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VLM Precoded SLM Technique for PAPR Reduction in OFDM Systems

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Abstract Orthogonal frequency division multiplexing (OFDM) is perhaps the most spectrally efficient, robust transmission technique discovered so far for communication systems, and it also mitigates the problem of multipath environment. High peak-to-average power ratio (PAPR) has always been a major drawback of the OFDM systems. In this article, a new precoding technique has been proposed based on Vandermonde-like matrix (VLM) and selective mapping (SLM) to reduce PAPR in OFDM systems. VLM precoding reduces the autocorrelation of the input sequences while SLM takes an advantage of the fact that the PAPR is very sensitive to phase shifts of the signal. The main advantage of this proposed scheme is to achieve a significant reduction in PAPR without increasing the system complexity. Computer simulations show that, the proposed method outperforms the existing precoding techniques without degrading the error performance of the system.

Keywords OFDM · PAPR · ACF · VLM · SLM

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a special case of frequency division multiplexing (FDM) where all the carrier signals are orthogonal to each other. OFDM offers a considerable high spectral efficiency, multipath delay spread tolerance, power efficiency and a very strong immunity to the frequency selective fading channels [1,2]. As a result, it has become the technology for the next generation of wireless and wireline digital communication systems and widely deployed in many wireless communication standards [3]. OFDM is a multicarrier transmission scheme where a high-rate serial data is transmitted in parallel at a slower rate via multiple narrowband orthogonal subcarriers. The linear combination of

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the data-modulated subcarriers can have a very large amplitude fluctuations resulting in a large PAPR which is a well-known limitation of OFDM that leads to a severe nonlinear distortion in practical implementations of high power amplifier (HPA) [4]. To overcome above mentioned serious limitation, several advances have been proposed in the literature including clipping, filtering, coding schemes, phase optimization, nonlinear companding transforms, tone reservation (TR) and tone injection (TI), constellation shaping, partial transmission sequence (PTS) and selective mapping (SLM). An excellent survey of PAPR reduction techniques along with the mathematical analysis is given in [5]. SLM and PTS can provide good performance for PAPR reduction, and this improvement requires a high computational complexity [6]. Several techniques have been proposed based on low-complexity SLM techniques [7]. SLM requires the transmission of several side information bits for each data block. These bits must generally be channel-encoded because they are particularly sensitive to the error performance of the system. This increases the system complexity and transmission delay, and decreases the data rate.

In this article, a VLM precoded SLM technique has been proposed to reduce the PAPR in OFDM signals. It has been shown that the proposed method can achieve a significant gain in PAPR without increasing the computational complexity or degrading the error performance of the system. The rest of the paper is organized as follows: A brief summary of the related works is presented in Sect. 2. An overview of PAPR in OFDM signals is given in Sect. 3. Section 4 introduces the proposed method. System model is given in Sect. 5. Section 6 presents the simulation results. Finally, Sect. 7 concludes the article.

2 Related Works

The precoding based PAPR reduction techniques show great promise as they are simple linear techniques to implement without the need of any side information to be sent to the receiver. Few recent works based on precoded OFDM systems are given in [8–14]. Walsh–Hadamard transform (WHT), is a very popular precoding in the literature, but unfortunately the PAPR gain is very less [8]. On the other hand, Discrete-Hartley transform (DHT) precoding is presented in [9] and Zadoff-Chu matrix transform (ZCMT) precoding in [10] which are very effective solutions for PAPR problem, [11]. But the computational complexity increases in ZCMT than WHT or DHT precoding as it involves some extra stages of multiplication.

Sun et.al [12] presented a DCT precoded PAPR reduction technique for MSE–OFDM system and it is shown that DCT based precoding technique can considerably reduce the PAPR without degrading the error performance. Baig and Jeoti [13] proposed another DCT precoder based SLM technique for PAPR reduction with less computational complex than other precoders and it does not require any complex optimization technique.

3 Overview: PAPR Problems in OFDM

In this section, we review the basics of OFDM transmitter and the definition of PAPR. OFDM signal may be generated by an N-point inverse fast Fourier transform (IFFT) in the transmitter, and the fast Fourier transform (FFT) is employed at the receiver to restore the signal. Let us consider a block of N symbols $\mathbf{X_N} = \{\mathrm{X_k}, \mathrm{k} = 0, 1, \ldots, \mathrm{N} - 1\}$ is formed with each symbol modulating the corresponding subcarrier from a set of orthogonal subcarriers, where $\mathrm{X_k}$ is the symbol carried by the kth sub-carrier. Therefore, the discrete-time complex OFDM symbol can be written as

$$x\,(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\,j\frac{2\pi}{N}kn}, \quad 0 \le n \le N-1 \tag{1}$$

Eq. (1) consists N number of independently modulated subcarriers, which can give a large PAPR when added up coherently. PAPR is the ratio between the maximum power and the average power of the complex signal. The PAPR computed from the L-times oversampled time domain OFDM signal samples can be defined as

$$\mathrm{PAPR} = \left[\frac{\max|x(n)|^2}{E\{|x\,(n)|^2\}}\right] \tag{2}$$

where $E\{\cdot\}$ denotes the expectation operation. PAPR increases proportionally with the number of subcarriers. Reducing $\max|x(n)|$ is the principle goal of PAPR reduction techniques.

4 Proposed Precoding: Vandermonde-like Matrix

The idea of PAPR reduction using VLM transform is to reduce the autocorrelation of the input sequence before the IFFT operation is applied so that PAPR of OFDM signal is also reduced. According to [18], there is a close relation between the PAPR of OFDM signals and the aperiodic autocorrelation function (ACF) of the input vector **X**. Let $\rho\,(i)$ be the ACF of the signal X, and thus

$$\rho\,(i) = \sum_{k=0}^{N-1-i} X_{k+i} X_k^* \quad \text{for} \quad i = 0, 1, \ldots, N-1 \tag{3}$$

where * denotes the complex conjugate. Then the PAPR of the transformed signal is bounded by

$$\mathrm{PAPR} \le 1 + \frac{2}{N} \times \lambda \tag{4}$$

where λ is defined as

$$\lambda = \sum_{i=1}^{N-1} |\rho(i)| \tag{5}$$

where $|\rho(i)|$ is the absolute aperiodic ACF. It is clear from Eqs. (4) and (5) that the input vector with a lower λ can yield a signal with a lower PAPR in OFDM systems. Contribution of Vandermonde-Like matrix in this article is verified in the simulation section.

In general, the Vandermonde matrix is generated by the monomials $1, x, \ldots, x^n$ at the given nodes. If the monomials are replaced in the Vandermonde matrix by polynomials $p_0\,(x), p_1\,(x), \ldots, p_n(x)$ the resulting matrix is a Vandermonde-like matrix. The reason of such interest in Vandermonde-like matrices is based on the fact that for certain polynomials they tend to be better conditioned than any classical Vandermonde matrices especially when $p_j(x)$ satisfy a recurrence relation [15]. According to the Theorem 2.1 in [16, p. 157], two confluent Vandermonde-like matrices can be given as

$$\mathbf{V}_1(i,j) = \sqrt{\frac{2}{N+1}}\left(\cos\left(\frac{(i-1)(j-1)\,\pi}{N-1}\right)\right) \tag{6}$$

$$\mathbf{V}_2(\mathrm{i},\mathrm{j}) = \sqrt{\frac{2}{\mathrm{N}+1}}\left(\cos\left(\frac{(\mathrm{i}-1)\left(\mathrm{j}-\frac{1}{2}\right)\pi}{\mathrm{N}}\right)\right) \tag{7}$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are two examples of Vandermonde-like matrices $\mathbf{V}(\alpha_1, \alpha_2, \ldots, \alpha_n) = [\mathrm{T}_{i-1}(\alpha_j)]$ based on the Chebyshev polynomials T_k. Scaling factor $\sqrt{2/(N+1)}$ is used to normalize the matrices. For $\mathbf{V}_1$ the points $\alpha_j = \cos((j-1)\pi/(n-1))$ are the extrema of T_{n-1}, and the points $\alpha_j = \cos\left(\left(j-\frac{1}{2}\right)\pi/n\right)$ are the zeros of T_n for $\mathbf{V}_{2.}$ The Chebyshev polynomials satisfy orthogonality conditions over both these sets of points [15, pp. 522–524]. Composite OFDM signals can exhibit a very high PAPR when the input sequences are highly correlated and subcarriers will be aligned in-phase. Orthogonal transformation like VLM, is a phase-rotation of the signal vector in N-dimensional space. So, the transformed signals will be very less likely aligned in-phase, which will reduce the high peak-power of the subcarriers.

5 System Model: VLM Precoded SLM

The system model of the VLM precoded SLM-OFDM transmitter and receiver is shown in the Fig. 1. VLM transform can reduce the autocorrelation of the input sequence while SLM scrambles the OFDM signal to generate alternative input symbol sequences. At the transmitter side, the serial data sequence is divided into block of length 'N', then mapped to constellation points by PSK to produce the modulated symbols **X**. Each of the modulated symbols is then scrambled using SLM. In SLM technique of PAPR reduction, each of the input data block $\mathbf{X} = [X(0), X(1), \ldots X(N-1)]$ is multiplied with U different phase sequences $\mathbf{P}^u$, where $\mathbf{P}^u_v = e^{j\varphi^u_v}$ and $\varphi^u_v \in [0, 2\pi]$ for $v = 0, 1, 2, \ldots, N-1$ and $u = 1, 2, 3, \ldots, U$ to produce a modified data block $\mathbf{X}^u = \left[X^u[0], X^u[1], \ldots, X^u[N-1]\right]^T$. Scrambled data blocks are then VLM transformed using the matrix given in Eqs. (6) or (7). Each of these transformed sequences is made IFFT operation to produce $\mathbf{x}^u = \left[x^u[0], x^u[1], \ldots, x^u[N-1]\right]^T$, and then the one with the lowest PAPR is selected for the transmission, as shown in Fig. 2. So, the implementation of SLM technique requires U number of IFFT operations. Considering $\mathbf{H}_N = \mathbf{V}_1$ or $\mathbf{V}_{2,}$ we get the transformed OFDM signal from Eq. (1) as $\mathbf{x_N} = \mathrm{IFFT}[\mathbf{X^u H_N}]$. At the receiver side, the selected scrambled sequence $\tilde{\mathbf{X}}^{\mathbf{u}}$ can be reconstructed by performing

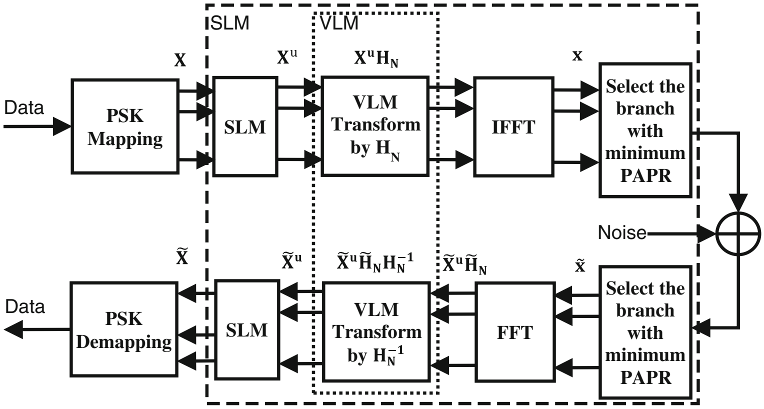

Fig. 1 Block diagram of VLM precoded SLM OFDM system

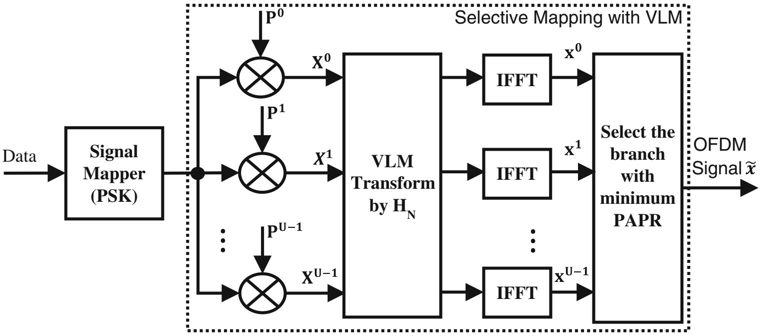

Fig. 2 OFDM transmitter of VLM precoded with SLM

an inverse VLM transform as $\tilde{\mathbf{X}}^{\mathbf{u}}\tilde{\mathbf{H}}_{\mathbf{N}}\mathbf{H}_{\mathbf{N}}^{-1}$. An effective fast algorithm for VLM inversion is given in [17]. Finally, the modulated symbols $\tilde{\mathbf{X}}$ can be recovered from $\tilde{\mathbf{X}}^{\mathbf{u}}$ using the side information about phase index ‘u’.

6 Simulation

The key idea of this article is to use VLM transformation to reduce the autocorrelation of the input sequence which will reduce PAPR of the OFDM systems according to Eqs. (4) and (5). Figure 3 shows the comparison of autocorrelation functions of original sequence, Hadamard, DCT and proposed VLM precoded sequences. It is obvious that, if the side-lobes of autocorrelation have higher values, then the input sequence is highly correlated and so is

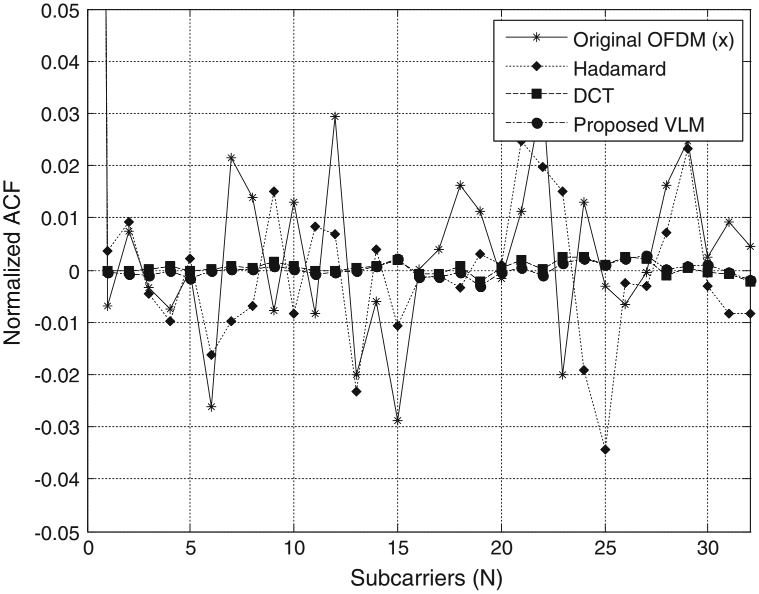

Fig. 3 The normalized autocorrelation function of original signal, signals precoded with existing techniques and proposed technique, for N = 32

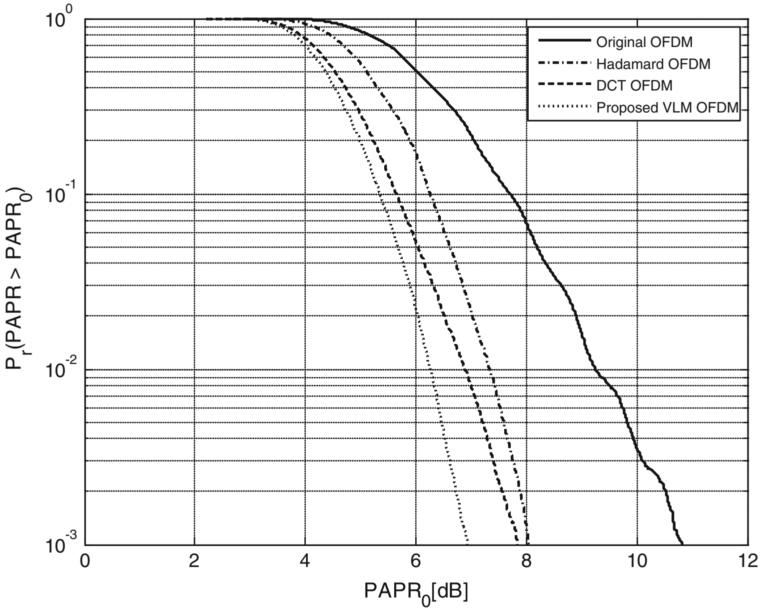

Fig. 4 CCDF of PAPR with conventional precoding techniques and the proposed method for BPSK OFDM, IEEE 802.11a

the PAPR. When highly correlated sequence is given to the IFFT input, the subcarriers will be aligned in-phase. After summing these in-phase functions the output will have a very high PAPR. Figure 3 shows that the side lobe value of the VLM precoded sequence is smaller than the original sequence or the sequences precoded with existing techniques. Therefore, it is concluded that VLM transform can decorrelate the input sequence and thus the PAPR is reduced.

The PAPR reduction performance was evaluated using the complementary cumulative distribution function (CCDF) of the PAPR, which indicates the probability that the PAPR of an OFDM sequence may exceed the threshold level $PAPR_0$. Figure 4 shows the CCDF of an OFDM system having no PAPR reduction scheme (denoted as ‘original OFDM’), the conventional precoding methods, and the proposed method. Following IEEE 802.11a standard [19], $N = 64$ FFT size, $N_V = 52$ number of valid subcarriers excluding the null carriers and binary PSK mapping are used for 10^7 randomly generated OFDM symbols. $\mathbf{V}_1$ in Eq. (6) is used as VLM precoder matrix $\mathbf{H}_N$. It is shown in Fig. 4 that the VLM precoding can achieve 1 dB PAPR gain over DCT precoding, 1.1 dB over WHT precoding and about 4 dB over normal OFDM systems at CCDF $= 10^{-3}$. According to DVB-T standard in [20], $N = 2,048$, $N_V = 1{,}706$, binary PSK for pilot symbols, quadrature PSK for data symbols and symbol length 224 μs are used in Fig. 5. It is also shown that VLM precoding achieves the same 4 dB PAPR gain in DVB-T standard.

Figure 6 shows the CCDF comparison of PAPR with conventional precoded SLM techniques and the proposed VLM precoded SLM for BPSK OFDM, IEEE 802.11a. For comparison, the SLM scheme (denoted ‘conventional SLM’) and the SLM scheme with VLM precoding (denoted ‘proposed method’) were considered together. Simulation shows that the proposed method can achieve 0.2 dB PAPR gain over DCT–SLM, 1.3 dB gain over WHT-SLM and 2.3 dB gain over conventional SLM at CCDF $= 10^{-3}$ with $U = 4$. Figure 7

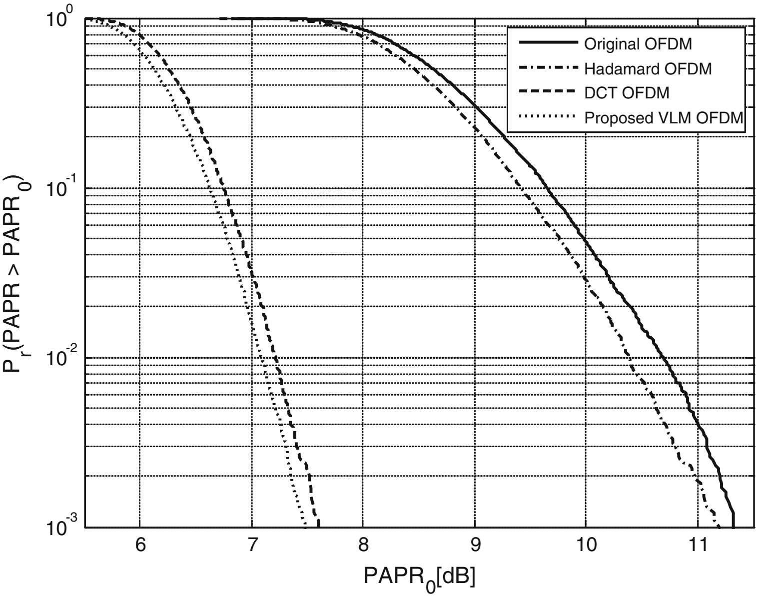

Fig. 5 PAPR characteristics of original OFDM, DCT, WHT and the proposed VLM precoded OFDM, QPSK mapped for DVB-T standard

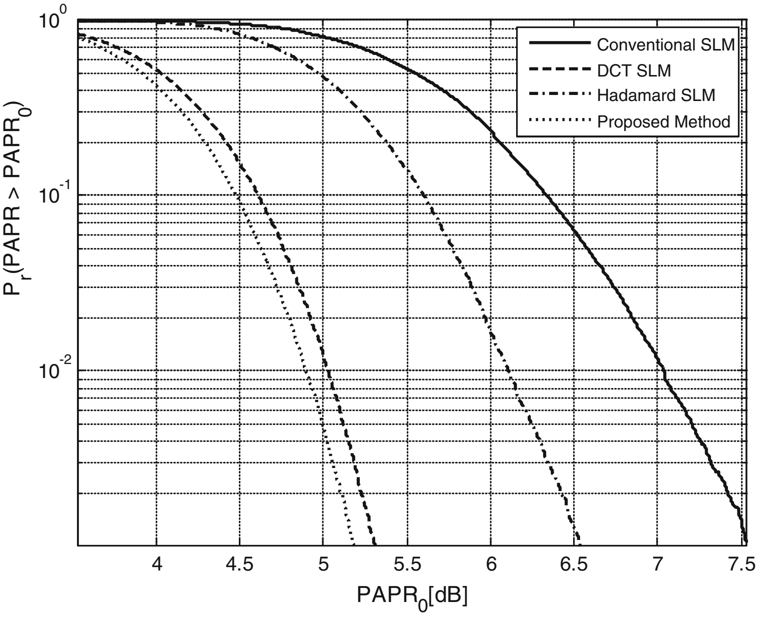

Fig. 6 CCDF of PAPR with conventional precoded SLM and proposed VLM precoded SLM for BPSK OFDM, IEEE 802.11a standard

extends the simulation for QPSK mapping, proposed method can achieve 0.3 dB PAPR gain over DCT–SLM, 0.7 dB gain over WHT-SLM and about 1.5 dB gain over conventional SLM systems at CCDF $= 10^{-3}$.

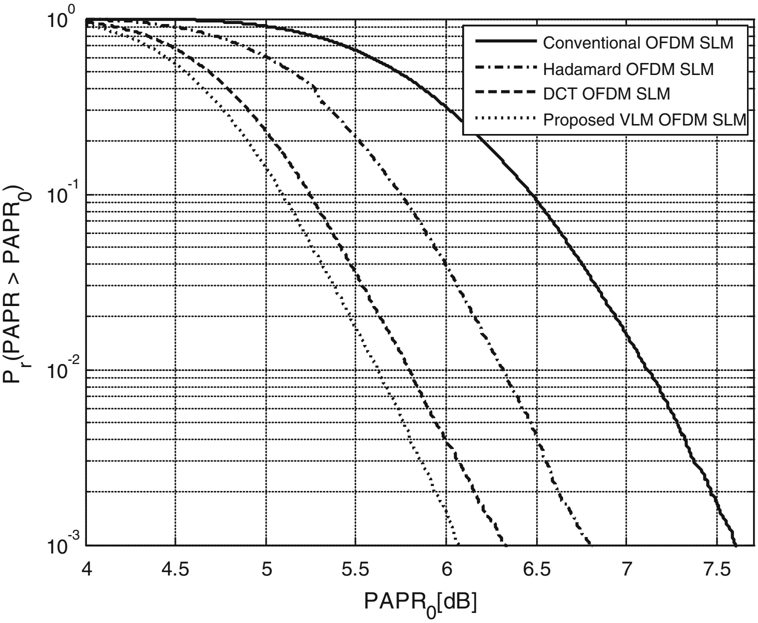

Fig. 7 CCDF of PAPR with conventional precoded SLM and proposed VLM precoded SLM for QPSK OFDM, IEEE 802.11a standard

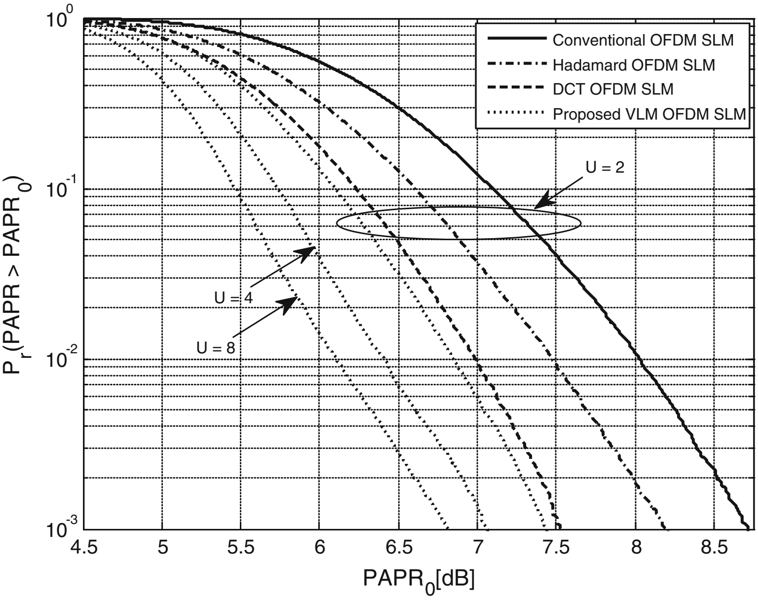

Fig. 8 CCDF of PAPR with conventional precoded SLM techniques and the proposed VLM precoded SLM for 32QAM OFDM, IEEE 802.11a standard

Figure 8 simulates the proposed method for OFDM systems with 32QAM and IEEE 802.11a standard. Simulation shows that the proposed method can achieve 0.1 dB PAPR gain over DCT-SLM, 0.9 dB gain over WHT–SLM and 1.4 dB gain over conventional SLM at

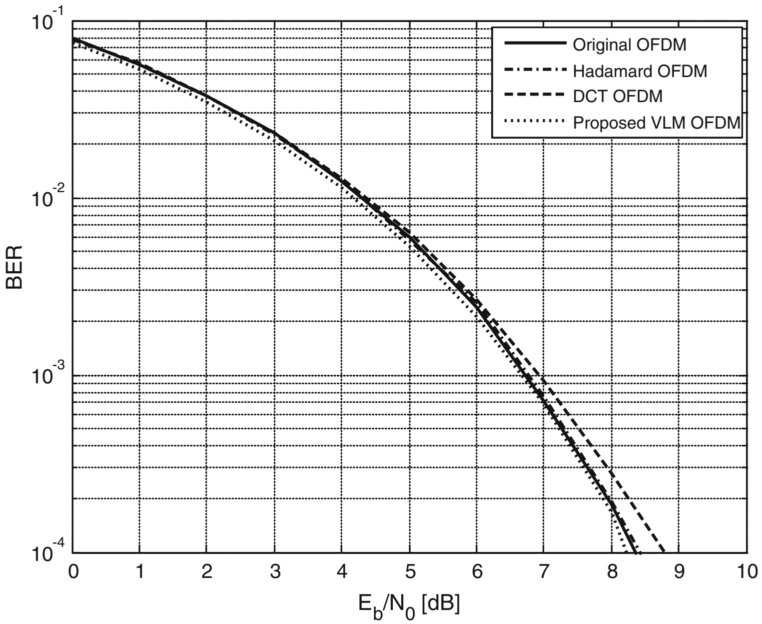

Fig. 9 BER performance of OFDM systems with conventional precoding schemes and the proposed method

CCDF $= 10^{-3}$ with U = 2. Simulation shows that, by employing more redundant bits the PAPR can be further reduced to 1.8 and 2 dB for U = 4 and 8 respectively. This reduction of PAPR can be achieved without degrading the Bit-Error-Rate (BER) performance as shown in Fig. 9.

The precoding techniques are very simple to implement, because the transformation is just the multiplication of the signal vector $\mathbf{X}_N$ and the precoder matrix $\mathbf{H}_N$. For a set of real numbers $\mathbb{R}$, the computational complexity of the proposed method is the complexity of multiplying $\mathbf{X} \in \mathbb{R}^{1\times N}$ and $\mathbf{H} \in \mathbb{R}^{N\times N}$, which is $O(N^3)$ according to schoolbook multiplication. Recently, Williams developed new tools for analyzing matrix multiplication in $O(N^{2.373})$ operations [21], which is faster than the standard matrix multiplication and can be very useful in large precoder matrices in OFDM systems like in DVB-T standard (N = 2,048, 8,192). SLM requires $UN\log_2 N$ number of additions and $U(1 + \frac{1}{2}N\log_2 N)$ number of multiplications, where U is the number of phase sequences [6,7]. Side information is an important issue in SLM as the receiver needs to be informed about the phase vector, and $\lfloor \log_2 U \rfloor$ bits are required to send this information increasing redundancy [6]. Therefore, the ability of PAPR reduction in SLM depends on the number of phase factors U, and the design of the phase factors. The issue of side information for SLM is further explored in [7] where the need for side information can be avoided.

7 Conclusion

Orthogonal frequency division multiplexing (OFDM) is a very attractive technique for communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high peak power when the input sequences are highly correlated. In this paper, author proposed a VLM precoding based SLM technique to reduce the high PAPR generated by multi carrier modulation in the OFDM systems. The PAPR reduction performances are evaluated by MATLAB simulation in terms of CCDF and BER. It was shown that proposed precoding method performs better than the conventional precoding techniques without increasing the complexity of the system or degrading the BER. Simulation results and mathematical analysis are given to support the statement.

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