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Performance Analysis and Enhancement for Opportunistic Analog Network Coding with Imperfect CSI

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Abstract Imperfect channel state information (CSI) is among the main factors that affect system performance in wireless networks. In this paper, we investigate the impact of imperfect CSI on the performance of analog network coding (ANC) for a two-way relaying system based on opportunistic relay selection (ORS). An exact and generalized closed-form expression for system outage probability is presented in a Rayleigh flat-fading environment. To provide more insights, the closed-form asymptotic expression is then obtained. It is shown that the presence of channel estimation error causes outage probability maintain a fixed level even when a noiseless channel is adopted. Therefore, to mitigate the negative impact of imperfect CSI, we deduce the power allocation to minimize the system outage probability based on the knowledge of instantaneous channel information. Numerical results validate the accuracy of the derived expressions and highlight the effect of proposed power allocation algorithm compared with conventional uniform power allocation.

Keywords Analog network coding (ANC) · Two-way relay · Outage probability · Relay selection · Imperfect channel state information (CSI)

1 Introduction

Owing to its ability to overcome the loss in the spectral efficiency due to half-duplex transmission [1], considerable attention has been concentrated on the two-way relaying [2,3]. In this scheme, two end nodes can transmit their signals to the relay simultaneously, and then the relay transmits the combined signal to both nodes. Each of the nodes can obtain its desired data by removing its own information, called self-interference. Therefore, only two time intervals are required for bi-directional transmission. Of particular interest is an analog network coding (ANC) protocol, which is a well-known amplify-and-forward (AF)-based two-way relaying protocol. The authors in [4,5] have explored the application of opportunistic relay

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selection (ORS) to the ANC and shown that the system performance is improved significantly. However, these benefits depend on knowledge of the channel state information (CSI). Clearly, with the impact of system noise and imperfect channel estimation algorithms, the relay and two end nodes hardly have the perfect knowledge of CSI. Therefore, performance analysis and enhancement that take into account such CSI uncertainties play an important role in the design of practical systems. Such a study is more challenging compared with the context of one-way relaying networks. For instance, when the channel estimation error exists, users cannot completely subtract their own information from the observations, which results in strong residual self-interference.

Very recently, some related works on ANC have taken into consideration the impact of imperfect CSI [6–8]. For single-relay scenario, lower bound and upper bound of system outage probability for ANC are evaluated in [6,7], respectively. For multi-relay scenario [8] based on ORS, the analyses are conducted by adopting an approximation which is originally proposed for the high SNR and only approximate expressions are derived. To the best of our knowledge, when CSI is not perfect, an exact performance analysis in terms of outage performance for ANC under ORS is still an open issue.

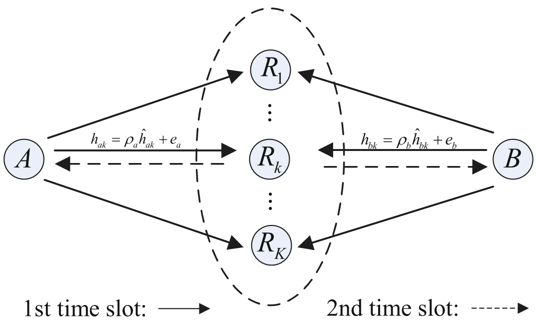
In this paper, we derive an exact and generalized closed-form outage probability expression of ORS algorithm for ANC protocol with channel estimation errors over independent and non-identically distributed (i.ni.d) Rayleigh links. Furthermore, to enhance the system performance, we perform power allocation that optimizes the system outage probability based on the knowledge of instantaneous channel information. Simulation results are in excellent agreement with the analytical results and show that the proposed power allocation algorithm offers considerable performance improvement over conventional uniform power allocation.

2 System Model

As shown in Fig. 1, we consider a two-way opportunistic relaying system in AF strategy, where two nodes A and B exchange information with each other using K relay nodes $S_{relay} = \{1, 2, \ldots, K\}$ over Rayleigh fading channels. We assume that all the nodes are equipped with a single antenna and operate in a half-duplex mode. There is no direct path between A and B, and the transmissions are subject to flat fading and additive noise. The transmit powers at A, B and relays are denoted by P_a, P_b and P_r.

In the first time slot, both A and B transmit their signals simultaneously to all the relays. Then, relay $k \in S_{relay}$ is selected as single forwarding node and the received signal at relay k can be written as $y_k = h_{ak}\sqrt{P_a}x_a + h_{bk}\sqrt{P_b}x_b + n_k$, where x_a and x_b are the unit

Fig. 1 System model for the two-way relay channel with channel estimation errors



energy transmit symbols from A and B, $n_k \sim \mathcal{CN}(0, 1)$[1] is the additive while Gaussian noise (AWGN) at relay k, $h_{ak} \sim \mathcal{CN}(0, \Omega_{h,ak})$ and $h_{bk} \sim \mathcal{CN}(0, \Omega_{h,bk})$ denote the channel coefficient between A, B and the relay k.

During the second time slot, the relay broadcasts the combined signal y_k after multiplying it with amplifying gain, G. We assume that both A and B have knowledge about their own symbols and can remove the *back-propagating self-interference* from the superimposed signals. However, as stated in the previous section, due to the channel estimation errors, the estimate of the channels gains, denoted as $\hat{h}_{ak}$ and $\hat{h}_{bk}$, are not necessarily the same as h_{ak} and h_{bk}. According to [9], the channel estimation error model can be modeled as $h_{ik} = \hat{h}_{ik} + e_{ik}$, where $i \in \{a, b\}$, $e_{ik} \sim \mathcal{CN}(0, \Omega_{e,ik})$ is the channel-estimation error. We assume that $\hat{h}_{ak}$ and $\hat{h}_{bk}$ are statistically independent of e_{ak} and e_{bk}, respectively. As a result, $\hat{h}_{ak} \sim \mathcal{CN}(0, \Omega_{\hat{h},ak})$ and $\hat{h}_{bk} \sim \mathcal{CN}(0, \Omega_{\hat{h},bk})$, where $\Omega_{\hat{h},ak} = \Omega_{h,ak} - \Omega_{e,ak}$ and $\Omega_{\hat{h},bk} = \Omega_{h,bk} - \Omega_{e,bk}$. We further assume that $\rho_a = \Omega_{e,ak}/\Omega_{h,ak}$ and $\rho_b = \Omega_{e,bk}/\Omega_{h,bk}$, which can be called as relative channel estimation error ($0 \leq \rho_i \leq 1$, where $\rho_i = 0$ means perfect CSI). In this corresponding, the power-scaled gain, G, can be written as

$$G = \sqrt{\frac{P_r}{P_a|\hat{h}_{ak}|^2 + P_b|\hat{h}_{bk}|^2 + P_a\Omega_{e,ak} + P_b\Omega_{e,bk} + 1}} \tag{1}$$

Since only $\hat{h}_{ak}$ and $\hat{h}_{bk}$ are known to node A, the resulting signal at node A can be evaluated as

$$\begin{aligned} y_{ak} &= G\hat{h}_{ak}\hat{h}_{bk}\sqrt{P_b}x_b + G\sqrt{P_a}e_{ak}(2\hat{h}_{ak} + e_{ak})x_a \\ &\quad + G\sqrt{P_b}(\hat{h}_{ak}e_{ak} + \hat{h}_{bk}e_{bk} + e_{ak}e_{bk})x_b \\ &\quad + G(\hat{h}_{ak} + e_{ak})n_k + n_a \end{aligned} \tag{2}$$

where the term $G\sqrt{P_a}e_{ak}(2\hat{h}_{ak}+e_{ak})x_a$ is the residual self-interference introduced by imperfect self-interference cancelation in the presence of imperfect CSI. Then the resultant instantaneous SNR for the link $B \to k \to A$ can be given as

$$\gamma_{ak} = \frac{P_r P_b|\hat{h}_{ak}|^2|\hat{h}_{bk}|^2}{\alpha_{1k}|\hat{h}_{ak}|^2 + \alpha_{2k}|\hat{h}_{bk}|^2 + I_{ak}} \tag{3}$$

where $\alpha_{1k} = 4P_r P_a\Omega_{e,ak} + P_r P_b\Omega_{e,bk} + P_r + P_a$, $\alpha_{2k} = P_r P_b\Omega_{e,ak} + P_b$ and $I_{ak} = 3P_r P_a\Omega_{e,ak}^2 + P_r P_b\Omega_{e,ak}\Omega_{e,bk} + (P_r + P_a)\Omega_{e,ak} + P_b\Omega_{e,bk} + 1$. The instantaneous SNR for the link $A \to k \to B$ can be obtained in a similar way:

$$\gamma_{bk} = \frac{P_r P_a|\hat{h}_{ak}|^2|\hat{h}_{bk}|^2}{\beta_{1k}|\hat{h}_{ak}|^2 + \beta_{2k}|\hat{h}_{bk}|^2 + I_{bk}} \tag{4}$$

where $\beta_{2k} = 4P_r P_b\Omega_{e,bk} + P_r P_a\Omega_{e,ak} + P_r + P_b$, $\beta_{1k} = P_r P_a\Omega_{e,bk} + P_a$ and $I_{bk} = 3P_r P_b\Omega_{e,bk}^2 + P_r P_a\Omega_{e,ak}\Omega_{e,bk} + (P_r + P_b)\Omega_{e,bk} + P_a\Omega_{e,ak} + 1$. The corresponding one-sided data-rates via the relay k are thus given as $R_{ak} = \frac{1}{2}\log_2(1+\gamma_{ak})$ and $R_{bk} = \frac{1}{2}\log_2(1+\gamma_{bk})$.

As clearly shown in (3) and (4), the forms of instantaneous SNR are not easily tractable due to the existence of imperfect CSI. For theoretical analysis simplicity, I_{ak} and I_{bk} are omitted in the sequel, since they are negligible compared with other terms [7].

[1] *Notations:* We use $x \sim \mathcal{CN}(a, b)$ to denote a complex Gaussian random variable x with mean a and variance b. $\mathbf{Pr}[\cdot]$ and $|\cdot|$ denote probability and absolute value respectively.

3 Outage Probability Analysis

In this paper, we adopt the max-min method for relay selection which has the best performance in minimizing the system outage probability [3]. Since the worse one of the two received SNRs is the bottleneck of the outage probability, the relay selection criterion is given by

$$k^* = \arg \max_{k \in \{1,2,\ldots,K\}} \min \left(\gamma_{ak},\ \gamma_{bk}\right) \tag{5}$$

where k^* denotes the best relay. With required data rate R_{th} at A and B, the system outage probability can be expressed as

$$\begin{aligned}
\mathcal{P}_{out} &= \mathbf{Pr}\left[R_{ak^*} \leqslant R_{th}\ or\ R_{bk^*} \leqslant R_{th}\right] \\
&= \mathbf{Pr}\left[\max_{k \in \{1,2,\ldots,K\}} \min \left(\gamma_{ak},\ \gamma_{bk}\right) \leqslant \gamma_{th}\right] \\
&= \prod_{k=1}^{K} \left\{\mathbf{Pr}\left[\min \left(\gamma_{ak},\ \gamma_{bk}\right) \leqslant \gamma_{th}\right] \triangleq \mathcal{P}_{out,k}\right\}
\end{aligned} \tag{6}$$

where $\gamma_{th} = 2^{2R_{th}} - 1$. According to (3) and (4), γ_{ak} and γ_{bk} are highly correlated, since they are both functions of variables $|\hat{h}_{ak}|^2$ and $|\hat{h}_{bk}|^2$. Moreover, the existence of channel estimation error makes (6) mathematically intractable. As a result, it is very difficult to evaluate (6) straightforwardly. In order to make the analysis feasible, numerous well known existing literatures take a bounding approach (e.g., $\frac{xy}{x+y} \leq \min(x,\ \ y)$) (see [10] and the references [7] therein). Unlike the current research activities, in this paper, we first decompose $\mathcal{P}_{out,k}$ into following three separated parts and focus on an exact analysis of it.

$$\mathcal{P}_{out,k} = \begin{cases}
\mathbf{Pr}\left[\gamma_{bk} \leqslant \gamma_{th}\right], & \frac{P_a}{P_b} \leqslant \frac{\beta_{1k}}{\alpha_{1k}} \\
\mathbf{Pr}\left[\min \left(\gamma_{ak},\quad \gamma_{bk}\right) \leqslant \gamma_{th}\right], & \frac{\beta_{1k}}{\alpha_{1k}} < \frac{P_a}{P_b} < \frac{\beta_{2k}}{\alpha_{2k}} \\
\mathbf{Pr}\left[\gamma_{ak} \leqslant \gamma_{th}\right], & \frac{P_a}{P_b} \geqslant \frac{\beta_{2k}}{\alpha_{2k}}
\end{cases} \tag{7}$$

Proof see “Appendix A”. □

When $\frac{P_a}{P_b} \leqslant \frac{\beta_{1k}}{\alpha_{1k}}$, since $|\hat{h}_{ak}|^2$ and $|\hat{h}_{bk}|^2$ are both exponentially distributed with parameters $1/\Omega_{\hat{h},ak}$ and $1/\Omega_{\hat{h},bk}$, $\mathcal{P}_{out,k}$ can be evaluated straightforward with the help of [11], which can be given as

$$\begin{aligned}
\mathcal{P}_{out,k} = 1 - 2\frac{\beta_{1k}\beta_{2k}\gamma_{th}}{P_r P_a}\sqrt{\frac{1}{\beta_{1k}\beta_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\, e^{-\left(\frac{1}{\beta_{1k}\Omega_{\hat{h},ak}}+\frac{1}{\beta_{2k}\Omega_{\hat{h},bk}}\right)\frac{\beta_{1k}\beta_{2k}\gamma_{th}}{P_r P_a}} \\
\times K_1\left(2\frac{\beta_{1k}\beta_{2k}\gamma_{th}}{P_r P_a}\sqrt{\frac{1}{\beta_{1k}\beta_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\right)
\end{aligned} \tag{8}$$

Similarly, when $\frac{P_a}{P_b} \geqslant \frac{\beta_{2k}}{\alpha_{2k}}$, $\mathcal{P}_{out}$ can be presented as

$$\begin{aligned}
\mathcal{P}_{out,k} = 1 - 2\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}\sqrt{\frac{1}{\alpha_{1k}\alpha_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\, e^{-\left(\frac{1}{\alpha_{1k}\Omega_{\hat{h},ak}}+\frac{1}{\alpha_{2k}\Omega_{\hat{h},bk}}\right)\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}} \\
\times K_1\left(2\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}\sqrt{\frac{1}{\alpha_{1k}\alpha_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\right)
\end{aligned} \tag{9}$$

When $\frac{\beta_{1k}}{\alpha_{1k}} < \frac{P_a}{P_b} < \frac{\beta_{2k}}{\alpha_{2k}}$, $\mathcal{P}_{out}$ is evaluated based on the corresponding intervals, i.e., $\frac{\beta_{1k}}{\alpha_{1k}} < \frac{P_a}{P_b}$ and $\frac{P_a}{P_b} < \frac{\beta_{2k}}{\alpha_{2k}}$, and can be presented as follows (*proof:* see "Appendix B")

$$\begin{aligned}
\mathcal{P}_{out,k} &= 1 - 2\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}\sqrt{\frac{1}{\alpha_{1k}\alpha_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\, e^{-\left(\frac{1}{\alpha_{1k}\Omega_{\hat{h},ak}}+\frac{1}{\alpha_{2k}\Omega_{\hat{h},bk}}\right)\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}} \\
&\quad \times K_1\left(2\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}\sqrt{\frac{1}{\alpha_{1k}\alpha_{2k}\Omega_{\hat{h},ak}\Omega_{\hat{h},bk}}}\right) \\
&\quad - \frac{P_r P_a w - \beta_{1k}\gamma_{th}}{\Omega_{\hat{h},bk} P_r P_a} e^{-\left(\frac{1}{\beta_{2k}\Omega_{\hat{h},bk}}+\frac{1}{\beta_{1k}\Omega_{\hat{h},ak}}\right)\frac{\beta_{1k}\beta_{2k}\gamma_{th}}{P_r P_a}} \\
&\quad \times \sum_{n=0}^{\infty}\left[\frac{1}{n!}\left(-\frac{\beta_{2k}\beta_{1k}\gamma_{th}\gamma_{th}}{\Omega_{\hat{h},ak}P_r P_a(P_r P_a w - \beta_{1k}\gamma_{th})}\right)^n E_n\left(\frac{P_r P_a w - \beta_{1k}\gamma_{th}}{\Omega_{\hat{h},bk}P_r P_a}\right)\right] \\
&\quad + \frac{P_r P_b w - \alpha_{1k}\gamma_{th}}{\Omega_{\hat{h},bk} P_r P_b} e^{-\left(\frac{1}{\alpha_{2k}\Omega_{\hat{h},bk}}+\frac{1}{\alpha_{1k}\Omega_{\hat{h},ak}}\right)\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}}{P_r P_b}} \\
&\quad \times \sum_{n=0}^{\infty}\left[\frac{1}{n!}\left(-\frac{\alpha_{1k}\alpha_{2k}\gamma_{th}\gamma_{th}}{\Omega_{\hat{h},ak}P_r P_b(P_r P_b w - \alpha_{1k}\gamma_{th})}\right)^n E_n\left(\frac{P_r P_b w - \alpha_{1k}\gamma_{th}}{\Omega_{\hat{h},bk}P_r P_b}\right)\right]
\end{aligned} \tag{10}$$

where $w = \frac{\gamma_{th}}{P_r}\frac{\alpha_{1k}\beta_{2k}-\alpha_{2k}\beta_{1k}}{P_b\beta_{2k}-P_a\alpha_{2k}}$ and $E_n(\cdot)$ is the exponential integral function defined in (5.1.4) of [12].

Note that (8)–10) are generalized expressions for system outage probability of ANC protocol, which encompass the outage probability performance with perfect CSI as a special case ($\rho_a = \rho_b = 0$).

For better insights, we can represent (8) and (9) in simple closed-form by using a high SNR and low channel estimation error approximation. By applying the approximations $K_1(x) \approx 1/x$ and $e^{-x} \approx 1 - x$ for $x \to 0$, (8) and (9) can respectively represented as (11 and (12).

$$\tilde{\mathcal{P}}_{out,k} \approx \left(\frac{\Omega_{e,ak}}{\Omega_{\hat{h},ak}} + \frac{\Omega_{e,bk}}{\Omega_{\hat{h},bk}}\right)\gamma_{th} + \left(\frac{4P_2\Omega_{e,bk}}{P_1\Omega_{\hat{h},ak}} + \frac{P_r+P_2}{P_r P_1\Omega_{\hat{h},ak}} + \frac{1}{P_r\Omega_{\hat{h},bk}}\right)\gamma_{th} \tag{11}$$

and

$$\tilde{\mathcal{P}}_{out,k} \approx \left(\frac{\Omega_{e,ak}}{\Omega_{\hat{h},ak}} + \frac{\Omega_{e,bk}}{\Omega_{\hat{h},bk}}\right)\gamma_{th} + \left(\frac{4P_1\Omega_{e,ak}}{P_2\Omega_{\hat{h},bk}} + \frac{1}{P_r\Omega_{\hat{h},ak}} + \frac{P_r+P_1}{P_r P_2\Omega_{\hat{h},bk}}\right)\gamma_{th} \tag{12}$$

By adopting additional approximation $E_n(z) \approx E_0(z) = e^{-z}/z$ for $z \to 0$, (10) can be approximated as

$$\begin{aligned}
\tilde{\mathcal{P}}_{out,k} &\approx \left(\frac{4P_2\Omega_{e,bk}}{P_1\Omega_{\hat{h},ak}} + \frac{4P_1\Omega_{e,ak}}{P_2\Omega_{\hat{h},bk}} + \frac{P_r+P_1}{P_r P_2\Omega_{\hat{h},bk}} + \frac{P_r+P_2}{P_r P_1\Omega_{\hat{h},ak}}\right)\gamma_{th} \\
&\quad + \left(\frac{\Omega_{e,ak}}{\Omega_{\hat{h},ak}} + \frac{\Omega_{e,bk}}{\Omega_{\hat{h},bk}}\right)\gamma_{th}
\end{aligned} \tag{13}$$

(11)–13) readily reveal that the outage probability will not approach zero even when the value of transmission SNR is infinite. The presence of channel estimation error causes the outage probability maintain a fixed level even noiseless channel is adopted. This fixed level cannot be eliminated totally. However, it can be reduced by adequate power control, which reveals the necessity of power allocation strategies.

4 Power Allocation

To mitigate the negative effect of imperfect CSI, we resort to adaptive power allocation by minimizing the outage probability under the total power consumed constraint. The optimization problem can be formulated under a total power constraint P_t as

$$P_r^*, P_a^*, P_b^* = \arg \min_{P_r, P_a, P_b} \mathcal{P}_{out}$$
$$s.t. \begin{cases} P_r + P_a + P_b \leqslant P_t \\ P_r, P_a, P_b \geqslant 0 \end{cases} \tag{14}$$

From (6), we can observe that optimizing $\mathcal{P}_{out}$ is equivalent to optimizing $\mathcal{P}_{out,k}$, where $k \in \{1, 2, \ldots, K\}$. Therefore, similar to [10], the optimization problem given in (14) is equivalent to the following one

$$P_{r,k}^*, P_{a,k}^*, P_{b,k}^* = \arg \max_{P_{r,k}, P_{a,k}, P_{b,k}} \min(\gamma_{ak}, \gamma_{bk})$$
$$s.t. \begin{cases} P_{r,k} + P_{a,k} + P_{b,k} \leqslant P_t \\ P_{r,k}, P_{a,k}, P_{b,k} \geqslant 0 \; and \; k \in \{1, 2, \ldots, K\} \end{cases} \tag{15}$$

As mentioned in [10], the above can be solved by first balancing $\gamma_{ak} = \gamma_{bk}$. Applying Kuhn-Tucker conditions [13] for Lagrangian optimality, we can obtain the following optimal solution after some involved manipulations:

$$\begin{cases} P_{a,k}^* = \frac{P_t}{2} \frac{\sqrt{|\hat{h}_{ak}|^2 |\hat{h}_{bk}|^2 (1+\Omega_{e,ak} P_t)(1+\Omega_{e,bk} P_t)} - |\hat{h}_{bk}|^2 (1+\Omega_{e,bk} P_t)}{|\hat{h}_{ak}|^2 (1+\Omega_{e,ak} P_t) - |\hat{h}_{bk}|^2 (1+\Omega_{e,bk} P_t)} \\ P_{b,k}^* = \frac{P_t}{2} \frac{|\hat{h}_{ak}|^2 (1+\Omega_{e,ak} P_t) - \sqrt{|\hat{h}_{ak}|^2 |\hat{h}_{bk}|^2 (1+\Omega_{e,ak} P_t)(1+\Omega_{e,bk} P_t)}}{|\hat{h}_{ak}|^2 (1+\Omega_{e,ak} P_t) - |\hat{h}_{bk}|^2 (1+\Omega_{e,bk} P_t)} \\ P_{r,k}^* = \frac{1}{2} P_t \end{cases} \tag{16}$$

When $\rho_a = \rho_b = 0, \ \Omega_{e,ak} = \Omega_{e,bk} = 0$, then (16) reduces to

$$\begin{cases} P_{a,k}^* = \frac{P_t}{2} \frac{|\hat{h}_{bk}|}{|\hat{h}_{ak}| + |\hat{h}_{bk}|} \\ P_{b,k}^* = \frac{P_t}{2} \frac{|\hat{h}_{ak}|}{|\hat{h}_{ak}| + |\hat{h}_{bk}|} \\ P_{r,k}^* = \frac{1}{2} P_t \end{cases} \tag{17}$$

which corresponds to the optimal power allocation provided in [10], indicating that (16) is the generalized power allocation form for ANC protocol. Substituting (16) into the objective function, we can obtain the optimum SNRs.

$$\begin{aligned} \gamma_{ak} &= \gamma_{bk} \\ &= \frac{P_t |\hat{h}_{ak}| |\hat{h}_{bk}|}{|\hat{h}_{ak}|(2 + \Omega_{e,bk} P_t) + |\hat{h}_{bk}|(2 + \Omega_{e,ak} P_t) + 4|\hat{h}_{ak}| \sqrt{1 + \Omega_{e,bk} P_t} \sqrt{1 + \Omega_{e,ak} P_t}} \\ &\triangleq \ \gamma_k^* \end{aligned} \tag{18}$$

On applying the proposed power allocation algorithm, the max-min relay selection criterion presented in (5) can be reformulated as

$$
\begin{aligned}
k^* &= \arg \max_{k \in \{1,2,\ldots,K\}} \max_{P_{r,k}+P_{a,k}+P_{b,k} \leqslant P_t} \min\left(\gamma_{ak},\ \gamma_{bk}\right) \\
&= \arg \max_{k \in \{1,2,\ldots,K\}} \gamma_k^*
\end{aligned} \tag{19}
$$

As shown in (19), we skillfully combine the ORS policy with the optimal power allocation. It is well known that opportunistic relay selection and power allocation for relaying protocols are both effective means to enhance system performance. Thereby, the combination of them can bring significant performance improvement. To provide guidelines for practical system design, we summarize the joint relay selection and power allocation scheme as follows

a) Select the optimal relay based on the following criterion
$k^* = \arg \max_{k \in \{1,2,\ldots,K\}} \frac{P_t |\hat{h}_{ak}||\hat{h}_{bk}|}{|\hat{h}_{ak}|(2+\Omega_{e,bk}P_t)+|\hat{h}_{bk}|(2+\Omega_{e,ak}P_t)+4|\hat{h}_{ak}|\sqrt{1+\Omega_{e,bk}P_t}\sqrt{1+\Omega_{e,ak}P_t}}$

b) Calculate the power levels of both nodes A and B according to (16).

c) Use the feedback channel to notify the selected relay and the power levels.

5 Numerical and Simulation Results

In this section, Monte Carlo simulations are performed to verify the accuracy of our analytical results and present the performance of the proposed power allocation algorithm. All channels in the cooperation model are described by Rayleigh fading. We consider a 2-D network topology and normalize the distance between A and B to unity. We let d_i denote the normalized distance between nodes i and relay R. Nodes A and B are located at points (0, 0) and (1, 0). We assume that the data rate requirement $R_{th} = 0.5$ bps/Hz and the path loss exponent $\alpha = 3$. Thus, $\Omega_{h,ik} = d_i^{-3}$ and $\Omega_{e,ik} = \rho_i d_i^{-3}$, where $i \in \{a, b\}$.

In Fig. 2, the system outage probability of two-way AF relay system based on opportunistic relay selection is presented by varying SNR with $\rho_a = \rho_b = 0.005$ and relay numbers $K = 1, 2, 3, 4, 5$. We consider uniform power allocation ($P_r = P_a = P_b$) and the relay locations (0.5, 0), (0.5, 0), (0.5, 0.25), (0.25, 0.25) and (0.25, – 0.25). As shown in Fig. 2, our theoretical curves are in excellent agreement with the Monte Carlo simulation results, which validates the accuracy of our derived expressions. Also, we can observe that the outage probability reaches a fixed level, called error floor, due to the imperfection in the channel estimation.

In Fig. 3, we make the comparisons in terms of the system outage probability between the proposed power allocation algorithm and the conventional uniform power allocation strategy. We assume that the three nodes are located in a straight line with $d_a + d_b = 1$ and consider $\rho_a = \rho_b = 0.005$ and relay numbers $K = 1, 2, 3$. As can be clearly seen from Fig. 3, the proposed power allocation brings significant performance improvement compared with the uniform power allocation, which is more remarkable when the relays are located close to either node.

To gain more insight into the impact of the imperfect CSI on the performance of ANC protocol for a two-way relaying system based on ORS, in Fig. 4, we present the overall system outage probability against the channel estimation error. We assume that $d_a = d_b = 0.5$ and vary $\rho_a = \rho_b = \rho$ from 0.001 to 0.1. From Fig. 4, we can observe that the proposed power allocation outperforms the uniform power allocation, where the performance improvement brought by the proposed algorithm increases with ρ. Therefore, it is necessary to apply the proposed power allocation to ANC protocol with imperfect CSI.

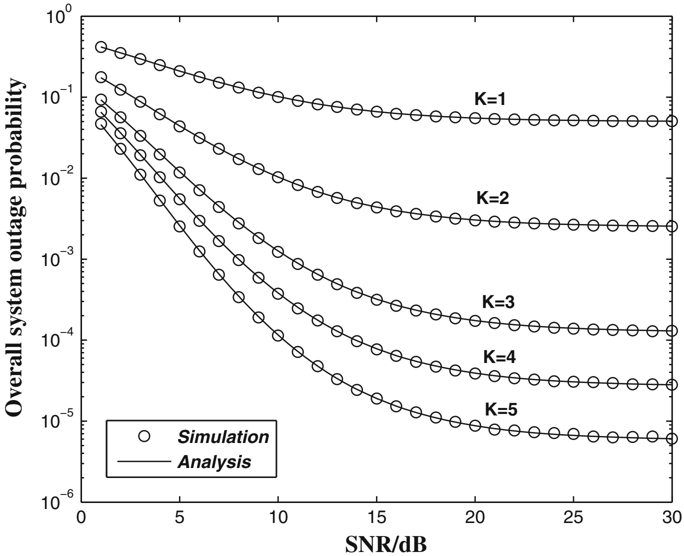


Fig. 2 System outage probability against SNR with $\rho_a = \rho_b = 0.005$ and relay numbers $K = 1, 2, 3, 4, 5$

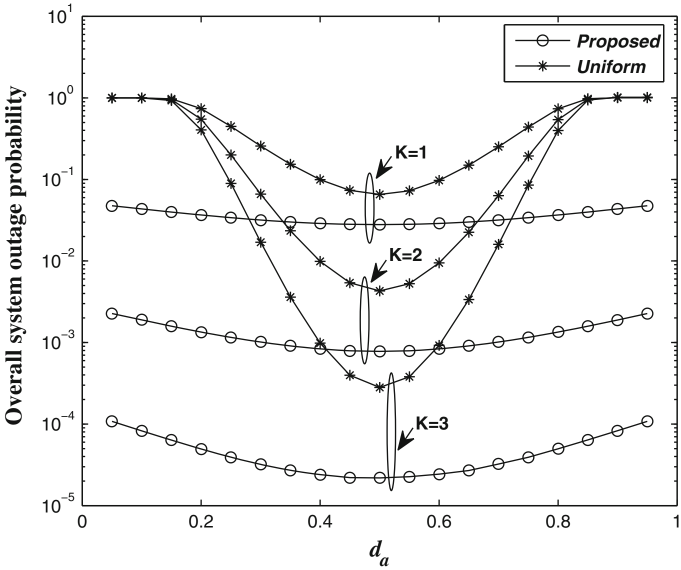


Fig. 3 System outage probability against d_a with $\rho_a = \rho_b = 0.005$, SNR=20dB and relay numbers $K = 1, 2, 3$

6 Conclusions

In this paper, we study the performance and instantaneous power allocation for two-way opportunistic AF relaying with imperfect CSI under flat Rayleigh fading channels. We have first derived a generalized closed-form expression of the system outage probability. Then, an optimum power allocation scheme is proposed so as to minimize the system outage

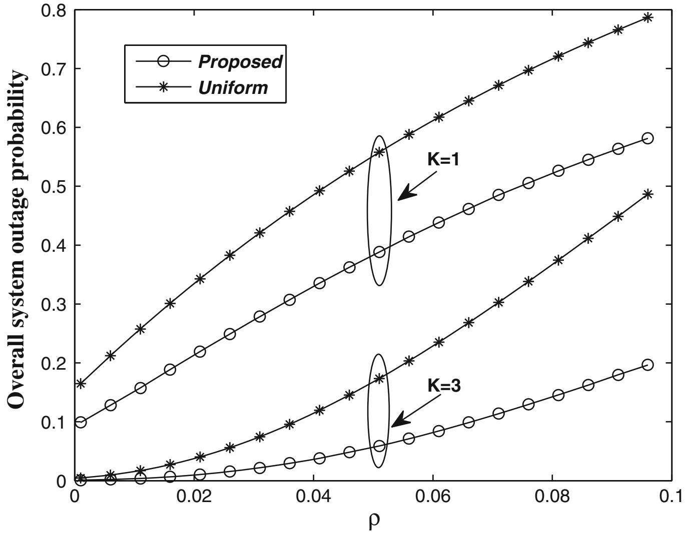


Fig. 4 System outage probability against ρ with $d_a = d_b = 0.5$, SNR=10dB and relay numbers $K = 1, 3$

probability, forming a joint relay selection and power allocation scheme. Simulation results verify that our derived outage probability expression is accurate and the proposed power allocation algorithm is greatly superior to conventional uniform power allocation, hence mitigating the negative impact of imperfect CSI. As further work, it should be interesting to analyze the ANC protocol for frequency selective channels.

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Appendix A

$$\begin{aligned}\gamma_{ak} - \gamma_{bk} &= \frac{P_r P_b |\hat{h}_{ak}|^2 |\hat{h}_{bk}|^2}{\alpha_{1k}|\hat{h}_{ak}|^2 + \alpha_{2k}|\hat{h}_{bk}|^2} - \frac{P_r P_a |\hat{h}_{ak}|^2 |\hat{h}_{bk}|^2}{\beta_{1k}|\hat{h}_{ak}|^2 + \beta_{2k}|\hat{h}_{bk}|^2} \\ &= \Phi\left((P_b\beta_{1k} - P_a\alpha_{1k})|\hat{h}_{ak}|^2 + (P_b\beta_{2k} - P_a\alpha_{2k})|\hat{h}_{bk}|^2\right)\end{aligned} \tag{20}$$

where $\Phi = \frac{P_r|\hat{h}_{ak}|^2|\hat{h}_{bk}|^2}{\left(\alpha_{1k}|\hat{h}_{ak}|^2+\alpha_{2k}|\hat{h}_{bk}|^2\right)\left(\beta_{1k}|\hat{h}_{ak}|^2+\beta_{2k}|\hat{h}_{bk}|^2\right)} \geq 0$.

a) Clearly, when $P_b\beta_{1k} - P_a\alpha_{1k} \geq 0$, i.e., $\beta_{1k}/\alpha_{1k} \geq P_a/P_b,\ P_b\beta_{2k} - P_a\alpha_{2k} \geq 0$. From (20), we can obtain $\gamma_{ak} \geq \gamma_{bk}$. Thus we have

$$\mathbf{Pr}\left[\min(\gamma_{ak},\ \gamma_{bk}) \leq \gamma_{th}\right] = \mathbf{Pr}\left[\gamma_{bk} \leq \gamma_{th}\right] \tag{21}$$

b) Similarly, when $P_b\beta_{2k} - P_a\alpha_{2k} \leq 0$, i.e., $\beta_{2k}/\alpha_{2k} \leq P_a/P_b,\ P_b\beta_{1k} - P_a\alpha_{1k} \leq 0$. From (20), we can obtain $\gamma_{ak} \leq \gamma_{bk}$. Thus we have

$$\mathbf{Pr}\left[\min(\gamma_{ak},\ \gamma_{bk}) \leq \gamma_{th}\right] = \mathbf{Pr}\left[\gamma_{ak} \leq \gamma_{th}\right] \tag{22}$$

According to the above discussions, we can arrive at the criterion in (7).

Appendix B

$$\mathcal{P}_{out,k} = 1 - \mathbf{Pr}\left[\gamma_{ak} > \gamma_{th},\ \gamma_{bk} > \gamma_{th}\right] \tag{23}$$

As γ_{ak} and γ_{bk} are not independent, it is hard to evaluate (23) straightforward. By using the fact that $\beta_{1k}/\alpha_{1k} < P_a/P_b$ and some manipulations, we can obtain

$$\mathcal{P}_{out,k} = 1 - \mathbf{Pr}\left[X > \max\left(\vartheta_1,\ \vartheta_2\right), Y > \frac{\alpha_{1k}\gamma_{th}}{P_r P_b}\right] \tag{24}$$

where $X = |\hat{h}_{ak}|^2$, $Y = |\hat{h}_{bk}|^2$, $\vartheta_1 = \frac{\alpha_{2k}\gamma_{th}Y}{P_r P_b Y - \alpha_{1k}\gamma_{th}}$ and $\vartheta_2 = \frac{\beta_{2k}\gamma_{th}Y}{P_r P_a Y - \beta_{1k}\gamma_{th}}$. Furthermore, by using $\beta_{2k}/\alpha_{2k} > P_a/P_b$, (24) can be rewritten as

$$\begin{aligned}\mathcal{P}_{out} &= 1 - \underbrace{\mathbf{Pr}\left[X > \vartheta_2,\ Y > \omega\right]}_{\xi_{ak}} \\ &\quad \underbrace{- \mathbf{Pr}\left[X > \vartheta_1,\ \frac{\alpha_{1k}\gamma_{th}}{P_r P_b} < Y \leqslant \omega\right]}_{\xi_{bk}}\end{aligned} \tag{25}$$

As X and Y are independent, we can evaluate ξ_{ak} as

$$\begin{aligned}\xi_{ak} &= \int_{\omega}^{\infty} f_Y(y) \int_{\vartheta_2}^{\infty} f_X(x)dxdy \\ &= \int_{\omega}^{\infty} f_Y(y) e^{-\frac{1}{\Omega_{\hat{h},ak}} \frac{\beta_{2k}\gamma_{th}y}{P_r P_a y - \beta_{1k}\gamma_{th}}} dy\end{aligned} \tag{26}$$

where $f_X(x)$ and $f_Y(y)$ denote the probability density function (PDF) of X and Y. Making the change of variable $t = P_r P_a y - \beta_{1k}\gamma_{th}$ within the integral and applying the Taylor series expansion for the exponential term and with the help of [12], we can obtain the final form.

Similarly, ξ_{bk} can be evaluated as

$$\begin{aligned}\xi_{bk} &= \int_{\frac{\alpha_{1k}\gamma_{th}}{P_r P_b}}^{\omega} f_Y(y) \int_{\vartheta_1}^{\infty} f_X(x)dxdy \\ &= \int_{\frac{\alpha_{1k}\gamma_{th}}{P_r P_b}}^{\infty} f_Y(y) \int_{\vartheta_1}^{\infty} f_X(x)dxdy \\ &\quad - \int_{\omega}^{\infty} f_Y(y) \int_{\vartheta_1}^{\infty} f_X(x)dxdy\end{aligned} \tag{27}$$

The first term of (27) can be evaluated straightforward and the second term can be deduced by following the same approach as carried out to obtain ξ_{ak}. Substituting ξ_{ak} and ξ_{bk} into (25) yields (10).

References

1. Rankov, B., & Wittneben, A. (2007). Spectral efficiency protocols for half-duplex fading relay channels. *IEEE Journal of Selected Areas in Communications*, *25*, 379–389.
2. Zhang S. L., Liew C. S. & Lam P. P. (2006). Hot topic: Physical-layer network coding. Proceedings of ACM Mobicom, 358–365.
3. Duy, T. T., & Kong, H.-Y. (2013). Performance Analysis of two-wayhybrid decode-and-amplify relaying scheme with relay selection for secondary spectrum access. *Wireless Personal Communications*, *64*, 857–878.

4. Zhang, J., Bai, B., & Li, Y. (2010). Outage-optimal opportunistic relaying for two-way amplify and forward relay channel. *Electronics letters*, *46*, 595–597.
5. Ubaidulla, P., & Aissa, S. (2012). Optimal relay selection and power allocation for cognitive two-way relaying networks. *Wireless Communications Letters*, *1*, 225–228.
6. Wang, L., Cai, Y. M., & Yang, W. W. (2012). On the finite-SNR DMT of two-way AF relaying with imperfect CSI. *IEEE Wireless Communications Letters*, *1*, 161–164.
7. Wu, Y., & Patzold, M. (2012). Outage probability and power allocation of two-way amplify-and-forward relaying with channel estimation errors. *IEEE Transaction on Wireless Communications*, *11*, 1985–1990.
8. Wang, C. Y., Lui, T. C.-K., & Dong, X. D. (2012). Impact of channel estimation error on the performance of amplify-and-forward two-way relaying. *IEEE Transaction on Vehicular Technology*, *61*, 1197–1207.
9. Han, S., Ahn, S., Oh, E., & Hong, D. (2009). Effect of channel-estimation error on BER performance in cooperative transmission. *IEEE Transaction on Vehicular Technology*, *58*, 4060–4070.
10. Yi, Z., Ju, M., & Kim, I.-M. (2011). Outage probability and optimum power allocation for analog network coding. *IEEE Transactions on Wireless Communications*, *10*, 407–412.
11. Gradshteyn, I. S., & Ryzhik, I. M. (1994). *Table of integrals, series, and products*. FL: Academic, Orlando.
12. Abramowitz, M., & Stegun, I. A. (1965). *Handbook of mathematical functions with formulas graphs, and mathematical tables*. New York: Dover.
13. Moon, T. K., & Stirling, W. C. (2000). *Mathematical methods and algorithms for signal processing*. Englewood Cliffs, NJ: Prentice-Hall.

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