Performance Analysis of MC-CDMA Systems Under Nakagami Hoyt Fading

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Abstract Based on an alternative expression for Q-function, a simple bit error rate expression is derived in this paper for multicarrier code division multiple access systems with maximal ratio combining in correlated Nakagami-*q* channels. Furthermore, in this paper, we derive bounds on the probability of error and ergodic capacity of spatially multiplexed MC-CDM systems with zero forcing unified successive interference cancellation technique. Closed-form expressions for Capacities per unit bandwidth and Outage probability using optimal power and rate adaptation policy are derived and plotted. Asymptotic approximations and upper bounds on spectrum efficiency are also derived and plotted. Numerical results for Symbol Error Rate are also derived and plotted using MATLAB.

Keywords Spatially multiplexed multicarrier code division multiplexing · Zero forcing unified successive interference cancellation · Nakagami Hoyt · Maximal ratio combining \cdot Optimal power and rate adaptation policy \cdot Outage probability

1 Introduction

The common feature of next generation wireless technologies will be the convergence of multimedia services such as speech, audio, video, image and data. This indicates that future wireless communications for high-quality multimedia and internet-based applications require high throughput and reliability under limited power and bandwidth resources. To improve reliability of communication, Orthogonal Frequency Division Multiplexing (OFDM) has been combined with spread spectrum to form a new transmission technique known as Multicarrier Code Division Multiplexing (MC-CDM) or OFDM-CDM [\[1\]](#page-11-0). There have been numerous

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research papers on the computation of bit error rate (BER) for multicarrier code division multiple access (MC-CDMA) systems in independent or correlated fading channels [\[2,](#page-11-1)[3](#page-11-2)].

In [\[4\]](#page-11-3), the capacity of Nakagami Multipath Fading (NMF) channels with an average power constraint for three power and rate adaptation policies is studied. Closed-form solutions for NMF channel capacity with and without diversity for each power and rate adaptation strategy are compared with the additive white Gaussian noise channel capacity. Karmani and Sivarajan found bounds and approximations for the capacity of mobile cellular communication networks based on CDMA in $[5]$. In $[6]$ $[6]$, the authors presented a study of the capacity evaluation of various multiuser MIMO schemes in cellular environments. This study provides vital information for applying multiuser MIMO schemes in multi-cell environments.

In [\[7](#page-12-1)], the channel capacity per unit bandwidth for different adaptation policies over Generalized Rayleigh fading channels has been computed. Closed-form expressions for the spectral efficiencies for the three adaptation policies were derived for the single antenna reception and maximal ratio combining (MRC) diversity reception cases. In [\[8\]](#page-12-2), closed-form expressions for the single user capacity of Selection Combining Diversity (SCD) system were derived, taking into account the effect of imperfect channel estimation at the receiver. Recently, there has been some work dealing with the channel capacity of Rayleigh fading channels employing different adaptive schemes such as [\[9\]](#page-12-3) and the references therein.

In this paper, we present a novel closed-form expression, upper and lower bounds for average BER for MC-CDMA systems with MRC in correlated Nakagami-*q* fading channels. Further, novel closed-form expressions for spectrum efficiency of MC-CDMA systems in the presence of Nakagami Hoyt fading are derived and plotted. The asymptotic expression and upper bound of spectrum efficiency are also derived and plotted. Further, bounds on the probability of error for spatially multiplexed MC-CDM (SM MC-CDM) are also derived and plotted.

2 BER of MC-CDMA

We consider an asynchronous MC-CDMA communication system in which K simultaneously active mobiles (users) communicate with a single Base Station (BS), and each of the active users employ *N* subcarriers and Binary Phase-Shift Keying (BPSK) modulation. For each user, we consider a slowly varying Nakagami-*q* fading channel. All subcarriers are assumed to experience flat but correlated fading. The fading amplitude, β*n*, for the *n*th subcarrier of user *k* is Nakagami-*q* distributed, and is given by

$$
f(\beta_{n,k}) = \frac{1+q^2}{q\Omega} \beta_{n,k} \exp\left(-\frac{(1+q^2)\beta_{n,k}^2}{4q^2\Omega}\right) I_0\left(\frac{1-q^2}{4q^2\Omega}\beta_{n,k}^2\right) \quad \forall \beta > 0,
$$
 (1)

where $\Omega = E[\beta_{n,k}^2]$, $q \in [0, 1]$ is the Hoyt fading parameter, and $I_0(x)$ is the modified Bessel function of the first kind and order zero [\[10](#page-12-4)].

Assume that the users are time synchronous. After demodulation and combining subcarrier signals, the decision variable can be written as $\vartheta_0 = S + I + \eta$, where S represents the desired signal term, I is the Multiple Access Interference (MAI) from other users, and η is the AWGN term. Decision variable is a Gaussian random variable conditioned on the fading amplitude β . Since η and *I* are mutually independent, the BER using BPSK modulation conditioned on fading is simply given by

$$
Pr(\text{error}|\beta) = Q\left(\sqrt{\frac{S^2}{\sigma_{\eta}^2 + \sigma_{I}^2}}\right).
$$
\n(2)

Average BER is obtained by statistically averaging [\(2\)](#page-2-0) over the joint PDF of fading amplitudes. An alternative representation for the Q-function was presented in [\[10\]](#page-12-4), and led to a convenient method for performance analysis. It is given as [\[10](#page-12-4)]

$$
Q(t) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \exp\left(-t^2/2\sin^2\theta\right) d\theta; \quad t \ge 0.
$$
 (3)

To evaluate the average BER, one must then average over the statistics of the fading amplitudes. Since in the *Q* function definition, the argument appears in the lower limit of the integral, it is analytically difficult to perform such averages.

Using the alternative *Q*-function in [\(3\)](#page-2-1) and the assumption of independent fading channels at different subcarriers, the average BER of the *k*th user can be expressed as [\[3](#page-11-2)]

$$
P_e(k) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \prod_{n=0}^{N-1} I_{k,n}(\Omega_{k,n}, \theta) d\theta,
$$
 (4)

where *N* is the number of subcarriers, and

$$
I_{k,n}(\Omega_{k,n}, \theta) = \frac{1+q^2}{2q\Omega} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m+1)}
$$

$$
\times \frac{(1-q^4)^{2m}}{4q^2\Omega} \left(\frac{1}{2\left(\left(\frac{N}{2}\right) + \frac{E_b}{N_0} \left(\frac{(k-1)}{2}\right) \right) \sin^2 \theta} + \frac{(1+q^2)^2}{(4q^2\Omega)} \right)^{(-2m-1)}
$$
(5)

Equation [\(5\)](#page-2-2) is the average Signal to Interference plus Noise Ratio (SINR) for the *n*th subcarrier of the *k*th user. If all *N* subcarriers are identically distributed with the same average SINR per bit, then [\(4\)](#page-2-3) simplifies to

$$
P_e(k) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} \left[I_{k,n}(\Omega_{k,n}, \theta) \right]^N d\theta.
$$
 (6)

Since a multiuser system is considered, the average BER is given as

$$
BER = \frac{1}{K} \sum_{k=0}^{K-1} P_e(k).
$$
 (7)

3 Performance of SM MC-CDM with ZF USIC

A ZF-USIC detector for SM-MC-CDM was proposed in [\[11\]](#page-12-5). The error probability of ZF-USIC is dominated by the detection error of the first layer. As a result, the error probability of the first layer could be considered as an upper bound of error probability of the SM-MC-CDM system with zero forcing unified successive interference cancellation (ZF USIC) [\[11\]](#page-12-5). An approximation for the Probability Density Function (PDF) of γ is obtained as (Equation (20) of $[11]$ $[11]$)

$$
f_Y(\gamma) = \left(\frac{\beta}{\zeta}\right)^{\alpha} \gamma^{\alpha-1} \exp\left(\frac{-\beta\gamma}{\zeta}\right) \frac{1}{\Gamma(\alpha)}; \quad r \ge 0,
$$
 (8)

where α is the shape parameter, β is the scale parameter, $\zeta = \frac{E_s}{M\sigma_n^2}$, *M* denotes the number of transmit antennas, E_s is the signal energy, and σ_n^2 represents the noise variance. The BER for QPSK signaling is then upper bounded by

$$
P_e = \int_{0}^{\infty} Q(\sqrt{\gamma}) f_{\gamma}(\gamma) d\gamma.
$$
 (9)

The three upper bounds of $Q(x)$ are given by Equation (3.35), (3.37) and (3.38), respectively, from [\[12\]](#page-12-6)

$$
Q(x) \le \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right); \quad x > 0,
$$
 (10)

$$
Q(x) \le \frac{1}{2} \exp\left(-\frac{x^2}{2}\right); \quad x > 0,\tag{11}
$$

$$
Q(x) \le \frac{1}{2} \exp\left(-x\sqrt{\frac{2}{\pi}}\right); \quad x \ge 0. \tag{12}
$$

Substituting the upper bounds of $Q(x)$, from [\(10\)](#page-3-0), [\(11\)](#page-3-0) and [\(12\)](#page-3-0) into [\(9\)](#page-3-1), the expressions for the upper bound 1, upper bound 2, and upper bound 3, respectively, are derived. The closed form expressions for these bounds follow:

Upper bound 1:
$$
P_e \le \frac{1}{\sqrt{2\pi}} \frac{\Gamma(\alpha - 0.5)}{\Gamma(\alpha)} \frac{\left(\frac{\beta}{\varsigma} + 0.5\right)^{1/2}}{\left(1 + \left(\frac{\varsigma}{2\beta}\right)\right)^{\alpha}}; \quad \alpha > 0.5,
$$
 (13)

Upper bound 2:
$$
P_e \le \frac{1}{2} \left(\frac{\beta}{5}\right)^{\alpha} \left(\frac{\beta}{5} + 0.5\right)^{-\alpha}; \quad \alpha > 0.5,
$$
 (14)

and Upper bound 3:
$$
P_e \le \left(\frac{\beta}{5}\right)^{\alpha} (2c_2)^{-\alpha/2} \exp\left(\frac{\alpha^2}{8c_2}\right) D_{-\alpha}\left(\frac{c_1}{c_2\sqrt{2}_2}\right).
$$
 (15)

In [\(15\)](#page-3-2), D is the Parabolic Cylinder function [\[13](#page-12-7)], and c_1 , c_2 are constants. Now, $Q(x)$ is lower bounded as (Equation (3.34) of [\[12\]](#page-12-6))

$$
Q(x) > \frac{1}{x\sqrt{2\pi}}(1 - x^{-2})e^{-\frac{x^2}{2}}; \quad x > 1.
$$
 (16)

Substituting (16) into (9) , the lower bound on the BER for ZF-USIC is obtained as

$$
P_e \ge \frac{1}{\sqrt{2\pi}} (2\beta)^{\alpha} \frac{\Gamma(\alpha - 0.5) - \Gamma(\alpha - 3/2)}{(\beta + 5)^{\alpha}}; \quad \alpha > 3/2.
$$
 (17)

The theoretical BER for ZF-USIC is expressed as

$$
P_e = \left(\frac{\beta}{\varsigma}\right)^{\alpha} \frac{1}{2\sqrt{\pi}\Gamma(\alpha)} \int_{0}^{\infty} \exp\left(-\frac{\beta t}{\varsigma}\right) t^{\alpha-1} \Gamma_{\text{CIGF}}\left(0.5, \frac{t}{2}\right) dt. \tag{18}
$$

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By definition, $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$ is the Gamma function, and $\Gamma_{\text{CIGF}}(\alpha, \gamma_0) = \int_{\gamma_0}^{\infty} t^{\alpha-1} e^{-t} dt$ is the Complementary Incomplete Gamma Function (CIGF) (section 8.35, ^{γ∞} *t*^{α−1}*e^{−t}dt* is the Complementary Incomplete Gamma Function (CIGF) (section 8.35, page 890 of $[13]$).

Ergodic capacity is computed by averaging over the PDF of instantaneous received SNR. This quantity is defined when there is only one sample function of a stochastic process. The ergodic capacity is given by Equation (56) of [\[14\]](#page-12-8)

$$
C = \int_{0}^{\infty} M \log_2(1+\gamma) f_{\gamma}(\gamma) d\gamma.
$$
 (19)

Substituting $f_{\gamma}(\gamma)$ into [\(19\)](#page-4-1), the ergodic capacity is expressed as

$$
C = \frac{M}{\ln 2\Gamma(\alpha)} \left(\frac{\beta}{\varsigma}\right)^{\alpha} I_{\alpha} \left(\frac{\beta}{\varsigma}\right),\tag{20}
$$

where $I_{\alpha} \left(\frac{\beta}{5} \right) = \int_0^{\infty} t^{\alpha-1} \ln(1+t) e^{-\left(\frac{\beta}{5} \right)t} dt$ (Equation (77) of [\[15\]](#page-12-9)). In [\(20\)](#page-4-0), capacity increases with the number of transmit antennas (M).

4 Capacity Adaptation Policies

Let us consider a SM-MC-CDM system with ZF-USIC employed at the receiver. We assume perfect channel knowledge at the receiver. The distribution of the received SNR of SM-MC-CDM with ZF-USIC is approximated to be the inverse Gamma distribution [\[11](#page-12-5)]

4.1 Optimal Simultaneous Power and Rate Adaptation (OPRA) Policy

Given an average transmit power constraint, the channel capacity of a fading channel with received CNR distribution and optimal power and rate adaptation, $\langle C \rangle_{\text{onra}}(b/s)$, is given in $[15]$ as

$$
\frac{\langle C \rangle_{\text{opra}}}{B} = \int_{\gamma_0}^{\infty} \log_2 \left(\frac{\gamma}{\gamma_0} \right) f_{\gamma}(\gamma) d\gamma, \tag{21}
$$

where *B* is the channel bandwidth and γ_0 is the optimal cut-off CNR level below which data transmission is suspended. This optimal cut off must satisfy the condition [\[15](#page-12-9)]

$$
\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_{\gamma}(\gamma) d\gamma = 1.
$$
\n(22)

To achieve the capacity in [\(20\)](#page-4-0), channel fading level must be tracked at both the receiver and the transmitter, and the transmitter has to adapt its power and rate accordingly, allocating higher power levels and rates for good channel conditions (γ large), and lower power levels and rates for unfavourable channel conditions (γ small). Since no data is sent when $\gamma < \gamma_0$, the optimal policy suffers a probability of outage, P_{out} , equal to the probability of no transmission given by [\[15\]](#page-12-9)

$$
P_{\text{out}} = P\left[\gamma \le \gamma_0\right] = \int\limits_0^{\gamma_0} f_\gamma(\gamma) d\gamma,\tag{23}
$$

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and substituting $f_{\gamma}(\gamma)$ from [\(8\)](#page-3-4) into the integral of [\(22\)](#page-4-2), it is found that γ_0 must satisfy

$$
\frac{\Gamma_{\text{ClGF}}(\alpha, \gamma_0)}{\gamma_0} - \Gamma_{\text{ClGF}}(\alpha - 1, \gamma_0) = \Gamma(\alpha). \tag{24}
$$

Substitute $x = y_0$ in [\(24\)](#page-5-0) and define

$$
g(x) = \frac{\Gamma_{\text{ClGF}}(\alpha, x)}{x} - \Gamma_{\text{ClGF}}(\alpha - 1, x) - \Gamma(\alpha). \tag{25}
$$

Note that $\frac{dg(x)}{dx} = -\frac{1}{x^2} \Gamma_{\text{ClGF}}(\alpha, x) < 0 \forall x > 0$. Moreover, from [\(25\)](#page-5-1), $\lim_{x \to 0^+} g(x) = +\infty$ and $\lim_{x\to+\infty} g(x) = -\Gamma(\alpha) < 0$. Thus, it can be concluded that there is a unique positive γ_0 for which $g(\gamma_0) = 0$, which satisfies [\(24\)](#page-5-0). MATLAB numerical results show that $\gamma_0 \in$ [0, 1]. An approximation for the PDF of post processing SNR of SM MC-CDM with ZF USIC is shown in (8) . Substituting (8) into (21) , we have

$$
\frac{\langle C \rangle_{\text{opra}}}{B} = \int_{\gamma_0}^{\infty} \log_2 \left(\frac{\gamma}{\gamma_0} \right) \left(\frac{\beta}{\zeta} \right)^{\alpha} \gamma^{\alpha - 1} \exp \left(-\frac{\beta}{\zeta} \gamma \right) \frac{1}{\Gamma(\alpha)} d\gamma.
$$

Making change of variables in the above integral by substituting $t = \frac{\gamma}{\gamma_0}$, and simplifying,

$$
\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2} \sum_{k=0}^{\alpha - 1} \frac{\Gamma_{\text{CIGF}}\left(k, \frac{\beta \gamma_0}{\varsigma}\right)}{k!}.
$$
 (26)

Equation (26) can also be written as [\[9\]](#page-12-3)

$$
\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2} \left(E_1 \left(\frac{\beta \gamma_0}{\varsigma} \right) + \sum_{k=1}^{\alpha - 1} \frac{\Gamma_{\text{CIGF}} \left(k, \frac{\beta \gamma_0}{\varsigma} \right)}{k!} \right),\tag{27}
$$

where $E_1(.)$ is the exponential integral of the first order function defined as $E_1(x)$ = $\int_1^{\infty} \frac{e^{-xt}}{t} dt$ [\[13\]](#page-12-7). Asymptotic approximation for capacity as a result of OPRA policy can be obtained as $\gamma \to \infty$ using the series expansion of exponential integral of the first order given by [\[8\]](#page-12-2)

$$
E_1(x) = -E - \ln(x) - \sum_{i=1}^{\infty} \frac{(-x)^i}{i \cdot i!},
$$
\n(28)

where $E = 0.5772156659$ is the Euler-Mascheroni constant. Then, the capacity per unit bandwidth (in bps/Hz) using OPRA policy is approximated asymptotically as

$$
\frac{\langle C \rangle_{\text{opra}}}{B} = \frac{1}{\ln 2} \left(\left(-E - \ln \left(\frac{\beta \gamma_0}{\varsigma} \right) + \left(\frac{\beta \gamma_0}{\varsigma} \right) \right) + \sum_{k=1}^{\alpha - 1} \frac{\Gamma_{\text{CIGF}} \left(\frac{k, \beta \gamma_0}{\varsigma} \right)}{k!} \right). \tag{29}
$$

The capacity per unit bandwidth expression of OPRA policy can be upper bounded by applying Jensen's inequality to (21) as follows

$$
\frac{\langle C \rangle_{\text{opra}}^{\text{UB}}}{B} = \ln(E[\gamma]) = \ln\left(\int_{0}^{\infty} \gamma f(\gamma) d\gamma\right) = \ln\left(\frac{\Gamma(\alpha+1)\varsigma}{\beta \Gamma(\alpha)}\right),\tag{30}
$$

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where $E[\cdot]$ is the expectation operator. The outage probability associated with this policy is obtained by substituting (8) into (23) and is given as

$$
P_{\text{out}} = \frac{1}{\Gamma(\alpha)} \left(\frac{\beta}{\zeta}\right)^{\alpha} \int_{0}^{\gamma_0} \gamma^{\alpha - 1} \exp\left(-\left(\frac{\beta}{\zeta}\right) \gamma\right) d\gamma = \frac{1}{\Gamma(\alpha)} \Gamma_{\text{IGF}}\left(\frac{\gamma_0 \beta}{\zeta}, \alpha\right), \quad (31)
$$

where $\Gamma_{IGF}(x, \alpha) = \int_0^x t^{\alpha-1} e^{-t} dt$ is the Incomplete Gamma Function (IGF) (section 8.36, page 890 of [\[13\]](#page-12-7)).

5 Numerical Results

Figure [1](#page-6-0) shows a decrease in error probability with increase in *q*. The fading parameter *q* shows the severity of fading. The Hoyt model can be approximated by a suitable Nakagami-m model. The relation between the fading parameter, *m*, of the Nakagami distribution, and the *q* fading parameter is given by [\[16\]](#page-12-10)

$$
m = \frac{(1+q^2)^2}{2(1+2q^4)}
$$

The analysis of the relation shows that the value of *m* increases as *q* increases till 0.7. After $q = 0.7$, *m* decreases with increase in q. A Nakagami-q model with $q = 0.25$, 0.4 and 0.5 can be approximated to a Nakagami-m fading environment with $m = 0.56, 0.64, 0.69$, respectively. Between $m = 0.5$ and $m = 1$, its performance is worse than Rayleigh (when $m = 1$, Nakagami-m is modelled as Rayleigh). So, the BER decreases as *q* increases.

Figure [2a](#page-7-0), b show BER performance improvement with decreases in the number of users, and increase in the number of subcarriers, respectively. As the number of subcarriers increases, the transmitted bits are decoded more accurately thereby decreasing the probability of error. As observed clearly from Fig. [2a](#page-7-0), with increase in the number of users, the probability of error increases. In the case of MC-CDMA, in addition to MAI, there is Inter

Fig. 1 Probability of error of MC-CDMA under Nakagami-*q* fading for varying *q*

Fig. 2 a Probability of error of MC-CDMA under Nakagami-*q* fading for varying number of users. **b** Probability of error of MC-CDMA under Nakagami-*q* fading for varying number of subcarriers

Carrier Interference (ICI). When the number of users increases, noise plus interference (MAI + ICI) term increases, thereby increasing the BER.

Figure [3](#page-8-0) clearly explains that for $\alpha = 6$ and $\beta = 2$, transmission is possible if the probability of error lies between that of upper bound 2 and lower bound. The theoretical result also lies between upper bound 2 and the lower bound. The lower bound gives the minimum measure of BER possible for this detector.

Figure [4a](#page-8-1)–c show the performance of ergodic capacity for varying α , M and β . From Fig. [4b](#page-8-1), it can be observed that ergodic capacity increases as the number of layers increases. Similarly, from Fig. [4a](#page-8-1), as α increases, ergodic capacity also increases. The shape parameter indicates fading severity or scintillation index. This indeed varies from deep fade to light fade. As α increases, the fluctuations in the signal diminishes, and this is in turn improves system performance.

Figure [5a](#page-9-0) shows a decrease in capacity for the OPRA policy with increase in β . The scale parameter, β , relates to the inverse of average fading power. When β increases the average fading power decreases, and correspondingly, the capacity also decreases. In Figs. [5b](#page-9-0) and [6,](#page-10-0) outage probability is plotted versus average SNR for varying $α$ and varying $β$, respectively. The outage probability is high when α is low as in Fig. [6,](#page-10-0) and outage probability is low when β is low as in Fig. [5b](#page-9-0). Outage probability is associated with OPRA policy since this policy adapts power and rate only if the instantaneous SNR of the channel is above the optimal cut-off SNR. The optimal cut-off SNR is obtained numerically using MATLAB and is found to be 0.516788 in both Figs. [5b](#page-9-0) and [6.](#page-10-0)

Figure [7](#page-10-1) shows the asymptotic approximation of OPRA policy. Figure [7](#page-10-1) clearly depicts the comparison between the theoretical result and asymptotic approximation associated with OPRA policy. This is plotted using closed form expressions, [\(27\)](#page-5-3) and [\(29\)](#page-5-4), respectively. The optimal cut-off SNR is found numerically using MATLAB and is 0.516788. This value of γ_0

Fig. 3 Comparison of upper bounds, lower bound and analytical probability of error of SM MC-CDM with ZF USIC

Fig. 4 a Ergodic capacity of SM MC-CDM with ZF USIC for varying α. **b** Ergodic capacity of SM MC-CDM with ZF USIC for varying number of layers (M). **c** Ergodic capacity of SM MC-CDM with ZF USIC for varying β

Fig. 5 a Capacity per unit bandwidth for OPRA policy for varying β. **b** Outage probability of OPRA policy for varying β

Fig. 6 Outage probability of OPRA policy for varying α

Fig. 7 Asymptotic approximation of OPRA policy for $\alpha = 2$ and $\beta = 3$

is used to plot expression (27) . Asymptote of a curve is a line such that the distance between the curve and the line approaches zero at infinity. Figure [8](#page-11-5) also shows the tightness between the theoretical result and asymptotic approximation at high SNRs.

Figure [8a](#page-11-5), b show the upper bounds and theoretical results for OPRA policy for varying values of $β$. Figure [8a](#page-11-5), b clearly depict that the theoretical result is well within the upper bound curve. The closed form expressions obtained in [\(27\)](#page-5-3) and [\(30\)](#page-5-5) are used to arrive at this result.

Fig. 8 a Comparison of upper bound and theoretical result of OPRA policy with $\alpha = 4$ and $\beta = 1.75$. **b** Comparison of upper bound and theoretical result of OPRA policy with $\alpha = 4$ and $\beta = 2$

6 Conclusions

A novel closed-form BER expression has been derived for the MC-CDMA systems with MRC in correlated Nakagami-*q* fading channels. Also, bounds on the probability of error and ergodic capacity of SM MC-CDM with ZF USIC are derived. Novel expressions for Spectrum efficiency and outage probability for optimal power and rate adaptation policy are also derived and plotted. As a future work, we would like to compare the spectrum efficiency performance of OPRA policy with those of other adaptation policies available in literature when the distribution of the post processing SNR is as given in this paper.

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