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Stackelberg Game on Space and Frequency Heterogeneity Analysis in an OFDMA-Based Cognitive Spectrum Leasing

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Abstract With the development of cognitive radio technologies, dynamic spectrum access (DSA) techniques are being regarded as a promising approach to increase the efficiency of spectrum utilization and to solve spectrum scarcity problem. This comes as a greater challenge in a cellular network where there are multiple primary users (PUs) who communicate with their access point while the other secondary users (SUs) want to use PU's spectrum. On the other hand, heterogeneity in terms of space and frequency can affect the primary users' decision to release their spectrum to the SUs. In this respect, the present paper is intended to address this issue and thus propose a solution with regard to the reward and punishment policy and equivalent revenue per unit transmission parameter. It has to be noted that both PUs and SUs aim to maximize their utilities in terms of their transmission rate and revenue/payment. Therefore, the proposed model is formulated as a Stackelberg Game, and a unique Nash Equilibrium Point is achieved by analytical procedure. Based on the analyses, the paper presents the conditions under which cooperation will enhance the performance of the whole system. Both analytical and numerical results reveal that the cooperative cognitive radio framework is a promising framework under which the utility of both the primary and secondary systems is maximized.

Keywords Space heterogeneity (SH) · Frequency heterogeneity (FH) · OFDMA · Spectrum leasing · Stackelberg game · Nash Equilibrium Point (NEP) · Cooperative cognitive radio network (CCRN)

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1 Introduction

Radio spectrum is one of the scarce and valuable communication resources. In fact, many users seek to make efficient use of spectrum by using DSA techniques as introduced by cognitive radio networks (CRNs) [1]. In this regard, SUs may follow two different approaches to take advantage of these valuable resources. Given the first approach, SUs recognize the PU's spectrum holes and use these opportunity for their transmissions. When PUs want to reuse their spectrum, SUs' have the choice to either hop to other frequencies (spectrum overlay) or reduce their power as much as noise level, by using the power control and spread spectrum techniques (spectrum underlay) [2, 3]. The second approach is based on an awareness spectrum leasing method from the PUs to the SUs. PUs in a beneficial action lease a portion of their spectrum to SUs. SUs compete with each other to access these resources for their own transmissions. The cost of access to these resources is given by SUs who have won the competition. They communicate primary signals via cooperative transmission or pay remuneration.

If PUs suffer bad performance due to the channel fading, or they have a heavy traffic, then suitable SUs are selected as the cooperative relays to improve the performance of primary transmission. However, for most primary services, when the required traffic demand is satisfied, primary systems have no interest to increase their transmission rate any more, instead, they want to achieve certain benefit in other format, for example revenue, which is more interesting to them [4, 5]. Increasing the PU's rate via cooperation, more opportunities are provided to SUs. Thus SUs can exploit these opportunities to their own transmissions and promote their QoS. Therefore, by exploiting cooperation between primary and secondary systems, both systems can increase their own interest and a win–win situation can be achieved [6, 7].

In the same vein in [8], authors considered the system, where a primary transmitter (PT) communicates with the intended receiver (PR). In the same spectrum band, a secondary (unlicensed) network composed of multiple transmitters receivers pairs{STi, SRi}, is seeking to exploit possible transmission opportunities. By comparing the cooperative and Non-cooperative transmission rates, PT decides whether to use the entire slot for direct transmission to PR or to employ cooperation. If PT chooses cooperative transmission, a portion of time frame belonging to PU, is leased to the suitable secondary relays which exploit decode and forward (DF) scheme and the remaining frame is divided into two subslots. The first and second subslots are dedicated for PT to ST and ST to PR transmissions, respectively. Since both SUs and PU are rational and selfish, which are interested into maximizing their own utilities, an optimization problem which is analyzed by the Stackelberg Game is proposed.

In [9] a mechanism which is focused amplify and forward (AF) cooperation protocol has been proposed. Therefore it is necessary to define power control policy on SUs, indicating how much power they are willing to spend for relaying the PUs signals. Thus the SU's are forced to have variable power.

To overcome the problems that we encountered in TDMA, a two dimensional time–frequency domain leasing for an OFDM system has been proposed. To achieve high rates, PT allocates portion of these resources in a fraction of time and frequency with the subset of SUs who have the highest bid. SUs allocate some subcarriers for relaying the primary data and use the rest of the subcarriers for their own transmissions [10, 11].

This spectrum leasing scenario will be more complicated in a cellular network where multiple PUs live in coexistence of multiple SUs [12, 13]. Due to emerge of heterogeneity in both spatial and frequency domains in a realistic scenario, different spectrum provided by different owners have different costs [14, 15]. The prior works done on the spectrum leasing scenario have assumed that spectrums are identical while frequency diversity may cause non-identical conflicts among spectrum buyers since frequencies have different communication ranges (FH) and Spectrum availability varies in different geo-locations (SH). Hence, existing spectrum leasing schemes cannot provide truthfulness or efficiency in the realistic scenario. This paper addresses this issue by developing multiple primary users' spectrum leasing scenario in presence of SH and FH. Since several PTs may simultaneously choose same secondary user, it is necessary to consider multi-antenna equipment on cooperative STs [16]. The main contributions of this work can be summarized as follows: (1) Multi primary users' spectrum leasing in presence of space and frequency heterogeneities is designed and implemented based on the analytical result. Therefore it can provide truthfulness and efficiency in the realistic scenario. (2) To implement this system, an OFDMA system based on Frequency Hopping algorithm has been applied to the CCRN for the first time. (3) Proposing a developed technique in CR network by introducing a new parameter (SUs transmission rate satisfaction degree) as well as Reward and Punishment policy in order to modeling and overcome SH and FH respectively. (4) This model is formulated as a Stackelberg game and a unique NEP is achieved in analytical format. (5) Numerical analysis reveal that under our framework, both primary and secondary systems achieve more reliable and truthful performance compared to previous works.

The rest of this paper is outlined as follows. Section 2 present the detailed system model, including the network structure, TF strategy adopted in this paper, is described. New utility functions by adding new parameter according to SH and FH are defined for primary and secondary networks In Sect. 3. Backward induction is adopted to analyze the formulated Stackelberg game and the NEP is given, whose property is demonstrated with numerical results in Sect. 4. Finally Sect. 5 concludes the paper.

2 System Model

In this section, the model of CCRN and the Frequency Hopping algorithm have been described. Consider a communication cell with normalized radius as shown in Fig. 1. This system composed of K_p number of $\{PT_i\}_{i=1}^{k_p}$ plan to communicate with their Primary Access Point (*PAP*) and K_s number of $\{STj, SRj\}_{j=1}^{k_s}$ are seeking spectrum holes In order to exploit for their own transmission. All nodes except the PAP are mobile in our scenario. It should also be considered a predefined traffic requirement in terms of transmission rate R_{0i} for each PT_i contrary to secondary network. Each ST accesses the channel in best-effort manner.

Each *PT* depending on it's utility, decides to transmits directly (Fig. 1a) or cooperate with subsets of STs (Fig. 1b–d). In the latter case PT select suitable STs as cooperative relays, and in return, give them the chance to access the channel which is belongs to primary system. To overcome FH phenomenon, a simple Frequency Hopping technique [17] during the time slots which is performed by PAP has been used in an OFDMA system sketched in Fig. 2. Otherwise it may increases the secondary attention to the specific PT due to FH. This will be described in more detail later. The channels between the nodes are modeled as independent proper complex Gaussian random variables, with frequency

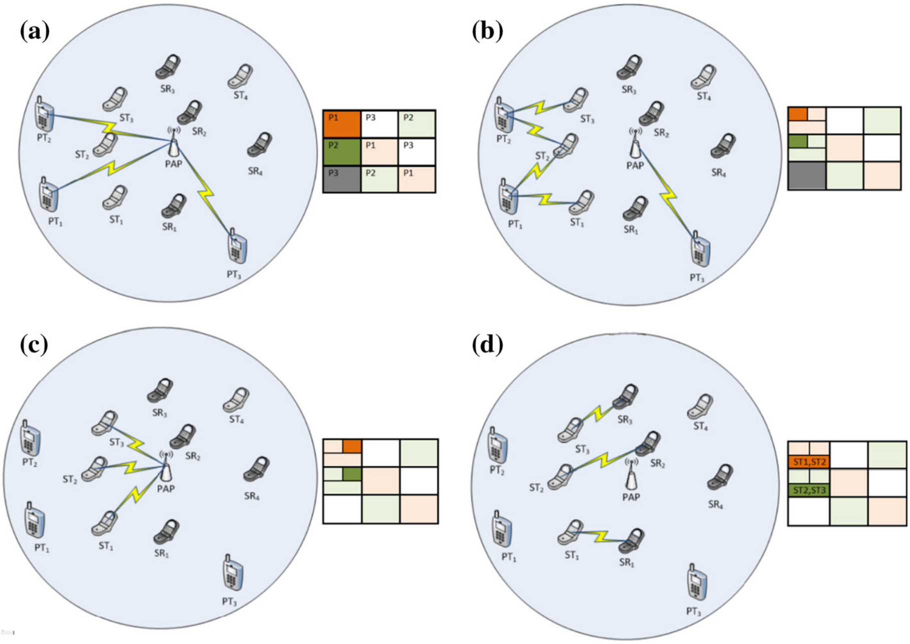
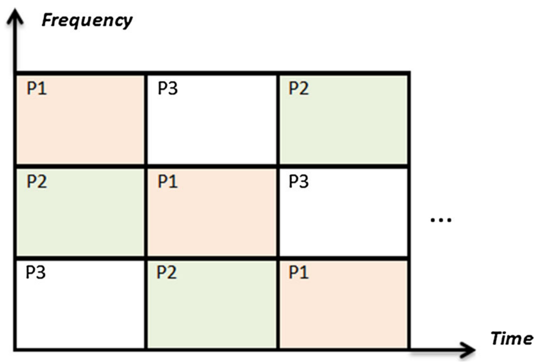


Fig. 1 Proposed system model for multi primary spectrum leasing scenario; **a** all PTs transmit directly; **b** To perform spectrum leasing PT_1 and PT_2 divide their own OFDM frame into two main parts and transmit to their selected subset ($S_1 = \{ST_1, ST_2\}$, $S_2 = \{ST_2, ST_3\}$) in 1st part; **c** Each ST_j uses a number of subcarriers for PT_i retransmission toward the PAP; **d** remained subcarriers are leased for ST_j intra-link communication to it's respective receiver

Fig. 2 OFDMA technique with simple Frequency Hopping



coefficients assumed to be constant in time within a block of OFDM symbols (i.e., Rayleigh block-fading channels).

The whole parameters used in this paper are gathered in Table 1. Therefore the non-cooperative transmission rate for each $\{PT_i\}_{i=1}^{k_p}$ in an OFDMA-based system is calculated as follow:

Table 1 Description of the symbols

Symbol	Description
$L(dB)$	Path loss in dB
f_i	Center frequency belongs to PT_i in MHz
γ	Distance power loss coefficient (constant)
$p_f(n)$	Floor loss penetration factor (constant)
p_{p_i}	PT_i's power
p_{s_i}	ST's power
N_0	Noise power
$h_{i,p}$	Channel coefficient between PT_i and APP
$h_{i,j}$	Channel coefficient between PT_i and ST_j
$h_{j,p}$	Channel coefficient between ST_j and APP
$h_{j,j}$	Channel coefficient between ST_j and SR_j
$R_{i,p}$	Transmission rate of PT_i to APP
$R_{i,j}$	Transmission rate of PT_i to ST_j
$R_{J_i,p}$	Transmission rate of ST_j to APP
R_{coop_i}	Cooperative transmission rate of PT_i
R_{jj_i}	Transmission rate of ST_j to SR_j due to PT_i's signal retransmission
$R_{jj_i}(S_i)$	Transmission rate of ST_j to SR_j due to cooperate with PTs which had chosen ST_j in several subsets (S_i)
$d_{i,j}$	Distance between PT_i and ST_j
k_p	Number of PTs
k_s	Number of STs
$\bar{c}$	Maximum payment by ST_j to motivate PT_i
τ	Data rate satisfactory parameter for PT_i
R_{0_i}	Predefined traffic requirement for PT_i
ω_{p_i}	Equivalent revenue per unit PT_i data rate satisfactory contribute to the overall utility (predefined constant parameter)
k_{s_m}	Number of STs which cooperate with PT_m

$$\left[R_{i,p}\right]_{k_p\times 1} = \log_2\left(1 + \frac{\left|h_{i,p}\right|^2 p_{p_i}}{N_0}\right) \tag{1}$$

Proposed System model for multi primary spectrum leasing scenario which has exploited DF relaying scheme has the following steps (Fig. 3): (1) Each PT_i divides it's own TF plane into two main parts and transmit to their selected subset S_i in the first subsection. The first portion $\alpha_i T_s$, $\left(0<\{\alpha_i\}_{i=1}^{k_p}<1\right)$ is devoted for PT_i transmission and the second $(1-\alpha_i)T_s$ dedicated for secondary intra link transmission (without loss of generality we assume $T_s = 1$) (2) PT_i transmits it's data to the ST_j, $j \in \{s_i\}$ in the first subsection $\alpha_i\beta_i T_s$, $\left(0<\{\beta_i\}_{i=1}^{k_p}<1\right)$. (3) Each ST_j, $j \in \{s_i\}$ uses a number of subcarriers for PT_i retransmission toward the PAP in the second subsection $\alpha_i(1-\beta_i)T_s$. (4) In the latter subsection, the selected STs access the channel in frequency-division multiplexing access (FDMA)

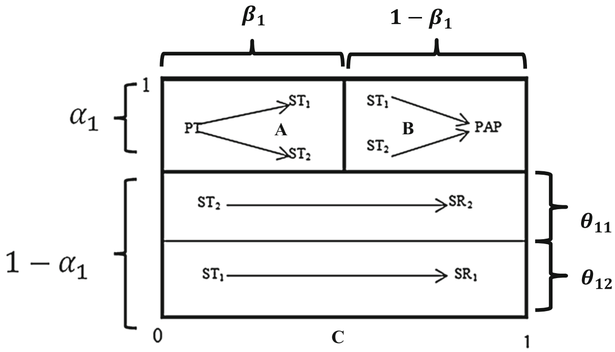


Fig. 3 Reward and punishment policy to overcome FH phenomenon

mode to communicate with their intended receivers. Each $\mathrm{ST_j}$, $j \in \{s_i\}$ takes advantage of $\mathrm{PT_i}$ resources (α_{ij}) proportional to the contribution it makes in the cooperative process (θ_{ij}), which is related to it's payment to $\mathrm{PT_i}$ (c_{ij}which $0 \le c_{ij} \le \overline{c}$).

$$\alpha_{ij} = (1-\alpha_i)\theta_{ij} = (1-\alpha_i)\frac{c_{ij}}{\sum_{j\in\{s_i\}} c_{ij}} \tag{2}$$

Therefore the $\mathrm{PT_i}$'s cooperative transmission rate to $\mathrm{ST_j}$ ($\mathrm{R_{i,j}}$) (subsection A. Fig. 3) which is dominated by the worst channel $\mathrm{h_{i,j}}$ in the subset $\mathrm{S_i}$ is:

$$R_{i,j}(s_i) = \log\left(1 + \frac{min_{j\in\{s_i\}}\left|h_{i,j}\right|^2 p_{p_i}}{N_0}\right) \tag{3}$$

For the subsection B, Assuming the PAP exploits maximum ratio combining before decoding the signal. Hence the effective SNR is equal to the sum of all the SNRs of each $\mathrm{ST_j}$. Therefore, the achievable rate of the cooperative link is given by:

$$R_{J_i,p} = \log\left(1 + \sum_{j\in\{s_i\}} \frac{\left|h_{i,j}\right|^2 p_{s_j}}{N_0}\right), \quad \forall j \in \{s_i\} \tag{4}$$

The overall achievable rate of the DF cooperative transmission equals to the minimum rate of the two above stages:

$$R_{coop_i} = min\left\{\beta_i R_{i,j}(s_i), (1-\beta_i) R_{J_i,p}(s_i)\right\} \tag{5}$$

Each $\mathrm{PT_i}$ allocates α_{ij} fraction of it's resources to the $\mathrm{ST_j}$ transmission, thus the achievable rate for each $\mathrm{ST_j}$, $j \in \{s_i\}$ is calculated as a function of their contributions to cooperative process with $\mathrm{PT_i}$ (α_{ij}). This parameter will be considered later in (9):

$$\left[R_{jj_i}\right]_{k_p\times 1} = \log_2\left(1 + \frac{\left|h_{j,j}\right|^2 p_{s_j}}{N_0}\right), \quad \forall j \in \{s_i\} \tag{6}$$

It should be noted that the spectrums are non-identical and the use of $\mathrm{PT_i}$ resources is affected by FH phenomenon. As is clear in (7), with the assumption of distance -invariant between nodes, FH states that the frequencies have different communication ranges [17] and different costs.

$$L(dB) = 10\log f_i^2 + \gamma \log d_{i,j} + p_f(n) - 28 \quad (7)$$

This issue affected the secondary users' interests and causes to emerge two problems: (1) As mentioned before it may increases the secondary attention to the PTs who have lower frequencies and We solve it by using Frequency Hopping technique. (2) If several SUs were selected by PT_i, Since SUs interest to lower frequencies it may cause to non-optimal use of TF plane belongs to the PT_i. We also solve this problem by using Reward and Punishment policy as shown in Fig. 3. Therefore if ST_j cooperate with PT_i at higher frequencies in subsections A and B, lower frequencies will be leased to it with lower prices in subsection C (Reward). Also if ST_j cooperate at lower frequencies, higher frequencies will be leased to it with higher prices (Punishment). Hence all frequencies will be identical.

Since PU is licensed user parameters announces α, β to selected subset ST_j, $j \in \{s_i\}$, so as to maximize its own utility in terms of both traffic rate and revenue. The ST_j selects it's strategy (c_{ij}) to exploiting PT_i's spectrum based on the amount of the leased resources from the PT_i ($\theta_{ij}(1-\alpha_i)$). Each ST_j by defining a utility function which expresses it's benefit in the cooperation process, tries to maximize it's utility function. Hence it is important for each ST_j to select the best strategy without making too much payment and suggest it to the PT_i in a competitive procedure. Each PT_i chooses the best relay for cooperation after listening to different suggestion coming from the STs. In the next section, we formulate these spectrum leasing interactions.

3 Utility Functions and Nash Equilibrium Point

Based on aforementioned description, it can be said we have two-stage leader–follower game which can be analyzed under Stackelberg game framework. Thus PT_i (game leader), optimizes its strategy $\{\alpha_i, \beta_i, \{s_i\}\}$ based on the knowledge of the effects of its decision on the behavior of the followers (STs). Therefore we first define the primary and secondary utility functions then achieve unique NEP by solving the game.

3.1 Primary Utility Function

The PT_i utility function consists of two components: (1) The PT_i—utility with respect to it's transmission rate satisfactory depending on chosen decision to cooperate or not. It has been shown that sigmoid function is a proper function to express user's satisfaction with respect to demand traffic [6]. This decision will be taken after comparing between utility of direct link and cooperative transmission which is denoted by D_i for each PT_i. (2) The overall revenues obtained from all ST_j, $j \in \{s_i\}$:

$$U_{p_i} = \frac{\omega_{p_i}}{1 + e^{-\tau\left(\alpha_i\left(D_i R_{p,i} + (1-D_i)R_{coop_i}\right) - R_{0_i}\right)}} + (1 - D_i)\sum_{j\in\{s_i\}} c_{ij}, \quad \forall i \in \{1, 2, \ldots, k_p\} \quad (8)$$

where all parameters are gathered in Table 1.

3.2 Secondary Utility Function

Since the ST_j may be existed in several subsets which are chosen by different PT_i, thus The ST_j utility function due to cooperate with PTs which had chosen ST_j in several subsets (S_i) can be defined as the sum of utility with respect to transmission rates they are able to achieve $R_{jj_i}(s_i)$ minus its payment to the primary network. Since SUs have no traffic requirement on their transmission, their utility functions are linear with $R_{jj_i}(s_i)$, which are proportional to the payment they are willing to pay.

$$U_{s_j} = \sum_i \left(\omega_{ij} \alpha_{ij} R_{jj_i}(s_i) - c_{ij} \right) \tag{9}$$

where ω_{ij} is the ST_j transmission rate satisfaction degree contributes to the overall utility. In [6–9] it is a predefined coefficient and has the same value for all STs. While frequency diversity may cause non-identical conflicts among spectrum buyers since frequencies have different communication ranges (FH) and spectrum availability varies in different geo-locations (SH). We already removed FH by exploiting reward and punishment policy. Therefore if PT_i cooperate with ST_j who is far from PT_i, primary utility will be reduced due to R_{coop_i} decreasing. To compensate this utility decreasing the PT_i will prefer to lease it's own resources with more expensive prices (c_{ij}) to the farther STs. We solve this spectrum heterogeneity due to variable distances by using ω_{ij} which is the level of ST_j transmission rate satisfactory contribute to the overall utility. Based on aforementioned description, it can be said ω_{ij} is inversely proportional to d_{ij}:

$$\omega_{ij} = \frac{d_{ij}^p}{\sum_{i,j} d_{ij}^p} \tag{10}$$

As is clear p should be negative and is a constant predefined parameter which will be obtained by plotting normalized utility difference (NUD) versus this parameter variation. Hence we define the NUD for each PT_i as below:

$$NUD = \frac{U_p(p) - U_p(p=0)}{U_p(p=0)} \times 100 \tag{11}$$

As can be observed in Fig. 4, when p is equal to -0.15 we have about 3 % improvement in the primary utility functions (PT_1 and PT_2). When $p = 0$, the space heterogeneity parameter ω_{ij} is the same for all STs. In other words, with this value of p the model is more robust to the space heterogeneity. It is noteworthy that for the rest of our experiment, p is assumed to be equal to -0.15.

3.3 Nash Equilibrium Point

As mentioned before $ST_j, j \in \{s_i\}$ compete with each other in a Non-cooperative Payment selection Game (NPG), $G = \left[\{s_i\}, \{c_{ij}\}, \{U_{s_j}(.)\}\right]$ based on the selected strategy by leader $\{\alpha_i, \beta_i, \{s_i\}\}$. Each ST_j chooses its strategy within the strategy space $\mathbf{C} = \left[C_{ij}\right]_{j \in s_i, i \in \{1,2,\ldots,K_p\}}$ which is given by solving gradient Eq. (11):

$$\nabla_{\underline{c}} U_{s_j} = \left[\frac{\partial U_{s_j}}{\partial c_{1j}} \frac{\partial U_{s_j}}{\partial c_{2j}} \cdots \frac{\partial U_{s_j}}{\partial c_{k_p j}} \right]^t = 0, \quad \forall j \in \{1, 2, \ldots, k_s\} \tag{12}$$

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & \cdots & c_{1k_s} \\ \vdots & \ddots & \vdots \\ c_{k_p 1} & \cdots & c_{k_p k_s} \end{bmatrix}_{k_p \times k_s} \tag{13}$$

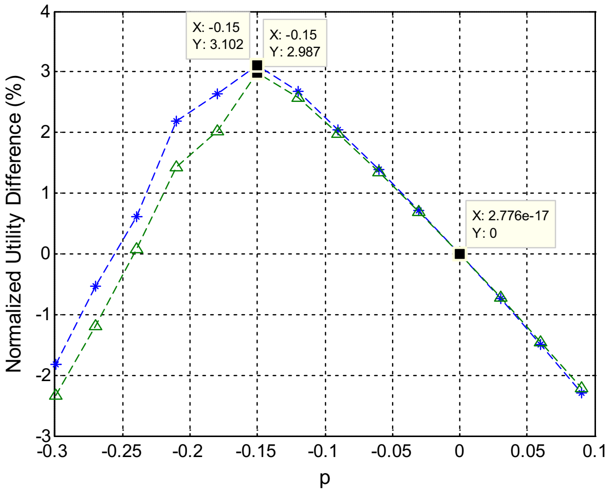


Fig. 4 Normalized utility difference (NUD) versus different value of *P*

It should be noted that solving $\frac{\partial U_{s_j}}{\partial c_{1j}} = 0$ yields the first column of the $\boldsymbol{C}$ contains $\left[c_{11}, c_{12}, \ldots, c_{1k_p}\right]^t$. If $\mathrm{ST_j}$ doesn't cooperate with any $\mathrm{PT_i}$ it leads to the corresponding matrix element will be equal to zero. It has been shown that necessary and sufficient conditions for this class of NPG game to demonstrate NE existence and uniqueness are satisfied [6]. Now solving (12) yields a unique NE for NPG game as:

$$c^*_{mn} = \frac{(1-\alpha_m)(k_{s_m}-1)\left[\left(\sum_{j\in\{s_m\}}\frac{1}{\omega_{mj}R_j}\right) - \frac{k_{s_m}-1}{\omega_{mn}R_n}\right]}{\left(\sum_{j\in\{s_m\}}\frac{1}{\omega_{mj}R_j}\right)^2}, \quad \forall m \in \left\{1, 2, \ldots, k_p\right\}, n \in \{1, 2, \ldots, k_s\} \tag{14}$$

Since we knew c_{ij} is bounded (i.e. $0 \le c_{ij} \le \bar{c}$), we should define new constraints which is adapted to this achieved optimal point (c^*_{mn}). These constraints will be used by the primary user to select optimal cooperative relay set.

$$\left(\sum_{j\in\{s_m\}}\frac{1}{\omega_{mj}R_j}\right) - \frac{k_{s_m}-1}{\omega_{mn}R_n} > 0 \tag{15}$$

$$\frac{(1-\alpha_m)(k_{s_m}-1)\left[\left(\sum_{j\in\{s_m\}}\frac{1}{\omega_{mj}R_j}\right) - \frac{k_{s_m}-1}{\omega_{mn}R_n}\right]}{\left(\sum_{j\in\{s_m\}}\frac{1}{\omega_{mj}R_j}\right)^2} < \bar{c} \tag{16}$$

Now the sign of $\mathrm{D_i}$ must be determined as follow:

$$U_{p_{iD}} = \frac{\omega_p}{1 + e^{-\tau\left(\alpha_i R_{i,p} - R_{0_i}\right)}} \tag{17}$$

$$U_{p_{i_{coop}}} = \frac{\omega_p}{1 + e^{-\tau\left(\alpha_i R_{coop_i} - R_{0_i}\right)}} + \sum_{j \in s_i} c_{ij} \tag{18}$$

$$D_i = \begin{cases} 0, & U_{p_{iD}} < U_{p_{i_{coop}}} \\ 1, & U_{p_{iD}} > U_{p_{i_{coop}}} \end{cases} \tag{19}$$

Now PT_i (game leader) can optimize its strategy $\{\alpha_i, \beta_i, \{s_i\}\}$ based on the analytical result of NPG game with substituting (14) into (18) and calculate the first order derivative of $U_{pi_{coop}}$ in respect with α_i. It is important to know that this decision will affect the behavior of the followers (STs).

$$\frac{dU_{p_{i_{coop}}}}{d\alpha_i} = 0 \overset{yields}{\Longrightarrow} \alpha_i^* = \frac{\left(-\frac{\ln(X)}{\tau} + R_{0_i}\right)}{R_{coop_i}} \tag{20}$$

where X is:

$$X = \frac{\pm\sqrt{(B-A) - 4B^2} - (2B - A)}{2B} \tag{21}$$

In which:

$$A = \tau\omega_p\beta_m R_{m,j}(s_m), \quad B = \frac{(k_{s_m} - 1)\left[\left(\sum_{j \in \{s_m\}} \frac{1}{\omega_{mj}R_j}\right) - \frac{k_{sm}-1}{\omega_{mn}R_n}\right]}{\sum_{j \in \{s_m\}} \frac{1}{\omega_{mj}R_j}} \tag{22}$$

We also should be aware that transmitted symbols allocated for PT_i to ST_j should be equal to the number of subcarriers in the ST_j to PAP link in our proposed cooperative model [11]. Hence we have:

$$\beta_i R_{i,j}(s_i) = (1 - \beta_i) R_{J_i,p}(s_i) \tag{23}$$

$$\beta_i^* = \frac{R_{J_i,p}(s_i)}{R_{i,j}(s_i) + R_{J_i,p}(s_i)} \tag{24}$$

4 Numerical Results

In this section, simulation results are presented to demonstrate the impact of different spectrum leasing characteristics on the optimal multi primary cooperation scheme. A cognitive network includes k_p number of PT_i and ks number of ST_j–SR_j pairs is considered in which the distance between the PT_i and PAP is assumed to be normalized to 1 and secondary nodes are all placed at approximately the same normalized distance d ($0 < d < 1$) from the PT and $1 - d$ from the PAP. All parameters used in the simulation are set as follows in Table 2. Both primary user and secondary users transmit at a fixed power level without power control. To demonstrate SH and PH phenomenon's separately, we consider two scenario in which the number of PTs are selected different so that k_p (SH) and k_p (FH) are set to 2 and 3 respectively. Therefore we present our simulation results in two subsections as follow:

Table 2 Description of the symbols

Symbol	Value
$E\{h_{i,p}\}$	1
$E\{h_{j,p}\}$	$\frac{1}{1-d_{ij}}$
k_s	8
$E\{h_{j,j}\}$	0.8
$\mathrm{E}\{h_{i,j}\}$	$\frac{1}{d_{ij}}$
$\bar{c}$	0.1
R_0	3.6
T	1
$k_p(SH)$	2
$k_p(FH)$	3
w_p	0.3
$\frac{p_{p_i}}{N_0} = \frac{p_{s_i}}{N_0}$	10 dB
P	−0.15

4.1 Multi Primary Spectrum Leasing Scenario with Considering Space Heterogeneity

In this subsection we only consider SH with respect to predefined parameter p which is calculated in subsection 3.2. Figure 5 shows the optimal parameters α_* and β_*, versus the distance between $\{PT_i\}_{i=1}^{2}$ and various numbers of ST_j, $1<j<8$ which are in subset $\{S_i\}_{i=1}^{k_p}$. As is expected, with the increase of d_{ij}, secondary utilities are reduced (Fig. 6). Hence it leads to decrease the number of ST_j which interest to cooperate with PT_i and the amount of leased resources from PT_i $(1-\alpha_i^*)$. Increasing d_{ij} also causes to decreasing the broadcast transmission rate from PT_i to ST_j ($R_{i,j}(s_i)$), while the cooperative transmission rate from ST_j to PAP ($R_{J_i,p}(s_i)$) is increased. To receive certain amount of data and forward the same amount to intended receiver (PAP), more time is needed for the first broadcast stage and less is needed for the second cooperation stage. Therefore, β^* increases when the normalized distance d becomes larger, which also agrees with the analysis result given in [6].

As is expected if PT_i cooperate with ST_j which is far from PT_i, primary utility will be reduced (Fig. 9) due to R_{coop_i} decreasing. In this case the PT_i will prefer to lease it's own resources with more expensive prices (c_{ij}) for the farther STs and not interested to increase it's rate. We solve this spectrum heterogeneity due to variable distances by using ω_{ij}. Hence if distance becomes larger, ω_{ij} should be decreased to prevent distant STs not to cooperate with PT_i. Figures 7 and 8 show the total secondary users utility which are located in subset S_1 and a specifc secondary utility (i.e. secondary utility 1) under different schemes versus the normalized distance d_{ij} respectively. To compare between different scheme we set $p = -0.15$ and $p = 0$ to introduce a system in precense of SH with and without considering ω_{ij} as a ST_j transmission rate satisfaction degree contributes to the overall utility, respectively. As can be observed a system with considering ω_{ij} is more robust against space variations.

Fig. 5 Optimal parameters α^* and β^* versus normalized distance between ST_j and PT_i

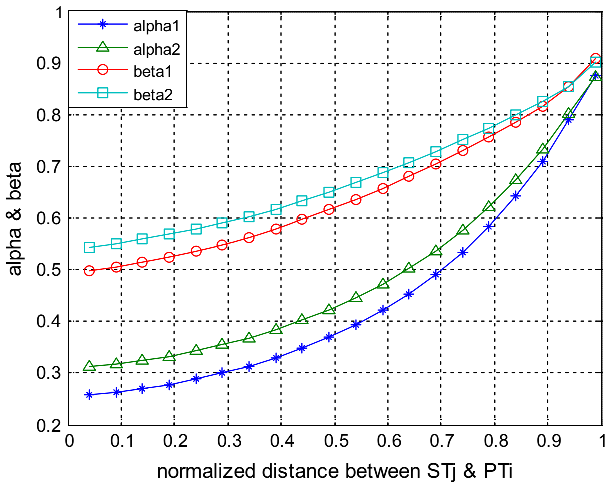


Fig. 6 Secondary utilities versus normalized distance between ST_j and PT_i

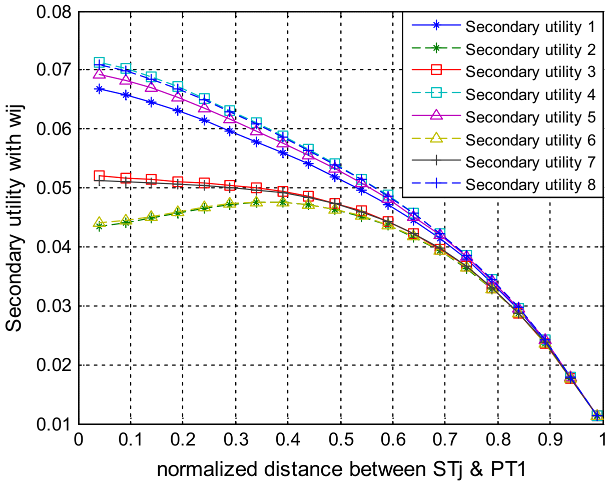


Figure 9 illustrates primary utility under different schemes versus distance between ST_j and PT_i. U_P denotes the utility function of the optimal scheme, in which primary user leases some portion of it's resources for secondary user and leverages them to transmit cooperatively. U_0 and U_p *with direct link* denotes the primary user's utility function when $\alpha = 0$ and $\alpha = 1$ respectively. U_0 implies that all the primary user's resources is given to the secondary users to receive payment without sending any of its own data while U_p with direct link refers to the primary utility when no cooperation is leveraged and all the channel is used for the licensed service. As we can see, the benefit is brought by the appropriate trade-off between two strategies ($\alpha = 0$ and $\alpha = 1$) and selecting the optimal amount of leased resources ($1 < \alpha_i^* < 8$). Increasing α_i^* causes to decreasing the leased resources from

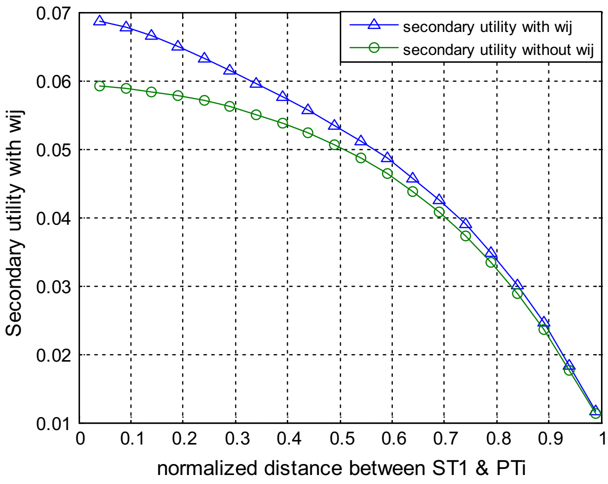


Fig. 7 Secondary utility 1 which cooperate with $\{PT_i\}_{i=1}^{2}$ versus normalized distance between ST_1 and PT_i

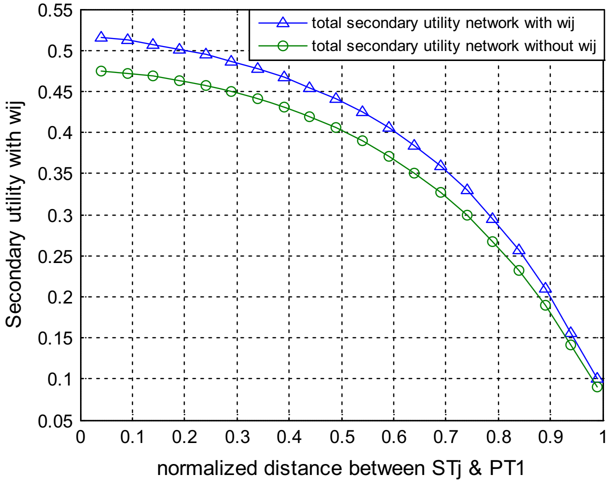


Fig. 8 Total secondary utility (S_1) which cooperate with PT_1 utility versus normalized distance between ST_j and PT_i

PT_i to ST_j and optimal scheme utility function U_p is decreased dramatically (Fig. 10). This also cause to secondary utility decreasing which are selected by PT_i (Fig. 11). Figure 12 also shows secondary utility 1 versus $\{\alpha_{ij}\}_{i=1\,and\,j=1}$.

4.2 Multi Primary Spectrum Leasing Scenario with Considering Space Heterogeneity

In this subsection we only consider FH which makes to emerge two problems. (1) It may affect the secondary users' interests to the specific PTs who have lower frequencies and we solved it by using Frequency Hopping technique. Figure 13a shows $\{PT_i\}_{i=1}^{3}$ which use OFDM subcarriers for their transmissions in $0 < t < 1$. At this time PT_3 which has lower

Fig. 9 Different scheme of primary utility versus normalized distance between ST_j and PT_i

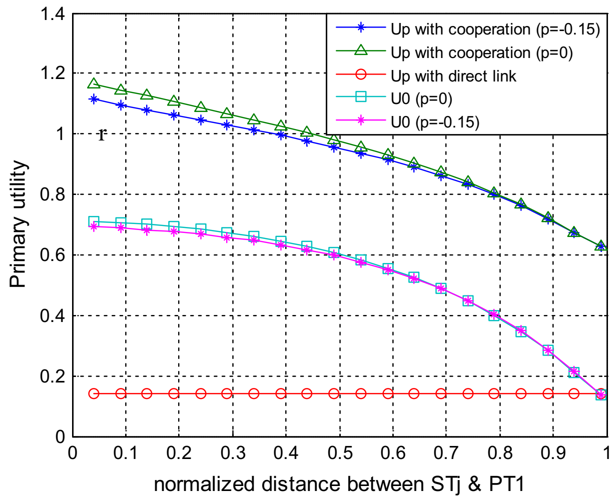
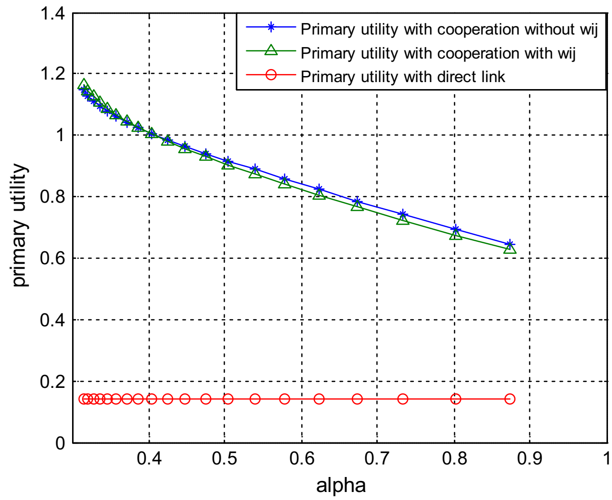


Fig. 10 Different scheme of primary utility versus normalized optimal parameter α_*



frequency attract more attentions of STs and achieve more utility via cooperation with subset $S_3 = \{ST1, ST3, ST5, ST8\}$. At the next time $(1 < t < 2)$ PT_2 has lower frequency and achieve more utility compare to other PTs (Fig. 13b).

(2) If several SUs were selected by PT_i, Since SUs interest to lower frequencies it may cause to non-optimal use of TF plane belongs to the PT_i. We also solve this problem by using Reward and Punishment policy which is described in Sect. 2. As we can observed in Figs. 14 and 15 this algorithm make system more robust against FH phenomenon.

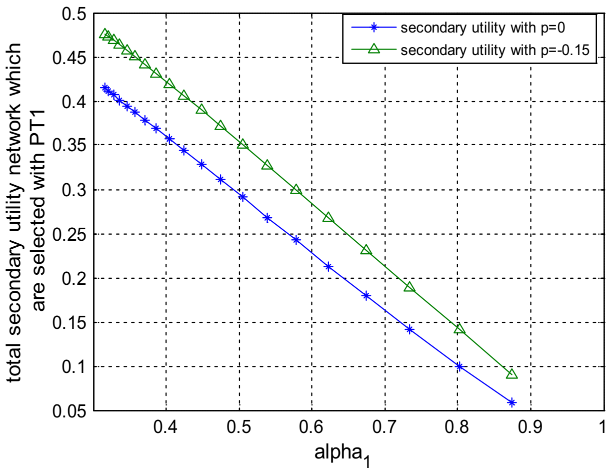


Fig. 11 Total secondary utility (S_1) which cooperate with PT_1 utility versus normalized optimal parameter α_1^*

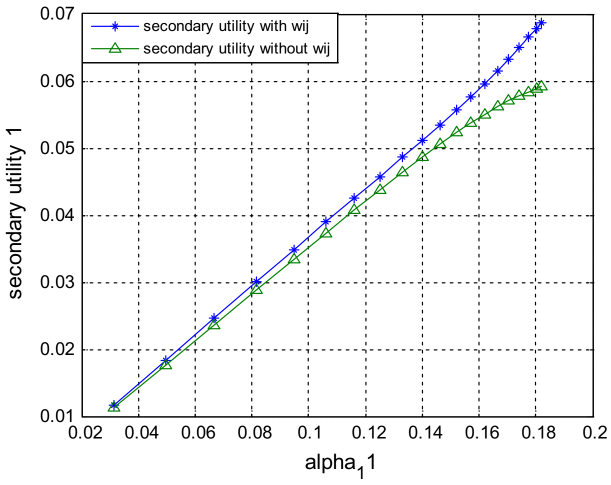


Fig. 12 Secondary utility 1 which cooperate with PT_1 versus optimal parameter α_{11}^*

5 Conclusions

The present paper was intended to propose a model to deal with the problem of space and frequency heterogeneity. To this end, multi primary users were utilized to state the space and frequency heterogeneity and subsequently, a new parameter ω_{ij} as well as the Frequency Hopping and Reward and Punishment policy were introduced in order to consider the Space and Frequency Heterogeneity respectively. Finally to achieve optimal strategies (unique Nash Equilibrium Point (NEP)) which are selected by leaders (primary users) and followers (secondary users), the Stackelberg Game is applied. Numerical analysis reveal that under our framework, both primary and secondary systems achieve more reliable and truthful performance.

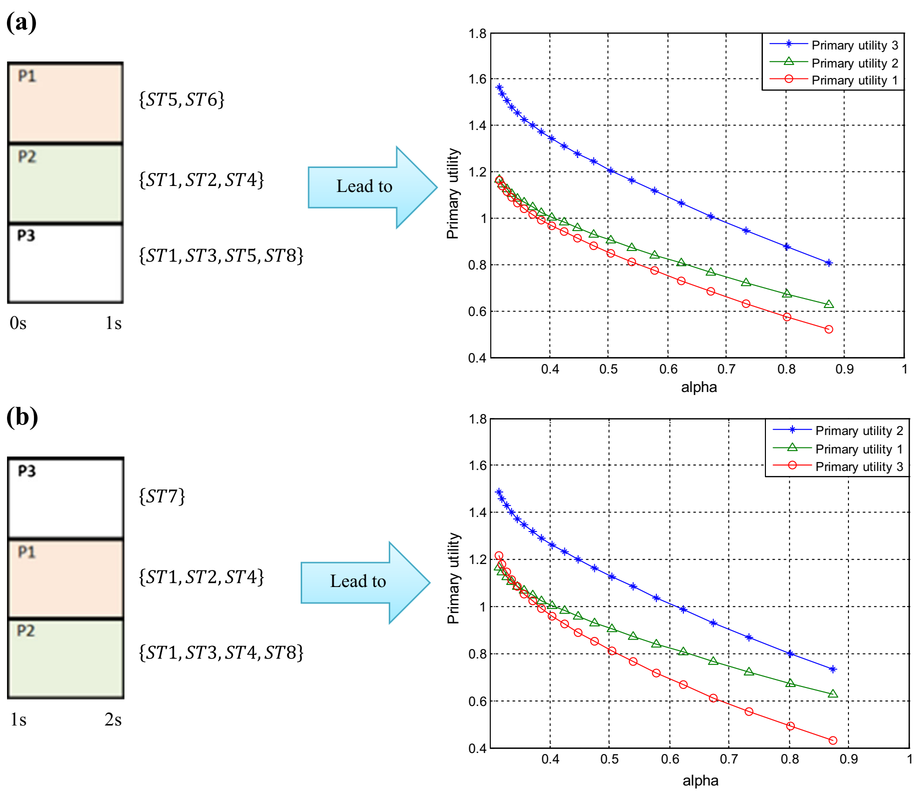
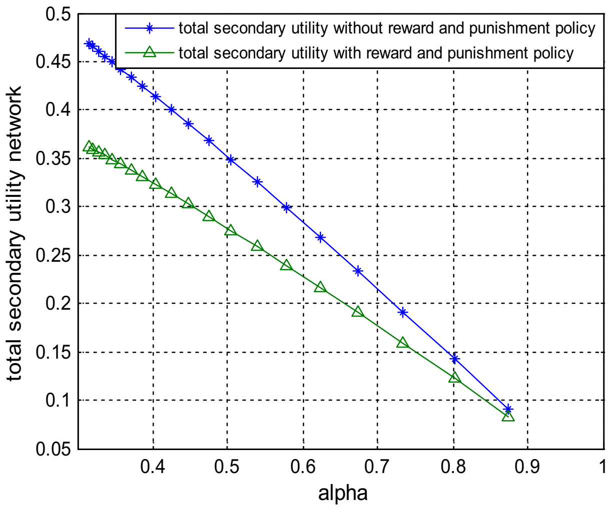


Fig. 13 Frequency Hopping technique to remove FH among PTs spectrums

Fig. 14 Total secondary utility (S_1) which cooperate with PT_1 utility versus normalized optimal parameter α_1^* with considering FH phenomenon



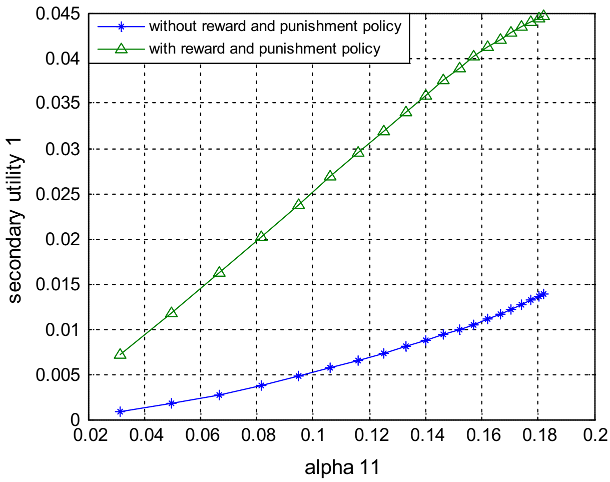


Fig. 15 Secondary utility 1 which cooperate with PT_1 versus optimal parameter $\alpha^*_{11}W$ with considering FH phenomenon

Appendix

Proof for (14).

$$\frac{\partial U_{s_j}}{\partial c_{mj}} = \frac{\omega_{mj}(1-\alpha_m)R_j\sum_{n\neq j}c_{mn}}{\left(\sum_{n\neq j}c_{mn}\right)^2} - 1 = 0, \quad \forall j \in \{1,\ldots,k_s\} \tag{25}$$

$$\omega_{mn}\overbrace{(1-\alpha_m)}^{A_m}R_n\overbrace{\sum_{j\neq n}c_{mj}}^{k_m-c_{mn}} = \overbrace{\left(\sum_j c_{mj}\right)^2}^{k_m} \tag{26}$$

$$c_{mn} = k_m\left(1 - \frac{k_m}{A_m\omega_{mn}R_n}\right) \tag{27}$$

$$\overbrace{\sum_n(c_{mn})}^{k_m} = \sum_n k_m\left(1-\frac{k_m}{A_m\omega_{mn}R_n}\right) \xRightarrow{yields} k_m = \frac{A_m(k_{s_m}-1)}{\sum_n\frac{1}{\omega_{mn}R_n}} \xRightarrow{substituting\ k_m\ in\ (31)\ yields} c^*_{mn} \tag{28}$$

References

1. Hossain, E., Niyato, D., & Han, Z. (2009). *Dynamic spectrum access and management in cognitive radio networks*. Cambridge: Cambridge University Press.
2. Wang, B., & Liu, K. R. (2011). Advances in cognitive radio networks: A survey. *Selected Topics in Signal Processing, IEEE Journal of, 5*(1), 5–23.
3. Moradkhani, M., Azmi, P., & Pourmina, M. A. (2014). Optimized reliable data combining cooperative spectrum sensing method in cognitive radio networks. *Wireless Personal Communications, 74*(2), 569–583.

4. Haqiqatnejad, A. R., Shahtalebi, K., & Forouzan, A. R. (2014). Spectrum leasing with disjoint secondary user selection in cognitive radio networks. In *Communication and Information Theory (IWCIT), 2014 Iran Workshop on* (pp. 1–6), IEEE.
5. Zhu, K., Niyato, D., Wang, P., & Han, Z. (2012). Dynamic spectrum leasing and service selection in spectrum secondary market of cognitive radio networks. *Wireless Communications, IEEE Transactions on, 11*(3), 1136–1145.
6. Zhang, J., & Zhang, Q. (2009). Stackelberg game for utility-based cooperative cognitive radio networks. In *Proceedings of the Tenth ACM International Symposium on Mobile Adhoc Networking and Computing* (pp. 23–32).
7. Yi, Y., Zhang, J., Zhang, Q., & Jiang, T. (2011). Spectrum leasing to multiple cooperating secondary cellular networks. In *Communications (ICC), 2011 IEEE International Conference on* (pp. 1–5), IEEE.
8. Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., & Pickholtz, R. (2008). Spectrum leasing to cooperating secondary ad hoc networks. *Selected Areas in Communications, IEEE Journal on, 26*(1), 203–213.
9. Wang, H., Gao, L., Gan, X., Wang, X., & Hossain, E. (2010). Cooperative spectrum sharing in cognitive radio networks: A game-theoretic approach. In *Communications (ICC), 2010 IEEE International Conference on* (pp. 1–5), IEEE.
10. Toroujeni, S. M. M., Sadough, S. S., & Ghorashi, S. A. (2010). Time-frequency spectrum leasing for OFDM-based dynamic spectrum sharing systems. In *Wireless Advanced (WiAD), 2010 6th Conference on* (pp. 1–5), IEEE.
11. Toroujeni, S. M. M., Sadough, S. M. S., & Ghorashi, S. A. G. (2013). Spectrum leasing for OFDM-based cognitive radio networks. *Vehicular Technology, IEEE Transactions on, 62*(5), 2131–2139.
12. Elkourdi, T., & Simeone, O. (2012). Spectrum leasing via cooperation with multiple primary users. *Vehicular Technology, IEEE Transactions on, 61*(2), 820–825.
13. Zhang, Y., Leng, S., & Zeng, M. (2012). An auction-theoretic spectrum leasing scheme for cognitive radio networks. In *Computational Problem-Solving (ICCP), 2012 International Conference on* (pp. 29–34), IEEE.
14. Pourmina, M. A., & MirMotahhary, N. (2012). Load balancing algorithm by vertical handover for integrated heterogeneous wireless networks. *EURASIP Journal on Wireless Communications and Networking, 2012*(1), 1–17.
15. Feng, X., Chen, Y., Zhang, J., Zhang, Q., & Li, B. (2012). TAHES: Truthful double auction for heterogeneous spectrums. In *INFOCOM, 2012 Proceedings IEEE* (pp. 3076–3080), IEEE.
16. Bahadori-Jahromi, F., Pourmina, M. A., & Masnadi-Shirazi, M. A. (2014). Performance of cooperative spatial multiplexing SISO/MIMO communication systems with constellation rearrangement technique. *Arabian Journal for Science and Engineering, 39*(2), 1067–1078.
17. Propagation data and prediction methods for the planning of indoor radiocommunication systems and radio local area networks in the frequency range 900 MHz to 100 GHz. Recommendation ITU-R P.1238-1, 1999.

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