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Adaptive Resource Allocation Algorithms for Multi-user MIMO-OFDM Systems

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Abstract Radio resource allocation to the system users is the key challenge issue in multi-user orthogonal frequency division multiplexing (MU-OFDM) systems. In this paper, an efficient and low-complexity proportional rate-adaptive radio resource (sub-carrier and POWER)

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allocation algorithm for MU-OFDM is proposed to maximize the sum-rate capacity of the system and achieve acceptable users' rates fairness. Three-dimensional (spatial, frequency, and multi-user) greedy power allocation (GPA) algorithms for the MU multi-input multi-output OFDM (MU MIMO-OFDM) systems are proposed. The proposed algorithms start with spatial sub-carrier allocation followed by an optimal two-dimensional spatial-frequency GPA (SFGPA) step, which exploits both the spatial and frequency diversities. Therefore, spatial, frequency, and multi-user diversities are exploited by the proposed three-dimensional GPA algorithms. Several experiments are carried out to test the performance of the proposed three-dimensional GPA algorithms in terms of sum-rate capacity. The optimal solution is achieved by the three-dimensional dynamic sub-carrier-SFGPA with average gain (DS-SFGPA-AG) algorithm, which achieves the best sum-rate capacity performance compared with the other power allocation algorithms. The performance of the DS-SFGPA-AG algorithm is also analyzed and compared for the four OFDM-based systems; single-user (SU) OFDM, SU MIMO-OFDM, MU OFDM, and MU MIMO-OFDM. The MU MIMO-OFDM system with adaptive sub-carrier and power allocation outperforms the other systems.

Keywords Multi-user · MIMO-OFDM · Greedy power allocation · Spectral efficiency

1 Introduction

The next-generation mobile communication systems are expected to provide high quality of service (QoS), high data rates, and high capacity [1]. However, high data rate transmission is limited by inter-symbol interference (ISI) due to the frequency selective fading in the wireless mobile radio channel. Orthogonal frequency division multiplexing (OFDM) is considered as one of the most effective multi-carrier transmission techniques for wireless communications, because it provides a large immunity to multi-path fading and impulsive noise, and eliminates the need for complicated equalizers [1–3].

Multi-User OFDM (MU-OFDM) and MIMO systems are the two leading technologies for achieving high downlink capacity in high-speed communication systems [4]. The problem of allocating the base station resources (sub-carriers, and powers) to MU-OFDM system users has been an area of active research in recent years. This problem has been studied from two perspectives; margin adaptive (MA) schemes that minimize the amount of transmit power [5,6], and rate adaptive (RA) schemes that maximize the sum-rate capacity of the system [7–9]. Since the fading parameters for different users are mutually independent in MU-OFDM system, the probability that a subcarrier is in deep fade for all users is very low, and thereby each subcarrier is likely to be in a good condition for some users in the system. Therefore, the adaptive subcarrier and power allocation is responsible for maximizing the total sum-rate capacity by using a multi-user diversity [10].

The MIMO-OFDM wireless communication system combines the simple equalization of the OFDM modulation with the capacity, diversity and array gain of the MIMO communication [11–13]. In the MU MIMO-OFDM systems, when different users communicate simultaneously, multi-user interference (MUI) occurs, which significantly degrades the performance. Transceiver techniques that deal with MUI are thus an essential part of the MIMO-OFDM wireless communication system. For more performance enhancement of the MIMO-OFDM wireless communication system, an adaptive scheme with respect to power allocation [14,15], modulation [16], beam forming [17], sub-carrier allocation [18,19] and transmission data rate [20] can be applied.

In the MU MIMO-OFDM Systems, efficient adaptive sub-carrier and power allocation algorithms are needed to improve the overall sum-rate capacity by exploiting time, frequency, spatial, and multi-user diversities. Recently, the need of an efficient radio resource (sub-carrier and power) allocation algorithm for the downlink MU OFDM based on spatial multiplexing has become more important [18,21]. The spatial multiplexing based MIMO-OFDM technique is one of the categories of the MIMO-OFDM techniques. It aims to increase the system capacity by exploiting the multiplexing gain in the spatial domain, i.e., transmitting independent data streams across antennas and tones. The optimal sub-channels allocation criterion and the optimal power allocation criterion for the downlink of the MIMO-OFDM wireless communication system was derived in [18]. Two sub-optimal sub-channels allocation schemes, product-criterion and sum-criterion, were also proposed to be used in high and low signal-to-noise ratio (SNR) regions, respectively. Senel and Love [22] proposed a low-complexity sub-optimal sub-channels allocation scheme and presented a lower bound on the system capacity and an upper bound on the bit error rate (BER), when using zero forcing (ZF) and minimum mean square error (MMSE) receivers are used.

Radio resource allocation in MU MIMO-OFDM systems has become an important research topic. In [7], the RA scheme to maximize the total sum-rate capacity over all users subject to power and bit error rate (BER) constraints was investigated. It was shown that the maximization of the total sum-rate capacity is achieved by allocating each subcarrier to the user with the best gain on it using the waterfilling algorithm for power allocation across the sub-carriers. However, no fairness among the users was considered in [7]. In [23], the fairness was considered, but computationally expensive iterative operations are required for solving non-linear equations. Therefore, the algorithm is not suitable for a cost-effective real-time implementation. The authors in [9] developed a subcarrier allocation scheme that linearizes the power allocation problem, while achieving approximate rate proportionality.

In [24], the downlink sum-rate capacity of MU MIMO-OFDM systems was analyzed, and a sub-carrier allocation algorithm based on product and sum principles was proposed to maximize the capacity. Zhenping et al. [25] proposed a sub-carrier and bit allocation algorithm based on minimum transmit power, distributing sub-carriers according to the result of a singular value decomposition (SVD) applied to the channel matrix. However, the above-mentioned algorithms limit each sub-carrier to only one user in one allocation time slot, therefore, they cannot make use of space division multiple access (SDMA) in MIMO systems well. As a result, the capacity performance they gained is largely limited. The combination of SDMA and MU OFDM results in a new technology called SDMA-OFDMA [18]. This paper uses the subcarrier allocation algorithm proposed in [4,21] and presents three-dimensional (spatial, frequency, and multi-user) GPA algorithms for MU MIMO-OFDM systems.

In this paper, SVD-based adaptive sub-carrier and power allocation algorithms are proposed to improve the spectral efficiency of the MU MIMO-OFDM wireless communication systems. The SVD technique is a matrix decomposition technique that decomposes the per-sub-carrier MIMO channel into parallel independent SISO spatial sub-channels with decreasing gains, and thereby eliminates the so-called inter-antenna interference (IAI). The MUI is mitigated by sub-carrier allocation in which each sub-carrier is allocated to only one user. The proposed power allocation algorithms exploit the three dimensions (frequency, space, and multi-user) of the MU MIMO-OFDM system to efficiently maximize the sum-rate capacity of the system. The proposed optimal power allocation algorithm is analyzed and compared for the four OFDM-based systems; SU OFDM, SU MIMO-OFDM, MU OFDM, and MU MIMO-OFDM. The MU MIMO-OFDM system with adaptive sub-carrier and power allocation outperforms the other systems, where all system diversities are fully exploited to maximize the system average sum-rate capacity and spectral efficiency.

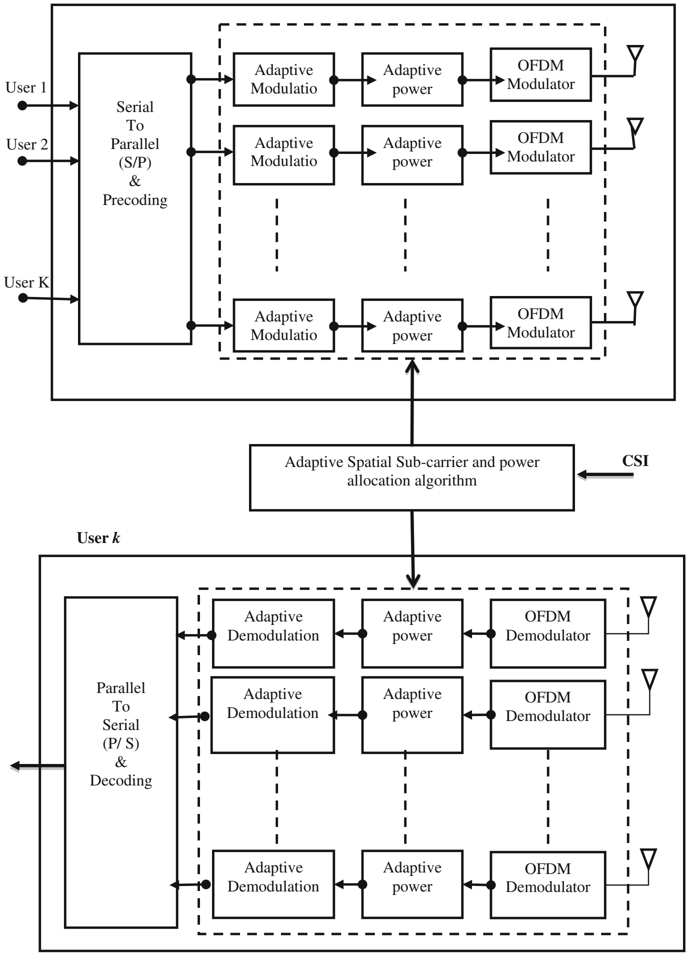

Fig. 1 The adaptive MU MIMO-OFDM system model

2 System Model and Problem Formulation

The MU MIMO-OFDM system employing the adaptive spatial-frequency power allocation algorithm is shown in Fig. 1. The MIMO system is assumed to be equipped with equal antennas at both the base station $M_{t,b}$ and the *kth* receiver $M_{r,k}$, i.e. $M_{t,b} = M_{r,k} = M$. The total available bandwidth B is splitted into N sub-carriers for the OFDM transmission system. For the *mth* spatial sub-channel $(1 \leq m \leq M)$, information bits are allocated on the *nth* sub-carrier of the *kth* user depending on the channel conditions in frequency domain. Specifically, the *nth* sub-carrier of the *kth* user delivers $R^m_{k,n}$. Then the allocated bits in the *nth* sub-carrier of the *kth* user are Gray mapped into one of $M_l = 2^{R^m_{k,n}}$ symbols in an L-level M-QAM signal set. Next, by applying the standard OFDM modulation and demodulation at the transmitter and the receiver, respectively, the demodulated signal of the *nth* sub-carrier of the *kth* user can be represented as

$$\mathbf{Y}_{k,n} = \mathbf{H}_{k,n}\mathbf{X}_{k,n} + \mathbf{Z}_{k,n} \tag{1}$$

where $\mathbf{X}_{k,n}$ and $\mathbf{Y}_{k,n}$ are the $M \times 1$ transmitted and received frequency-domain signal vectors of the kth user at the nth sub-carrier ($n = 1, 2, 3, \ldots, N$), respectively, $\mathbf{Z}_{k,n}$ is an $M \times 1$ AWGN vector of the kth user at the nth sub-carrier, and it is assumed as $\mathbf{Z}_{k,n} \sim CN(0, I)$, and $\mathbf{H}_{k,n}$ denotes the $M \times M$ frequency-domain channel response matrix of the kth user at the nth sub-carrier as

$$\mathbf{H}_{k,n} = \begin{bmatrix} H_{k,n}^{11} & H_{k,n}^{12} & H_{k,n}^{1M} \\ H_{k,n}^{21} & H_{k,n}^{22} & H_{k,n}^{2M} \\ & & \\ H_{k,n}^{M1} & H_{k,n}^{M2} & H_{k,n}^{MM} \end{bmatrix} \tag{2}$$

where $H_{k,n}^{mm}$ denotes the channel component between mth transmitter and receiver antennas of the kth user at nth sub-carrier. Here, $\mathbf{H}_{k,n}$ is assumed to be constant during the whole transmission frame.

By applying the SVD operation on $\mathbf{H}_{k,n}$, the MIMO channel matrix is decomposed as follows:

$$\mathbf{H}_{k,n} = \mathbf{U}_{k,n} \Sigma_{k,n} \mathbf{V}_{k,n}^{H} \tag{3}$$

where $\mathbf{U}_{k,n}$ and $\mathbf{V}_{k,n}$ are $M \times M$ unitary matrices of the kth user at the nth sub-carrier. $\Sigma_{k,n}$ is an $M \times M$ diagonal matrix of the kth user at the nth sub-carrier with $M \times 1$ diagonal singular values vector represented by $\rho_{k,n}$.

$$\mathbf{\Sigma}_{k,n} = \begin{bmatrix} \rho_{k,n}^{1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \rho_{k,n}^{M} \end{bmatrix} \tag{4}$$

Stacking up the singular values of all users at all sub-carriers, a $KM \times KN$ singular values matrix $\mathbf{\Sigma}$ can be represented as follows

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{1,1} & \cdots & \mathbf{\Sigma}_{1,N} \\ \vdots & \ddots & \vdots \\ \mathbf{\Sigma}_{K,1} & \cdots & \mathbf{\Sigma}_{K,N} \end{bmatrix} \tag{5}$$

The overall singular values matrix $\mathbf{\Sigma}$ is the input of the rate adaptive (RA) optimization algorithms to efficiently allocate the system sub-carriers and power to all activated users. The spatial sub-carrier and power allocation matrices are given, respectively as

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{1,1} & \cdots & \boldsymbol{C}_{1,N} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{K,1} & \cdots & \mathbf{C}_{K,N} \end{bmatrix} \tag{6}$$

and

$$\boldsymbol{P} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \tag{7}$$

where $\mathbf{C}_{k,n} = ones(M, 1)$ if and only if the nth sub-carrier is allocated to the kth user, and $\boldsymbol{P}_1$ is an $M \times 1$ power allocation vector. The mth spatial sub-channel gain of the kth user at the nth sub-carrier in its singular value is $\rho_{k,n}^{m}$ as in Eqs. (3) and (4), with AWGN power with $\sigma^2 = \mathrm{N}_0\ B/N$, where N_0 is the noise power spectral density. The corresponding spatial sub-channel SNR is thus denoted as

$$CNR_{k,n}^{m} = \frac{\left(\rho_{k,n}^{m}\right)^2}{\sigma^2} \tag{8}$$

and the *mth* spatial sub-channel received SNR of the *kth* user at the *nth* sub-carrier is

$$\gamma_{k,n}^{m} = p_{k,n}^{m}\, CNR_{k,n}^{m} \tag{9}$$

The slowly time-varying channel assumption is crucial, since it is also assumed that each spatial sub-channel state information can be estimated perfectly, and these estimates are known to the transmitter via a dedicated feedback channel. These channel estimates are then used as inputs to the resource allocation algorithms. In order that the QoS constraints are met, the effective SNR has to be adjusted, accordingly. The BER of a square L-level M-QAM with Gray bit mapping as a function of received SNR $\gamma_{k,n}^{m}$ and number of bits $R_{k,n}^{m}$ can be approximated to within 1 dB for $R_{k,n}^{m} \geq 4 \quad \mathrm{BER} \leq 10^{-3}$ [26].

$$BER_{MQAM}\left(\gamma_{k,n}^{m}\right) \cong 0.2\exp\left(\frac{-1.6\gamma_{k,n}^{m}}{2^{R_{k,n}^{m}} - 1}\right) \tag{10}$$

Solving for $R_{k,n}^{m}$

$$R_{k,n}^{m} = \log_2\left(1 + \frac{\gamma_{k,n}^{m}}{\Gamma}\right) \tag{11}$$

where $\Gamma = -\ln(5BER)/1.6$ is a constant SNR gap, and $H_{k,n}^{m} = h_{k,n}^{m}/\Gamma$ is the effective *mth* spatial sub-channel of the *kth* user at the *nth* sub-carrier.

The problem of the MU MIMO-OFDM spatial sub-carrier and power allocation with QoS constraint is formulated as follows:

Objective:

$$\max_{p_{k,n}^{m}, c_{k,n}^{m}} \frac{1}{N} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} c_{k,n}^{m} R_{k,n}^{m} \tag{12}$$

Constraints

$$\begin{aligned}
&\text{C1: } c_{k,n}^{m} \in \{0, 1\}, \quad \forall k, n, m \\
&\text{C2: } \sum_{k=1}^{K} \sum_{m=1}^{M} c_{k,n}^{m} = M, \quad \forall n \\
&\text{C3: } p_{k,n}^{m} \geq 0, \quad \forall k, n, m \\
&\text{C4: } \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} p_{k,n}^{m} \leq P_t \\
&\text{C5: } \mathcal{P}_{e,k,n}^{m} = \mathcal{P}_{e,\text{target}} \\
&\text{C6: } R_{k,n}^{m} \leq R_{\max}
\end{aligned} \tag{13}$$

where the objective is to maximize the average sum-rate capacity (the average system spectral efficiency) as in Eq. (12) within the total power budget p_t and QoS constraints of the system presented in Eq. (13). The kth user spectral efficiency is given by

$$R_k = \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{M} c_{k,n}^{m} R_{k,n}^{m} \tag{14}$$

The first two constraints are on spatial sub-carrier allocation to ensure that each sub-carrier is assigned to only one user. C4 is only effective in problems, where there is a power constraint on the total transmit power of the system P_t (e.g., rate adaptive algorithms). C5 and C6 determine the BER and maximum sub-carrier throughput constraints, respectively.

3 The Proposed Adaptive Spatial Sub-carrier and Power Allocation Algorithms

In this section, adaptive radio resource (sub-carrier and power) allocation algorithms for the spatial multiplexing-based MU MIMO-OFDM system are considered to maximize the average sum-rate capacity formulated in Eq. (12) under the constraints presented in Eq. (13). The proposed algorithms divide the radio resource allocation into two stages. Firstly, in the sub-carriers allocation stage, the system sub-carriers are spatially allocated to the activated users according to their spatial sub-channel gains. Then, an optimal two-dimensional SFGPA algorithm exploiting both the spatial and frequency diversities for efficient power allocation is applied.

3.1 Dynamic Spatial Sub-carrier Allocation (DSSA) Algorithm

The available system sub-carriers, N, are firstly adaptively allocated to the system users according to their spatial sub-channel gains, i.e. the sub-carrier allocation algorithm uses the spatial sub-channel gains, as in Eq. (5), and outputs the sub-carrier indicator matrix, as in Eq. (6). In the following sub-sections, the spatial sub-carrier allocation is classified into two algorithms. The first algorithm uses the maximum spatial sub-channel gains to allocate the sub-carriers to the users, whereas the second algorithm uses the average spatial sub-channel gains to allocate the sub-carriers to the users. The second algorithm benefits from the spatial dimension to more improve the spectral efficiency of the system as will be stated later in the simulation results section.

3.1.1 DSSA with Maximum Gain (DSSA-MG) Algorithm

The MU spatial sub-channel gains produced by applying the SVD on the MU MIMO channel matrix, stated in Eq. 5, are used to efficiently allocate power to the system activated users. In this algorithm, the maximum spatial sub-channel gain is used to allocate the system sub-carriers as shown in Table 1.

In the DSSA-MG algorithm, the input is the multi-user spatial sub-channel gains matrix, $\mathbf{\Sigma}$ which is used to calculate the spatial sub-channel power,

$$P_{k,n}^{m} = \left|\rho_{k,n}^{m}\right|^{2} \tag{15}$$

This spatial sub-channel power of the *kth* user at the *nth* sub-carrier is used to efficiently allocate the sub-carriers as in the initialization and recursion steps of the algorithm. In the initialization step, the spatial sub-channel gain is calculated for every sub-carrier. Also the per-sub-carrier indicator vector, $\mathbf{C}_{k,n}$ is initialized to zeros. In the recursion step, the *nth* sub-carrier that has the maximum spatial sub-channel power for the*kth* user is allocated for this user, and thereby it maximizes the overall sum-rate capacity and spectral efficiency. The output of the algorithm, which is the sub-carrier allocation matrix $\mathbf{C}$, is used in the power allocation algorithms. In this algorithm, the spatial dimension is not fully exploited, since the only spatial sub-channel that has maximum power is selected in the sub-carrier allocation.

Table 1 The DSSA-MG algorithm

Step	Operation
Input	MU MIMO spatial sub-channels gains, $\boldsymbol{\Sigma}$ as in Eq. (5)
Initialization	For $k = 1 : K$ For $n = 1 : N$ $\mathrm{P}_{ch,k,n} = \left\|\Sigma_{k,n}\right\|^2$ $\mathbf{C}_{k,n} = diag(zeros(1, M)$ end end
Recursion	For $n = 1 : M : N$ $k = \arg\max_n \mathrm{P}_{ch,k,n}$ $\mathbf{C}_{k,n} = diag(ones(1, M)$ end
Output	**C**

Table 2 The DSSA-AG algorithm

Step	Operation
Input	MU MIMO spatial sub-channels gains, $\boldsymbol{\Sigma}$ as in Eq. (5)
Initialization	For $k = 1 : K$ For $n = 1 : N$ $\mathrm{P}_{k,n} = mean(\left\|\Sigma_{k,n}\right\|^2)$ $\mathbf{C}_{k,n} = diag(zeros(1, M)$ end end
Recursion	For $n = 1 : M : N$ $k = \arg\max_n \mathrm{P}_{k,n}$ $\mathbf{C}_{k,n} = diag(ones(1, M)$ end
Output	**C**

3.1.2 DSSA with Average Gain (DSSA-AG) Algorithm

In this algorithm, the spatial dimension is fully exploited in the sub-carrier allocation process, where all spatial sub-channel powers are averaged and used in the sub-carrier allocation. The DSSA-AG algorithm is summarized in Table 2. The steps of the DSSA-AG is similar to those of the DSSA-MG algorithm except that the mean or average of all spatial sub-channel powers for each sub-carrier is used in the initialization and recursion steps of the algorithm. All spatial sub-channels are exploited in the DSSA-AG algorithm in addition to the frequency and multi-user dimension to really optimize the system sum-rate capacity and spectral efficiency of the MU MIMO-OFDM system as will be stated in the simulation results section.

3.2 GPA-Based Spatial Power Allocation Algorithms

When the available system sub-carriers are efficiently allocated to the activated system users, the spatial power allocation problem of the MU MIMO-OFDM system stated in Eqs. (12) and (13) is converted into a spatial power allocation problem of the single-user (SU) MIMO-OFDM system [27], which is solved by the power allocation algorithms described as follows.

The problem of the spatial multiplexing-based MIMO-OFDM power allocation with a system power budget and QoS constraints is formulated as follows:

Objective:

$$\max_{p_{m,n}} \frac{1}{N} \sum_{m=1}^{M} \sum_{n=1}^{N} R_{m,n} \tag{16}$$

Constraints:

$$\begin{aligned} &\text{C1: } p_{m,n} \geq 0, \quad \sum_{m=1}^{M} \sum_{n=1}^{N} p_{m,n} \leq P_t \\ &\text{C2: } R_{m,n} \leq R_{\max}, \quad \forall m, n \\ &\text{C3: } BER_{m,n} = BER_{\text{target}}, \quad \forall m, n \end{aligned} \tag{17}$$

where the objective is to maximize the average system spectral efficiency within the total power budget p_t and QoS constraints of the system. The average *mth* spatial sub-channel spectral efficiency is given by:

$$R_m = \frac{1}{N} \sum_{n=1}^{N} R_{m,n} = \frac{1}{N} \sum_{n=1}^{N} \log(1 + p_{m,n}\, CNR_{m,n}) \tag{18}$$

Note that constraints $c_2$and c_3 in Eq. (17) ensure the correct values for the sub-carrier throughput and QoS, respectively.

3.2.1 Spatial Power Allocation Algorithms for Spatial-Multiplexing SU MIMO-OFDM Communication Systems

In this sub-section, the adaptive spatial power allocation algorithms for the spatial multiplexing-based SU MIMO-OFDM system are considered. The optimal two-dimensional spatial frequency GPA (SFGPA) algorithm is proposed.

A. Optimal SFGPA Algorithm for SU MIMO-OFDM Communication Systems
The one-dimensional optimal GPA algorithm proposed for SU OFDM [28] is developed for the SU MIMO-OFDM. The spatial dimension is introduced in the process of power allocation to increase the average system spectral efficiency; the GPA algorithm operates on all sub-carriers in all spatial sub-channels to perform the two-dimensional GPA algorithm. The SFGPA algorithm of the SU MIMO-OFDM system can be described with the following steps:
Step 1 The transmit power is uniformly allocated for all system sub-carriers as follows

$$p_{m,n} = \frac{P_t}{M \times N} \tag{19}$$

Step 2 The problem of two-dimensional power allocation for the SU MIMO-OFDM system in Eqs. (17) and (18) is reformulated to a one-dimensional power allocation problem as follows

$$\max_{p_n} \frac{1}{N} \sum_{n}^{MN} \log_2(1 + p_n CNR_n), \quad \forall n \in \{1, 2, \ldots, MN\} \tag{20}$$

with total power budget, target BER, and maximum permissible QAM modulation order. These constraints are formulated in Eq. (18).
Step 3 The problem of the power allocation for the SU MIMO-OFDM system is solved by the two-dimensional PSGPA algorithm that is summarized in [27].

Step 4 The average system spectral efficiency and the excess power produced from the SFGPA algorithm is represented by Eqs. (21) and (22), respectively.

$$\mathrm{R}^{SFGPA} = \frac{1}{N}\sum_{n=1}^{NM} R_{m,n}^{SFGPA} \tag{21}$$

$$p_{\text{excess}}^{SFGPA} = \sum_{n=1}^{MN} \frac{\gamma_{l_{m,n}+1}^{QAM} - \gamma_{l_{m,n}}^{QAM}}{H_{m,n}} \tag{22}$$

In the subsequent sub-sections, the proposed three-dimensional GPA-based spatial power allocation algorithms for the MU MIMO-OFDM systems are described and compared.

3.2.2 Dynamic Sub-carrier Spatial Frequency Uniform Power Allocation (DS-SFUPA) Algorithm

In this algorithm, the DSSA algorithm is followed by a repeated SFUPA algorithm for each activated user. There are two versions of this algorithm according to the kind of the used DSSA algorithm. These versions are DS-SFUPA-MG algorithm, which refers to the DSSA-MG algorithm with the repeated SFUPA algorithm, and DS-SFUPA-AG algorithm, which refers to the DSSA-AG algorithm with the repeated SFUPA algorithm. The total power is uniformly allocated such that the *kth* user has the following power

$$P_k = \sum_{n=1}^{N}\sum_{m=1}^{M} c_{k,n}^{m} p_{k,n}^{m} \tag{23}$$

Figure 2 shows the flowchart of the general DS-SFUPA algorithm. The average system sum-rate capacity is calculated as

$$R^{\mathrm{DS-SFUPA}} = \frac{1}{N}\sum_{k=1}^{K} R_k^{\mathrm{SFUPA}} \tag{24}$$

3.2.3 Dynamic Sub-carrier SFGPA (DS-SFGPA) Algorithm

In this algorithm, the two-dimensional SFGPA algorithm is applied for each activated user with its allocated sub-carriers. The total power is allocated by the three-dimensional DS-SFGPA algorithm that is summarized by the flowchart depicted in Fig. 3. The average system sum-rate capacity is calculated as

$$R^{\mathrm{DS-SFGPA}} = \frac{1}{N}\sum_{k=1}^{K} R_k^{\mathrm{SFGPA}} \tag{25}$$

4 Simulation Results

Simulations experiments have been carried out to evaluate and compare the performance of the proposed spatial power allocation algorithms for the MU spatial multiplexing-based MIMO-OFDM systems. The parameters used in the simulations are summarized in Table 3. The FS and DS refer to fixed spatial sub-carrier allocation (FSSA) and DSSA, respectively.

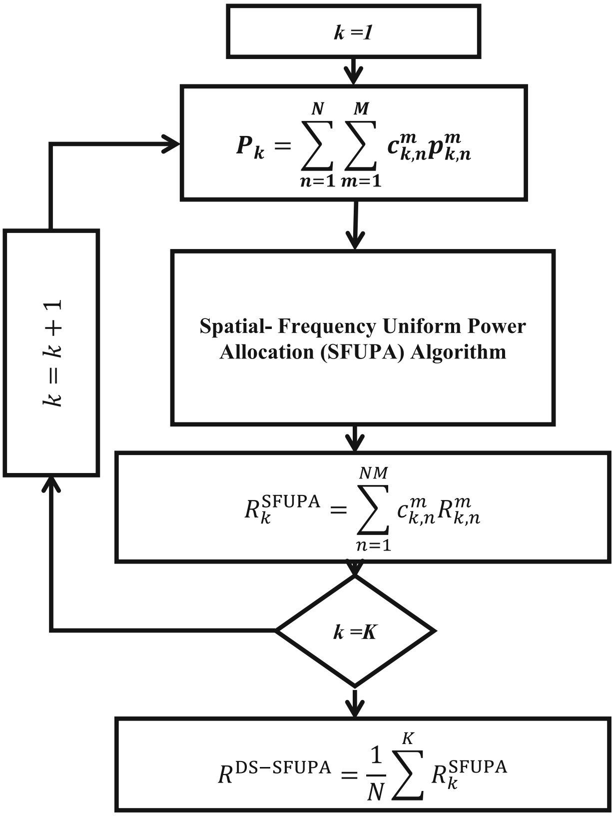

Fig. 2 Flowchart of the DS-SFUPA algorithm for MU MIMO-OFDM system

The average system sum-rate capacity is computed from Eq. (24) for DS-SFUPA algorithm and from Eq. (25) for DS-SFGPA algorithm.

In Fig. 4, the average sum-rate capacity versus number of system users, for the spatial multiplexing-based MU MIMO-OFDM systems, is depicted for the various proposed spatial power allocation algorithms. The systems are studied at an SNR of 20 dB and a number of system spatial sub-channels of $M = 4$. It is clear that the DSSA-AG based SFGPA algorithm (DS-SFGPA–AG) exhibits the maximum sum-rate capacity performance compared with the other power allocation algorithms. This is attributed to the full exploitation of the three diversities; frequency, spatial, and multi-user, to maximize the system sum-rate capacity. The DS-SFGPA-AG algorithm satisfies a sum-rate capacity of about 3.2 bps/Hz better than that of the DSSA-AG based SFUPA (DS-SFUPA-AG) algorithm, which achieves only spatial and multi-user diversity gains. The DS-SFGPA-AG algorithm also satisfies a sum-rate capacity of about 5.1 bps/Hz better than that of the traditional FS-SFUPA algorithm. The DSSA-MG SFGPA (DS-SFGPA-MG) algorithm performs less than the DS-SFGPA-AG algorithm because of the ignorance of the other spatial sub-channels except that of the maximum gain in the spatial sub-carriers allocation process.

Figure 5 shows the average sum-rate capacity versus number of system spatial sub-channels number, for the spatial multiplexing-based MU MIMO-OFDM systems, for the various proposed spatial power allocation algorithms. The system is studied at an SNR of

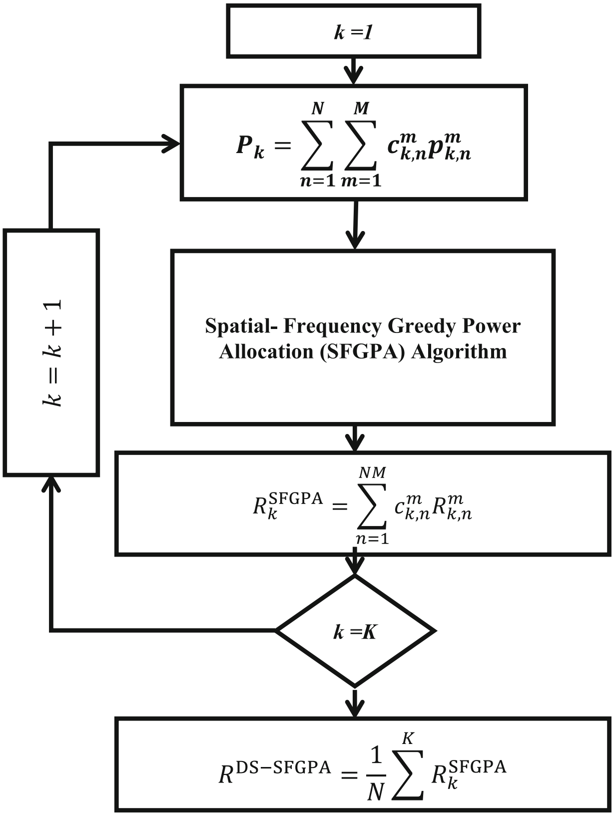

Fig. 3 Flowchart of the DS-SFGPA algorithm for MU MIMO-OFDM system

Table 3 Simulation parameters

Parameter	Value
Multipath channel	6-tap exponentially decaying power profile with Rayleigh fading
BER	10^{-3}
$\mathcal{K}$	{1, 2, 4, 8, 16, 32} user
N	64 sub-carrier
$\mathcal{M}$	{1, 2, 4, 8} antenna (spatial sub-channel)
$\mathcal{L}$	{4, 16, 64, 256} M-QAM Level

20 dB and a number of system users of K = 8. It is clear that the DS-SFGPA-AG algorithm exhibits the best sum-rate capacity performance compared with the other power allocation algorithms. The average system sum-rate capacity is increased linearly with the number of spatial sub-channels. In the case of the MU SISO-OFDM (M = 1), the two versions of the proposed DS-SFGPA algorithm coincide in performance. On the other hand, in the case of the MU MIMO-OFDM (M > 1), the DS-SFGPA-AG algorithm still outperforms the DS-SFGPA-MG algorithm. Increasing the number of system antennas leads to increasing the system spatial sub-channels, and therefore, the spatial multiplexing gain is increased,

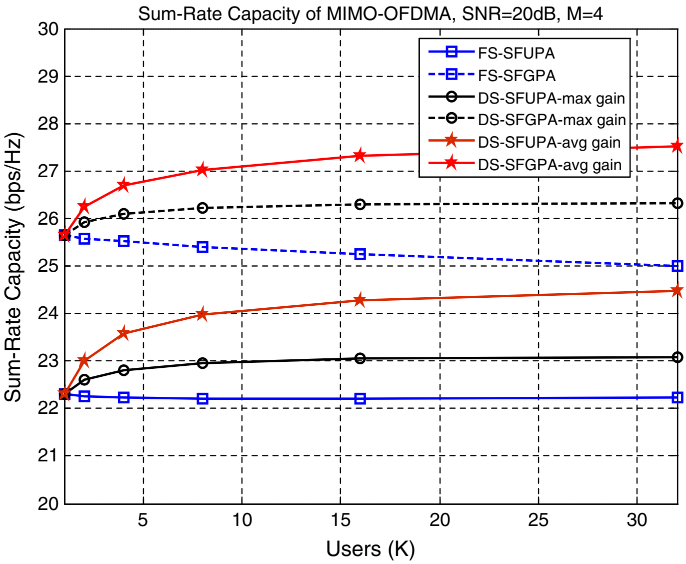

Fig. 4 Average sum-rate capacity versus number of users at SNR = 20 dB and number of antennas M = 4

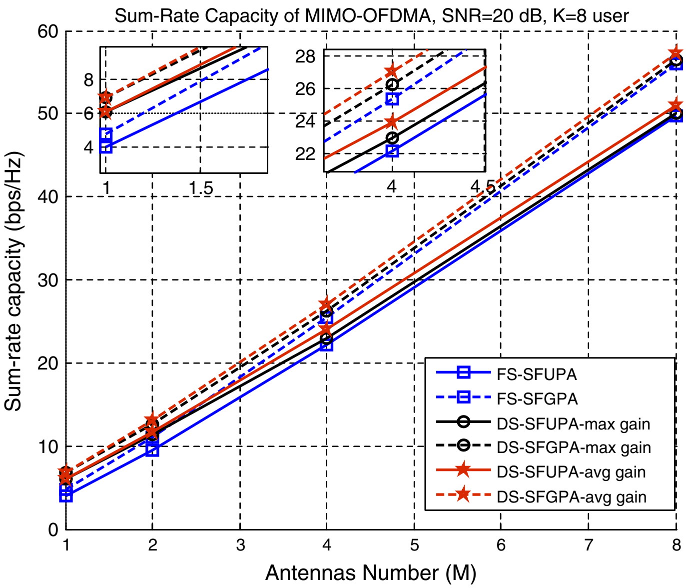

Fig. 5 Average sum-rate capacity versus number of antennas M at SNR = 20 dB and number of users K = 8

and spatial diversity can be achieved by the proposed SFGPA-based algorithms, which are represented in Fig. 5 with dotted curves.

In Fig. 6, the average sum-rate capacity versus the SNR (dB), for the spatial multiplexing-based MU MIMO-OFDM systems, is depicted for the various proposed spatial power allocation algorithms. The system is studied with number of system users, K = 8 and number of system spatial sub-channels, M = 4. It is clear that the DS-SFGPA-AG algorithm exhibits the

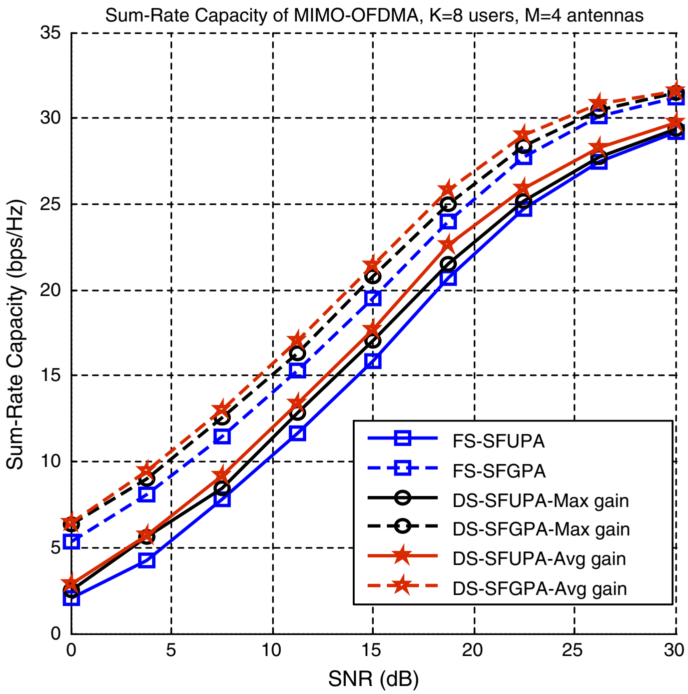

Fig. 6 Average sum-rate capacity versus SNR at a number of users, K = 8 and M = 4

best sum-rate capacity performance compared with the other power allocation algorithms. The average system sum-rate capacity is increased, when the SNR is increased. The spatial and multi-user diversity gains are fixed, whereas the frequency diversity gain is increased with SNR, i.e. the increase in SNR affects only the exploitation of the frequency diversity by the proposed spatial power allocation algorithms to maximize the average system sum-rate capacity.

In Fig. 7, the average sum-rate capacity versus the SNR (dB) of the DS-SFGPA-AG algorithm is depicted for various OFDM-based systems. These systems are studied at number of system users, K = 8, number of system spatial sub-channels, M = 4, and N = 64. Intuitively, the sum-rate capacity is increased with the increase of the SNR. It is evident that the DS-SFGPA-AG algorithm for the MU MIMO-OFDM (MIMO-OFDMA) system outperforms that of the other OFDM-based systems. In the MU MIMO-OFDM system, there are three diversities; frequency, spatial, and multi-user, which are fully exploited by the DS-SFGPA-AG algorithm to optimize the sum-rate capacity of the system transmission. In the MIMO-based systems the sum-rate capacity is largely increased, since the spatial multiplexing-based MIMO is used, and hence the spectral efficiency and sum-rate capacity is increased linearly with the number of the system antennas. In the MIMO-based systems, the sum-rate capacity is better than that of SISO-based systems with about 20 bps/Hz at SNR of 20 dB. This is attributed to the spatial multiplexing and spatial diversity gains of the MIMO-based systems. In MU systems, the sum-rate capacity is better than that of the SU systems with about 2–3 bps/Hz at an SNR of 20 dB. This is attributed to the multi-user diversity of the MU systems. At high SNR values, the average sum-rate capacities of the MU-based systems reach their maximum values of 8 and 32 bps/Hz for the SISO and MIMO schemes, respectively.

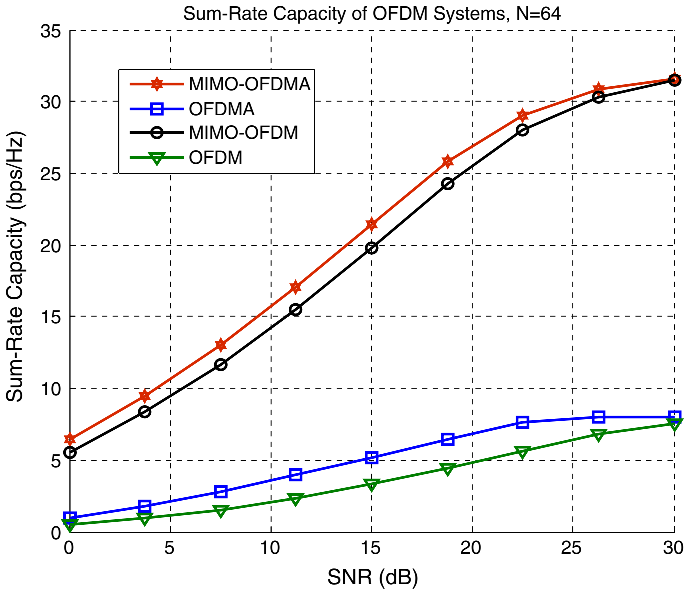

Fig. 7 Average sum-rate capacity comparison for the different OFDM-based systems

5 Conclusion

In this paper, the spatial power allocation problem of the MU MIMO-OFDM system has been discussed and analyzed. Efficient and low-complexity spatial power allocation algorithms have been proposed to solve the three-dimensional (frequency, spatial, and multi-user) resource management problem of the MU MIMO-OFDM system. The optimal solution is achieved by the three dimensional DS-SFGPA-AG algorithm, in which the spatial dimensions of the MU MIMO-OFDM system are fully exploited by proposing an efficient spatial sub-carrier allocation algorithm; DSSA-AG, which depends on the average gain of all system spatial sub-channels per sub-carrier. This proposed efficient spatial sub-carrier allocation is followed by an optimal two-dimensional SFGPA algorithm to fully exploit all available diversities of frequency, spatial, and multi-user. The optimal DS-SFGPA-AG algorithm has been analyzed and compared for the four OFDM-based systems; SU OFDM, SU MIMO-OFDM, MU OFDM, and MU MIMO-OFDM. The MU MIMO-OFDM system with adaptive sub-carrier and power allocation outperforms the other systems, where all system diversities are fully exploited to maximize the system average sum-rate capacity and spectral efficiency.

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