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Maximal Ratio Combining (MRC) in Shadowed Fading Channels in Presence of Shadowed Fading Cochannel Interference (CCI)

P. M. Shankar

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Abstract Simultaneous existence of multipath fading and shadowing leads to worsening conditions in wireless channels. This is further compounded by the interference from other base stations operating at the same frequency. The effect of this cochannel interference (CCI) and shadowed fading in error rates is studied when maximal ratio combining is used to mitigate short term fading. The CCI channels were also treated as undergoing shadowed fading. The generalized K distribution was used to model the signal-to-noise ratio of composite shadowed fading channel. The probability density functions of the signal-to-noise ratio taking into account the presence of multipath fading, shadowing and CCI were derived and used for the estimation of error rates. Results demonstrated the existence of degradation in the channel manifested as increased error rates and higher error floors. The improvements in the channel obtained through diversity were also demonstrated. The approach presented here can be easily adapted to the analysis of other diversity schemes in shadowed fading channels.

Keywords Shadowed fading · GK distribution · Cochannel interference · Error rates · Maximal ratio combining

1 Introduction

Multipath fading and shadowing pose serious problems in wireless communications [1]. They lead to channel degradations resulting in increased outages and higher error rates. The channel conditions are further adversely impacted by the existence of cochannel interference (CCI) caused by channels operating from different locations received along with the signal of interest. The cochannels also undergo fading and shadowing [1–4]. The presence of CCI can degrade the channel further and lead to error floors making data transmission less reliable [3–6]. Researchers have examined the effects of multipath fading and CCI on the error rates [2,3,7]. Reports are also available on error rates when diversity is implemented to mitigate

P. M. Shankar (✉)
Department of Electrical and Computer Engineering, Drexel University, Philadelphia, PA 19104, USA
e-mail: pshankar@coe.drexel.edu

the effects of multipath fading [5,7]. Because of the complexity introduced by the existence of shadowing, results on error rates in shadowed fading channels in presence of CCI are not readily available when diversity is implemented. This is primarily due to the fact that cochannels also undergo shadowed fading making the analysis cumbersome [6].

In this work, the error rates are estimated in presence of CCI in shadowed fading channels when the interfering channels also undergo fading and shadowing. The maximal ratio combining (MRC) algorithm to mitigate fading is also taken into consideration.

Following the introduction, the background on multipath fading, shadowing and CCI is presented in terms of the models for the shadowed fading channels and the density functions of the signal-to-noise ratio incorporating the cochannel interference. Using the generalized K (GK) model for shadowed fading in place of the Nakagami-lognormal model, the density functions of the SNR are derived [6,8]. This is followed by the inclusion of the MRC algorithm fading mitigation. Using the recently published results on the densities of the sum of GK variables, the pdf of the overall SNR is derived [9,10]. The error rates are then estimated and results are discussed. The potential applications of the results and the approach for the analysis of more complicated wireless channel conditions are also provided.

2 Theoretical Background

The phenomenon of multipath fading has been studied and analyzed using several models [1]. The most commonly used model is based on the Nakagami-m density function. When shadowing is present along with short term fading, probability density function (pdf) of the signal-to-noise ratio (SNR) X in a shadowed Nakagami faded channel can be expressed as [1,6,8]

$$f_X\left(x|\,w\right) = \left(\frac{m}{w}\right)^m \frac{x^{m-1}\exp\left(-\frac{m}{w}x\right)}{\Gamma\left(m\right)}, \quad m \geq \frac{1}{2},\ x > 0. \tag{1}$$

In Eq. (1), m is the Nakagami parameter and $\Gamma(.)$ is the gamma function. The conditioning in Eq. (1) reflects the existence of shadowing represented by w. In the absence of any shadowing, w would the average SNR in a Nakagami channel. Generally, a lognormal model is used for shadowing which results in an integral form of the density function for the SNR in shadowed fading channels. It was shown that a gamma pdf is a sufficient replacement for the lognormal pdf for shadowing and such an approach leads to an analytical form of the density function of the SNR [6,8,11]. Using a gamma pdf of order ν for shadowing in place of the traditional lognormal one, the density function of the SNR in a shadowed fading channel becomes [6,8]

$$f\left(x\right) = \frac{2}{\Gamma\left(m\right)\Gamma\left(\nu\right)}\left(\sqrt{\frac{m\nu}{\Omega_0}}\right)^{\nu+m} x^{\left(\frac{\nu+m}{2}\right)-1} K_{m-\nu}\left(2\sqrt{\frac{m\nu}{\Omega_0}x}\right). \tag{2}$$

In Eq. (2), Ω_0 is the average SNR in a shadowed fading channel and $K(.)$ is the modified Bessel function [12]. The pdf in Eq. (2) is also known as the generalized K (GK) pdf or gamma–gamma pdf for the shadowed fading channels [10,11]. When the order υ representing the shadowing levels becomes infinite, shadowing no longer exists and Eq. (2) becomes a simple gamma density function associated with a Nakagami-m density for the envelope [6,13].

While the received signal of interest undergoes fading and shadowing, the wireless environment is further affected by the existence of interfering signals. The signal-to-noise ratio Z in presence of interference can be expressed in simple form as [1,2]

$$Z = \frac{S}{N_0 + \sum_{k=1}^{N} P_k}. \tag{3}$$

In Eq. (3), S is the desired signal (signal of interest) power. The two terms in the denominator are the thermal noise power (N_0) and total power of the N interferers, each with a power of P_k. Equation (3) can be written by dividing all terms on the right hand side by N_0 resulting in

$$Z = \frac{X}{1 + \sum_{k=1}^{N} Y_k} = \frac{X}{1+U}. \tag{4}$$

In Eq. (4), X is the SNR of the desired signal and Y's are the SNRs of each of the interferers. The total SNR of the interfering components is represented by U. Equation (4) takes cognizance of the fact that the noise is not negligible as it was assumed in some cases [11]. To estimate the bit error rate, we need to derive the density function of Z which can be expressed in terms of $f_X(x)$ and $f_U(u)$, the densities of X and U respectively. Using the transformation of variables, the density function of Z becomes [14]

$$f_Z(z) = \int_0^\infty (1+u) f_X\left[(1+u)z\right] f_U(u) du. \tag{5}$$

In Eq. (5), we have assumed that the desired signal and the interferers are independent. The average error rate in shadowed fading channels taking into account the existence of CCI can now be expressed as

$$p_e(\Omega_0) = \int_0^\infty \frac{1}{2}\text{erfc}\left(\sqrt{z}\right) f(z)\, dz. \tag{6}$$

In Eq. (6), erfc(.) is the complementary error function and we have assumed that we have a coherent BPSK modem [1].

The error rates in Nakagami channels in presence of Nakagami distributed interferers have been studied when maximal ratio combining (MRC) is used to mitigate short term fading [1,13]. To the best understanding of the author, currently no results are available when both the desired signal and the cochannels undergo shadowed fading taking the existence of noise also into account as in Eq. (4).

Let us look at the case of the density function of the total SNR of the cochannels first. We assume that interferers undergo short term fading (Nakagami-m) and shadowing (gamma in place of lognormal) and that they are independent and identically distributed each with a pdf [6,8]

$$f(y_k) = \frac{2}{\Gamma(m_s)\Gamma(\nu_s)} \left(\sqrt{\frac{m_s \nu_s}{\Omega_s}}\right)^{\nu_s + m_s} x^{\left(\frac{\nu_s + m_s}{2}\right) - 1} K_{m_s - \nu_s}\left(2\sqrt{\frac{m_s \nu_s}{\Omega_s} y_k}\right). \tag{7}$$

The fading factor is m_s and the shadowing level for the interferers is ν_s. The average SNR in each interfering channel is Ω_s. The density function of U, the sum of SNRs of the interferers requires the N-fold convolution of the densities of the type in Eq. (7) and no simple analytical expression exists for the pdf of U. The pdf of U can be obtained by assuming that shadowing is the same in all the interfering channels as it was done earlier [11,13]. However, in practice, each interfering channels needs to be treated separately with distinct fading and shadowing requiring an N-fold convolution as mentioned above.

Recently two groups independently reported that the density function of the sum of GK random variables could be approximated to another GK distribution [9,10]. Their results vary only slightly and we follow the approach where the parameters of the GK pdf have been fitted using the method of least squares [10]. The density function of U will also be a GK pdf and it can be written as

$$f_U(u) = \frac{2}{\Gamma(m_n)\Gamma(\nu_n)}\left(\sqrt{\frac{m_n\nu_n}{N\Omega_s}}\right)^{\nu_n+m_n} u^{\left(\frac{\nu_n+m_n}{2}\right)-1} K_{m_n-\nu_n}\left(2\sqrt{\frac{m_n\nu_n}{N\Omega_s}u}\right). \tag{8}$$

The parameters of the GK pdf in Eq. (8) are given by [10]

$$m_n = Nm_s + (N-1)\left[\frac{-0.127 - 0.95m_s - 0.0058\nu_s}{1 + 0.00124m_s + 0.98\nu_s}\right], \tag{9}$$

and

$$\nu_n = N\nu_s. \tag{10}$$

Next we examine the fading mitigation through maximal ratio combining (MRC). We assume that we have an M-branch diversity and the branches are independent and identically distributed. The SNR output of the MRC combiner will be

$$R = \sum_{j=1}^{M} X_j, \tag{11}$$

and the overall signal-to-noise ratio Z now becomes

$$Z = \frac{R}{1+U}. \tag{12}$$

In Eq. (11), X's are independent identically distributed GK variables each having a pdf of the form in Eq. (2). We can obtain the pdf of the output of the MRC combiner R assuming that shadowing is the same over all the branches as it was reported earlier [11,13]. But, a more general approach is to look at the case where multipath fading and shadowing from branch to branch are distinct just as it was the case with the interferers. Proceeding analogously, the density function of the SNR at the output of the MRC combiner is

$$f_R(r) = \frac{2}{\Gamma(m_m)\Gamma(\nu_m)}\left(\sqrt{\frac{m_m\nu_m}{M\Omega_0}}\right)^{\nu_m+m_m} r^{\left(\frac{\nu_m+m_m}{2}\right)-1} K_{m_m-\nu_m}\left(2\sqrt{\frac{m_m\nu_m}{M\Omega_0}r}\right). \tag{13}$$

The parameters of the GK pdf in Eq. (13) will be [10]

$$m_m = Mm + (M-1)\left[\frac{-0.127 - 0.95m - 0.0058\nu}{1 + 0.00124m + 0.98\nu}\right], \tag{14}$$

and

$$\nu_m = M\nu. \tag{15}$$

The density function of the SNR of the MRC combiner incorporating the existence of cochannel interference now becomes

$$f_Z(z) = \int_0^\infty (1+u) f_R[(1+u)z] f_U(u) du. \tag{16}$$

Note that $f_X(x)$ in Eq. (5) has been replaced by $f_R(r)$ in Eq. (16). The average probability of error $p_e(\Omega_0)$ for a BPSK modem in a shadowed fading channel following MRC diversity and in presence of interferers which also undergo fading and shadowing can written as

$$p_e(\Omega_0) = \int_0^\infty \frac{1}{2} erfc\sqrt{z} \int_0^\infty (1+u) \frac{2[(1+u)z]^{\left(\frac{\nu_m+m_m}{2}\right)-1}}{\Gamma(m_m)\Gamma(\nu_m)} \left(\sqrt{\frac{m_m \nu_m}{M\Omega_0}}\right)^{\nu_m+m_m} K_{m_m-\nu_m}\left(2\sqrt{\frac{m_m\nu_m}{M\Omega_0}(1+u)z}\right)$$
$$\times \frac{2}{\Gamma(m_n)\Gamma(\nu_n)} \left(\sqrt{\frac{m_n\nu_n}{N\Omega_s}}\right)^{\nu_n+m_n} u^{\left(\frac{\nu_n+m_n}{2}\right)-1} K_{m_n-\nu_n}\left(2\sqrt{\frac{m_n\nu_n}{N\Omega_s}u}\right) du dz \quad (17)$$

Equation (17) can be easily reduced to a simpler case when the cochannels undergo only short term fading. This corresponds to the case where ν_s goes to infinity in Eq. (8) and it reduces to the case of the pdf of the sum of N gamma random variables, each independent and identically distributed with an order m_s and average SNR Ω_s. The double integral in Eq. (17) can be converted to single integrals using [12,15] and Maple (Version 14, Maplesoft, Waterloo, ON, Canada) as

$$I_1 = \int_0^\infty \frac{1}{2}\mathrm{erfc}\left(\sqrt{z}\right) \frac{2(1+u)}{\Gamma(m_m)\Gamma(\nu_m)} \left(\sqrt{\frac{m_m\nu_m}{M\Omega_0}}\right)^{\nu_m+m_m} [z(1+u)]^{\left(\frac{\nu_m+m_m}{2}\right)-1} K_{m-\nu}$$
$$\times \left(2\sqrt{\frac{m_m\nu_m}{M\Omega_0}z(1+u)}\right) dz. \quad (18)$$

We have

$$I_1 = \frac{1}{2} - \frac{1}{2}\left\{\pi^2 \frac{\csc(\pi m_m)\csc(\pi\nu_m)}{\Gamma(1-m_m)\Gamma(1-\nu_m)\Gamma(m_m)\Gamma(\nu_m)} - V\right\}, \quad (19)$$

$$V = \left[\frac{m_m\nu_m}{M\Omega_0}(1+u)\right]^{m_m} \frac{{}_2F_2\left(\left[m_m, m_m+\frac{1}{2}\right], [1+m_m, 1+m_m-\nu_m], \frac{m_m\nu_m}{M\Omega_0}(1+u)\right)}{\sqrt{\pi}\Gamma(m_m+1)\Gamma(\nu_m)}$$
$$\times\Gamma\left(m_m+\frac{1}{2}\right)\Gamma(-m_m+\nu_m)$$
$$+\left[\frac{m_m\nu_m}{M\Omega_0}(1+u)\right]^{\nu_m} \frac{{}_2F_2\left(\left[\nu_m, \nu_m+\frac{1}{2}\right], [1+\nu_m, 1+\nu_m-m_m], \frac{m_m\nu_m}{M\Omega_0}(1+u)\right)}{\sqrt{\pi}\Gamma(\nu_m+1)\Gamma(m_m)}$$
$$\times\Gamma\left(\nu_m+\frac{1}{2}\right)\Gamma(-\nu_m+m_m) \quad (20)$$

In Eq. (19), csc(.) is the cosecant(.) function and $F(.)$ is the confluent hypergeometric function [12]. The error rate in Eq. (17) now becomes

$$p_{sh}(\Omega_0) = \int_0^\infty I_1 \frac{2}{\Gamma(m_n)\Gamma(\nu_n)} \left(\sqrt{\frac{m_n\nu_n}{N\Omega_s}}\right)^{\nu_n+m_n} u^{\left(\frac{\nu_n+m_n}{2}\right)-1} K_{m_n-\nu_n}\left(2\sqrt{\frac{m_n\nu_n}{N\Omega_s}u}\right) du. \quad (21)$$

We will also look at the error rate when cochannel interference is absent. The average error rate in the absence of any CCI can be obtained easily from Eqs. (18) or (19) by choosing $u = 0$. Furthermore, when $M = 1$, we obtain the error rates in shadowed fading channels in the absence of any CCI when diversity is not implemented.

3 Results

The performance of wireless channels in shadowed fading in presence of cochannels which also undergo shadowed fading has been evaluated in terms of the error rates. The density functions and error rates were studied in terms of their dependence on the number of CCI channels, number of diversity branches and signal-to-interference ratio (SIR) per interfering channel defined as

$$\mathrm{SIR} = \frac{\Omega_0}{\Omega_s}. \tag{22}$$

Note that all the computations in this work were undertaken using Matlab (Version R2009 b, Mathworks Inc., Natick, MA, USA). The first step in the study of the effects of CCI is the examination of the probability density functions of SNR. The density function of SNR in presence of CCI given in Eq. (5) is shown in Fig. 1 in the absence of any diversity for the case of a shadowed fading channel ($m = 1.4$, $\nu = 2.3$, $m_s = 2.3$, $\nu_s = 3.5$, SIR $= 20$ dB, $\Omega_0 = 10$ dB). As the number of CCI channels goes up, the peak of the density function moves towards lower values of the SNR suggesting the existence of higher outage probabilities and higher error rates. Figure 2 shows the plots of the density functions in Eq. (16) when MRC is implemented ($m = 1.4$, $\nu = 2.3$, $m_s = 2.3$, $\nu_s = 3.5$, $N = 4$, SIR $= 10$ dB, $\Omega_0 = 10$ dB). As the number of diversity branches M goes up, the peak of the densities move to higher SNR values point to the likely enhancement in channels conditions. Figure 3 shows the densities for a few values of the SIR ($m = 1.4$, $\nu = 2.3$, $m_s = 2.3$, $\nu_s = 3.5$, $N = 4$, $M = 3$, $\Omega_0 = 10$ dB). As the value of SIR decreases, the peak of the density functions moves to lower SNR values indicating potential decline in performance with declining values of SIR. The deleterious effect of CCI in shadowed fading channels is seen in the plots of the density functions in terms of the shift of the peaks to lower values of SNR with increasing values of N and decreasing value of the SIR. The MRC algorithm shows improvement as seen by the shift of the peaks to increasing values of the SNR with M.

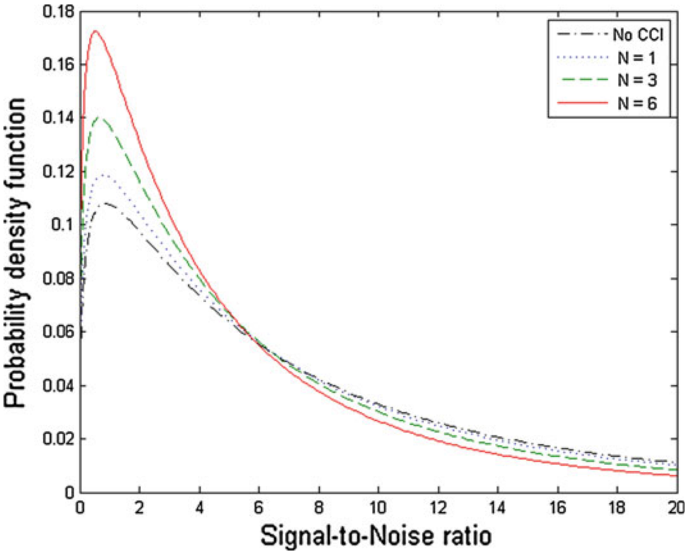

Fig. 1 Probability density functions of the SNR in presence of CCI (SIR = 20 dB) in shadowed fading channels with no diversity

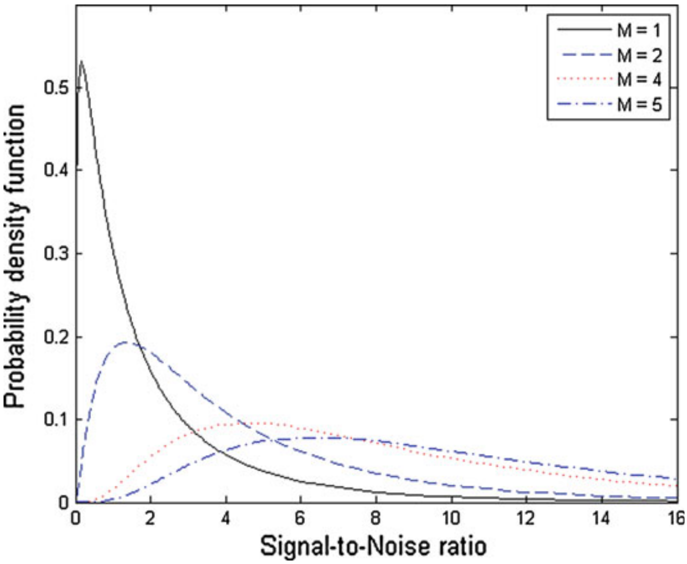

Fig. 2 Probability density functions of the SNR in presence of CCI ($N = 4$, SIR $= 10$ dB) in shadowed fading channels when MRC is implemented

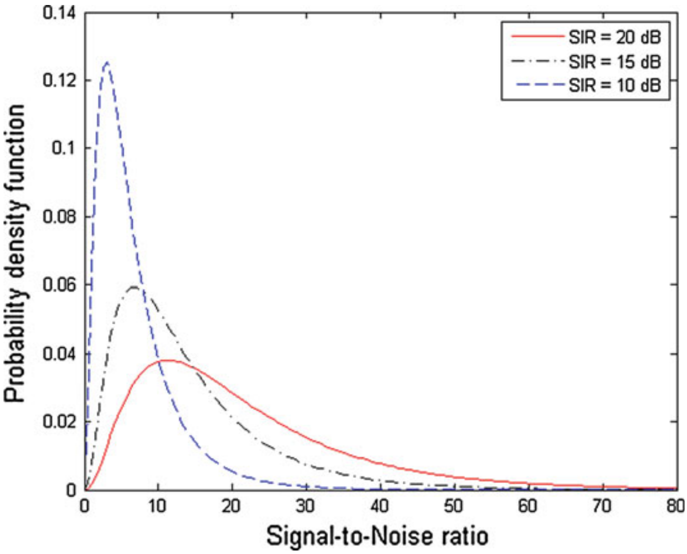

Fig. 3 Probability density functions of the SNR in shadowed fading channels when MRC is implemented ($M = 3$) for three levels of SIR ($N = 4$)

The average error rates were estimated next. Numerical integration was used to calculate the error rates in Eq. (21) for the set of values, $m = 1.5, \nu = 1.1, m_s = 1.2, \nu_s = 3.5$. The error rates in dual diversity ($M = 2$) are plotted in Fig. 4 as a function of the number of CCI channels N (including the case of $N = 0$ indicating the absence of any interference, SIR $= \infty$). The presence of error floors is evident when one compares the cases of the

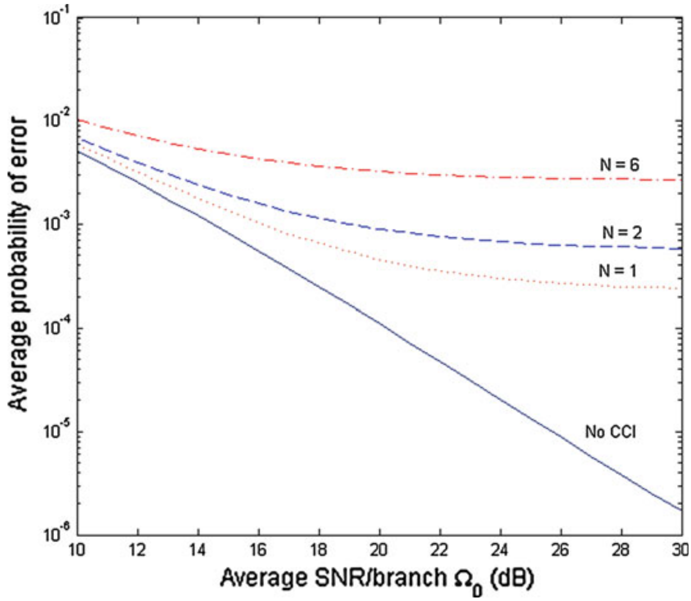

Fig. 4 Average error rates in shadowed fading channels for $M = 2$ and SIR $= 20$ dB

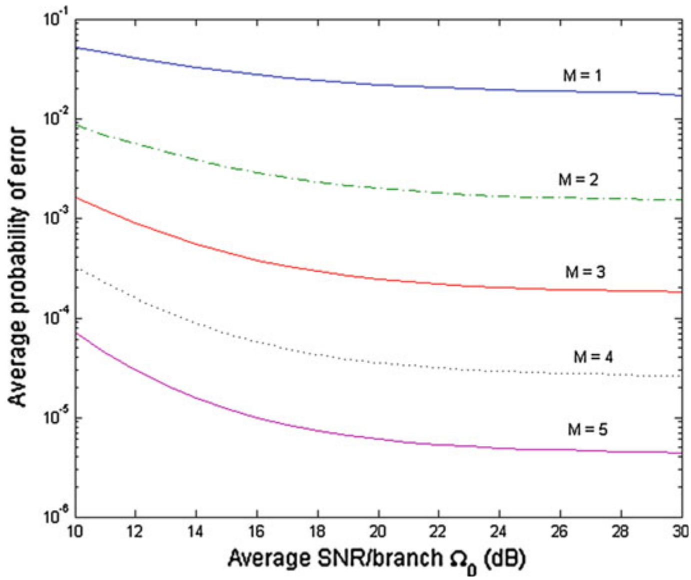

Fig. 5 Average error rates in shadowed fading channels with increasing order of diversity ($N = 4$, SIR $=$ 20 dB)

presence of cochannels to the case when cochannels are absent. Also, the error floors increase with increasing values of N posing serious problems in data transmission since increase in SNR does not lead to any further decline in error rates. Figure 5 shows the plots of average error rates for a fixed number of cochannels ($N = 4$) and fixed values of SIR (20 dB) with

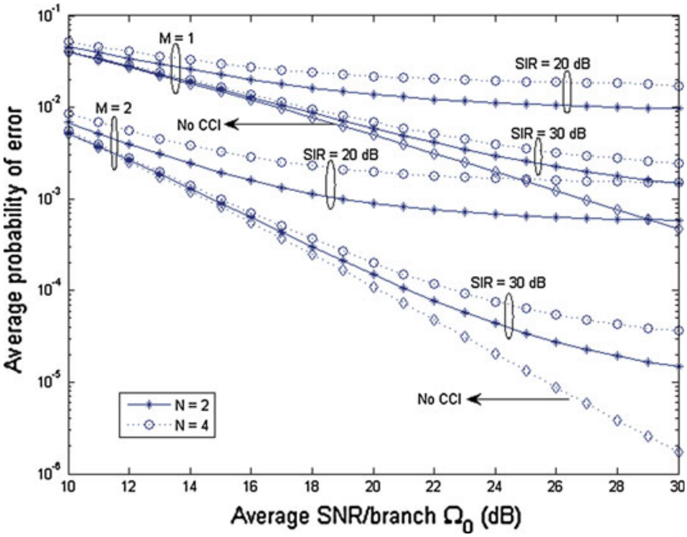

Fig. 6 Average error rates for $M = 1, 2$ for $N = 2, 4$ and SIR $= 20, 30$ dB

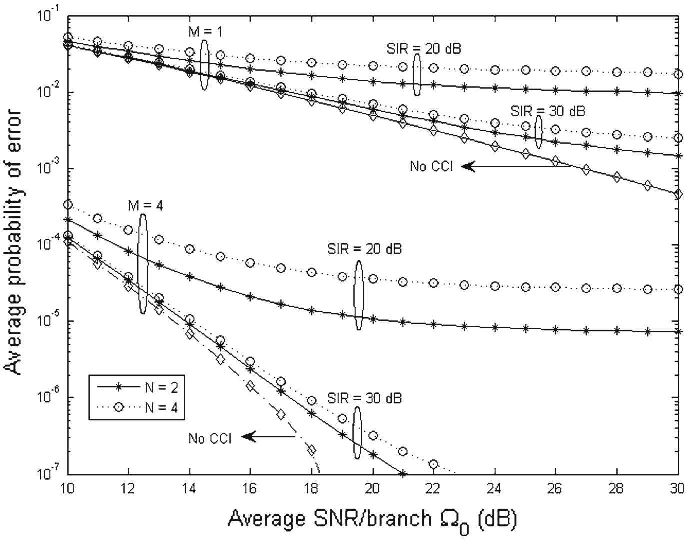

Fig. 7 Average error rates for $M = 1, 4$ for $N = 2, 4$ and SIR $= 20, 30$ dB

increasing values of M. As the order of diversity increases, the error rates come down along with error floors demonstrating the improvement gained through diversity.

Figures 6 and 7 show the error rates for two cases of $N = 2, 4$ for two levels of SIR (20 and 30 dB). In Figure 6, the results for the case of $M = 2$ are compared to those of $M = 1$ (no diversity) while the error rates are examined for $M = 1$ and $M = 4$ in Fig. 7. In both figures, the error rates in the absence of CCI are also shown. The benefits of diversity are clearly seen in these plots through the lowering of the error floors with increasing orders of diversity. One can also observe the lowering of the error floors with the increase in SIR. As the SIR increases, the error rates approach the error rates in the absence of any CCI.

4 Discussion and Conclusions

The performance of wireless channels operating in a shadowed fading environment has been analyzed in the general case when the desired channel and the cochannels undergo short term fading and shadowing Using the density function of the sum of the GK variables, it was possible to estimate the densities and error rates in an environment where shadowing in each branch can be treated as separate and independent. Just as in the case of MRC in any diversity processing scenario, it has been assumed that the channel characteristics are known so that the algorithm can be implemented. The analysis is general since it does not make any assumptions on the type of diversity (spatial, frequency, etc.) except that the branches are uncorrelated.

The error rate calculations demonstrated the existence of error floors, often unacceptably high ones which could be lowered through the use of diversity. Still, the error floors need to be reduced further and one needs to include additional mitigation approaches such as combining the MRC outputs from multiple base stations and implementing the diversity at the macrolevel to account for the shadowing seen. The approach enunciated in this work can be expanded to include such studies involving diversity at the microlevel and macrolevel.

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Author Biography

P. M. Shankar received his M.Sc. (1972) in Physics from Kerala University, India, M.Tech. (1975) in Applied Optics and Ph.D. in Electrical Engineering (1980) from Indian Institute of Technology, Delhi, India. He was a visiting scholar at the School of Electrical Engineering, University of Sydney, Australia, from 1981 to 1982. He joined Drexel University in 1982 and is currently the Allen Rothwarf Professor of Electrical and Computer Engineering. He is the author of the textbook 'Introduction to Wireless Systems', published by John Wiley & Sons, 2002. His research interests are in Statistical signal processing for medical applications, Modeling of fading channels, Wireless communications, Medical ultrasound, and Optical fiber sensors.