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Outage Probability Analysis of Dual-Hop Decode-and-Forward Relaying Over Mixed Rayleigh and Generalized Gamma Fading Channels

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Abstract In this letter, outage probability of dual-hop decode-and-forward (DF) relaying scheme is analyzed over mixed Rayleigh and generalized Gamma fading channels. Cooperation model considered in this work consists of a source, a relay and a destination. It is assumed that source-relay and relay-destination channels experience Rayleigh fading and generalized Gamma fading, respectively. Exact outage probability expression is derived and outage performance is illustrated for both direct transmission and DF relaying scheme.

Keywords Decode-and-forward · Outage probability · Asymmetric fading

1 Introduction

Wireless communications suffer from random signal attenuation which is called multipath fading. Fading condition can be characterized by some mathematical distribution functions such as Rayleigh, Nakagami-*m* and Weibull. Another fading distribution called generalized Gamma distribution which includes Rayleigh, Nakagami-*m* and Weibull as special cases was introduced by Stacy in [1]. Generalized Gamma distribution was shown to provide a good fit to measured and simulated data in multipath fading environments [2].

To date, several studies have presented both on the signal estimation and the performance of wireless relay networks operating over fading channels [3–18]. In [4], symbol error rate (SER) analysis and optimum power allocation were investigated for cooperative diversity networks with either amplify-and-forward (AF) or decode-and-forward (DF) cooperation techniques over Rayleigh fading channels. It was shown that an equal power strategy is good unless it is not optimum for both the DF and AF cooperation systems. The authors in [5] proposed a relay selection method and they provided the bit error probability (BEP) and outage

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probability of the proposed relay selection method in independent non-identically distributed Rayleigh fading channels. As a result, the proposed relay selection in their work outperforms the conventional relay selection scheme. Samb and Yu [6] developed optimal power allocation methods for addressing the problem of network lifetime limitation in AF relay systems. They proposed a tight SER lower bound and an optimal power technique for reducing the SER. In [7], outage probability of AF cooperative diversity networks was presented in Weibull and Weibull-lognormal fading channels with a tight lower bound approximation. Simulation results in [7] showed that the lower bound of outage probability is tight at high SNR region. Sun et al. [8] improved a theoretical framework on security performance in DF relay network in terms of both the secrecy outage probability and the average secrecy rate. In [9] and [10], the performance and the optimization problem of dual-hop AF relaying were analyzed in symmetric Weibull channels. They presented that optimum power allocation provides only coding gain, while optimum relay positioning turns out diversity gains.

In practical relay systems, channels between cooperating nodes can experience asymmetric fading conditions. In [11], DF cooperative diversity system was analyzed over different asymmetric fading conditions. Suraweera et al. [12] derived exact and lower bound expressions for the outage probability and average bit error probability (ABEP) of two hop AF relaying in Rayleigh/Rician fading channels. They presented the positive effect of the Rician factor on the performance of a dual-hop AF relaying system. The authors in [13] investigated the AF system when source-relay and relay-destination nodes experience Rayleigh/Rician and Rician/Rayleigh fading scenarios, respectively. In [14], a dual-hop AF relay system was examined over mixed Nakagami-*m* and Rician fading channels. Anastasov et al. [15] studied the performance analysis of dual-hop AF relaying over Rayleigh/Rician fading channels with interference at the relay. The performance of a two-hop opportunistic DF relaying system was investigated in terms of outage probability where the first and second hop links subject to different fading conditions in [16]. They observed that the same diversity can be achieved and diversity order is equal to the relay number. In [17], the authors proposed an analytical framework for the block error rate (BLER) performances of general hybrid automatic repeat request (HARQ) transmission in DF relaying systems under quasi-static Rayleigh fading channels. However, to the best of our knowledge, the outage probability of a dual-hop DF relaying over mixed Rayleigh and generalized Gamma fading channels has not been studied in the literature yet. In this letter, a mixed Rayleigh and generalized Gamma fading channel model is introduced and exact outage probability expression is derived. It is shown that through varying the parameters of generalized Gamma fading channel in the derived analytical expression, second hop link can reduce to other known fading channel models such as Rayleigh, Nakagami-*m* and Weibull as special cases.

The rest of this paper is organized as follows. Section 2 introduces the system and channel models. In Sect. 3, exact outage probability expression is obtained for a dual-hop DF relaying over asymmetric fading channels. The simulation results are illustrated in Sect. 4 and finally conclusion is presented in Sect. 5.

2 System Model

Consider a dual-hop DF relaying system as illustrated in Fig. 1. The source (S) communicates to the destination (D) with the help of relay (R). It is considered that R is operated in the half-duplex mode. In DF cooperation scheme, S transmits the message to the R and the R decodes, re-encodes and then transmits it to the D. $\gamma_1 = |h_{SR}|^2 P_1/N_0$ and $\gamma_2 = |h_{RD}|^2 P_2/N_0$ are instantaneous signal-to-noise ratios (SNRs) between S–R and R–D links, respectively. h_{SR}

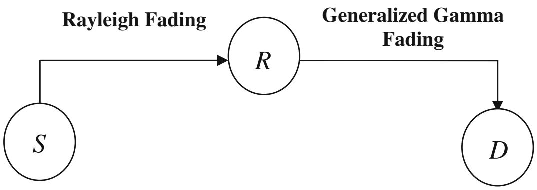

Fig. 1 Illustration of DF relaying scheme

denotes Rayleigh fading coefficient and h_{RD} denotes generalized Gamma fading coefficient. P_1 and P_2 are transmission powers at S and R, respectively. N_0 is the power of the additive white Gaussian noise (AWGN). S–R link is subject to Rayleigh fading and the probability density function (PDF) of instantaneous SNR can be given as

$$p_{\gamma_1}(\gamma_1) = \frac{1}{\bar{\gamma}_1} \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1}\right) \tag{1}$$

where $\bar{\gamma}_1 = E\left(|h_{SR}|^2\right) P_1/N_0$ is the average value of γ_1 and $E(\cdot)$ is the statistical average operator. Since R–D link is subject to generalized Gamma fading, the PDF of instantaneous SNR can be presented as

$$p_{\gamma_2}(\gamma_2) = \frac{v\,(\beta/\bar{\gamma}_2)^{mv}}{\Gamma(m)} \gamma_2^{mv-1} \exp\left\{-\left(\frac{\beta\gamma_2}{\bar{\gamma}_2}\right)^v\right\}, \quad \gamma_2 \geq 0 \tag{2}$$

where $\beta = \Gamma(m+1/v)/\Gamma(m)$, m is the fading parameter, v is the shape parameter, $\bar{\gamma}_2 = E\left(|h_{RD}|^2\right) P_2/N_0$ is the average value of γ_2 and $\Gamma(\cdot)$ is the gamma function [19]. The generalized Gamma fading has a flexible distribution which includes Rayleigh $(m = 1, v = 1)$, Nakagami-m $(v = 1)$ and Weibull $(m = 1)$ as special cases.

3 Outage Probability

Outage probability (P_{out}) is an important and widely used performance metric in wireless communication systems. In some cases, since there may not be suitable conditions for communication between S and D, it may be useful to communicate via a relay. In the DF cooperation, outage occurs when the instantaneous SNR of one or two of the hops fall below the outage threshold, γ_{th}. So, we can write P_{out} as

$$\begin{aligned} P_{out} &= \Pr\{\min(\gamma_1, \gamma_2) \leq \gamma_{th}\} \\ &= 1 - \Pr(\gamma_1 > \gamma_{th}) \Pr(\gamma_2 > \gamma_{th}) \end{aligned} \tag{3}$$

$\Pr(\gamma_1 > \gamma_{th})$ can be calculated according to Rayleigh fading case for S–R link as

$$\begin{aligned} \Pr(\gamma_1 > \gamma_{th}) &= \int_{\gamma_{th}}^{\infty} p_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right) \end{aligned} \tag{4}$$

For R–D link over generalized Gamma fading scenario, $\Pr(\gamma_2 > \gamma_{th})$ can be calculated as follows

$$\Pr(\gamma_2 > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} p_{\gamma_2}(\gamma_2) d\gamma_2$$
$$= \int_{\gamma_{th}}^{\infty} \frac{v(\beta/\bar{\gamma}_2)^{mv}}{\Gamma(m)} \gamma_2^{mv-1} \exp\left\{-\left(\frac{\beta\gamma_2}{\bar{\gamma}_2}\right)^v\right\} d\gamma_2 \quad (5)$$

Using [19, (3.381.9)], $\Pr(\gamma_2 > \gamma_{th})$ can be derived as

$$\Pr(\gamma_2 > \gamma_{th}) = \frac{\Gamma\left(m, \frac{\beta^v \gamma_{th}^v}{\bar{\gamma}_2^v}\right)}{\Gamma(m)} \quad (6)$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [19]. Based on Eqs. (3), (4) and (6), we propose following equation for P_{out}

$$P_{out} = 1 - \frac{\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right)\Gamma\left(m, \frac{\beta^v \gamma_{th}^v}{\bar{\gamma}_2^v}\right)}{\Gamma(m)} \quad (7)$$

4 Simulation Results

In this section, some simulation results of the proposed analytical expression are shown. In addition to DF relaying transmission, the outage probability of direct transmission under Rayleigh and generalized Gamma fading scenarios are also considered. The outage threshold value (γ_{th}) and generalized Gamma fading parameters (m and v) are the key criterias for performance design in this work. Figure 2 shows the outage performance of direct transmission

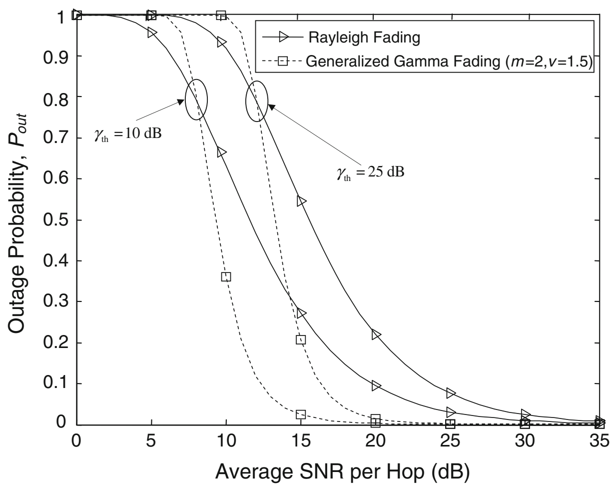

Fig. 2 Outage probability of direct transmission over Rayleigh and generalized Gamma fading channels ($\bar{\gamma}_1 = \bar{\gamma}_2$)

Table 1 Outage probability values of direct transmission for some arbitrary values of the average SNR per hop

Average SNR per hop (dB)	Rayleigh		G. Gamma	
	P_{out} ($\gamma_{th} = 10$ dB)	P_{out} ($\gamma_{th} = 25$ dB)	P_{out} ($\gamma_{th} = 10$ dB)	P_{out} ($\gamma_{th} = 25$ dB)
8	0.795	0.981	0.811	0.998
12	0.468	0.793	0.112	0.807
16	0.222	0.466	0.016	0.118
20	0.095	0.221	0.003	0.015

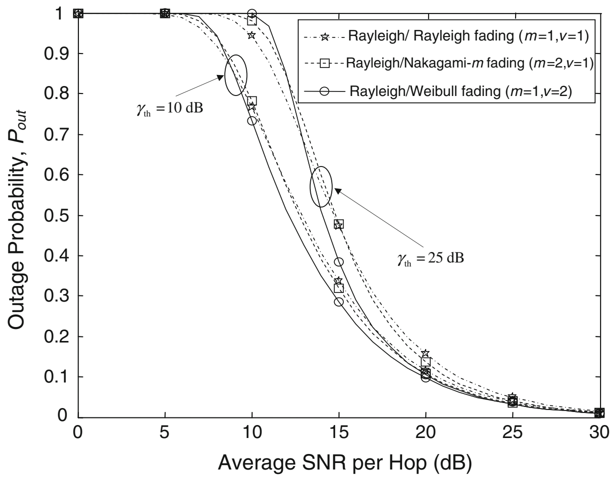

Fig. 3 Outage probability of a dual-hop DF relaying over mixed Rayleigh and generalized Gamma fading channels ($\bar{\gamma}_1 = \bar{\gamma}_2$)

scenario for several values of outage threshold ($\gamma_{th} = 10$ dB and $\gamma_{th} = 25$ dB). As can be seen from Fig. 2, better outage performance is obtained when outage threshold value is low for both of Rayleigh and generalized Gamma cases. It can be seen that the performance of direct transmission over generalized Gamma fading channel is better when the average SNR per hop is greater than 8 dB for $\gamma_{th} = 10$ dB. For the case of $\gamma_{th} = 25$ dB, similar behavior is carried out when the average SNR per hop is greater than 12 dB. In Table 1, the outage probability values of direct transmission are listed for arbitrary values of the average SNR per hop. As shown in Table 1, when $\gamma_{th} = 10$ dB, the outage probability which is obtained at 8 dB can be approximately provided at 12 dB for $\gamma_{th} = 25$ dB. It can be seen that similar behavior is observed for other values presented in Table 1.

In Fig. 3, the outage performance of a dual-hop DF relaying scheme are demonstrated for Rayleigh/Rayleigh ($m = 1,\ v = 1$), Rayleigh/Nakagami-m ($m = 2, v = 1$) and Rayleigh/Weibull ($m = 1,\ v = 2$) cases while the outage threshold γ_{th} is set to 10 dB and 25 dB. $(\cdot/\cdot)$ denotes the S–R link/R–D link. In Fig. 3, it can be seen that the best outage performance is obtained when the second hop link experiences Weibull fading conditions in high SNR region. In contrast to this, the worst outage performance is observed when the second hop link is subject to Rayleigh fading. The outage probability values over different fading conditions

Table 2 Outage probability values of a dual-hop DF relaying over asymmetric fading channels for some arbitrary values of the average SNR per hop when $\gamma_{th} = 10\,\text{dB}$

Average SNR per hop (dB)	Rayleigh/ Rayleigh	Rayleigh/Nakagami-*m*	Rayleigh/Weibull
	P_{out}	P_{out}	P_{out}
7	0.964	0.980	0.990
14	0.408	0.391	0.349
21	0.093	0.085	0.077

Table 3 Outage probability values of a dual-hop DF relaying over asymmetric fading channels for some arbitrary values of the average SNR per hop when $\gamma_{th} = 25\,\text{dB}$

Average SNR per hop (dB)	Rayleigh/ Rayleigh	Rayleigh/Nakagami-*m*	Rayleigh/Weibull
	P_{out}	P_{out}	P_{out}
7	0.999	0.999	0.999
14	0.574	0.599	0.511
21	0.124	0.106	0.082

are listed for the cases of $\gamma_{th} = 10$ dB and $\gamma_{th} = 25$ dB in Tables 2 and 3, respectively. From Tables 2 and 3, it is shown numerically that the value of the outage threshold γ_{th} should be set to low values to get better performance.

5 Conclusion

An exact outage probability expression is derived for a dual-hop DF transmission system over asymmetric fading channel in which *S*–*R* and *R*–*D* links experience Rayleigh and generalized Gamma fading, respectively. The outage performance is shown for several asymmetric channel models by using the flexibility of generalized Gamma distribution. The analytical expression proposed in this work is useful to evaluate the system performance of DF relaying over different asymmetric fading channel models such as Rayleigh/Nakagami-*m* and Rayleigh/Weibull. This analytical expression can be exploited to evaluate the system performance of relay application scenarios.

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