

# An Outage Analysis of Multibranch Diversity Receivers with Cochannel Interference in $\alpha$ - $\mu$ , $\kappa$ - $\mu$ , and $\eta$ - $\mu$ Fading Scenarios

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**Abstract** Wireless communications systems in a frequency reuse environment are subject to cochannel interference. In order to improve the system performance, diversity techniques are deployed. Among the practical diversity schemes used, Equal-Gain Combining (EGC) appears as a reasonably simple and effective one. Unfortunately, the exact analysis of the outage probability of EGC receivers is rather intricate for it involves the evaluation of multifold nested integrals. It becomes mathematically intractable with the increase of the number of diversity branches and/or interferers. For example, for  $N_B$  diversity branches and  $N_I$  arbitrary independent cochannel interferers, the exact formulation using the convolutional approach requires  $2 + N_B + (N_B \times N_I)$  nested integrals, which, very quickly, and for any practical system, turns out to be mathematically intractable. In this paper, we propose accurate approximate formulations for this problem, whose results are practically indistinguishable from the exact solution. In our model, the system is composed by  $N_B$  branches and  $N_I$  interferers so that the desired signals are coherently summed, whereas the interfering signals are incoherently summed at the EGC receiver. Three sets of fading scenarios, namely  $\alpha$ - $\mu$ ,  $\kappa$ - $\mu$ , and  $\eta$ - $\mu$ , are investigated. The proposed approach is indeed flexible and accommodates a variety of mixed fading scenarios for desired and interfering signals.

**Keywords** Cochannel interference · Diversity · Moment-based estimators · Outage probability · Generalized fading scenarios

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## 1 Introduction

Sums of random variables (RVs) arise in a number of wireless communications applications, such as maximal ratio combining, equal-gain combining (EGC), signal detection, phase jitter, intersymbol interference, outage probability, and others [1–3,26]. In several circumstances, obtaining the exact formulation of some statistics for such sums may be mathematically cumbersome. As an attempt to circumvent this, a number of approaches concerning approximation methods have been proposed in the literature for the well-known RVs [4–11]. In a pioneering work [4], a Nakagami- $m$  approximation to the sum of independent identically distributed (i.i.d.) Nakagami- $m$  RVs was proposed. Anchored on that idea, the parameters of the approximate Nakagami- $m$  distribution of the sum of two correlated identically distributed Nakagami- $m$  RVs were obtained in [5]. In [6,7], closed-form approximations to Rayleigh and Rice sum probability density functions (PDFs) were derived, based either on a modification of the small argument approximation (Rayleigh case) or on a modification of the sum distribution of squared Ricean RVs (Rice case). In such cases, the coefficients of the approximate formulations were optimized using the non-linear squares method based on the interior reflective Newton method [12]. A number of works employing moment-based estimators were successfully presented in [8–11] with the aim at approximating sums of Weibull, Nakagami- $m$ , Rice, and Hoyt (Nakagami- $q$ ) RVs.

The rapid and thriving expansion of wireless communication systems along the recent years, with the demand for new services growing in an incredible pace, has propelled the urge for an efficient use of the radio spectrum. In this sense, frequency reuse has certainly proved to be an effective and practical strategy. However, as is well known, such a technique gives rise to the so-called cochannel interference (CCI). Possible means of combating the deleterious effects of CCI are cell sectoring and diversity combining techniques. A number of researchers have investigated CCI-limited communication systems [1,2,13–15], but only a few examined CCI in an EGC scenario. EGC by itself is a rather intricate technique as far as its performance analysis is concerned, because it involves sums of fading envelopes. Including CCI to the model renders the intractability of the problem even more accentuated, because in such a case the required statistics are those of a ratio of sums of envelopes. To the best of the authors' knowledge, the outage probability (OP) of signals undergoing  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ , or  $\kappa$ - $\mu$  fading with CCI is not yet available in the technical literature.

A pioneering work in the CCI performance analysis was carried out by Abu-Dayya and Beaulieu in [16] for a Nakagami- $m$  fading scenario and assuming equal interfering powers. In that case, a method for computing the OP was presented for EGC as well as for other diversity techniques. In [17], an exact formulation for the OP of Rayleigh channels was derived taking into account distinct interfering powers and EGC. More recently [18], the OP and other performance metrics were derived for EGC considering a Rayleigh environment. All of these works make use of the Beaulieu series in order to obtain the statistics of the sum of RVs. Such a methodology is certainly very useful, but it is approximate and does require a careful choice of the parameters in order to achieve the necessary accuracy [19]. On the other hand, for  $N_B$  branches and  $N_I$  interferers, the exact formulation using the convolutional approach requires  $2 + N_B + (N_B \times N_I)$  nested integrals, which, very quickly, and for any practical system, turns out to be mathematically intractable.

Recently, accurate approximate expressions to the sum of  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ , and  $\kappa$ - $\mu$  RVs were proposed in [20,21]. In the present paper, we make use of the approach applied in [20,21] and present accurate, approximate expressions for the OP of multibranch EGC receivers undergoing  $\alpha$ - $\mu$ ,  $\kappa$ - $\mu$ , and  $\eta$ - $\mu$  fading [22,23] and subject to CCI. Such an approach relies on moment-based estimators and employs multinomial expansion to obtain the required

moments. In addition, the proposed expressions allow for arbitrary fading parameters and for distinct desired and interfering powers at the input branches. The approach proposed here is very flexible and accommodates different combinations of distinct fading scenarios. To attest the accuracy of our approximations, Monte Carlo simulation data are provided and an excellent match between approximate and simulation curves is observed. The reader is referred to [20, 21] for explanations and justification on the idea behind the proposed approximations.

The remainder of this paper is organized as follows. In Sect. 2, the three general fading distributions examined in this paper and their respective associated fading models are described. In Sect. 3, the system model under study is introduced. In Sect. 4, the arduousness inherent in the exact calculation of the OP in EGC receivers subject to  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ , and  $\kappa$ - $\mu$  fading is depicted, while in Sect. 5 accurate easy-to-compute approximate expressions are proposed. In Sect. 6, we delineate a way to obtain Nakagami- $m$ , Hoyt (Nakagami- $q$ ), Weibull, and Rice fading models as special cases of the more general  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ , and  $\kappa$ - $\mu$  fading models. Section 7 compares the approximated numerical plots with Monte Carlo simulations and a perfect agreement is attested. Finally, Sect. 8 presents some concluding remarks.

## 2 The $\alpha$ - $\mu$ , $\kappa$ - $\mu$ , and $\eta$ - $\mu$ Fading Distributions

The  $\alpha$ - $\mu$  [22],  $\kappa$ - $\mu$ , and  $\eta$ - $\mu$  distributions fading distributions [23] arise from general fading models, which have recently been proposed in the literature with the aim at providing a better understanding of the physical phenomena involved and, as a consequence, better adjustment to field measurement data when compared to other known fading distributions. The  $\alpha$ - $\mu$  distribution comprises both Weibull and Nakagami- $m$  as special cases. The  $\kappa$ - $\mu$  distribution encompasses both Rice and Nakagami- $m$  as special cases. And the  $\eta$ - $\mu$  one includes Hoyt (Nakagami- $q$ ) and Nakagami- $m$ . In the sequel, a brief description regarding these three new fading models is carried out.

### 2.1 The $\alpha$ - $\mu$ Fading Model

The fading model for the  $\alpha$ - $\mu$  distribution considers a signal composed of clusters of multipath waves propagating in a non-homogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay-time spreads of different clusters being relatively large. The clusters of multipath waves are assumed to have the scattered waves with identical powers. The resulting envelope is obtained as a nonlinear function of the modulus of the sum of the multipath components. Such a nonlinearity is manifested in terms of a power parameter, so that the resulting signal intensity is obtained not simply as the modulus of the sum of the multipath components, but as this modulus to a certain given exponent.

The PDF  $f_R(r)$  of the envelope  $R$  is given by

$$f_R(r) = \frac{\alpha \mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left(-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right), \quad (1)$$

where  $\hat{r} = \sqrt[\alpha]{E(R^\alpha)}$ ,  $\alpha > 0$  (which describes to the non-linearity of the medium), and  $\mu > 0$  (which is associated to the number of multipath clusters),  $\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t) dt$  is the gamma function, and  $E(\cdot)$  denotes expectation. The cumulative distribution function (CDF) of  $R$  can be found

$$F_R(r) = \frac{\Gamma(\mu, \mu r^\alpha / \hat{r}^\alpha)}{\Gamma(\mu)}, \quad (2)$$

where  $\Gamma(z, y) = \int_0^y t^{z-1} \exp(-t) dt$  is the incomplete gamma function. The  $k$ -th moment  $E(R^k)$  can be expressed as

$$E(R^k) = \hat{r}^k \frac{\Gamma(\mu + k/\alpha)}{\mu^{k/\alpha} \Gamma(\mu)}. \quad (3)$$

For  $\alpha = 2$ , (1) particularizes to the Nakagami- $m$  PDF, whereas for  $\mu = 1$ , (1) reduces to the Weibull one.

## 2.2 The $\kappa$ - $\mu$ Fading Model

The signal is composed by multipath clusters propagating in a non-homogeneous environment where, within each cluster, a dominant component is found. The inphase and quadrature components of each cluster are independent from each other. The powers of the scattered waves of each quadrature component are identical but those of the dominant components are arbitrary. As its name implies, this fading model is written in terms of two parameters, namely  $\kappa$  and  $\mu$ . The parameter  $\kappa > 0$  is given by the ratio between the total power of the dominant components and the total power of the scattered waves, whereas the parameter  $\mu > 0$  is defined as [23]

$$\mu = \frac{E^2(R^2)}{V(R^2)} \frac{(1 + 2\kappa)}{(1 + \kappa)^2}, \quad (4)$$

which is related to the number of multipath clusters. In (4),  $V(\cdot)$  denotes variance.

The PDF  $f_R(r)$  of the  $\kappa$ - $\mu$  envelope  $R$  is given by [23]

$$f_R(r) = \frac{2\mu(1 + \kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)} \frac{r^\mu}{\hat{r}^{\mu+1}} \exp\left(-\frac{\mu(1 + \kappa)r^2}{\hat{r}^2}\right) I_{\mu-1}\left(\frac{2\mu\sqrt{\kappa(1 + \kappa)}r}{\hat{r}}\right), \quad (5)$$

where  $\hat{r} = \sqrt{E(R^2)}$  and  $I_\nu(\cdot)$  is the modified Bessel function of the first kind and arbitrary order  $\nu$  [24, Eq. 9.6.20]. The  $n$ -th moment  $E(R^n)$  can be expressed as

$$E(R^n) = \frac{\Gamma(\mu + \frac{n}{2}) \exp(-\kappa\mu) \hat{r}^n}{\Gamma(\mu) ((1 + \kappa)\mu)^{\frac{n}{2}}} {}_1F_1\left(\mu + \frac{n}{2}; \mu; \kappa\mu\right), \quad (6)$$

where  ${}_1F_1(\cdot; \cdot; \cdot)$  is the confluent hypergeometric function [24, Eq. 13.1.2]. The CDF is obtained in closed form as

$$F_R(r) = 1 - Q_\mu\left(\sqrt{2\kappa\mu}, \frac{\sqrt{2\mu(1 + \kappa)}r}{\hat{r}}\right), \quad (7)$$

where

$$Q_\nu(a, b) = \frac{1}{a^{\nu-1}} \int_b^\infty x^\nu \exp\left(-\frac{x^2 + a^2}{2}\right) I_{\nu-1}(ax) dx, \quad (8)$$

is the generalized Marcum  $Q$ -function [23, Eq. 4], with  $I_\nu[\cdot]$  denoting the modified Bessel function of the first kind and arbitrary order  $\nu$  [24, Eq. 9.6.20]. For  $\kappa \rightarrow 0$  and  $\mu = m$ , where  $m$  stands for the Nakagami fading parameter, (5) reduces to the Nakagami- $m$  PDF and for  $\mu = 1$  and  $\kappa = k$ , where  $k$  denotes the Rice factor, (5) reduces to the Rice PDF.

### 2.3 The $\eta$ - $\mu$ Fading Model

The multipath clusters are composed of scattered waves only. The  $\eta$ - $\mu$  fading model appears in two formats, corresponding to two physical fading models. Although this paper investigates only the Format 1, our results can be easily extended to the Format 2. In fact, as pointed out in [23], these formats can be converted into each other by a simple transformation formula. In Format 1, the inphase and quadrature components within each multipath cluster are independent from each other and have different powers. The ratio between these powers is given by the parameter  $\eta$  and the parameter  $\mu$  is defined as [23]

$$\mu = \frac{E^2(R^2)}{2 V(R^2)} \left[ 1 + \left( \frac{H^2}{h} \right) \right], \tag{9}$$

which is related to the number of multipath clusters. Above,  $h = (2 + \eta^{-1} + \eta)/4$  and  $H = (\eta^{-1} - \eta)/4$ .

The PDF  $f_R(r)$  of the  $\eta$ - $\mu$  envelope  $R$  is given by [23]

$$f_R(r) = \frac{4\sqrt{\pi} \mu^{\mu+\frac{1}{2}} h^\mu r^{2\mu}}{\Gamma(\mu) H^{\mu-\frac{1}{2}} \hat{r}^{2\mu+1}} \exp\left(-\frac{2\mu hr^2}{\hat{r}^2}\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu Hr^2}{\hat{r}^2}\right), \tag{10}$$

where  $\hat{r} = \sqrt{E(R^2)}$ . The  $n$ -th moment  $E(R^n)$  can be expressed as

$$E(R^n) = \frac{\Gamma(2\mu + \frac{n}{2}) \hat{r}^n}{h^{\mu+\frac{n}{2}} (2\mu)^{\frac{n}{2}} \Gamma(2\mu)} {}_2F_1\left(\mu + \frac{n}{4} + \frac{1}{2}, \mu + \frac{n}{4}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^2\right), \tag{11}$$

where  ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$  is the confluent hypergeometric function [24, Eq. 15.1.1]. The CDF is shown as

$$F_R(r) = 1 - Y_\mu\left(\frac{H}{h}, \frac{\sqrt{2h\mu}r}{\hat{r}}\right), \tag{12}$$

where [23, Eq. 20]

$$Y_\nu(\lambda, \beta) \triangleq \frac{2^{-\nu+\frac{3}{2}} \sqrt{\pi} (1-\lambda^2)^\nu}{\Gamma(\nu) \lambda^{\nu-\frac{1}{2}}} \int_\beta^\infty x^{2\nu} \exp(-x^2) I_{\nu-\frac{1}{2}}(\lambda x^2) dx. \tag{13}$$

It is worth noting that, very recently, a closed-form expression for  $Y_\nu(\lambda, \beta)$  that is efficiently computable has been found in [25].

For  $\mu = 0.5$ , (10) particularizes into the Hoyt PDF, in which the Hoyt parameter is given by  $b = -\frac{1-\eta}{1+\eta}$ . The Nakagami- $m$  PDF can be attained from (10) in an exact manner by setting  $\mu = m/2$  and  $\eta \rightarrow 1$  or, in the same way, by setting  $\mu = m$  and  $\eta \rightarrow 0$  or  $\eta \rightarrow \infty$ .

### 3 System Model

In our analysis, we consider an EGC receiver composed of  $N_B$  antennas conveniently spaced so that the signals arriving at them are independent. In EGC, the received signals are cophased, equally weighted, and added to give the resultant desired signal. We assume that there are  $N_I$  cochannel interferers. Desired signals are added in a coherent manner whereas the addition of interfering signals are performed in an incoherent way [26]. In such a case, the SIR of these systems are modeled as

$$Z = \left( \frac{X}{Y} \right)^2, \quad (14)$$

where

$$X = \sum_{i=1}^{N_B} X_i, \quad (15)$$

represents the sum of the desired signals  $X_i$  at the diversity branches and

$$Y^2 = \sum_{j=1}^{N_I} \sum_{i=1}^{N_B} Y_{i,j}^2, \quad (16)$$

stands for the sum of the powers of the interference signals  $Y_{i,j}$  at the diversity branches. Note that  $X_i$  denotes the desired envelope at branch  $i$  and  $Y_{i,j}$  stands for the  $j$ -th interfering envelope at branch  $i$ .

Considering the  $\alpha$ - $\mu$  scenario,  $X_i$  and  $Y_{i,j}$  are  $\alpha$ - $\mu$  distributed with PDF given by (1) using the parameters  $(\alpha_i, \mu_i, \hat{x}_i)$  and  $(\alpha_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ , respectively. In this case,  $\hat{x}_i = \sqrt[{\alpha_i}]{E(X_i^{\alpha_i})}$  and  $\hat{y}_{i,j} = \sqrt[{\alpha_{i,j}}]{E(Y_{i,j}^{\alpha_{i,j}})}$ . Regarding the  $\kappa$ - $\mu$  case,  $X_i$  and  $Y_{i,j}$  are  $\kappa$ - $\mu$  distributed with a PDF given by (5) using the respective parameters  $(\kappa_i, \mu_i, \hat{x}_i)$  and  $(\kappa_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ . In this case,  $\hat{x}_i = \sqrt{E(X_i^2)}$  and  $\hat{y}_{i,j} = \sqrt{E(Y_{i,j}^2)}$ . Equivalently, in the  $\eta$ - $\mu$  fading scenario,  $X_i$  and  $Y_{i,j}$  are  $\eta$ - $\mu$  distributed with PDF given by (10) using the respective parameters  $(\eta_i, \mu_i, \hat{x}_i)$  and  $(\eta_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ . Again,  $\hat{x}_i = \sqrt{E(X_i^2)}$  and  $\hat{y}_{i,j} = \sqrt{E(Y_{i,j}^2)}$ . The readers may refer to [22,23] for further details on these distributions.

#### 4 Outage Probability: Exact Formulation

The OP is defined as the probability that the SIR goes beneath a given threshold,  $z_{th}$ . Thus

$$P_{out} = Pr[Z < z_{th}], \quad (17)$$

which is the CDF  $F_Z(z)$  evaluated at  $Z = z_{th}$ . In [27], an unified approach for computing the OP in wireless system is proposed, which is given by

$$F_Z(z_{th}) = \int_0^\infty \int_0^{y\sqrt{z_{th}}} f_X(x) f_Y(y) dx dy = \int_0^\infty F_X(y\sqrt{z_{th}}) f_Y(y) dy, \quad (18)$$

where  $f_W(\cdot)$  and  $F_W(\cdot)$  denote the PDF and CDF, respectively, of an arbitrary RV  $W$ . From (18), note that a tractable exact expression for the OP is very difficult to attain, if not impossible in case the number of branches and interferers grow. This is because the PDF of  $X$  and the PDF of  $Y$  cannot be obtained in a simple manner for the general case. One of the possible exact solutions involves multifold integrals or integral of the product of moment generating functions, certainly non attractive approaches as the number of diversity branches and interfering signals increases. For instance, for  $N_B$  branches and  $N_I$  interferers the number of nested integrals necessary to solve this problem is  $2 + N_B + N_B \times N_I$ , which turns out to be intractable for practical applications (e.g.,  $N_B = 2$  and  $N_I = 6$ , then 16 integrals are necessary). Using the convolutional approach, (18) can be expressed in an exact manner as

$$\begin{aligned}
 F_Z(z_{th}) &= \int_0^\infty \int_0^{y\sqrt{z_{th}}} \left( \int_0^x \int_0^{x-r_{N_B}} \dots \int_0^{x-\sum_{i=3}^{N_B} r_i} f_{R_1} \left( x - \sum_{i=2}^{N_B} r_i \right) \right. \\
 &\quad \times \left. \prod_{i=2}^{N_B} f_{R_i}(r_i) dr_2 \dots dr_{N_B-1} dr_{N_B} \right) \times \left( \int_0^y \int_0^{\sqrt{y^2-r_{N_B, N_I}}} \dots \int_0^{\sqrt{y^2-\sum_{i=3}^{N_B} \sum_{j=3}^{N_I} r_{i,j}}} \right. \\
 &\quad \times \left. f_{R_{1,1}} \left( \sqrt{y^2 - \underbrace{\sum_{i=1}^{N_B} \sum_{j=1}^{N_I} r_{i,j}}_{\text{except } i=j=1}} \right) \prod_{i=1}^{N_B} \prod_{j=1}^{N_I} f_{R_{i,j}}(r_{i,j}) dr_{1,2} \dots dr_{N_B, N_I} \right) dx dy
 \end{aligned} \tag{19}$$

In what follows, we propose an approximate formulation for the OP given in (18). Besides being very simple, it is highly accurate, as shall be seen from the numerical examples given in Sect. 7.

### 5 Outage Probability: Approximate Formulations

As is well-known, the OP of the SIR Z is defined as the probability that Z falls below a given threshold,  $z_{th}$ . Equivalently, it represents the CDF  $F_Z(z)$  evaluated at  $z = z_{th}$ , as given in (18). In the following, accurate approximate expressions for the OP in EGC receivers subject to  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ , and  $\kappa$ - $\mu$  fading will be proposed.

#### 5.1 $\alpha$ - $\mu$ Fading

Assume that both desired and interfering signals are  $\alpha$ - $\mu$  faded. We propose to approximate  $f_Y(y)$  and  $F_X(y\sqrt{z_{th}})$  by the PDF and CDF of a single  $\alpha$ - $\mu$  variate, as shown in (1) and (2), respectively. The motivation for this comes from the fact that the sum of independent  $\alpha$ - $\mu$  variates can be very well approximated by a single  $\alpha$ - $\mu$  variate [20]. The procedure is detailed next.

The CDF  $F_X(y\sqrt{z_{th}})$  is approximated by the CDF of an  $\alpha$ - $\mu$  variate, i.e.,

$$F_X(y\sqrt{z_{th}}) \approx \frac{\Gamma(\mu_S, \mu_S (y\sqrt{z_{th}})^{\alpha_S} / \hat{x}_S^{\alpha_S})}{\Gamma(\mu_S)} \tag{20}$$

In order to render (20) a good approximation, we use moment-based estimators to calculate  $\alpha_S$ ,  $\mu_S$ , and  $\hat{x}_S = \sqrt[\alpha_S]{E(X^{\alpha_S})}$  from the exact moments of X. Assume, for the moment, the knowledge of  $E(X)$ ,  $E(X^2)$ , and  $E(X^4)$ . Then, considering EGC receivers, moment-based estimators for  $\alpha_S$ ,  $\mu_S$ , and  $\hat{x}_S$  can be written as [20, Eqs. 4–6]

$$\frac{\Gamma^2(\mu_S + 1/\alpha_S)}{\Gamma(\mu_S) \Gamma(\mu_S + 2/\alpha_S) - \Gamma^2(\mu_S + 1/\alpha_S)} = \frac{E^2(X)}{E(X^2) - E^2(X)}, \tag{21}$$

$$\frac{\Gamma^2(\mu_S + 2/\alpha_S)}{\Gamma(\mu_S) \Gamma(\mu_S + 4/\alpha_S) - \Gamma^2(\mu_S + 2/\alpha_S)} = \frac{E^2(X^2)}{E(X^4) - E^2(X^2)}, \tag{22}$$

$$\hat{x}_S = \frac{\mu_S^{1/\alpha_S} \Gamma(\mu_S) E(X)}{\Gamma(\mu_S + 1/\alpha_S)}, \tag{23}$$

The systems of transcendental equations (21–22) must be numerically solved for  $\alpha_S$  and  $\mu_S$ . For this, the required function in *MATHEMATICA* is simply FindRoot. Having obtained  $\alpha_S$  and  $\mu_S$ , then  $\hat{x}_S$  is estimated as in (23). The exact moments  $E(X)$ ,  $E(X^2)$ , and  $E(X^4)$  required in (21), (22), and (23) can be written in terms of the individual moments of the  $\alpha$ - $\mu$  summands as [20, Eq. 7]

$$E(X^n) = \sum_{n_1=0}^n \sum_{n_2=0}^{n_1} \dots \sum_{n_{N_B-1}=0}^{n_{N_B-2}} \binom{n}{n_1} \binom{n_1}{n_2} \dots \binom{n_{N_B-2}}{n_{N_B-1}} E(X_1^{n-n_1}) E(X_2^{n_1-n_2}) \dots E(X_{N_B}^{n_{N_B-1}}), \tag{24}$$

where the individual moments above are given in (3) for the respective parameters  $(\alpha_i, \mu_i, \hat{x}_i)$ .

Now, the PDF  $f_Y(y)$  can be approximated by the PDF of an  $\alpha$ - $\mu$  variate, i.e,

$$f_Y(y) \approx \frac{\alpha_I \mu_I^{\mu_I} y^{\alpha_I \mu_I - 1}}{\hat{y}_I^{\alpha_I \mu_I} \Gamma(\mu_I)} \exp\left(-\mu_I \frac{y^{\alpha_I}}{\hat{y}_I^{\alpha_I}}\right), \tag{25}$$

Moment-based estimators for  $\alpha_I, \mu_I$ , and  $\hat{y}_I$  can be written as [20, Eqs. 22–24]

$$\frac{\Gamma^2(\mu_I + 2/\alpha_I)}{\Gamma(\mu_I) \Gamma(\mu_I + 4/\alpha_I) - \Gamma^2(\mu_I + 2/\alpha_I)} = \frac{E^2(Y^2)}{E(Y^4) - E^2(Y^2)}, \tag{26}$$

$$\frac{\Gamma^2(\mu_I + 4/\alpha_I)}{\Gamma(\mu_I) \Gamma(\mu_I + 8/\alpha_I) - \Gamma^2(\mu_I + 4/\alpha_I)} = \frac{E^2(Y^4)}{E(Y^8) - E^2(Y^4)}. \tag{27}$$

$$\hat{y}_I = \left[ \frac{\mu_I^{2/\alpha_I} \Gamma(\mu_I) E(Y^2)}{\Gamma(\mu_I + 2/\alpha_I)} \right]^{\frac{1}{2}} \tag{28}$$

On the other hand, the exact moments  $E(Y^2), E(Y^4)$ , and  $E(Y^8)$  required in (26), (27), and (28) can be obtained from [20, Eq. 25]

$$E(Y^{2n}) = \sum_{n_1=0}^n \sum_{n_2=0}^{n_1} \dots \sum_{n_{N_B-1}=0}^{n_{N_B-2}} \binom{n}{n_1} \binom{n_1}{n_2} \dots \binom{n_{N_B-2}}{n_{N_B-1}} E\left(Y_1^{2(n-n_1)}\right) \times E\left(Y_2^{2(n_1-n_2)}\right) \dots E\left(Y_{N_B}^{2(n_{N_B-1})}\right), \tag{29}$$

in which

$$E(Y_m^{2n}) = \sum_{n_1=0}^n \sum_{n_2=0}^{n_1} \dots \sum_{n_{N_I-1}=0}^{n_{N_I-2}} \binom{n}{n_1} \binom{n_1}{n_2} \dots \binom{n_{N_I-2}}{n_{N_I-1}} E\left(Y_{m,1}^{2(n-n_1)}\right) \times E\left(Y_{m,2}^{2(n_1-n_2)}\right) \dots E\left(Y_{m,N_I}^{2(n_{N_I-1})}\right), \tag{30}$$

where  $m = 1, 2, \dots, N_B$  and the individual moments in (30) are given in (3) for the respective parameters  $(\alpha_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ . Then, by substituting (20) and (25) in (18), with the appropriate substitutions as far as the evaluation of the exact moments are concerned, we arrive at approximate expressions for the OP of EGC receivers with CCI in  $\alpha$ - $\mu$  fading channels.



### 5.2 $\kappa$ - $\mu$ Fading

As before, for the  $\kappa$ - $\mu$  scenario we propose to approximate  $f_Y(y)$  and  $F_X(y\sqrt{z_{th}})$ , required in (18), by the PDF and CDF of a  $\kappa$ - $\mu$  variate, as shown in (5) and (7), respectively. The motivation for this stems from the fact that the sum of independent  $\kappa$ - $\mu$  variates can be very well approximated by a single  $\kappa$ - $\mu$  variate, as shown in [21].

Here, we approximate  $F_X(y\sqrt{z_{th}})$  by the CDF of a  $\kappa$ - $\mu$  variate, i.e.,

$$F_X(y\sqrt{z_{th}}) \approx 1 - Q_{\mu_S} \left( \sqrt{2\kappa_S\mu_S}, \sqrt{2\mu_S(1+\kappa_S)} \frac{y\sqrt{z_{th}}}{\hat{x}_S} \right). \tag{31}$$

Moment-based estimators for  $\kappa_S$ ,  $\mu_S$ , and  $\hat{x}_S$  can be expressed from [21, Eqs. 8–10] as

$$\kappa_S^{-1} = \frac{\sqrt{2} (E(X^4) - E(X^2)^2)}{\sqrt{2E^2(X^4) - E(X^2)^2E(X^4) - E(X^2)E(X^6)}} - 2, \tag{32}$$

$$\mu_S = \frac{E(X^2)^2}{E(X^4) - E(X^2)^2} \frac{1 + 2\kappa_S}{(1 + \kappa_S)^2}, \tag{33}$$

$$\hat{x}_S = \sqrt{E(X^2)}. \tag{34}$$

The exact moments  $E(X^2)$ ,  $E(X^4)$ , and  $E(X^6)$  required in (32), (33), and (34) can be calculated as (24), with the individual moments of the  $\kappa$ - $\mu$  obtained from (6) for the respective parameters  $(\kappa_i, \mu_i, \hat{x}_i)$ .

In the same way,  $f_Y(y)$  is approximated by the PDF of a  $\kappa$ - $\mu$  variate, i.e.,

$$f_Y(y) \approx \frac{2\mu_I(1+\kappa_I)^{\frac{\mu_I+1}{2}}}{\kappa_I^{\frac{\mu_I-1}{2}} \exp(\mu_I\kappa_I)} \frac{y^{\mu_I}}{\hat{y}_I^{\mu_I+1}} \exp\left(-\mu_I(1+\kappa_I) \left(\frac{y}{\hat{y}_I}\right)^2\right) \times I_{\mu_I-1} \left( 2\mu_I\sqrt{\kappa_I(1+\kappa_I)} \frac{y}{\hat{y}_I} \right), \tag{35}$$

and the procedure here follows the same steps as described previously, in which the parameters  $\kappa_I$ ,  $\mu_I$ , and  $\hat{y}_I$  are attained from (32) to (34) using  $Y$  instead of  $X$ . In this case, the individual moments needed in (30) are obtained from (6) for the respective parameters  $(\kappa_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ . Then, by substituting (31) and (35) in (18), with the appropriate substitutions as far as the evaluation of the exact moments are concerned, we arrive at approximate expressions for the OP of EGC receivers with CCI in  $\kappa$ - $\mu$  fading channels.

### 5.3 $\eta$ - $\mu$ Fading

Using a procedure similar to the one used before, an accurate approximate expression for the OP of multibranch EGC systems subject to  $\eta$ - $\mu$  fading is now presented. In particular, our formulations consider Format 1 of the  $\eta$ - $\mu$  distribution. Therefore,  $F_X(y\sqrt{z_{th}})$  and  $f_Y(y)$ , as required in (18), will be well approximated by (12) and (10), respectively, in which a good approximation is achieved by the suitable choice of the parameters of the CDF and PDF of an  $\eta$ - $\mu$  variate, as performed in the previous subsections. This rationale was also applied in [21] for approximating the sums of independent  $\eta$ - $\mu$  variates. Thus, it follows that

$$F_X(y\sqrt{z_{th}}) \approx 1 - Y_{\mu_S} \left( \frac{H_S}{h_S}, \sqrt{2h_S\mu_S} \frac{y\sqrt{z_{th}}}{\hat{x}_S} \right), \tag{36}$$

$$f_Y(y) \approx \frac{4\sqrt{\pi} \mu_I^{\mu_I + \frac{1}{2}} h_I^{\mu_I} y^{2\mu_I}}{\Gamma(\mu_I) H_I^{\mu_I - \frac{1}{2}} \hat{y}_I^{2\mu_I + 1}} \exp\left(-2\mu_I h_I \left(\frac{y}{\hat{y}_I}\right)^2\right) I_{\mu_I - \frac{1}{2}} \left[2\mu_I H_I \left(\frac{y}{\hat{y}_I}\right)^2\right], \quad (37)$$

Now, turning our attention for the calculation of the required parameters in the equations above, we have that moment-based estimators for  $\eta_S$ ,  $\mu_S$ , and  $\hat{x}_S$  can be written as [23]

$$\eta_S = \frac{\sqrt{2c_S} - \sqrt{3 - 2c_S \pm \sqrt{9 - 8c_S}}}{\sqrt{2c_S} + \sqrt{3 - 2c_S \pm \sqrt{9 - 8c_S}}}, \quad (38)$$

$$\mu_S = \frac{E(X^2)^2}{E(X^4) - E(X^2)^2} \frac{(1 + \eta_S^2)}{(1 + \eta_S)^2}, \quad (39)$$

$$\hat{x}_S = \sqrt{E(X^2)}, \quad (40)$$

where  $c_S$  in (38) is defined as

$$c_S \triangleq \frac{\frac{E(X^6)}{E(X^2)^3} - \frac{3E(X^4)}{E(X^2)^2} + 2}{2 \left( \frac{E(X^4)}{E(X^2)^2} - 1 \right)}. \quad (41)$$

Regarding the interfering signals, i.e., those associated to the PDF of  $Y$ , their respective parameters  $\eta_I$ ,  $\mu_I$ , and  $\hat{y}_I$  are obtained from (38) to (40) using  $Y$  in the place of  $X$ .

As in the two others fading scenarios, the exact moments of  $X$  and  $Y$  (orders 2, 4, and 6), necessary for the calculation of the parameters of (36) and (37), can be obtained from (24) and (29). As before, the individual moments inherent to these expressions can be found from (11) for the respective parameters  $(\eta_i, \mu_i, \hat{x}_i)$  and  $(\eta_{i,j}, \mu_{i,j}, \hat{y}_{i,j})$ . Then, by substituting (36) and (37) in (18), we arrive at an approximate expression for the OP of EGC systems subject to CCI in  $\eta$ - $\mu$  fading channels. Note that two pairs of estimators for  $\eta_S$  and  $\mu_S$  are found. To decide which one is more suitable, we select those that lead to the smallest  $|E(X) - E(R)|$ , by considering  $\eta_S$  and  $\mu_S$ , in which case  $E(R)$  is attained from [23, Eq. 21]. Concerning  $\eta_I$  and  $\mu_I$ , Eqs. (38–40) have led to the same values in all cases.

## 5.4 Mixed Fadings

The approach used here is indeed flexible and can be easily applied to cases in which desired and interfering signals undergo different fading conditions. For instance, desired signals may experience  $\alpha$ - $\mu$  or  $\eta$ - $\mu$  or  $\kappa$ - $\mu$  fading and interfering signals may be  $\alpha$ - $\mu$  or  $\eta$ - $\mu$  or  $\kappa$ - $\mu$  distributed. The procedure simply involves calculating  $F_X(y\sqrt{z_{th}})$  in one condition and  $f_Y(y)$  in the other, following the steps given above. Interestingly, because these distributions comprise many other fading conditions, several combinations of fading scenarios can be exercised. For instance, if desired and interfering signals are, respectively,  $\alpha$ - $\mu$  and  $\kappa$ - $\mu$  distributed, then desired signals may comprise different numbers of Rayleigh, Weibull, Nakagami- $m$ , and  $\alpha$ - $\mu$  faded signals, whereas interfering signals may comprise different numbers of Rayleigh, Nakagami- $m$ , Rice, and  $\kappa$ - $\mu$  faded signals. The same can be said with respect to  $\eta$ - $\mu$ , for which Hoyt, Rayleigh, Nakagami- $m$ , and  $\eta$ - $\mu$  itself are contemplated.

## 6 Special Cases of Our Formulations

As discussed in the previous sections, the fading scenarios investigated in this paper are indeed general and include as special cases important other fading models. The Hoyt (Nakagami- $q$ )

fading model can be obtained from the  $\eta$ - $\mu$  one in an exact manner by setting  $\mu = 0.5$ . In this case, the Hoyt parameter is given by  $b = -(1 - \eta)/(1 + \eta)$  in Format 1 or  $b = -\eta$  in Format 2. The Rice fading model can be attained from the  $\kappa$ - $\mu$  one by setting  $\mu = 1$  and  $\kappa = k$ , with  $k$  denoting the Rice factor. The Weibull fading model can be obtained from the  $\alpha$ - $\mu$  one for  $\mu = 1$ , where in this case  $\alpha$  will represent the non-linearity Weibull parameter. The Nakagami- $m$  fading model can be obtained from anyone of the three fading models analyzed here. From the  $\eta$ - $\mu$  one, it is attained by setting  $\mu = m/2$  and  $\eta \rightarrow 1$  in Format 1 or  $\eta \rightarrow 0$  in Format 2. In the same way, it can be attained by setting  $\mu = m$  and  $\eta \rightarrow 0$  or  $\eta \rightarrow \infty$  in Format 1 or  $\eta \rightarrow \pm 1$  in Format 2. In addition, the Nakagami- $m$  fading model can be achieved from the  $\kappa$ - $\mu$  one using  $\kappa = 0$  and  $\mu = m$ . Finally, the Nakagami- $m$  fading model can also be obtained from the  $\alpha$ - $\mu$  one when  $\alpha = 2$  and  $\mu = m$ . Therefore, in what follows, after performing the appropriate substitutions, each one of the fading scenarios cited above can be easily investigated.

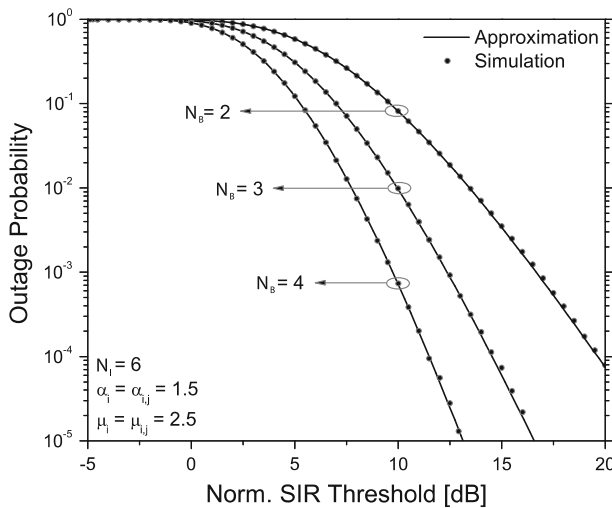
### 7 Numerical Results and Discussions

In this Section, we check our approximations against Monte Carlo simulation. As shall be seen, in all the cases, an excellent match between the respective results is found. A myriad of other comparisons have been checked by the authors and, in all of them, an excellent concordance was attested. For the plots, we define a normalized SIR threshold [16]

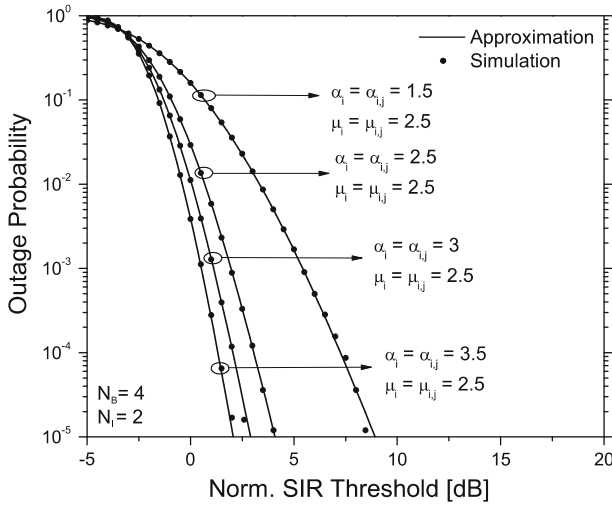
$$\text{Norm.SIRth} = \frac{\Omega_s}{\Omega_I z_{th}}, \tag{42}$$

where  $\Omega_s$  is the total average desired signal power and  $\Omega_I$  is the total average single interferer power.

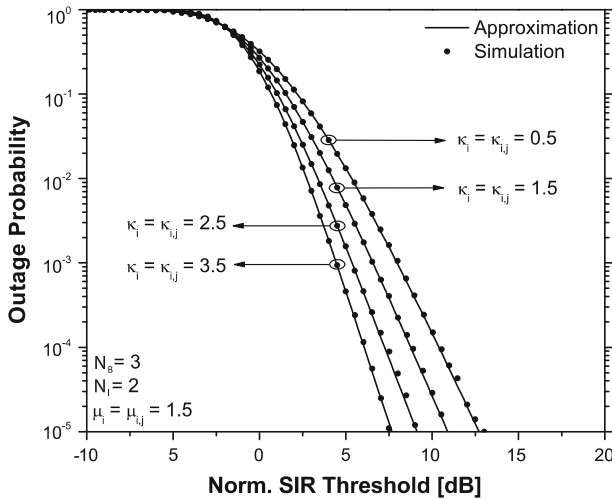
By considering the  $\alpha$ - $\mu$  fading channels, Fig. 1 illustrates the OP versus normalized SIR threshold for several values of  $N_B$  using  $\mu_i = \mu_{i,j} = 2.5, \alpha_i = \alpha_{i,j} = 1.5$  and



**Fig. 1** OP versus normalized SIR threshold in EGC receivers for  $\mu_i = \mu_{i,j} = 2.5, \alpha_i = \alpha_{i,j} = 1.5, N_I = 6$ , and varying  $N_B$



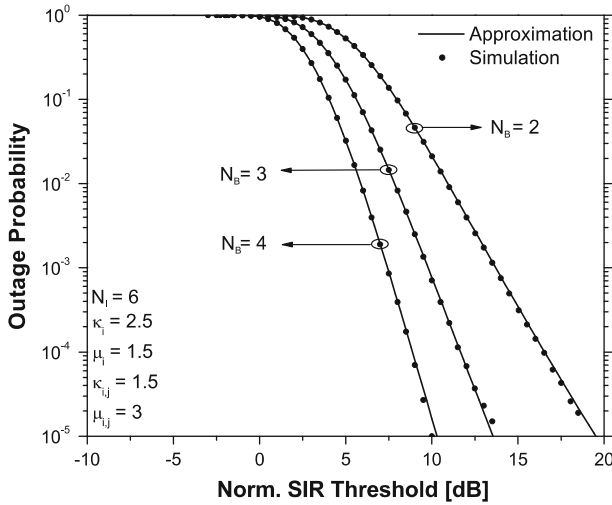
**Fig. 2** OP versus normalized SIR threshold in EGC receivers for  $\mu_i = \mu_{i,j} = 2.5$ ,  $N_B = 4$ ,  $N_I = 2$ , and varying  $\alpha_i = \alpha_{i,j}$



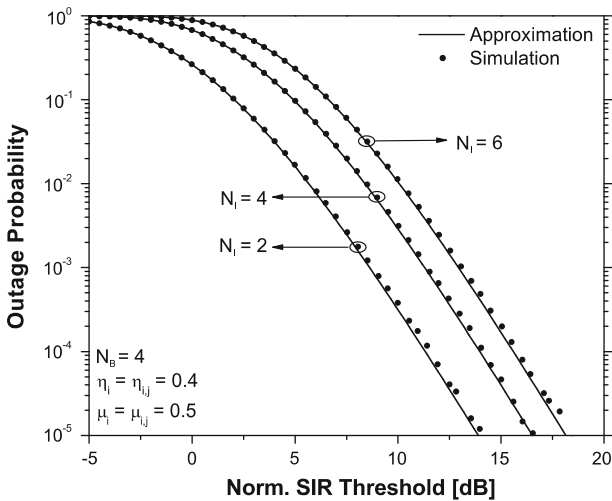
**Fig. 3** OP versus normalized SIR threshold in EGC receivers for  $\mu_i = \mu_{i,j} = 1.5$ ,  $N_I = 2$ ,  $N_B = 3$ , and varying  $\kappa_i = \kappa_{i,j}$

$N_I = 6$ . More specifically, this figure analyzes the effect of interference in a non-homogeneous environment with a non-linearity parameter (described by the parameter  $\alpha$ ) in which the signal is composed by clusters of multipath waves (expressed by the parameter  $\mu$ ). As expected, an improvement of the performance is noticed as  $N_B$  increases. Still regarding the  $\alpha$ - $\mu$  scenario, Fig. 2 outlines, for different values of  $\alpha_i = \alpha_{i,j}$ , the case in which there are  $N_B = 4$  diversity branches,  $N_I = 2$  cochannel interferers and  $\mu_i = \mu_{i,j} = 2.5$ .

Figures 3 and 4 depict the OP in  $\kappa$ - $\mu$  fading channels. The former employs  $N_B = 3$  diversity branches,  $N_I = 2$  interfering carriers, and varies  $\kappa_i = \kappa_{i,j}$  by keeping constant



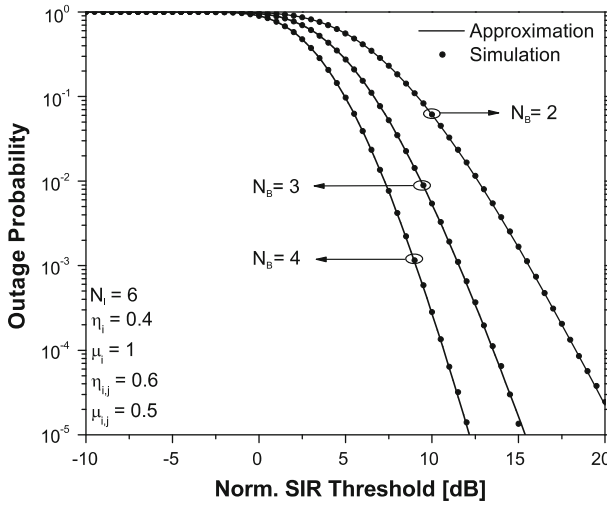
**Fig. 4** OP versus normalized SIR threshold in EGC receivers for  $\mu_i = 1.5, \mu_{i,j} = 3, \kappa_i = 2.5, \kappa_{i,j} = 1.5, N_I = 6$ , and varying  $N_B$



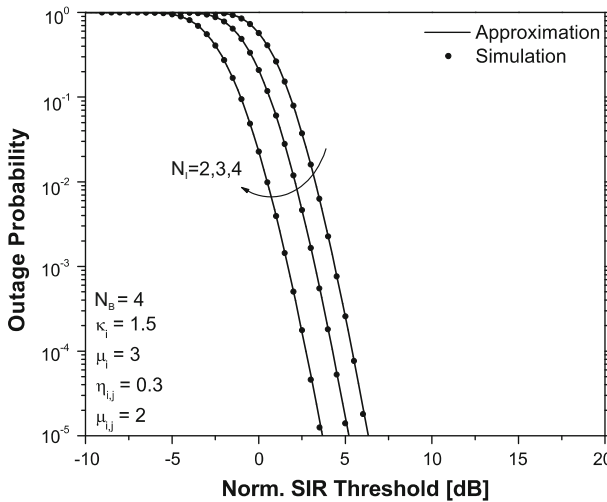
**Fig. 5** OP versus normalized SIR threshold in EGC receivers for  $N_B = 4, \eta_i = \eta_{i,j} = 0.4, \mu_i = \mu_{i,j} = 0.5$ , and varying  $N_I$

$\mu_i = \mu_{i,j} = 1.5$ , whereas the latter explores a scenario with  $N_I = 6$  cochannel interferers. Note that here we investigate the OP in an interference scenario where the signal is composed by multipath clusters and, within each cluster, a dominant component (line-of-sight condition) is found. The performance is shown to deteriorate as the value of  $\kappa_i = \kappa_{i,j}$  decreases.

Assuming an interference scenario where the inphase and quadrature components within each multipath cluster are independent from each other and have different powers, Fig. 5 portrays the OP of  $\eta$ - $\mu$  channels for different values of  $N_I$  and using  $\eta_i = \eta_{i,j} = 0.4, \mu_i = \mu_{i,j} = 0.5$ , while Fig. 6 illustrates the case with  $N_I = 6$  cochannel interferers for different



**Fig. 6** OP versus normalized SIR threshold in EGC receivers for  $N_I = 6$ ,  $\eta_i = 0.4$ ,  $\eta_{i,j} = 0.6$ ,  $\mu_i = 1$ ,  $\mu_{i,j} = 0.5$ , and varying  $N_B$



**Fig. 7** OP versus normalized SIR threshold in EGC receivers for  $N_B = 4$ ,  $\kappa_i = 1.5$ ,  $\eta_{i,j} = 0.6$ ,  $\mu_i = 3$ ,  $\mu_{i,j} = 2$ , and varying  $N_I$

numbers of diversity branches. Noticeably, the system becomes less reliable as we increase the number of interfering carriers.

Finally, we consider in Fig. 7 a mixed-fading scenario in which the desired signals undergo  $\kappa$ - $\mu$  fading, whereas the interfering signals are considered to be  $\eta$ - $\mu$  faded. Such an environment is an example for which the desired signals are subject to a *line-of-sight* fading, whereas the interfering signals are experimenting a *non-line-of-sight* fading. As anticipated, the performance deteriorates as  $N_I$  increases.

Although the numerical values will certainly vary from one case to the other, the overall outage performance basically depends on the fading conditions of the channel, as expected

and as illustrated in the plots given here. For the  $\alpha$ - $\mu$  case, this happens with the increase of  $\alpha$  and/or  $\mu$ . For the  $\kappa$ - $\mu$  scenario, this occurs with the increase of  $\kappa$  and/or  $\mu$ . And for the  $\eta$ - $\mu$  environment, this is characterized by having  $\eta$  moving towards 1 and/or  $\mu$  increase. A performance degradation occurs otherwise.

Note in all the cases how exact (simulated) and approximate curves are, in practice, indistinguishable from each other. Note further that the use of these generalized distributions in conjunction with the proposed approximations provide an important and efficient tool, useful in the analysis of a multi-branch multi-interferer wireless systems.

## 8 Conclusions

In this paper, accurate approximate expressions for the OP of multibranch EGC receivers subject to multiple cochannel interferers were provided. Three different generalized fading scenarios were analyzed, namely,  $\alpha$ - $\mu$ ,  $\kappa$ - $\mu$ , and  $\eta$ - $\mu$ . The proposed approximations were obtained in terms of a singlefold integral, which replaces the multifold nested integrals required into the exact solution. In all the comparisons, an excellent match between the approximate and simulation results were observed. The approach shown here is simpler than the available alternatives and also flexible, in the sense that it admits a wide range of fading environments. Moreover, it allows for mixed-fading scenarios in which many different combinations of fading conditions of interest may be exercised.

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