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Impact of Quality of Service Constraints on the Performance of Spectrum Sharing Cognitive Users

Abdallah K. Farraj · **Eman M. Hammad**

Abstract This article investigates the impact of choosing a specific quality of service constraint on the performance of the cognitive users in a spectrum sharing cognitive environment. To communicate over a wireless channel reserved to a primary user, the cognitive user has to satisfy the primary user's quality of service constraint. Cognitive systems under interference temperature and outage probability constraints are investigated in this work. The outage probability of the primary user in an interference temperature-constraint environment is analyzed, and the performance measures of the cognitive users are developed. A comparative study of the cognitive user's performance under equivalent outage probability and interference temperature constraints is conducted as well. Numerical results are presented to verify the theoretical analysis and compare the performance measures under these constraints. Results of this work illustrate the effects of the communication environment parameters on the cognitive users and detail the performance differences between the equivalent outage probability-constraint and interference temperature-constraint cognitive systems.

Keywords Cognitive communications · Spectrum sharing · Outage probability · Interference temperature · Transmit power · Bit error rate · Channel capacity

1 Introduction

Extensive use of new technologies and applications has made the frequency spectrum a very limited resource for wireless communication systems. Although the spectrum is limited, many studies indicate that it is under-utilized [1]. Cognitive communication systems [2–5] have been proposed in order to achieve more efficient spectrum utilization. In a cognitive

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communication environment, a cognitive user, also referred to as secondary user (SU), adapts its communication settings in order to transmit its data over a channel that is licensed to a primary user (PU). However, the cognitive usage of the channel should not violate a specific quality of service (QoS) constraint. The QoS constraints include, to mention some, limiting the maximum or average interference experienced by the primary user, having a minimum value of the primary user's signal-to-interference plus noise ratio, or limiting the primary user's outage probability to some threshold.

Cognitive communication can be carried out using spectrum sensing or spectrum sharing modes. In a spectrum sensing mode [6–11], a cognitive user can only transmit data over a channel when it detects that the primary user is not using that channel. On the other hand, in a spectrum sharing mode, a cognitive user transmits data simultaneously, along with the primary user, over the same channel as long as the cognitive user's communication does not violate the QoS constraint of the primary user. In order to make a successful cognitive communication, the channel under consideration has to be detected, the channel gain between the cognitive user and the receiver unit has to be estimated, and the cognitive communication settings are to be chosen in a way that meets the QoS requirements of the primary user of the channel.

In conventional spectrum sharing cognitive systems, and in order to maintain the QoS requirement of the primary user, the peak received power of the cognitive user's signal, measured at the primary user's receiver, must be below a predefined threshold. This maximum cognitive interference value is often referred to as the interference temperature (IT) level. Because the IT constraint limits the interference experienced by the primary user to a certain limit, this QoS has gained popularity in the research community, and this interest has resulted in an abundant research activity in the past few years. A Few examples include [12–20]. Much of these studies have focused on power and resource allocation. In addition, the capacity of the cognitive channel, assuming various channel fading types and users' configurations, were investigated as well.

The outage probability (OP) of the primary user was proposed as the QoS requirement in spectrum sharing cognitive systems in [21–24]. The optimal power allocation strategies to achieve the ergodic capacity of the cognitive user's fading channel was derived in [21]. In addition, the channel capacity of uplink cognitive networks with opportunistic scheduling of multiple cognitive users was analyzed in [22]; in the phase of opportunistic scheduling, the cognitive user that has the weakest interference channel with the receiver unit was selected to share the channel with the primary user. Moreover, a study of the performance measures of the cognitive queueing system was conducted in [23], and the queue service rate was assumed to equal the channel capacity of the cognitive channel in a spectrum sharing system under the primary user's outage probability constraint. The performance of the primary users under the outage probability constraint was investigated in [24] as well.

To the extent of the authors' knowledge, the outage probability of the primary user under the interference temperature constraint has not been found yet. Moreover, a comparative study of the performance of the cognitive users under equivalent outage probability and interference temperature constraints is lacking as well. The motivation of this article is to shed a light on these issues; first, we investigate the outage probability of the primary user under the interference temperature constraint. In addition, for a specific outage probability, we develop an equivalent interference temperature-constraint cognitive system and find the parameters of the communication environment. Then, we develop and compare the performance measures of the cognitive users under these two constraints.

The contributions of this work include developing the analytical expressions for the probability density function (PDF) and the cumulative distribution function (CDF) of the signal-to-interference-plus-noise ratio (SINR) of the primary user's and the cognitive user's signals under the interference temperature constraint. We also find an exact expression that relates the maximum value of the cognitive received power (i.e., the IT level) with the resultant primary user's outage probability. Moreover, we study the performance measures of the cognitive user under these two constraints. Specifically, we develop the analytical expressions for the average transmit and received powers, average bit error rate, and average channel capacity. In addition, we investigate the effects of changing the parameters of the communication environment on the mentioned performance measures; these parameters include the primary user's outage probability constraint, the transmit power and rate of the primary user, and the cognitive user's maximum transmit power.

The rest of the paper is organized as follows: the system model and the parameters of the communication environment are introduced in Sect. 2. The cognitive user's performance measures are developed in Sect. 3, and the probability density function of the signal-to-interference plus noise ratio for the primary and cognitive users are found as well. Section 4 shows numerical results that display the average transmit and received powers, bit error rate, and channel capacity of the cognitive user under the interference temperature and outage probability constraints. Finally, conclusions are discussed in Sect. 5.

2 System Model

In this section, we investigate a spectrum sharing cognitive communication system, introduce the setup terminology, name the parameters of the environment, and develop the SINR values of the PU and the SU.

In this environment, there is one PU that uses the wireless channel to transmit its information to the receiver unit (RU). At the same time, there is one SU that wishes to share this channel with the PU using spectrum sharing method. This SU intends to transmit its data to the RU as well, and the cognitive communication of the SU will take place while the PU is using the channel. As a consequence of the concurrent usage of the channel, the signal of the PU will experience an extra interference due to the presence of the SU's signal, and vice versa. The described system setup is illustrated in Fig. 1.

The RU acts as a receiver for both the PU and the SU in this environment. As an example, the RU works similar to a base station that is dedicated to support licensed users, but at the same time it has the capability to act as a receiver for lower-priority unlicensed users. The RU is also assumed to have the capability to decode both the signals of the PU and the SU and totally separate them. Moreover, the noise at the RU is assumed to be an additive white Gaussian noise with zero mean with a variance of σ^2.

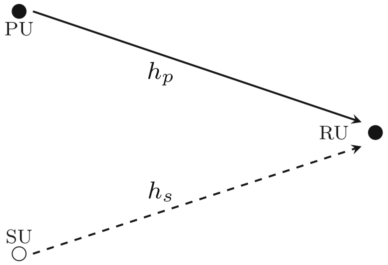


Fig. 1 Cognitive system model

In this environment, the transmit power of the primary user is assumed to be constant and is equal to P_p, its normalized data rate is denoted as R_p (i.e., the primary user communicates at a bit rate equal to $R_p B$ where B is the system bandwidth), its SINR is denoted as γ_p, and its outage probability requirement is bounded by δ. Moreover, let's denote the transmit power of the cognitive user as P_s. The usage of the channel by the SU (i.e., the transmission of its data concurrently with the PU) should not increase the PU's outage probability above a certain specified limit (i.e., δ). The outage probability of the PU is defined as $\Pr\left\{\log_2(1+\gamma_p) \leq R_p\right\} \leq \delta$ [25].

The channel gain between the PU and the RU is denoted as h_p, and the channel gain between the SU and the RU is denoted as h_s. Both channels are assumed to undergo independent and identically distributed (i.i.d.) block fading processes, each having a Rayleigh distribution. The channel power gain, denoted as G, is defined as $G = |h|^2$. Because h_p and h_s are assumed to have a Rayleigh distribution, both G_p and G_s have exponential distributions with mean equal to one.

The received power of the primary user's signal is $P_p G_p$, while that of the cognitive user is $P_s G_s$. Accordingly, the SINR of the PU is defined as $\gamma_p = \frac{P_p G_p}{P_s G_s + \sigma^2}$. In an interference-limited communication environment, the interference signal is much stronger than the noise (i.e., $P_s G_s \gg \sigma^2$), then the noise term can be ignored in γ_p, and so $\gamma_p \approx \frac{P_p G_p}{P_s G_s}$. Similarly, the SINR of the SU is expressed as $\gamma_s = \frac{P_s G_s}{P_p G_p + \sigma^2}$. The noise term can be ignored in an interference-limited environment, and so we get $\gamma_s \approx \frac{P_s G_s}{P_p G_p}$.

3 Performance Analysis

In this section, we develop the outage probability of the primary user under the interference temperature constraint, and we link the outage probability constraint to the resultant interference temperature. We also develop the performance measures of the cognitive user under this constraint. Moreover, we compare the performance findings with that of a cognitive user under an equivalent outage probability constraint.

3.1 Interference Temperature QoS

In the interference temperature-constraint (ITC) environment, the QoS requirement the cognitive user has to satisfy is the maximum interference power, denoted Ω. In other words, the power of the cognitive received signal is bounded by Ω; this means that $P_s G_s \leq \Omega$. Let's also assume that the maximum transmit power of the SU is denoted P_m. In addition, let's define $\alpha = \frac{\Omega}{P_m}$ and $\beta = \frac{P_m}{P_p}$. In this setup, the transmit power of the SU is $\min\left(P_m, \frac{\Omega}{G_s}\right)$, which is also expressed as

$$P_s = \begin{cases} P_m & P_m G_s \leq \Omega \\ \frac{\Omega}{G_s} & P_m G_s > \Omega \end{cases},$$

or, equivalently

$$P_s = \begin{cases} P_m & G_s \leq \alpha \\ \frac{\Omega}{G_s} & G_s > \alpha. \end{cases} \tag{1}$$

The received power of the cognitive signal, denoted as P_r, is equal to $P_s G_s$. This quantity can be expressed as

$$P_r = \begin{cases} P_m G_s & P_m G_s \leq \Omega \\ \Omega & P_m G_s > \Omega \end{cases},$$

or,

$$P_r = \begin{cases} P_m G_s & G_s \leq \alpha \\ \Omega & G_s > \alpha \end{cases}. \tag{2}$$

Note that $P_r \leq \Omega$, and so the cognitive user satisfies the QoS constraint. The SINR of the primary user, denoted as γ_p, is expressed in this case as

$$\gamma_p = \begin{cases} \frac{P_p G_p}{P_m G_s} & G_s \leq \alpha \\ \frac{P_p G_p}{\Omega} & G_s > \alpha \end{cases}. \tag{3}$$

Similarly, the SINR value of the cognitive user, γ_s, is found as

$$\gamma_s = \begin{cases} \frac{P_m G_s}{P_p G_p} & G_s \leq \alpha \\ \frac{\Omega}{P_p G_p} & G_s > \alpha. \end{cases} \tag{4}$$

To develop the cognitive performance measures and compare this environment with an equivalent outage-constraint system, we first need to find the PDF and CDF of γ_p and γ_s. Going back to Eq. (3), let $\gamma_{p_1} = \frac{P_p G_p}{P_m G_s}$ and $\gamma_{p_2} = \frac{P_p G_p}{\Omega}$. Accordingly, the PDF of γ_p, denoted as f_{γ_p}, is equal to $f_{\gamma_{p_1}} + f_{\gamma_{p_2}}$. In this case, $f_{\gamma_{p_1}}(z)$ is expressed as

$$\begin{aligned} f_{\gamma_{p_1}}(z) &= \int_0^\alpha f_{\gamma_{p_1}}(z/x) f_X(x) dx \\ &= \int_0^\alpha \beta x e^{-\beta x z}\, e^{-x} dx \\ &= \frac{\beta}{(\beta z+1)^2} \left(1 - (\alpha(\beta z+1)+1)\, e^{-\alpha(\beta z+1)}\right). \end{aligned} \tag{5}$$

In a similar way, $f_{\gamma_{p_2}}(z)$ can be found to equal to

$$\begin{aligned} f_{\gamma_{p_2}}(z) &= \int_\alpha^\infty \alpha\beta e^{-\alpha\beta z}\, e^{-x} dx \\ &= \alpha\beta e^{-\alpha(\beta z+1)}. \end{aligned} \tag{6}$$

Accordingly, the PDF of γ_p is expressed as

$$f_{\gamma_p}(z) = \beta e^{-\alpha(\beta z+1)} \left(\alpha - \frac{\alpha(\beta z+1)+1}{(\beta z+1)^2}\right) + \frac{\beta}{(\beta z+1)^2}. \tag{7}$$

The CDF of γ_p, denoted as F_{γ_p}, is developed as

$$F_{\gamma_p}(z) = \int_0^z \frac{\beta}{(\beta w+1)^2}\left(1-(\alpha(\beta w+1)+1)\,e^{-\alpha(\beta w+1)}\right)dw + \int_0^z \alpha\beta e^{-\alpha(\beta w+1)}\,dw. \tag{8}$$

It can be shown that the first part of the integral is equal to $\frac{e^{-\alpha(\beta z+1)}}{\beta z+1} - e^{-\alpha} + 1$. In addition, $\int_0^z \alpha\beta e^{-\alpha(\beta w+1)}dw = e^{-\alpha}(1-e^{-\alpha\beta z})$. Accordingly, the CDF of γ_p is expressed as

$$F_{\gamma_p}(z) = \frac{\beta z}{\beta z+1}\left(1-e^{-\alpha(\beta z+1)}\right). \tag{9}$$

As the cognitive transmission will generate an extra interference over the primary user's signal, the primary user suffers channel outage. Let's assume the resultant outage probability is denoted as δ. Using the definition of outage probability, we link the CDF of γ_p with δ as

$$\begin{aligned} \Pr\{\log_2(1+\gamma_p) \le R_p\} &\le \delta \\ \Pr\{\gamma_p \le 2^{R_p}-1\} &\le \delta \\ F_{\gamma_p}(2^{R_p}-1) &\le \delta. \end{aligned} \tag{10}$$

Plugging the results of Eq. (9), we get $\frac{\beta(2^{R_p}-1)}{\beta(2^{R_p}-1)+1}\left(1-e^{-\alpha(\beta(2^{R_p}-1)+1)}\right) \le \delta$, or

$$\alpha \le \frac{-1}{\beta(2^{R_p}-1)+1}\log\left(1-\delta\frac{\beta(2^{R_p}-1)+1}{\beta(2^{R_p}-1)}\right).$$

If we replace α with $\frac{\Omega}{P_m}$ and β with $\frac{P_m}{P_p}$, the relation between the interference temperature level and the primary user's outage probability can be expressed as

$$\Omega \le \frac{P_m P_p}{P_m(2^{R_p}-1)+P_p}\log\left(\frac{P_m(2^{R_p}-1)}{P_m(2^{R_p}-1)-\delta(P_m(2^{R_p}-1)+Pp)}\right),$$

or

$$\Omega \le \frac{P_m P_p}{P_m(2^{R_p}-1)+P_p}\log\left(\frac{1}{1-\delta(1+\frac{Pp}{P_m(2^{R_p}-1)})}\right). \tag{11}$$

When $P_m \gg \frac{P_p}{2^{R_p}-1}$, Ω can be approximated as $\frac{P_p}{2^{R_p}-1}\log\left(\frac{1}{1-\delta}\right)$. We will be using the formula in Eq. (11) to map a cognitive system under interference temperature constraint into an equivalent outage probability-constraint environment.

Now, let's develop the PDF of γ_s. Because $\gamma_s = \frac{1}{\gamma_p}$, the PDF of γ_s, denoted as f_{γ_s}, is found as $f_{\gamma_s}(z) = \frac{1}{z^2} f_{\gamma_p}(\frac{1}{z})$. In this case, it will be equal to

$$f_{\gamma_s}(z) = \beta e^{-\alpha\left(\frac{\beta}{z}+1\right)}\left(\frac{\alpha}{z^2} - \frac{\alpha\left(\frac{\beta}{z}+1\right)+1}{(\beta+z)^2}\right) + \frac{\beta}{(\beta+z)^2}\,. \tag{12}$$

Moreover, the CDF of γ_s, denoted as F_{γ_s}, can be expressed as

$$
\begin{aligned}
F_{\gamma_s}(z) &= \int_0^z \frac{\beta}{(\beta+w)^2} + \frac{\alpha\beta}{w^2} e^{-\alpha\left(\frac{\beta}{w}+1\right)} - \frac{\beta\left(\alpha\left(\frac{\beta}{w}+1\right)+1\right)}{(\beta+w)^2} e^{-\alpha\left(\frac{\beta}{w}+1\right)}\, dw \\
&= \frac{z}{\beta+z} + e^{-\alpha\left(\frac{\beta}{z}+1\right)} - \frac{1}{\frac{\beta}{z}+1} e^{-\alpha\left(\frac{\beta}{z}+1\right)} \\
&= \frac{z}{\beta+z} + \frac{\beta}{\beta+z} e^{-\alpha\left(\frac{\beta}{z}+1\right)}.
\end{aligned}
\tag{13}
$$

3.1.1 Performance Measures

The average transmit power of the cognitive user, denoted as $\bar{P}_s$, is found as $\bar{P}_s = \mathbb{E}[P_s]$, where $\mathbb{E}[\cdot]$ is the expectation operator. In this case, it is expressed as

$$
\begin{aligned}
\bar{P}_s &= \int_0^\alpha P_m e^{-x} dx + \int_\alpha^\infty \frac{\Omega}{x} e^{-x} dx \\
&= P_m(1 - e^{-\alpha}) + \Omega \mathrm{E}_1(\alpha) \\
&= P_m(1 - e^{-\frac{\Omega}{P_m}}) + \Omega \mathrm{E}_1(\frac{\Omega}{P_m}),
\end{aligned}
\tag{14}
$$

where $\mathrm{E}_1(\cdot)$ is the exponential integral function which is defined as $\mathrm{E}_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$.

The average received power of the cognitive user's signal, denoted as $\bar{P}_r$, is found as $\bar{P}_r = \mathbb{E}[P_s G_s]$. In this case, it can be found as

$$
\begin{aligned}
\bar{P}_r &= \int_0^\alpha P_m x e^{-x} dx + \int_\alpha^\infty \Omega e^{-x} dx \\
&= P_m - e^{-\alpha}(P_m(\alpha+1) - \Omega) \\
&= P_m(1 - e^{-\frac{\Omega}{P_m}}).
\end{aligned}
\tag{15}
$$

We note that $\bar{P}_r < \bar{P}_s$ for this environment. By plugging the value of Ω from Eq. (11), the average value of P_r is expressed as

$$
\bar{P}_r = P_m - P_m \left(1 - \delta - \frac{\delta P_p}{P_m(2^{R_p}-1)}\right)^{\frac{P_p}{P_p + P_m(2^{R_p}-1)}}.
\tag{16}
$$

Assuming that the SU uses Binary Phase Shift Keying (BPSK) modulation scheme, and for an SINR of γ_s, the instantaneous bit error rate is $\mathrm{Q}(\sqrt{2\gamma_s})$ [26], where $\mathrm{Q}(\cdot)$ is the Gaussian Q-function which is defined as

$$
\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{1}{2}y^2\right) dy.
$$

The average bit error rate of the SU, denoted as $\bar{P}_e$, is found as $\bar{P}_e = \mathbb{E}[\mathrm{Q}(\sqrt{2\gamma_s})]$. Accordingly,

$$
\bar{P}_e = \int_0^\infty \mathrm{Q}(\sqrt{2z}) \left(\beta e^{-\alpha\left(\frac{\beta}{z}+1\right)} \left(\frac{\alpha}{z^2} - \frac{\alpha\left(\frac{\beta}{z}+1\right)+1}{(\beta+z)^2}\right) + \frac{\beta}{(\beta+z)^2\, dz}\right) dz.
\tag{17}
$$

It can be shown that the average bit error rate of the cognitive user will equal to

$$\bar{P}_e = \frac{1}{2} + \frac{1}{2}e^{-(\alpha+2\sqrt{\alpha\beta})} - \sqrt{\pi\beta}\, e^{\beta}\, \mathrm{Q}(\sqrt{2\beta}) - \Xi_{\alpha\beta}, \tag{18}$$

where $\Xi_{\alpha\beta}$ is equal to $\int\limits_0^\infty \mathrm{Q}(\sqrt{2z})\, (\alpha(\frac{\beta}{z}+1)+1)\, \frac{\beta}{(\beta+z)^2}\, e^{-\alpha(\frac{\beta}{z}+1)}\, dz$.

For an SINR of γ_s, the instantaneous, bandwidth-independent, channel capacity of the cognitive user is $\log_2(1+\gamma_s)$ [27]. Accordingly, the average capacity of the cognitive channel, denoted as $\bar{C}_s$, is found as $\bar{C}_s = \mathbb{E}[\log_2(1+\gamma_s)]$. Using integration by parts, it can also be shown that $\bar{C}_s = \log_2(e) \int_0^\infty \frac{1-F_{\gamma_s}(z)}{1+z}\, dz$ [19]. In this case, the average channel capacity is expressed as

$$\begin{aligned} \bar{C}_s &= \log_2(e) \int\limits_0^\infty \frac{\beta}{(\beta+z)(1+z)} \left(1 - e^{-\alpha\left(\frac{\beta}{z}+1\right)}\right) dz \\ &= \log_2(e) \begin{cases} 1 + \alpha \mathrm{E}_1(\alpha) - e^{-\alpha} & \beta = 1 \\ \frac{\beta}{\beta-1}\left(\log(\beta) - \mathrm{E}_1(\alpha) + \mathrm{E}_1(\alpha\beta)e^{\alpha(\beta-1)}\right) & \beta \neq 1. \end{cases} \end{aligned}$$

Plugging the values of α and β, then

$$\bar{C}_s = \log_2(e) \begin{cases} 1 + \frac{\Omega}{P_m}\mathrm{E}_1\left(\frac{\Omega}{P_m}\right) - e^{\frac{-\Omega}{P_m}} & P_m = P_p \\ \frac{P_m}{P_m-P_p}\left(\log\left(\frac{P_m}{P_p}\right) - \mathrm{E}_1\left(\frac{\Omega}{P_m}\right) + \mathrm{E}_1\left(\frac{\Omega}{P_p}\right)e^{\frac{\Omega}{P_p}-\frac{\Omega}{P_m}}\right) & P_m \neq P_p, \end{cases} \tag{19}$$

where Ω is found from Eq. (11).

3.2 Outage Probability QoS

In the outage probability-constraint (OPC) environment, the usage of the channel by the cognitive user should not increase the outage probability of the primary user above δ. The performance of the cognitive user in this setup has been studied in [23]. The PDF of γ_p and γ_s can be expressed as [23]

$$\begin{aligned} f_{\gamma_p}(z) &= \frac{P_p}{P_s} \frac{1}{\left(z + \frac{P_p}{P_s}\right)^2} \\ f_{\gamma_s}(z) &= \frac{P_s}{P_p} \frac{1}{\left(z + \frac{P_s}{P_p}\right)^2}, \end{aligned} \tag{20}$$

and the CDF of γ_p and γ_s can be shown to equal

$$\begin{aligned} F_{\gamma_p}(z) &= \frac{z}{z + \frac{P_p}{P_s}} \\ F_{\gamma_s}(z) &= \frac{z}{z + \frac{P_s}{P_p}}. \end{aligned} \tag{21}$$

The cognitive transmit power required to satisfy, on average, the outage probability requirement of the primary user can also be found as [23]

$$P_s = \frac{P_p}{2^{R_p} - 1} \frac{\delta}{1-\delta}. \tag{22}$$

3.2.1 Performance Measures

Because the transmit power of the cognitive user is constant, then the average transmit power is equal to $\bar{P}_s = P_s = \frac{P_p}{2^{R_p}-1}\frac{\delta}{1-\delta}$. On the other hand, the average received power is found as

$$\begin{aligned}\bar{P}_r &= \mathbb{E}[P_s G_s] = P_s \mathbb{E}[G_s] = P_s \\ &= \frac{P_p}{2^{R_p}-1}\frac{\delta}{1-\delta}.\end{aligned} \tag{23}$$

For this constraint, we note that $\bar{P}_r = \bar{P}_s$. Moreover, the average bit error rate of the cognitive user can be shown to equal [23]

$$\bar{P}_e = \frac{1}{2} - \sqrt{\frac{\pi P_s}{P_p}}\, e^{\frac{P_s}{P_p}}\, \mathrm{Q}\left(\sqrt{\frac{2P_s}{P_p}}\right),$$

or

$$\bar{P}_e = \frac{1}{2} - \sqrt{\frac{\pi\delta}{(2^{R_p}-1)(1-\delta)}}\, e^{\frac{\delta}{(2^{R_p}-1)(1-\delta)}}\, \mathrm{Q}\left(\sqrt{\frac{2\delta}{(2^{R_p}-1)(1-\delta)}}\right). \tag{24}$$

Finally, the average channel capacity is easily derived as [23]

$$\bar{C}_s = \begin{cases} \log_2(e) & P_s = P_p \\ \frac{P_s \log_2\left(\frac{P_p}{P_s}\right)}{P_p - P_s} & P_s \neq P_p \end{cases},$$

and by plugging the value of P_s, the average channel capacity is expressed as

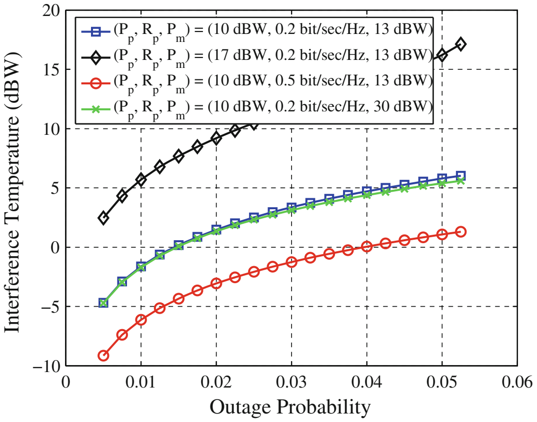
$$\bar{C}_s = \begin{cases} \log_2(e) & 2^{R_p} = \frac{1}{1-\delta} \\ \frac{\log_2\left((2^{R_p}-1)\frac{(1-\delta)}{\delta}\right)}{(2^{R_p}-1)\frac{(1-\delta)}{\delta}-1} & 2^{R_p} \neq \frac{1}{1-\delta}. \end{cases} \tag{25}$$

4 Numerical Results

In this section, we investigate the performance of the cognitive user under different communication settings, and we look into the performance differences between the interference temperature and outage probability QoS constraints. We study the relation between the outage probability in a system and the corresponding interference temperature level, then we investigate the cognitive user's average transmit and received powers, bit error rate, and channel capacity.

For the following results, the values of the primary user's transmit power, P_p, are 10 and 17 dBW. Likewise, the values of the primary user's transmit rate, R_p, are 0.2 and 0.5 bit/s/Hz. Moreover, the values of the cognitive maximum transmit power, P_m, are 13 and 30 dBW. The value of the primary user's outage probability, δ, is varied between 0.005 and 0.055 with increments of 0.0025. Each figure displays the performance measure as a response to changing the value of δ for the four combinations of P_p, R_p, and P_m. Accordingly, each figure shows four curves; the first curve is the base one where the values of P_p, R_p, and P_m are set to 10 dBW, 0.2 bit/s/Hz, and 13 dBW respectively. The second curve represents the case when the value of P_p increases from 10 to 17 dBW. The third curve indicates the response when the value of R_p is increased from 0.2 to 0.5 bit/s/Hz. Finally, the fourth curve shows the result of changing the value of P_m from 13 to 30 dBW.

Fig. 2 Interference temperature



4.1 Interference Temperature

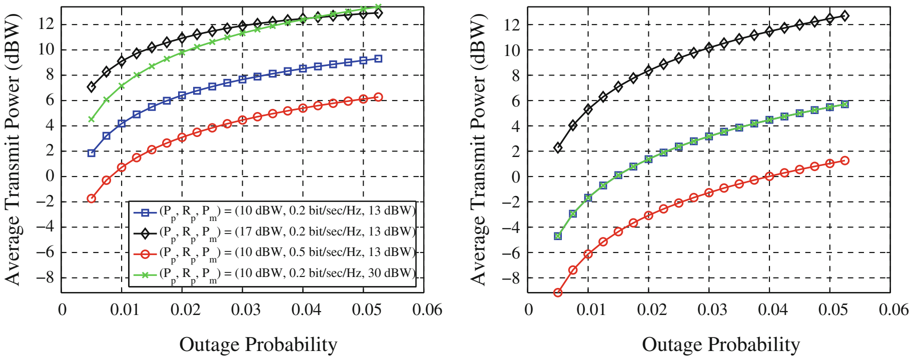
In this subsection, we investigate the relation between the interference temperature constraint, Ω, and the equivalent primary user's outage probability constraint, δ, as outlined in Eq. (11). Figure 2 illustrates the relation between Ω and δ. In addition, the figure also displays the effect of the primary transmit power, P_p, primary transmission rate, R_p, and the cognitive maximum transmit power, P_m, on this relation. The results of this subsection provide a tool to map an OPC cognitive system into an equivalent ITC system.

The relation between Ω and δ appears to follow, roughly, a logarithmic trend, in which an increase in one term causes the other term to increase as well. This can be traced back to Eq. (11) where $\Omega \propto log\left(\frac{1}{1-\delta\zeta}\right)$, where ζ is a positive number. The figure also displays that for the same value of δ, the value of Ω increases when P_p increases. This behavior can be explained by noting that for a high value of P_m, the relation between Ω and P_p is linear. On the other hand, we observe that Ω decreases with increasing R_p. To explain this result, we note that Ω can be approximated as $\frac{P_p}{2^{R_p}-1}\log\left(\frac{1}{1-\delta}\right)$ for a high value of P_m, which means it is inversely proportional to R_p. Finally, the effect of P_m is noted to be marginal, with Ω slightly decreasing for a great increase in P_m. As explained earlier, for high values of P_m, the value of Ω is approximately independent of P_m.

4.2 Transmit Power

We now study the average cognitive transmit power, $\bar{P}_s$, that is required to satisfy the QoS constraint. Figure 3 displays this value for the two environments under consideration. However, both systems satisfy the same primary user's outage probability. The IT system maps the value of δ into an equivalent value of Ω as seen in Fig. 2 and Eq. (11). We first note that $\bar{P}_s$ increases with increasing δ, and this is explained by examining Eq. (22) where the transmit power is proportional to $\frac{\delta}{1-\delta}$. In addition, it is obvious that $\bar{P}_s$ is proportional to P_p, and from Eq. (22) we note that $\bar{P}_s$ and P_p are linear, at least in the OPC systems. Moreover, the figure shows that the cognitive transmit power is decreasing when the primary rate is increasing, and this evident by noting that $\bar{P}_s$ is inversely proportional with $\left(2^{R_p}-1\right)$ in Eq. (22).

Comparing the performance differences of the two equivalent constraint systems, we note that the average transmit power in OPC systems is independent of the value of P_m. This



(a) **(b)**

Fig. 3 Transmit power. **a** Interference temperature constraint. **b** Outage probability constraint

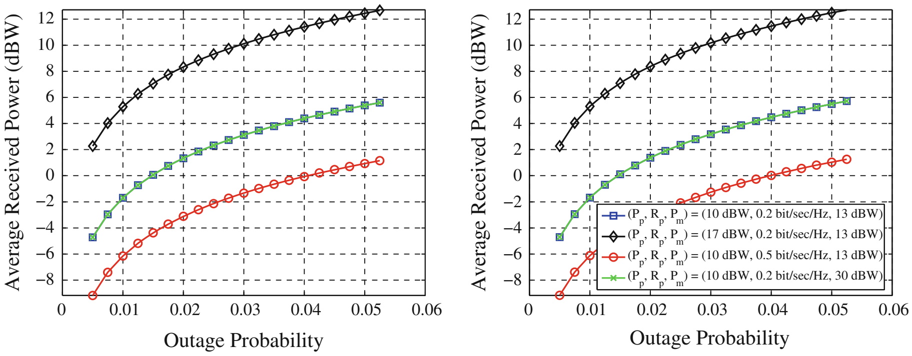
result is obvious from the fact that in this environment, the transmit power is constant and independent of the channel fluctuations. In addition, P_m has a mixed effect on the ITC systems; for low values of δ, the value of $\bar{P}_s$ decreases with increasing P_m, and the opposite occurs for higher values of δ. Finally, we note that the OPC systems have a lower average transmit power compared to the ITC systems. Although both systems cause the same value of outage probability on the primary user, an OPC system needs less transmit power to do that compared to an equivalent ITC system. However, the difference in $\bar{P}_s$ is more pronounced for lower values of δ.

4.3 Received Power

The average received power of the cognitive signal is an important measure as it resembles the cognitive interference on the primary user's signal. For OPC systems, we know that $\bar{P}_r = \bar{P}_s$, but $\bar{P}_r < \bar{P}_s$ for ITC systems. The interesting observation from Fig. 4 is that the average received power is the same for the two systems. To explain this result, we know that the two systems have to satisfy the same outage probability, so we expect that the cognitive interference (i.e., the cognitive received power) be the same. Moreover, the value of P_m seems to have no effect on the value of $\bar{P}_r$ for ITC systems. Actually, Eq. (2) shows the mixed effect of P_m on P_r; if the value of P_m increases, then $\alpha = \frac{\Omega}{P_m}$ decreases. Accordingly, the effect of higher value of P_m diminishes. In addition, the relation between the value of $\bar{P}_r$ and the values of P_p, R_p, and δ has a similar trend to that between $\bar{P}_s$ and these parameters.

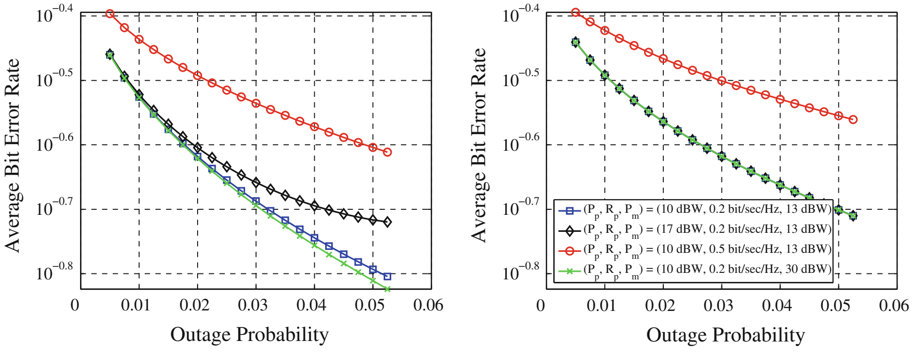
4.4 Bit Error Rate

Figure 5 displays the average bit error rate values. We first note that the ITC systems have lower $\bar{P}_e$ values than the equivalent OPC systems. From Eq. (24) we expect that $\bar{P}_e$ in OPC systems to be dependent on R_p and δ only. As the figure also displays, the average bit error rate decreases (i.e., the system is becoming better) with increasing the outage probability and/or decreasing the primary data rate. On the other hand, the values of P_p and P_m do not affect the average bit error rate in the OPC systems. The explanation to these results is that with increasing the outage probability and/or decreasing the primary data rate, the cognitive user has more room to operate, and so it can achieve better results. This performance measure behaves a little bit differently for the ITC systems; for example, increasing the value of P_m



(a) **(b)**

Fig. 4 Received power. **a** Interference temperature constraint. **b** Outage probability constraint



(a) **(b)**

Fig. 5 Bit error rate. **a** Interference temperature constraint. **b** Outage probability constraint

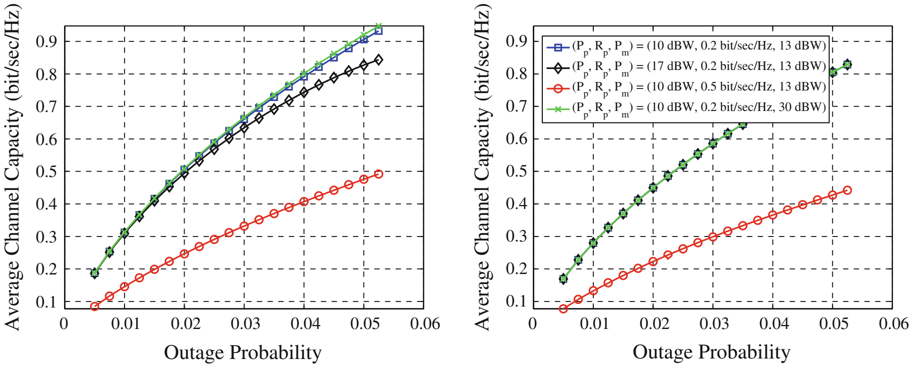
reduces $\bar{P}_e$ slightly. In addition, increasing the value of P_p increases the average bit error rate, especially in higher values of δ.

4.5 Channel Capacity

The average capacity of the cognitive channel is shown in Fig. 6. We first note that the ITC systems have slightly higher $\bar{C}_s$ values than those of the OPC systems. As evident in Eq. (25), increasing the value of R_p causes $\bar{C}_s$ to decrease, and increasing the value of δ causes the average channel capacity to increase. Moreover, Eq. (25) indicates that P_p and P_m have no effect on $\bar{C}_s$ for the OPC systems. On the other hand, similar to the results in Fig. 5, increasing the value of P_p causes this performance measure to deteriorate in the ITC systems. Similarly, increasing the value of P_m slightly enhances the average channel capacity.

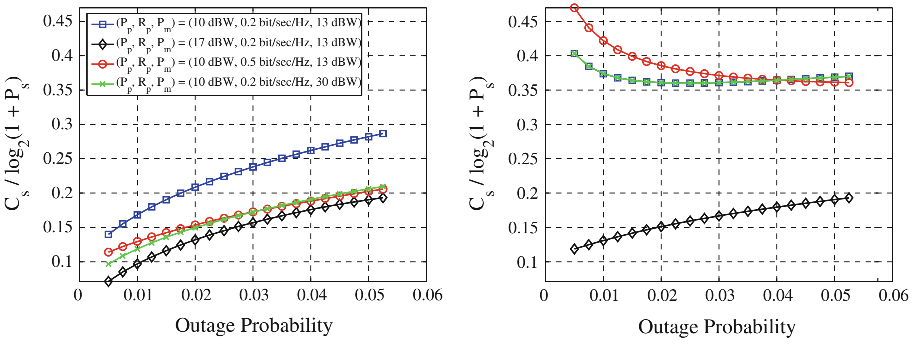
4.6 Ratio of Channel Capacity to Log Transmit Power

The average channel capacity to log transmit power ratio, $\bar{C}_s / \log_2 (1 + \bar{P}_s)$, is studied in this subsection. This measure is an indication of how much the average cognitive channel capacity is compared to the average capacity of a single-user Gaussian channel with the same



(a) **(b)**

Fig. 6 Channel capacity. **a** Interference temperature constraint. **b** Outage probability constraint



(a) **(b)**

Fig. 7 Ratio of channel capacity to log transmit power. **a** Interference temperature constraint. **b** Outage probability constraint

average transmit power. Apparently, as this ratio increases, more cognitive channel capacity is achieved due to the same transmit power. The results for this measure are shown in Fig. 7.

This ratio increases, in most cases, with increasing the value of the primary user's outage probability requirement. In addition, it is obvious that OPC systems have better results compared to the equivalent ITC systems. Although the average channel capacity is slightly lower, the average transmit power is much lower for the OPC systems, and this makes this ratio higher compared to the ITC systems. Moreover, increasing the primary user's transmit power, primary transmission rate, and/or the cognitive maximum transmit power decreases the value of this measure for the ITC systems.

5 Conclusions

This article investigated the impact of the quality of service constraint on the performance measures of the cognitive users in a spectrum sharing environment. A cognitive user wants to share a channel dedicated to a primary user in order to transmit its data using spectrum sharing techniques. The usage of the channel by the cognitive user should not violate the primary user's quality of service constraint. Two constraints were discussed in this work; the peak cognitive received power, also called the interference temperature level, and the primary user's outage probability.

In this work, the outage probability of the primary user under the interference temperature constraint was developed. Then for a specific outage probability requirement, an equivalent interference temperature-constraint cognitive system was found. The performance measures of the cognitive user were also developed; these measures include the average transmit and received powers, average bit error rate, and average channel capacity. In addition, the performance measures were compared to those in an equivalent outage probability-constraint environment.

Results of this work indicate that for the same outage probability requirement, the interference temperature-constraint systems have higher average transmit power, higher average channel capacity, and lower average bit error rate. The ratio of channel capacity to log transmit power is higher in outage probability-constraint systems. On the other hand, the average received power is the same for the two systems. In addition, the impact of changing the parameters of the communication environment (i.e., the primary user's transmit power and rate, outage probability requirement, and cognitive user's maximum transmit power) on the performance of the cognitive users was investigated in detail as well.

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