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Efficient Decoding for Generalized Quasi-Orthogonal Space–Time Block Codes

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Abstract Space–time coding can achieve transmit diversity and power gain over spatially uncoded systems without sacrificing bandwidth. There are various approaches in coding structures, including space–time block codes. A class of space–time block codes namely quasi-orthogonal space–time block codes can achieve the full rate, but the conventional decoders of these codes experience interference terms resulting from neighboring signals during signal detection. The presence of the interference terms causes an increase in the decoder complexity and a decrease in the performance gain. In this article, we propose a modification to the conventional coding/decoding scheme that will improve performance, reduce decoding complexity, and improve robustness against channel estimation errors as well.

Keywords Space–time block code · QOSTBC · Decoder complexity · Transmit diversity · Zero-forcing decoding

1 Introduction

In multiple antenna systems, space–time coding provides an effective way to achieve transmit diversity and power gain without sacrificing bandwidth. There are various approaches in space–time coding structures, including *space–time block codes* (STBC), *space–time trellis*

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codes (STTC), *space–time turbo trellis codes* and *layered space–time codes* (LST). STBC gained a lot of interests after their introduction by Alamouti [1], and by other researchers [2–4].

The STBC proposed by Alamouti for two transmit antenna provides a full rate and full diversity and allows for a simple linear *maximum likelihood* (ML) decoding. The rate of a space–time block code is defined as the ratio between the number of symbols the encoder takes as its input and the number of space–time coded symbols transmitted from each antenna. It is given by $R = \frac{k}{p}$, where k is the number of symbols the encoder takes as its input and p is the number of space–time symbols transmitted from each antennas for each block of k input symbols. These kinds of STBC are known as *orthogonal space time block codes* (OSTBC). Tarokh et al. proposed generalized OSTBC for more than two antennas using the theory of orthogonal designs in [5–7]. The objective in [7] was to design codes that provide full diversity. It was shown that STBC from a complex orthogonal design, which provides full diversity and full rate, is not possible for more than two antennas. An STBC that has a full diversity provides a maximum rate of $3/4$ for three and four transmit antennas without linear processing [6,8,9]. It is also difficult to construct STBC with rate higher than $1/2$ for more than four transmit antenna [6,8]. Moreover, OSTBC from orthogonal designs provides STBC with full diversity and enables linear decoding.

In order to achieve the advantages of OSTBC schemes with properties close to such optimal codes providing full rate, the so called *quasi-orthogonal space–time block codes* (QOSTBC) were proposed in [10–14]. These STBC were developed from quasi-orthogonal designs, where the orthogonality is relaxed to provide higher rate. QOSTBC allows a trade-off between higher rate and maximum diversity. The full rate QOSTBC provides only half of the maximum diversity for four transmit antennas. The decoder of QOSTBC processes pairs of transmitted symbols instead of a single symbol. Two maximum likelihood detectors are used in parallel to decode pairs of transmitted symbols in QOSTBC. The decision metric of [12, eqn.(3)] is shown as the sum of two terms; thus minimizing the decision metric is equivalent to minimizing two terms independently. Two maximum likelihood detectors are used either in sequence or in parallel. Therefore, decoding pairs for QOSTBC is more complex than decoding single symbols for space–time block codes [6], [12, p.44]. This results in higher complexity decoding at the receiver. Specifically, the decoding complexity increases with the modulation level. In order to minimize the decision metric [12, eqn.(3)] using maximum likelihood method, the receiver computes the decision metric over all possible symbols of a constellation or modulation level and decides in favor of the constellation symbols that minimize the decision metric. As the size of the constellation increases, the receiver must minimize the decision metric over large number of symbols. This will, subsequently, increase the transmission delay when high modulation schemes or more antennas are employed.

In this paper, based on the quasi-orthogonal code structures in the Jafarkhani scheme, the Tirkkonen–Boariu–Hottinen scheme, and the Papadias–Foschini scheme, we propose a modification for the generalized QOSTBC that allows linear decoding at the receiver. We propose an implementation technique that reduces computational load at the receiver. Diversity order of the proposed QOSTBC with full rate and linear processing will be 1. The proposed method demonstrates robustness to imperfect knowledge of the channel as well.

The rest of the paper is organized as follows: Sect. 2 provides an introduction to Quasi-Orthogonal Space Time Block Codes. Section 3 explains some of the famous schemes of QOSTBC. Section 4 explains the proposed linear decoding solution. A numerical simulation that validates the effectiveness of the proposed solution is provided in Sect. 5. Finally, conclusions are discussed in Sect. 6.

2 Quasi-Orthogonal Space Time Block Code System Model

For a given *multiple-input single-output* (MISO) wireless communication system, let the number of transmit antennas be m. A space–time block code encodes the input symbols vector of length l, $\mathbf{s} = [s_1, s_2, \ldots, s_l]^T$, into $k \times m$ matrix $\mathbf{S}$, where k is the number of time slots over which the transmission occurs, and T stands for the transpose operator. The code rate in this case is $r = \frac{l}{k}$. The received signal model is given as

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{w}, \tag{1}$$

where $\mathbf{h}$ is the $m \times 1$ complex channel vector, $\mathbf{y}$ and $\mathbf{w}$ are $k \times 1$ received vector and noise vector respectively. The entries of $\mathbf{h}$ and $\mathbf{w}$ are assumed to be independent samples of a zero-mean complex Gaussian random variable with variance of 1 and σ_w^2 respectively. The channel is assumed to be quasi-static, meaning that the channel coefficients will not change during one codeword transmission (i.e., for k time samples). The average energy of the symbol transmitted from each antenna is normalized to be 1, so the average signal-to-noise ratio (SNR) is $\frac{1}{\sigma_w^2}$.

We will continue our overview mostly on the basis of four antennas at the transmitter and one antenna at the receiver side. However, we would like to point out that the statements we give are equivalently true for more antennas. Further, four antennas are more likely to be used in the near future than for example 8 or 16 antennas. For the case of four transmit antennas, k will be equal to 4. Therefore, let the code matrix $\mathbf{S}$, in general form, be given by a 4×4 matrix, where the elements of $\mathbf{S} \in \{\pm s_1, \pm s_2, \pm s_3, \pm s_4, \pm s_1^*, \pm s_2^*, \pm s_3^*, \pm s_4^*\}$. Let us define the received signal vector $\mathbf{y} = \left[r_1\ r_2\ r_3\ r_4\right]^T$ and the complex channel vector as $\mathbf{h} = \left[h_1\ h_2\ h_3\ h_4\right]^T$. Then Eq. (1) will be

$$\mathbf{y} = \mathbf{S}\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \mathbf{w}. \tag{2}$$

Complex conjugating of some rows of $\mathbf{S}$ leads to the equivalent highly structured MIMO channel matrix $\mathbf{H}$. Then Eq. (1) can be transformed into

$$\widetilde{\mathbf{y}} = \mathbf{H}\mathbf{s} + \widetilde{\mathbf{w}}, \tag{3}$$

where $\mathbf{s} = \left[s_1\ s_2\ s_3\ s_4\right]^T$, $\mathbf{H}$ is a channel matrix with space (in columns) and time (in rows) dimensions defined as a 4×4 matrix, the elements of $\mathbf{H} \in \{\pm h_1, \pm h_2, \pm h_3, \pm h_4, \pm h_1^*, \pm h_2^*, \pm h_3^*, \pm h_4^*\}$, and $\widetilde{\mathbf{y}}$ and $\widetilde{\mathbf{w}}$ are transformed versions of $\mathbf{y}$ and $\mathbf{w}$ respectively.

The Grammian matrix, also known as detection matrix, is essential for analyzing the decoder performance. The Grammian matrix is defined as

$$\mathbf{G} = \mathbf{H}^H\mathbf{H}, \tag{4}$$

where $\mathbf{H}^H$ is the complex conjugate transpose of $\mathbf{H}$. The diagonal elements of $\mathbf{G}$ represent the overall channel gain and the non-diagonal elements represent the channel interference elements. For OSTBC schemes, the Grammian matrix $\mathbf{G}$ is proportional to the identity matrix. This means simple linear decoding can be applied at the receiver, and an estimate of $\mathbf{s}$, denoted as $\widetilde{\mathbf{s}}$, can be found as [15]

$$\begin{aligned} \widetilde{\mathbf{s}} &= \mathbf{H}^H\widetilde{\mathbf{y}} \\ &= \mathbf{H}^H\mathbf{H}\mathbf{s} + \mathbf{H}^H\widetilde{\mathbf{w}}. \end{aligned} \tag{5}$$

The interference terms in $\mathbf{G}$ in the QOSTBC case result in a more complex decoding method to estimate $\widetilde{s}$. In this case, the estimate can be found as [16]

$$\begin{aligned}\widetilde{\mathbf{s}} &= (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\widetilde{\mathbf{y}} \\ &= \mathbf{s} + (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\widetilde{\mathbf{w}}.\end{aligned} \tag{6}$$

The need to compute the inverse of the Grammian matrix in order to estimate the transmitted symbols makes this method computationally expensive. If we can transform $\mathbf{G}$ such that it is proportional to a diagonal matrix, then simple linear decoding can be applied. We propose a modification to the way the signal is formed at the transmitter side in order to achieve a linear decoding for QOSTBC at the receiver side, this modification is based on an important observation of Grammian matrix of most popular QOSTBC in literature.

3 Popular QOSTBC Schemes

3.1 Jafarkhani Code

Jafarkhani proposed STBCs from quasi-orthogonal designs in [12]. For four antennas, a QOSTBC was constructed from the Alamouti scheme as follows

$$\mathbf{S}_J = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}, \tag{7}$$

and the corresponding equivalent channel matrix $\mathbf{H}_J$ is given by

$$\mathbf{H}_J = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}. \tag{8}$$

Jafarkhani proposed different versions of the same code but our result is applicable to all versions with slight modifications. For Jafarkhani code given in Eq. (7), the Grammian matrix can be computed using Eq. (4) as

$$\mathbf{G}_J = \mathbf{H}_J^H\mathbf{H}_J = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix}, \tag{9}$$

where $\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$ and $\beta = 2\, Re(h_1h_4^* - h_2h_3^*)$. Notice that $\mathbf{G}_J$ has off diagonal terms which are interference terms, and this will affect the performance and lead to complex and computationally expensive decoding as well. Since the terms α and β are real, $\mathbf{G}_J$ is a real symmetric matrix (i.e., $\mathbf{G}_J^T = \mathbf{G}_J$).

3.2 Tirkkonen Code

Tirkkonen et al. proposed QOSTBC structure for four antennas as follows [10]

$$\mathbf{S}_T = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 \\ s_4^* & -s_3^* & s_2^* & -s_1^* \end{bmatrix}, \tag{10}$$

and similarly, the corresponding equivalent channel matrix $\mathbf{H}_T$ is given by

$$\mathbf{H}_T = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ -h_4^* & h_3^* & -h_2^* & h_1^* \end{bmatrix}. \tag{11}$$

For Tirkkonen code given above, the Grammian matrix can be computed using Eq. (4) and is given as

$$\mathbf{G}_T = \mathbf{H}_T^H \mathbf{H}_T = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix}, \tag{12}$$

where $\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$, and $\beta = 2\ Re(h_1 h_3^* + h_2 h_4^*)$. Since the terms α and β are real, $\mathbf{G}_T$ is also a real symmetric matrix.

3.3 Papadias and Foschini Code

And finally, we consider QOSTBC code proposed by Papadias and Foschini as [13]

$$\mathbf{S}_P = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}, \tag{13}$$

and the corresponding equivalent channel matrix $\mathbf{H}_P$ is given by

$$\mathbf{H}_P = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3 & h_4 & h_1 & -h_2 \\ -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix}. \tag{14}$$

For this code, the Grammian matrix is given as follows

$$\mathbf{G}_P = \mathbf{H}_T^H \mathbf{H}_T = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & -\beta \\ -\beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix}, \tag{15}$$

where $\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$, and $\beta = 2\ Im(h_1 h_3^* + h_2 h_4^*)$. Since the terms α and β are real and imaginary respectively, $\mathbf{G}_P$ is a Hermitian matrix (i.e., $\mathbf{G}_P^H = \mathbf{G}_P$).

4 Proposed Linear Decoding

The presence of the interference terms in each of the Grammian, or detection, matrices described above requires complex decoding given by Eq. (6) instead of the linear decoding given by Eq. (5). We present a method to eliminate the interference terms in order for the linear decoding to be applied. We will consider each case at a time.

4.1 Jafarkhani Code

The Grammian matrix given in Eq. (9) is a real symmetric matrix. There is a theorem in linear algebra that can help us diagonalize a real symmetric matrix as follows

Theorem 1 *Let* $\mathbf{A}$ *be a symmetric real* $n \times n$ *matrix. Then there exists an* $n \times n$ *real unitary matrix* $\mathbf{U}$ *such that* $\mathbf{U}^T\mathbf{A}\mathbf{U} = \mathbf{U}^{-1}\mathbf{A}\mathbf{U}$ *is a diagonal matrix* [17, (p. 221)].

Proof Any symmetric real $n \times n$ matrix $\mathbf{A}$ can be expressed in the form of

$$\mathbf{A} = \mathbf{Q}\mathbf{D}\mathbf{Q}^T,$$

where $\mathbf{Q}$ is an $n \times n$ real unitary matrix and $\mathbf{D}$ is a diagonal matrix. For unitary matrix $\mathbf{Q}$ we know that $\mathbf{Q}^{-1} = \mathbf{Q}^T$. Pre and post multiplying the above equation by $\mathbf{Q}^T$ and $\mathbf{Q}$ we get

$$\begin{aligned} \mathbf{Q}^T\mathbf{A}\mathbf{Q} &= \mathbf{Q}^T\mathbf{Q}\mathbf{D}\mathbf{Q}^T\mathbf{Q} \\ &= \mathbf{D}, \end{aligned} \tag{16}$$

and this concluded our proof. □

A unitary matrix of the form given by

$$\mathbf{U}_J = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}, \tag{17}$$

yields the desired result. By pre and post multiplying $\mathbf{G}_J$ with $\mathbf{U}_J$ and $\mathbf{U}_J^T$ respectively, we get a diagonal matrix represented as

$$\mathbf{U}_J^T\mathbf{G}_J\mathbf{U}_J = \mathbf{G}'_J = \begin{bmatrix} \alpha+\beta & 0 & 0 & 0 \\ 0 & \alpha-\beta & 0 & 0 \\ 0 & 0 & \alpha+\beta & 0 \\ 0 & 0 & 0 & \alpha-\beta \end{bmatrix}. \tag{18}$$

The new channel matrix can be evaluated as

$$\begin{aligned} \mathbf{G}'_J &= \mathbf{U}_J^T\mathbf{G}_J\mathbf{U}_J \\ &= \mathbf{U}_J^T\mathbf{H}_J^T\mathbf{H}_J\mathbf{U}_J \\ &= (\mathbf{H}_J\mathbf{U}_J)^T(\mathbf{H}_J\mathbf{U}_J). \end{aligned} \tag{19}$$

Thus, $\mathbf{H}'_J = \mathbf{H}_J\mathbf{U}_J$ and is given as

$$\mathbf{H}'_J = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 + h_4 & h_2 + h_3 & -h_2 + h_3 & -h_1 + h_4 \\ -h_2^* + h_3^* & h_1^* - h_4^* & -h_1^* - h_4^* & h_2^* + h_3^* \\ -h_3^* + h_2^* & -h_4^* + h_1^* & h_4^* + h_1^* & h_3^* + h_2^* \\ h_4 + h_1 & -h_3 - h_2 & h_3 - h_2 & -h_4 + h_1 \end{bmatrix}, \tag{20}$$

and the corresponding $\mathbf{S}'_J$ can be shown to equal

$$\mathbf{S}'_J = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 - s_4 & s_2 - s_3 & s_2 + s_3 & s_1 + s_4 \\ s_2^* - s_3^* & -s_1^* + s_3^* & s_1^* + s_3^* & -s_2^* - s_3^* \\ -s_2^* + s_3^* & s_1^* + s_4^* & -s_1^* + s_4^* & -s_2^* + s_3^* \\ s_1 + s_4 & -s_2 - s_3 & -s_2 + s_3 & s_1 - s_4 \end{bmatrix}. \tag{21}$$

The new encoding matrix given in Eq. (21) is quasi-orthogonal rather than orthogonal. Nevertheless, since its channel matrix $\mathbf{H}'_J$ is orthogonal, decoding can be achieved via simple linear decoding, and the transmitted symbols estimate can be found as

$$\begin{aligned} \widetilde{\mathbf{s}} &= (\mathbf{H}'^H_J \mathbf{H}'_J)\mathbf{s} + \mathbf{H}'^H_J \widetilde{\mathbf{w}} \\ &= \mathbf{D}\,\mathbf{s} + \mathbf{H}'^H_J \widetilde{\mathbf{w}}, \end{aligned} \tag{22}$$

where $\mathbf{D}$ is a diagonal matrix.

4.2 Tirkkonen Code

The Tirkkonen QOSTBC structure is given by Eqs. (10) and (11), and the corresponding Grammian matrix is given by Eq. (12). In this case, $\mathbf{G}_T$ is a real symmetric matrix as well, thus there exists a unitary matrix which diagonalizes the detection matrix. The unitary matrix which will diagonalize $\mathbf{G}_T$ is given as

$$\mathbf{U}_T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}. \tag{23}$$

By pre and post multiplying $\mathbf{G}_T$ with $\mathbf{U}_T^T$ and $\mathbf{U}_T$ respectively, we get a diagonal Grammian matrix

$$\mathbf{U}_T^T \mathbf{G}_T \mathbf{U}_T = \mathbf{G}'_T = \begin{bmatrix} \alpha - \beta & 0 & 0 & 0 \\ 0 & \alpha - \beta & 0 & 0 \\ 0 & 0 & \alpha + \beta & 0 \\ 0 & 0 & 0 & \alpha + \beta \end{bmatrix}. \tag{24}$$

The new channel matrix can be evaluated as

$$\begin{aligned} \mathbf{G}'_T &= \mathbf{U}_T^T \mathbf{G}_T \mathbf{U}_T \\ &= \mathbf{U}_T^T \mathbf{H}_T^T \mathbf{H}_T \mathbf{U}_T \\ &= (\mathbf{H}_T \mathbf{U}_T)^T (\mathbf{H}_T \mathbf{U}_T). \end{aligned} \tag{25}$$

Thus, $\mathbf{H}'_T = \mathbf{H}_T \mathbf{U}_T$ and is given as

$$\mathbf{H}'_T = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 - h_3 & h_2 - h_4 & h_1 + h_3 & h_2 + h_4 \\ h_2^* - h_4^* & h_3^* - h_1^* & h_2^* + h_4^* & -h_1^* - h_3^* \\ h_3 - h_1 & h_4 - h_2 & h_1 + h_3 & h_2 + h_4 \\ h_4^* - h_2^* & h_1^* - h_3^* & h_2^* + h_4^* & -h_1^* - h_3^* \end{bmatrix}, \tag{26}$$

and the corresponding $\mathbf{S}'_T$ will be

$$\mathbf{S}'_T = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + s_3 & s_2 + s_4 & -s_1 + s_3 & -s_2 + s_4 \\ -s_2^* - s_4^* & s_1^* + s_3^* & s_2^* - s_4^* & -s_1^* + s_3^* \\ -s_1 + s_3 & -s_2 + s_4 & s_1 + s_3 & s_2 + s_4 \\ s_2^* - s_4^* & -s_1^* + s_3^* & -s_2^* - s_4^* & s_1^* + s_3^* \end{bmatrix}. \tag{27}$$

The decoding will be accomplished using simple linear decoding similar to that in Eq. (22).

4.3 Papadias and Foschini Code

For Papadias and Foshchini code, $\mathbf{G}_P$ is Hermitian matrix. The previous theorem can be applied for a complex symmetric matrix with a slight change.

Theorem 2 *Let $\boldsymbol{A}$ be a Hermitian $n \times n$ matrix then there exists an $n \times n$ complex unitary matrix such that $\boldsymbol{U}^H \boldsymbol{A} \boldsymbol{U} = \boldsymbol{U}^{-1} \boldsymbol{A} \boldsymbol{U}$ is a diagonal matrix.*

The proof for this theorem is similar to that of the previous theorem.

The unitary matrix, which will diagonalize $\mathbf{G}_P$, is as follows

$$\mathbf{U}_P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & j\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & j\frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & j\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}. \tag{28}$$

Pre and post multiplying $\mathbf{G}_P$ with $\mathbf{U}_P^H$ and $\mathbf{U}_P$ respectively, we get a diagonal matrix represented as

$$\mathbf{U}_P^H \mathbf{G}_P \mathbf{U}_P = \mathbf{G}'_P = \begin{bmatrix} \alpha - \beta & 0 & 0 & 0 \\ 0 & \alpha + \beta & 0 & 0 \\ 0 & 0 & \alpha + \beta & 0 \\ 0 & 0 & 0 & \alpha - \beta \end{bmatrix}. \tag{29}$$

Thus, $\mathbf{H}'_P = \mathbf{H}_P \mathbf{U}_P$ will be equal to

$$\mathbf{H}'_P = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 + jh_3 & h_2 + jh_4 & jh_1 + h_3 & jh_2 + h_4 \\ -h_2^* - jh_4^* & h_1^* - jh_3^* & -jh_2^* - h_4^* & h_1^* + jh_3^* \\ -h_3 + jh_1 & h_4 - jh_2 & -jh_3 + h_1 & jh_4 + h_2 \\ -h_4^* + jh_2^* & -h_3^* + jh_1^* & -jh_4^* + h_2^* & -jh_3^* + h_1^* \end{bmatrix}, \tag{30}$$

and the corresponding $\mathbf{S}'_P$ will be

$$\mathbf{S}'_P = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + js_3 & s_2 + js_4 & js_1 + s_3 & js_2 + s_4 \\ -s_2^* - js_4^* & s_1^* - js_3^* & -js_2^* - s_4^* & s_1^* + js_3^* \\ -s_3 + js_1 & s_4 - js_2 & -js_3 + s_1 & js_4 + s_2 \\ -s_4^* + js_2^* & -s_3^* + js_1^* & -js_4^* + s_2^* & -js_3^* + s_1^* \end{bmatrix}. \tag{31}$$

4.4 Proposed Encoder and Decoder

The multiple-input-single-output system under consideration is usually deployed for the downlink (base station to mobile unit). Base stations usually have more processing capability and surplus power whereas mobile units have limited processing capability and limited power. If the decoding computations on the mobile unit can be reduced, the cost of the decoder will be reduced and the battery life will be extended. Therefore, to minimize the computations done on the mobile unit during the decoding process, we propose to change the way the transmitted signal is formed in order to enable the use of a linear decoder at the mobile unit side.

As was seen in the previous subsections and described in Eq. (22); pre and post multiplying the Grammian matrix by the unitary matrix **U** will result in a diagonal matrix **D**. In order to achieve the pre multiplication step, we propose to change the way the transmitted signal is formed. If we multiply the transmitted vector **s** by **U**, then the modified transmitted vector will be **Us** instead of **s**. Because of the way **U** is structured, the power of the modified transmitted signal will not change, (i.e., $\mathbb{E}\{|\mathbf{Us}|^2\} = \mathbb{E}\{|\mathbf{s}|^2\}$), where $\mathbb{E}$ represents the expectation operator.

With **Us** being transmitted over the channel **H**, the received signal, $\widetilde{\mathbf{y}}$, in the proposed scheme will be

$$\widetilde{\mathbf{y}} = \mathbf{HUs} + \widetilde{\mathbf{w}}. \tag{32}$$

The decoder will only need to multiply the received signal by $(\mathbf{H}\,\mathbf{U})^H$, and by doing so it will get the data estimate as

$$\begin{aligned}\widetilde{\mathbf{s}} &= (\mathbf{H}\,\mathbf{U})^H \mathbf{H}\,\mathbf{U}\,\mathbf{s} + (\mathbf{H}\,\mathbf{U})^H \widetilde{\mathbf{w}} \\ &= \mathbf{U}^H \mathbf{G}\,\mathbf{U}\,\mathbf{s} + \mathbf{U}^H \mathbf{H}^H \widetilde{\mathbf{w}} \\ &= \mathbf{D}\,\mathbf{s} + \mathbf{U}^H \mathbf{H}^H \widetilde{\mathbf{w}}.\end{aligned} \tag{33}$$

In this case **D** is a diagonal matrix with positive elements and can be found in Eq. (18) for the Jafarkhani case.

The proposed scheme decodes the received signal by a standard, simple, linear decoding process in contrast to the needed matrix inversion in the conventional schemes. As a result, the decoding complexity will be reduced in the proposed scheme compared to those in the standard QOSTBC decoding schemes.

5 Numerical Results

In this section we display some numerical results in order to compare the performance of the proposed scheme with the conventional ones. As an example, a Monte Carlo simulation was conducted for the proposed scheme, described in Eqs. (32) and (33), and Jafarkhani scheme, described in Eqs. (7) and (8). Quadrature Phase-Shift Keying (QPSK) was chosen as the modulation format.

Figure 1 displays the symbol error rate results for the two schemes; here we see that the proposed scheme outperforms the Jafarkhani scheme when the SNR is less than 9.3 dBs for the QPSK case. As the SNR exceeds that value, Jafarkhani scheme performs better. As obvious from the figure, the SNR value at which the two schemes have similar performance increase with increasing the constellation order. For example, at SNR of 20 dB, the two schemes have comparable symbol error rate for the 16-PSK modulation scheme. In either

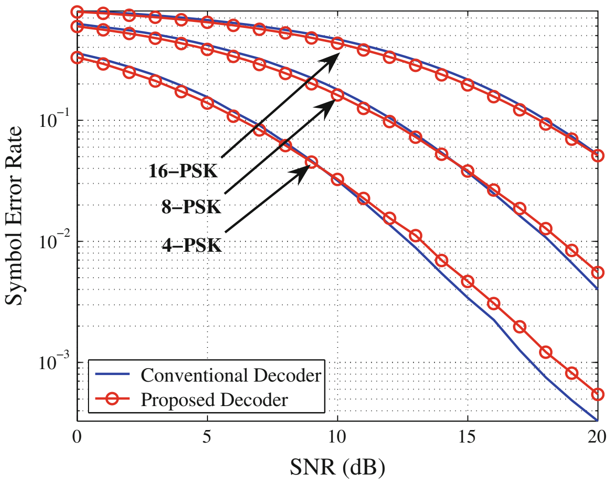

Fig. 1 Symbol error rate versus SNR

case, the difference in performance is marginal. This result indicates that the two schemes, at least for the practical SNR range, have comparable error rates. The loss in performance in the proposed algorithm in higher SNR values is believed to be due the loss of the diversity order.

It was shown that STBC that provides full diversity and full rate is not possible for more than two antennas [7]. So for the case of 4 antennas, there are three design parameters: diversity order, rate, and linear processing at the decoder. An STBC with full diversity provides a maximum rate of $3/4$ for four antennas without linear processing [8]. In conventional QOSTBC, diversity is relaxed in order to obtain higher rate without linear processing. Maximum diversity in conventional QOSTBC is half the maximum diversity (i.e., 2 in the case of 4 antenna). In the scheme proposed in this work, in order to achieve linear processing and full rate, diversity order is further sacrificed. Thus, the diversity order of our scheme is 1.

Next, we will study the case when the decoder has imperfect knowledge of the channel. This study is concerned about the robustness of the decoder when there are errors in estimating the channel gain coefficients. Usually in practice, the decoder has to estimate the channel gain, and estimation may not be perfect. In this measure, we prefer a scheme that achieves a better job when the decoder has estimation errors.

Let's assume that the true channel matrix is denoted $\mathbf{H}_c$ and the decoder estimated channel matrix is denoted as $\mathbf{H}_d$. Then, the two matrices can be related as $\mathbf{H}_d = \mathbf{H}_c + \mathbf{H}_e$. In this case, $\mathbf{H}_e$ represents the estimation errors which are considered random. Assuming the estimation errors are normally distributed with a variance of σ^2, we need to compare the performance of the two QOSTBC schemes with varying σ^2. When σ^2 is lower, the decoder has better estimation to the channel, but when σ^2 becomes higher, the decoder estimation errors increase.

Figure 2 displays the symbol error rate with varying σ^2 for SNR values of 4.5, 7, 9.5, and 12 dB under the QPSK modulation format. Different SNR values were selected to compare the performance of the two schemes against channel estimation errors or imperfect channel knowledge. The SNR of 9.5 dB was chosen specifically because the two schemes have a very close symbol error rate at this SNR value for the perfect channel estimation case, so any deviation in performance will be mainly to the way the decoder behaves when there are

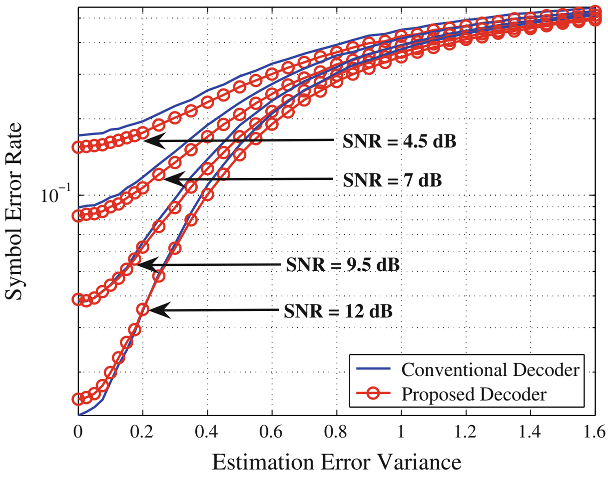

Fig. 2 Symbol error rate versus estimation error variance

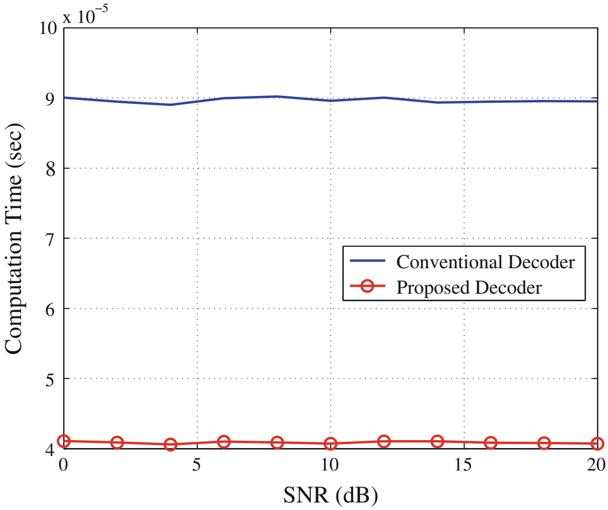

Fig. 3 Computation time versus SNR

estimation errors. As obvious from the figure, the proposed scheme outperforms the conventional scheme. This result proves that the proposed algorithm is more robust to the channel estimation errors.

Finally, we want to investigate the decoding complexity of the two schemes. As shown in Eq. (6), the conventional decoder needs to conduct a matrix inversion in order to decode the signal. On the other hand, the proposed scheme needs to conduct a simple linear operation in order to accomplish that job. So we expect the proposed scheme to have a lower computational load on the decoder.

Figure 3 displays the decoder average computational time, this is a measure of how much time the decoder spends in order to decode the received signal. As expected, the proposed algorithm has lower computation time than the conventional one. Actually, as shown in the figure, the conventional scheme needs around 230% more computation time than the proposed scheme. The gain in the computation time can translate to more battery time, less latency, and lower complexity and cost for the decoder.

If we compare the decoder equations of (6) for the conventional algorithm and (33) for the proposed algorithms, we notice that the operations in the convectional decoder include an extra $n \times n$ matrix multiplication and an $n \times n$ matrix inversion, where n is the number of antennas. As we know, computational complexity order for a matrix multiplication is $\mathcal{O}(n^3)$ using regular matrix multiplication method, and the computational complexity order is $\mathcal{O}(n^3)$ for matrix inversion using Gauss-Jordan elimination method. Apparently, the conventional decoder complexity is higher than that of the proposed algorithm. Consequently, the results in Fig. 3 confirm that the proposed algorithm needs less computation time compared to the conventional scheme.

6 Conclusion

In this paper, we proposed a modification to the conventional coding/decoding scheme for full rate QOSTBC. The decoding complexity is a concern in the conventional QOSTBC schemes. Making use of the properties of the Grammian matrix, the proposed modification eliminates the need to compute matrix inversion at the decoder, and it results in a simple linear decoding at the receiver side. The decoder complexity of the proposed scheme is reduced without sacrificing the performance.

Numerical results indicate that the proposed scheme has comparable error rate, is more robust against channel estimation errors, and has a much lower computation load than that of the conventional decoder schemes.

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