantage of OPA becomes less significant and in fact decreases to about 0.1 dB.

Fig. 14 The probability region for Γ_4 ; **a** $\frac{R_2}{\lambda_4} < \frac{\lambda_3}{1+\lambda_3}R_3, \frac{R_1}{\lambda_3} \leq \frac{\lambda_4}{1+\lambda_4}R_3$; **b** $\frac{R_2}{\lambda_4} \leq \frac{\lambda_3}{1+\lambda_3}R_3, \frac{\lambda_4}{1+\lambda_4}R_3 < \frac{R_1}{\lambda_3} \leq \frac{1}{1+\lambda_3}R_3$; **c** $\frac{R_2}{\lambda_4} \leq \frac{\lambda_3}{1+\lambda_3}R_3, \frac{1}{1+\lambda_3}R_3 < \frac{R_1}{\lambda_3}$; **d** $\frac{\lambda_3}{1+\lambda_3}R_3 < \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4}R_3, \frac{R_1}{\lambda_3} \leq \frac{\lambda_4}{1+\lambda_4}R_3$; **e** $\frac{\lambda_3}{1+\lambda_3}R_3 < \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4}R_3, \frac{\lambda_4}{1+\lambda_4}R_3 < \frac{R_1}{\lambda_3} \leq \frac{1}{1+\lambda_3}R_3$; **f** $\frac{\lambda_3}{1+\lambda_3}R_3 < \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4}R_3, R_3 - \frac{R_2}{\lambda_4} < \frac{R_1}{\lambda_3} \leq \frac{1}{\lambda_3\lambda_4}R_2$; **g** $\frac{\lambda_3}{1+\lambda_3}R_3 < \frac{R_2}{\lambda_4} \leq \frac{1}{1+\lambda_4}R_3, \frac{1}{\lambda_3\lambda_4}R_2 < \frac{R_1}{\lambda_3}$; **h** $\frac{R_2}{\lambda_4}, \frac{R_1}{\lambda_3} \leq R_2$; **i** $\frac{1}{1+\lambda_4}R_3 < \frac{R_2}{\lambda_4}, R_2 < \frac{R_1}{\lambda_3} \leq \frac{1}{\lambda_3\lambda_4}R_2$; **j** $\frac{1}{1+\lambda_4}R_3 < \frac{R_2}{\lambda_4}, \frac{1}{\lambda_3\lambda_4}R_2 < \frac{R_1}{\lambda_3}$

Another important result has been depicted by Fig. 11. This figure indicates that as the sum-rate value $(R_a + R_b)$ falls below about 4 bits/s/Hz, the OPA gives no noticeable gain over EPA scheme. It is so because as the sum-rate decreases, the objective function

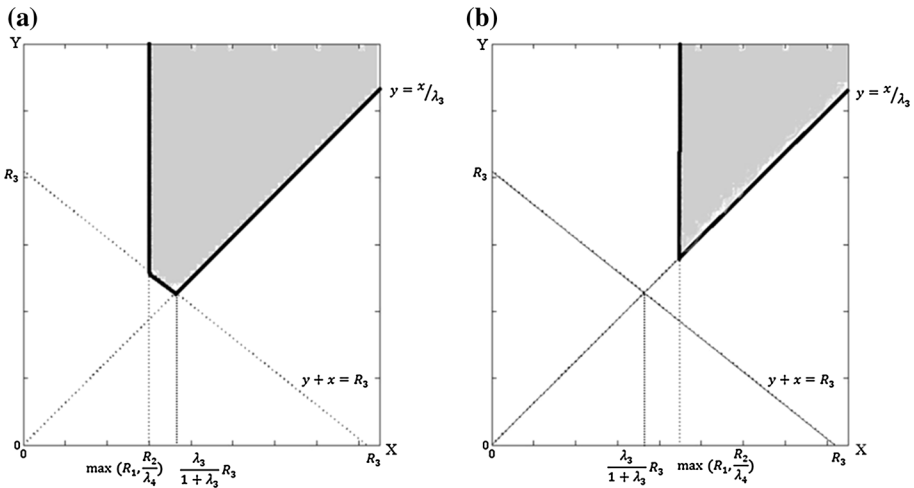


Fig. 12 The probability region for Γ_2 ; **a** $\max(R_1, \frac{R_2}{\lambda_4}) < \frac{\lambda_3}{1+\lambda_3}R_3$; **b** $\max(R_1, \frac{R_2}{\lambda_4}) > \frac{\lambda_3}{1+\lambda_3}R_3$

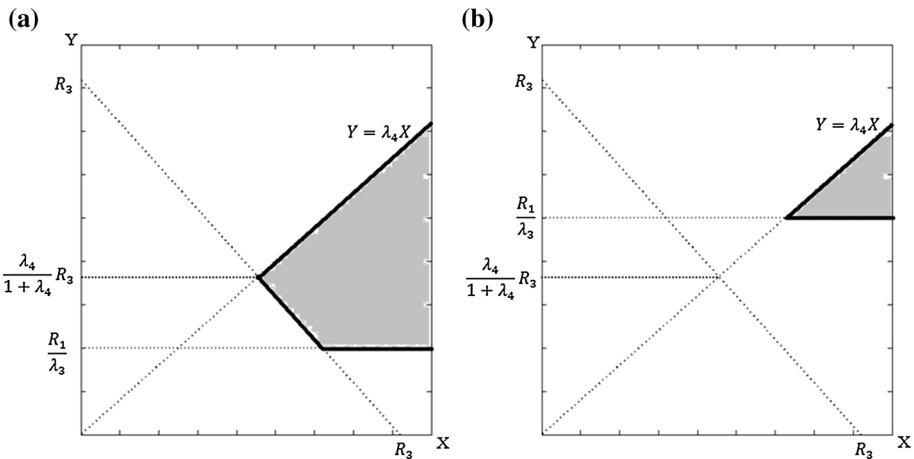
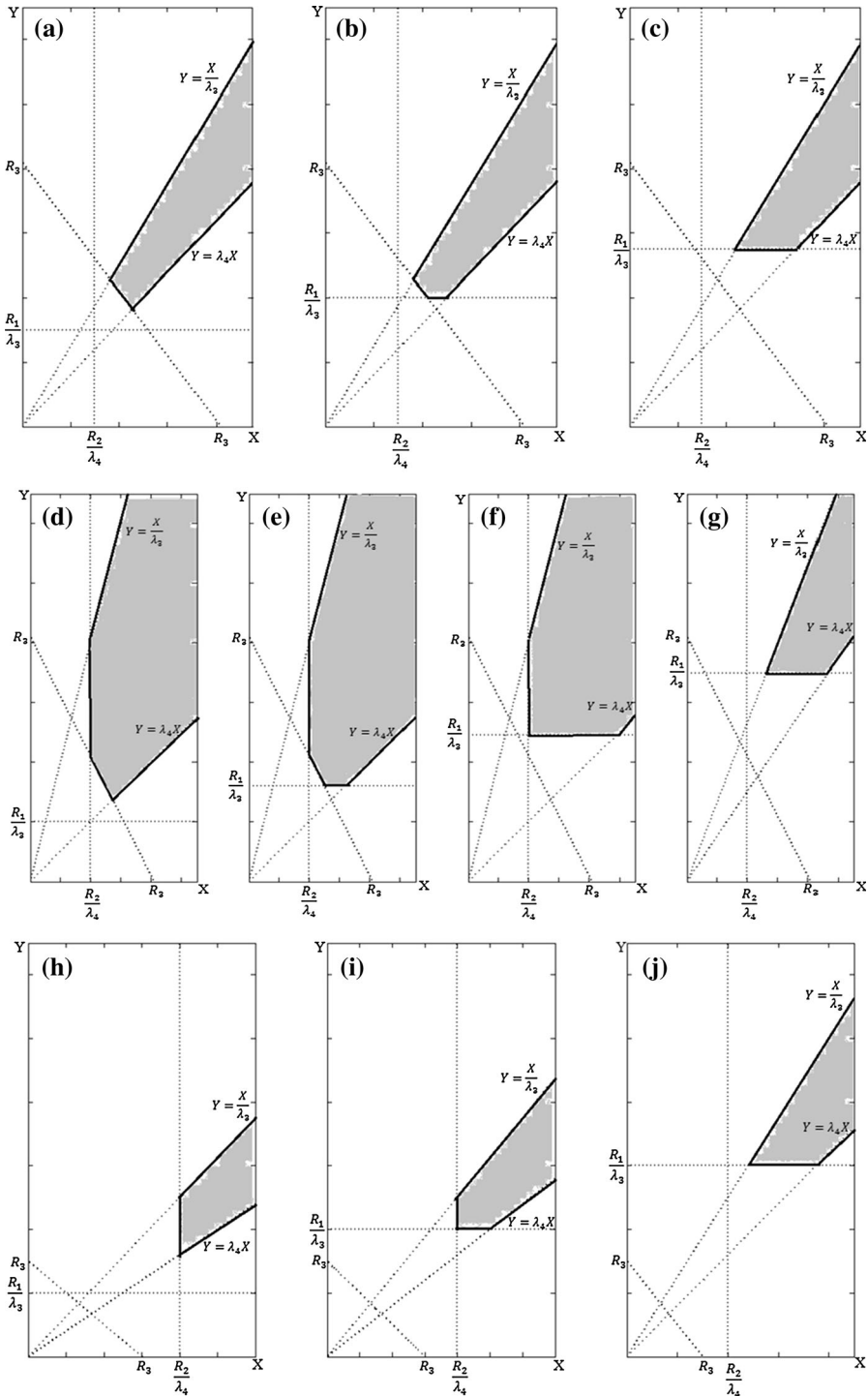


Fig. 13 The probability region for Γ_3 ; **a** $\frac{R_1}{\lambda_3} < \frac{\lambda_4}{1+\lambda_4}R_3$; **b** $\frac{R_1}{\lambda_3} > \frac{\lambda_4}{1+\lambda_4}R_3$



$\Gamma(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ becomes more smoother and as a result trying to find optimal solution will not result any advantage over EPA.

5 Conclusion

In this paper, the outage probability for a TWRN was obtained in terms of a closed mathematical expression for the first time. Specifically this expression was obtained in the more general case without imposing any constraints on the rate of the nodes. The only limiting constraint is that the relay transmit power is less then transmit power of end terminals. Moreover a new power allocation scheme was proposed based on minimizing the obtained outage probability. The only assumption made about the links between nodes was Rayleigh fading channel without any knowledge about the instantaneous CSI.

We also examined the validity of the mathematical expression for outage probability. Specifically, Monte Carlo simulation technique has been conducted and the correctness of mathematical expressions verified. The curves of the outage probability for simulation and analytical schemes exactly coincide with each other. At last, we investigated the effectiveness of the proposed OPA. Our proposed OPA scheme led to about 1 dB gain in outage probability for average and large sum-rate and average SNR values.

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Appendix: Determining the Probability Regions

In this section, we explore the probability regions for Γ_i s in (15). They are regions in the X–Y plane ($X = P_a \frac{|h_a|^2}{\sigma^2}$ and $Y = P_b \frac{|h_b|^2}{\sigma^2}$ as explained in Sect. 3.1) where integrating the probability distribution function (in this article $f_X(x)f_Y(y)$) over that region results in the Γ_i .

Probability Region For Γ_1

We can simplify Γ_1 in (14) as

$$\Gamma_1 = pr \left\{ R_1 \leq X, R_2 \leq Y, \frac{X}{\lambda_3} < Y < \lambda_4 X, R_3 \leq X + Y \right\}. \tag{22}$$

As explained in Sect. 3, we assume that $P_r \leq P_a, P_b$ and Consequently $\lambda_3, \lambda_4 \leq 1$. This results in $\Gamma_1 = 0$, since $\lambda_4 X \leq \frac{X}{\lambda_3}$ and the condition $\frac{X}{\lambda_3} < Y < \lambda_4 X$ in (22) will not hold.

Probability Region For Γ_2

Considering $\lambda_3, \lambda_4 \leq 1$, Γ_2 in (14) can be simplified as

$$\begin{aligned} \Gamma_2 &= pr \left\{ \max \left(R_1, \frac{R_2}{\lambda_4} \right) \leq X \leq \min \left(\lambda_3 Y, \frac{Y}{\lambda_4} \right), R_3 \leq X + Y \right\} \\ &= pr \left\{ \max \left(R_1, \frac{R_2}{\lambda_4} \right) \leq X \leq \lambda_3 Y, R_3 \leq X + Y \right\}. \end{aligned} \tag{23}$$

S_2 (the probability region for Γ_2) is either the shaded region in Fig. 12a (if $\max\left(R_1, \frac{R_2}{\lambda_4}\right) < \frac{\lambda_3}{1+\lambda_3}R_3$) or Fig. 12b (if $\max\left(R_1, \frac{R_2}{\lambda_4}\right) > \frac{\lambda_3}{1+\lambda_3}R_3$).

Probability Region For Γ_3

Γ_3 in (14) can be reformulated as

$$\begin{aligned}\Gamma_3 &= pr\left\{\max\left(R_2, \frac{R_1}{\lambda_3}\right) \leq Y \leq \min\left(\lambda_4 X, \frac{X}{\lambda_3}\right), R_3 \leq X + Y\right\} \\ &= pr\left\{\max\left(R_2, \frac{R_1}{\lambda_3}\right) \leq Y \leq \lambda_4 X, R_3 \leq X + Y\right\};\end{aligned}\quad (24)$$

where the constraints $\lambda_3, \lambda_4 \leq 1$ have been used in the second equality. S_3 , the probability region for Γ_3 has been depicted in Fig. 13 for both $\frac{R_1}{\lambda_3} < \frac{\lambda_4}{1+\lambda_4}R_3$ and $\frac{R_1}{\lambda_3} > \frac{\lambda_4}{1+\lambda_4}R_3$.

Probability Region For Γ_4

Simplifying Γ_4 in (14) results in

$$\Gamma_4 = pr\left\{\frac{R_1}{\lambda_3} \leq Y, \frac{R_2}{\lambda_4} \leq X, \lambda_4 X < Y < \frac{X}{\lambda_3}, R_3 \leq X + Y\right\}.\quad (25)$$

Depending on the quantities $\frac{R_1}{\lambda_3}$ and $\frac{R_2}{\lambda_4}$, different 10 types of probability region for Γ_4 will arise which are depicted in Fig. 14.

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