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Adaptive Channel Estimation for Multiple Antenna OFDM Systems

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Abstract We propose a simple adaptive algorithm for channel estimation of multiple antenna OFDM systems to provide competitive performance with low computational complexity and tracking ability. The derivation of the mean-squared error (MSE) for the proposed scheme is presented and the optimum selection of the parameter in the scheme is analyzed by utilizing the derived MSE expression.

Keywords Multiple antenna systems · Channel estimation · OFDM

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has been shown to be a very promising technology for high-speed data transmission and to be robust to frequency selective fading over wireless channels. Multiple antenna systems have been shown to provide a significant increase in capacity through the use of transmitter and receive diversity. The task of channel estimation is necessary before the demodulation of transmit signals, that is, we have to know the channel state information (CSI) at the receiver. For systems with multiple transmit antennas and multiple receiver antennas, various types of channel estimation schemes can be found in [1–5]. The pioneer research work based on minimizing mean-square error cost function for the channel estimation is shown in [1] by using a training sequence. However,

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the computational complexity is high due to the operation of matrix inversion with large dimensionality. The research topics were focused on reducing its complexity or improving performance. In [6], the optimum training sequences design and a simplified channel estimator has been proposed to reduce the complexity. By exploiting the correlation of adjacent subchannel response, the sizes of the matrix inversion and Fast Fourier Transform (FFT) needed in the channel estimation for every OFDM data symbol are reduced by half in [7]. In [8], by utilizing continuously transmitted pilot tons, a recursive least-squares (RLS)-based adaptive channel estimation algorithm is developed. However, this approach requires pilot sequence design to form an orthogonal matrix for the feasibility of the derived RLS-based adaptive algorithm. Without continuously transmitted pilot tones to save the bandwidth, the demodulated symbols passing cyclic redundancy check (CRC) may be utilized to take the role of the pilot symbols in a decision feedback fashion. However, the random transmit data may not provide the orthogonal matrix property which is required by the algorithm in [8]. Alternatively, without the requirement of the orthogonal matrix property for the purpose in reducing the complexity, we focus on a low-complexity scheme design and propose a simple adaptive algorithm to reduce computational complexity with competitive performance. Due to the derived adaptive form, it provides tracking capability in a decision feedback fashion without the need in continuously transmitting training symbols and the requirement of the orthogonal matrix property, which is different from these schemes in [6–8]. The derivation of the MSE for the proposed adaptive algorithm is presented and the selection of the coefficient in the scheme is analyzed by utilizing the derived MSE expression for given parameters.

2 System Model

The signal model of an OFDM system with two transmit and two receive antennas is presented as in [1]. Two different modulated symbols, $\{t_i[n,k] : k = 0, 1, \ldots K-1\}$, are transmitted at a transmission time n for $i = 1, 2$, where K, k, and i are the number of subcarriers of the OFDM systems, subcarrier index, and transmit antenna index, respectively. Each of these signals forms an OFDM block. At the receiver, the discrete Fourier transform (DFT) of the received signal at each receive antenna is the superposition of two distorted transmitted signals. The received signal at the yth receive antenna can be expressed as

$$r_y[n,k] = \sum_{i=1}^{2} H_{iy}[n,k]t_i[n,k] + w_y[n,k], \tag{1}$$

where $H_{iy}[n,k]$ is the channel frequency response for the kth subcarrier at time n, corresponding to the i-th transmitter antenna and the y-th receive antenna; $w_y[n.k]$ is the additive complex Gaussian noise with zero mean and variance σ_n^2. By combining the received signal from two receive antennas, (1) can be rewritten in a matrix form

$$\mathbf{r}[n,k] = \mathbf{H}[n,k]\mathbf{t}[n,k] + \mathbf{w}[n,k] \tag{2}$$

where

$$\mathbf{r}[n,k] = \begin{pmatrix} r_1[n,k] \\ r_2[n,k] \end{pmatrix}, \quad \mathbf{H}[n,k] = \begin{pmatrix} H_{11}[n,k] & H_{21}[n,k] \\ H_{12}[n,k] & H_{22}[n,k] \end{pmatrix} \text{ and } \mathbf{t}[n,k] = \begin{pmatrix} t_1[n,k] \\ t_2[n,k] \end{pmatrix}.$$

The channel frequency response with the discrete time complex baseband representation can be described by

$$H_{iy}[n,k] = \sum_{l=0}^{K_0-1} h_{iy}[n,l]W_K^{kl}, \tag{3}$$

where $W_K = \exp(-j(2\pi/K))$. $h_{iy}[n,l]$'s, for $l = 0, 1, \ldots, K_0 - 1$, are the coefficients of the channel impulse response, which are modeled as wide-sense stationary complex zero-mean Gaussian processes. The average power of $h_{iy}[n,l]$ and K_0 are related by the delay profiles of the wireless channels.

For the yth receive antenna, if the transmitted $t_i[n,k] i = 1, 2$ are known through the use of a training block, the temporal estimation of $h_{iy}[n,l]$ can be found by minimizing an MSE cost function in [1]. The index y for different antennas is omitted from $r_y[n,k]$ and $h_{iy}[n,k]$ for the following discussion. By defining $p_i[n,l] = \sum_{l=0}^{K-1} r[n,k]\,t_i^*[n,k]W_K^{-kl}$ and $q_{ij}[n,l] = \sum_{l=0}^{K-1} t_i[n,k]\,t_j^*[n,k]W_K^{-kl}$, the channel estimation has the form as

$$\tilde{\mathbf{h}}[n] = \mathbf{Q}^{-1}[n]\mathbf{P}[n] \tag{4}$$

based on the MMSE criterion and the training data in [1], where

$$\widetilde{\mathbf{h}}[n] = \begin{pmatrix} \mathbf{h}_1[n] \\ \mathbf{h}_2[n] \end{pmatrix}, \mathbf{p}[n] = \begin{pmatrix} \mathbf{p}_1[n] \\ \mathbf{p}_2[n] \end{pmatrix}, \quad \text{and} \quad \mathbf{Q}[\mathrm{n}]=\begin{pmatrix} \mathbf{Q}_{11}[n] & \mathbf{Q}_{21}[n] \\ \mathbf{Q}_{12}[n] & \mathbf{Q}_{22}[n] \end{pmatrix}$$

with

$$\begin{aligned} \widetilde{\mathbf{h}}_i[n] &= (\widetilde{h}_i[n,0], \;\; \widetilde{h}_i[n,1], \;\; \ldots, \;\; \widetilde{h}_i[n,K_0-1])^T, \\ \mathbf{p}_i[n] &= (p_i[n,0], \;\; p_i[n,1], \;\; \ldots, \;\; p_i[n,K_0-1])^T \end{aligned}$$

and

$$\mathbf{Q}_{ij}[\mathrm{n}]=\begin{pmatrix} q_{ij}[n,0] & q_{ij}[n,-1] & \ldots & q_{ij}[n,-K_0+1] \\ q_{ij}[n,1] & q_{ij}[n,0] & \ldots & q_{ij}[n,-K_0+2] \\ \vdots & \ddots & \ddots & \vdots \\ q_{ij}[n,K_0-1] & q_{ij}[n,K_0-2] & \ldots & q_{ij}[n,0] \end{pmatrix}.$$

3 Adaptive Channel Estimation

In order to get the information of channel parameters, $\mathbf{Q}[n]$ in (4) has to be invertible and it requires computational complexity $\mathrm{O}(K_0^3)$. Normally, since training data are known to the receiver, the inverse can be pre-computed and training data are typically chosen to satisfy certain orthogonality constrains further facilitating easy inversion of the $\mathbf{Q}$ matrix. However, if we would like to utilize the detected data to achieve the channel estimation task, the computation of the inverse for the $\mathbf{Q}$ matrix is still required. Therefore, we propose an adaptive algorithm of channel estimation by extending the original algorithm in [1] to avoid the matrix inversion operation and provide competitive performance as follows. We also utilize our derived MSE expression to select the parameter used in the adaptive channel estimation scheme as will be discussed latter.

Note that the adaptive channel estimation with tracking capability is performed in the time domain. Again, the index y for different receive antennas is omitted for the following discussion for simplicity of the notation. For the yth receiver antenna, the channel impulse response, $\mathbf{h}_i[n] = [h_i[n,0], h_i[n,1], \ldots, h_i[n,K_0-1]]^T$, corresponding to the ith (for $i = 1, 2$) transmit antenna, can be estimated by

$$\hat{\mathbf{h}}[n+1] \triangleq \begin{bmatrix} \hat{\mathbf{h}}_1[n+1] \\ \hat{\mathbf{h}}_2[n+1] \end{bmatrix} = \frac{\lambda \cdot \hat{\mathbf{h}}[n] + \lambda^2 \cdot \hat{\mathbf{h}}[n-1] + \lambda^3 \cdot \hat{\mathbf{h}}[n-2] + \ldots + \lambda^{n+1} \cdot \hat{\mathbf{h}}[0]}{1 + \lambda + \lambda^2 + \lambda^3 + \ldots + \lambda^{n+1}} + \frac{\mathbf{Q}^{-1}[n+1]\mathbf{P}[n+1]}{1 + \lambda + \lambda^2 + \lambda^3 + \ldots + \lambda^{n+1}}, \tag{5}$$

where $\lambda < 1$ is the forgetting factor. Assuming that channels are stationary, the channel estimation may be formulated as

$$\hat{\mathbf{h}}[n+1] = \lambda \frac{1-\lambda^{n+1}}{1-\lambda^{n+2}} \hat{\mathbf{h}}[n] + \frac{1-\lambda}{1-\lambda^{n+2}} \mathbf{Q}^{-1}[n+1]\,\mathbf{P}[n+1] \tag{6}$$

The computational complexity is the same as $O(K_0^3)$. However, it offers better performance as revealed from the results by conducting various simulations. For reducing its complexity, we assume that the power of the transmitted signal for each antenna is unit with constant modulus modulation format. By utilizing the expression

$$\begin{aligned} \mathbf{Q}^{-1}[n+1]\,\mathbf{P}[n+1] = \begin{bmatrix} \hat{\mathbf{h}}_1[n+1] \\ \hat{\mathbf{h}}_2[n+1] \end{bmatrix} &= \begin{bmatrix} \frac{1}{K}\left(\mathbf{p}_1[n+1] - \mathbf{Q}_{21}[n+1] \cdot \hat{\mathbf{h}}_2[n+1]\right) \\ \frac{1}{K}\left(\mathbf{p}_2[n+1] - \mathbf{Q}_{12}[n+1] \cdot \hat{\mathbf{h}}_1[n+1]\right) \end{bmatrix} \\ &\approx \begin{bmatrix} \frac{1}{K}\left(\mathbf{p}_1[n+1] - \mathbf{Q}_{21}[n+1] \cdot \hat{\mathbf{h}}_2[n]\right) \\ \frac{1}{K}\left(\mathbf{p}_2[n+1] - \mathbf{Q}_{12}[n+1] \cdot \hat{\mathbf{h}}_1[n]\right) \end{bmatrix} \\ &= \frac{1}{K}\left(\mathbf{P}[n+1] - \mathbf{Q}_0[n+1] \cdot \hat{\mathbf{h}}[n]\right) \end{aligned} \tag{7}$$

With $\mathbf{Q}_{ii}[n] = K \cdot \mathbf{I}$ for $i = 1, 2$ and

$$\mathbf{Q}_0[n+1] = \begin{pmatrix} \mathbf{0} & \mathbf{Q}_{21}[n+1] \\ \mathbf{Q}_{12}[n+1] & \mathbf{0} \end{pmatrix}, \tag{8}$$

Eq. 6 becomes

$$\hat{\mathbf{h}}[n+1] = \frac{1}{1-\lambda^{n+2}} \left\{ \left[\lambda\left(1-\lambda^{n+1}\right)\mathbf{I} - \frac{1-\lambda}{K}\mathbf{Q}_0[n+1] \right] \hat{\mathbf{h}}[n] + \frac{1-\lambda}{K}\mathbf{P}[n+1] \right\} \tag{9}$$

This reduces the computational complexity of the channel estimation to $O(K_0^2)$ and provides tracking capability. During the training period, we assume that the transmitted signals from each antenna are known and we may use the optimum training sequence as in [6,8]. When the system is in a data transmission mode, decoded data passing CRC are used to play the role of the training symbols to take advantage of the adaptive form as in a decision feedback fashion.

According to the more detailed analysis of the computational cost, the derived equation can be rewritten as

$$\hat{\mathbf{h}}[n+1] = \frac{\lambda\left(1-\lambda^{n+1}\right)}{1-\lambda^{n+2}} \hat{\mathbf{h}}[n] - \frac{1-\lambda}{(1-\lambda^{n+2})K} \left\{ \mathbf{Q}_0[n+1]\hat{\mathbf{h}}[n] - \mathbf{P}[n+1] \right\} \tag{10}$$

Regarding the evaluation of the first term in (10), λ^{n+2} can be computed in an update form as $\lambda^{n+1} \bullet \lambda$ where λ^{n+1} has been calculated in the previous time instant. It requires one multiplication operation. Therefore, to compute $\frac{\lambda(1-\lambda^{n+1})}{1-\lambda^{n+2}} = \frac{(\lambda-\lambda^{n+2})}{1-\lambda^{n+2}}$ requires one multiplication, one division, and two subtractions. To evaluate $\frac{\lambda(1-\lambda^{n+1})}{1-\lambda^{n+2}} \hat{\mathbf{h}}[n]$ requires additional $2K_0$ multiplications. Regarding the evaluation of the second term in (10), to calculate $\frac{1-\lambda}{(1-\lambda^{n+2})K}$ needs one additional multiplication and division by utilizing the evaluation results of the first term. In the calculation of $\mathbf{Q}_0[n+1]\hat{\mathbf{h}}[n]$, it needs $2K_0 \bullet K_0$ multiplications and $2K_0 \bullet (K_0 - 1)$ additions. Therefore, to compute $\frac{1-\lambda}{(1-\lambda^{n+2})K}\left\{\mathbf{Q}_0[n+1]\hat{\mathbf{h}}[n] - \mathbf{P}[n+1]\right\}$ requires additional $2K_0$ subtractions and multiplications. The total computational cost has $(2K_0^2 + 4K_0 + 2)$ multiplications, 2 divisions, $(2K_0^2 - 2K_0)$ additions, and $(2K_0 + 2)$ subtractions.

On the other hand, to compute $\tilde{\mathbf{h}}[n] = \mathbf{Q}^{-1}[n]\mathbf{P}[n]$ in the original form requires the matrix inversion of $\mathbf{Q}[n]$ and the evaluation of the matrix $\mathbf{Q}^{-1}[n]$ post-multiplied by the vector $\mathbf{P}[n]$. The evaluation of $\mathbf{Q}^{-1}[n]\mathbf{P}[n]$ needs $2K_0 \bullet 2K_0 = 4K_0^2$ multiplications and $2K_0 \bullet (2K_0 - 1) = 4K_0^2 - 2K_0$ additions, which has the approximate the same computational complexity of $O(K_0^2)$ compared to the proposed algorithm. However, the matrix inversion are the additional computational operation which has a complexity of $O(K_0^3)$. The algorithms for computing the inverse of a matrix are varied and the Big-Oh notation is adopted here.

4 MSE Expression

The determination of the forgetting factor is an issue to solve for the proposed adaptive scheme. In this section, we derive the MSE expression for the adaptive algorithm. We may utilize the derived MSE for the selection of the forgetting factor. In the derivation, we assume the demodulated symbols are correct for those passing CRC. The amplitude of each path varies independently of the others, according to a Rayleigh distribution and Jakes Doppler power spectral density as in [9]. The number of transmit antennas is two. However, the derivation can be developed similarly for the case of a general number of transmit antennas.

4.1 MSE Expression

The MSE of the estimator is expressed as

$$\begin{aligned}\text{MSE}[n] &= \frac{1}{2K_0} E\left\{\left\|\hat{\mathbf{h}}[n] - \mathbf{h}[n]\right\|^2\right\}\\ &= \frac{1}{2K_0} E\left\{\left\|\frac{1}{1-\lambda^{n+1}}\left\{\left[\lambda\left(1-\lambda^n\right)\mathbf{I} - \frac{1-\lambda}{K}\mathbf{Q}_0[n]\right]\hat{\mathbf{h}}[n-1] + \frac{1-\lambda}{K}\mathbf{P}[n]\right\} - \mathbf{h}[n]\right\|^2\right\}.\end{aligned} \tag{11}$$

With

$$\mathbf{P}[n] = \mathbf{Q}[n]\cdot\mathbf{h}[n] - \mathbf{W}[n] = (\mathbf{Q}_0[n] + K\cdot\mathbf{I})\cdot\mathbf{h}[n] - \mathbf{W}[n];$$

$$\mathbf{W}[n] = \begin{bmatrix}\mathbf{W}_1[n]\\ \mathbf{W}_2[n]\end{bmatrix};\ \mathbf{W}_i = \left[\tilde{w}_i[n,0],\ldots,\tilde{w}_i[n,K_0-1]\right]^T;$$

$$\text{and}\quad \tilde{w}_i[n,l] = \sum_{k=0}^{K-1} w[n,k]t_i^*[n,k]W_K^{-kl},$$

we have

$$\text{MSE}[n] = \frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}$$

$$\times E\left\{\left\|\left[\lambda\left(1-\lambda^{n}\right)\mathbf{I}-\frac{1-\lambda}{K}\mathbf{Q}_0[n]\right]\cdot\left(\mathbf{h}[n]-\hat{\mathbf{h}}\left[n-1\right]\right)+\frac{1-\lambda}{K}\mathbf{W}[n]\right\|^2\right\}$$

Here we assumed that the noise and the estimation error are orthogonal. Therefore,

$$\begin{aligned}
\mathrm{MSE}[n] =& \frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\left[\lambda\left(1-\lambda^{n}\right)\mathbf{I}-\frac{1-\lambda}{K}\mathbf{Q}_0[n]\right]\cdot\left(\mathbf{h}[n]-\hat{\mathbf{h}}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\frac{1-\lambda}{K}\mathbf{W}[n]\right\|^2\right\}\\
=& \frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\left[\lambda\left(1-\lambda^{n}\right)\mathbf{I}-\frac{1-\lambda}{K}\mathbf{Q}_0[n]\right]\cdot\left(\mathbf{h}[n]-\mathbf{h}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\left[\lambda\left(1-\lambda^{n}\right)\mathbf{I}-\frac{1-\lambda}{K}\mathbf{Q}_0[n]\right]\cdot\left(\mathbf{h}\left[n-1\right]-\hat{\mathbf{h}}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}\left(\lambda^2\left(1-\lambda^{n}\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right)\\
&\times\frac{\lambda\left(1-\lambda^{n}\right)}{1-\lambda\left(1-\lambda^{n}\right)}E\left\{\left\|\mathbf{h}[n]-\mathbf{h}\left[n-1\right]\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\frac{1-\lambda}{K}\mathbf{W}[n]\right\|^2\right\}\\
=& \frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\lambda\left(1-\lambda^{n}\right)\left(\mathbf{h}[n]-\mathbf{h}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\frac{1-\lambda}{K}\mathbf{Q}_0[n]\left(\mathbf{h}[n]-\mathbf{h}[n-1]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\lambda\left(1-\lambda^{n}\right)\left(\mathbf{h}\left[n-1\right]-\hat{\mathbf{h}}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\frac{1-\lambda}{K}\mathbf{Q}_0[n]\left(\mathbf{h}\left[n-1\right]-\hat{\mathbf{h}}\left[n-1\right]\right)\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}\left(\lambda^2\left(1-\lambda^{n}\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right)\\
&\times\frac{\lambda\left(1-\lambda^{n}\right)}{1-\lambda\left(1-\lambda^{n}\right)}E\left\{\left\|\mathbf{h}[n]-\mathbf{h}\left[n-1\right]\right\|^2\right\}\\
&+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}E\left\{\left\|\frac{1-\lambda}{K}\mathbf{W}[n]\right\|^2\right\}\\
\approx& \frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}\left[\lambda^2\left(1-\lambda^{n}\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right.\\
&\left.+1\cdot\left(\lambda^2\left(1-\lambda^{n}\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right)\cdot\frac{\lambda\left(1-\lambda^{n}\right)}{1-\lambda\left(1-\lambda^{n}\right)}\right]\omega_d^2
\end{aligned}$$

$$+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}\left[\lambda^2\left(1-\lambda^n\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right]\cdot 2K_o\cdot \text{MSE}[n]$$
$$+\frac{1}{2K_0\cdot\left(1-\lambda^{n+1}\right)^2}\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\cdot 2\sigma_n^2$$

where $\omega_d = 2\pi f_d T_f$, f_d is the maximum Doppler frequency of the channel, and T_f is the OFDM block duration. In the last step of the derivation, we adopt the same channel model and the results in [10] for the evaluation of the term $E\left\{\|\mathbf{h}[n]-\mathbf{h}[n-1]\|^2\right\}$ (see (A4) of [10]. After some algebraic manipulation, the final MSE expression is

$$\begin{aligned}\text{MSE}[n] \approx \Bigg\{\Bigg[&\lambda^2\left(1-\lambda^n\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\\ &+\left(\lambda^2\left(1-\lambda^n\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right)\cdot\frac{\lambda\left(1-\lambda^n\right)}{1-\lambda\left(1-\lambda^n\right)}\Bigg]\omega_d^2\\ &+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\cdot\sigma_n^2\Bigg\}\Bigg/\Bigg\{K_0\cdot\left(1-\lambda^{n+1}\right)^2\\ &-K_0\cdot\left[\lambda^2\left(1-\lambda^n\right)^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right]\Bigg\}\end{aligned} \quad (12)$$

As $n\to\infty$, we have

$$\begin{aligned}\text{MSE}[n] \approx \Bigg\{\Bigg[&\lambda^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o+\left(\lambda^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right)\cdot\frac{\lambda}{1-\lambda}\Bigg]\omega_d^2\\ &+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\cdot\sigma_n^2\Bigg\}\Bigg/\Bigg\{K_0-K_0\cdot\left[\lambda^2+\left(\frac{1-\lambda}{K}\right)^2\cdot K\cdot K_o\right]\Bigg\}.\end{aligned} \quad (13)$$

4.2 Optimum Forgetting Factor λ

By utilizing Eq. 12, if we set the OFDM block tones $K = 128$, the maximum Doppler frequency $f_d = 5\,\text{Hz}$, and the number of channel taps $K_0 = 5$, the MSE can be calculated as the result displayed in Fig. 1(a) for taking SNR = 20 dB as the regular operation point. The minimum MSE is achieved at $\lambda = 0.64$ for the steady state in this example. Similar results can be observed in Fig. 1(b) for other parameters. Under an operation environment, it indicates that an optimum λ exists in a particular Doppler frequency to achieve the minimum MSE as shown in Fig. 1. With this property, we can select the best λ in a given operation environment. For the practical application, we may divide the range of the possible Doppler frequencies (5 ∼ 100 Hz as an example) into several zones and a value of λ is assigned to each divided zone. The λ associated with the Doppler frequency zone for a given noise power is employed for the adaptive channel estimation task with the Doppler frequency estimated by an auxiliary detection device in the application.

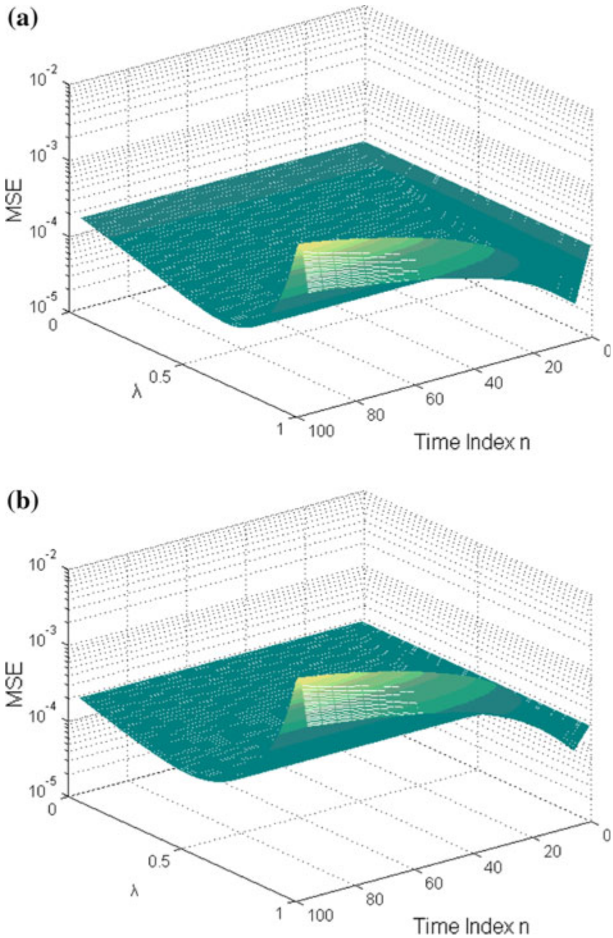


Fig. 1 Derived MSE of the proposed method versus time index n and the forgetting factor λ, with **a** the Doppler frequency $f_d = 5$ Hz, SNR = 20 dB, $K_0 = 5$, and $K = 128$. **b** The Doppler frequency $f_d = 20$ Hz, SNR = 20 dB, $K_0 = 17$, and $K = 128$

5 Simulation Results

In the simulation, we assume that two transmit antennas and two received antennas for diversity in OFDM systems, and its carrier frequency is 2 GHz. QPSK is used to transmit the signals. We also assume the synchronization in the receiver is perfect. The channel bandwidth is 800 kHz, divided into 128 subcarriers. The number of virtual carriers are 8, and 120 subcarriers are used to transmit data. The symbol duration is 160 and 40 μs is the guard interval. The maximum delay spreads ($\tau_{\max}$) of the multipath fading channel are 5 and 20 μs. The maximum Doppler frequencies f_d are 5, 20, and 100 Hz. The channel has K_0 paths with path delays of 0,1,..., $K_0 - 1$ samples . The amplitude A_k of each path varies independently of the others, according to a Rayleigh distribution with an exponential power delay profile.

$$E\{A_k^2\} = \exp(-k/5), k = 0, 1, \ldots, K_0 - 1.$$

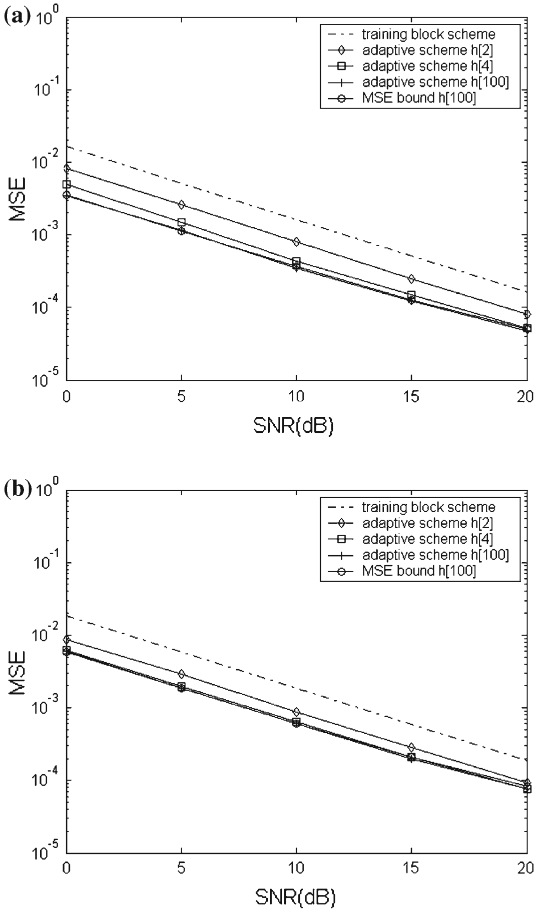


Fig. 2 MSE versus SNR with **a** $f_d = 5$ Hz and $\boldsymbol{\tau}_{\max} = 5\,\mu$s. **b** $f_d = 20$ Hz and $\boldsymbol{\tau}_{\max} = 20\,\mu$s

The channel lengths are $K_0 = 5$ and 17, respectively. In the simulation, the Rayleigh fading signal of each path is generated by adopting the model in [9] and we normalize the total power contained in the channel impulse response equal to one.

Figure 2(a), (b) present the performance comparison between the adaptive estimator and the training-block-based scheme equivalent to the estimator in [1] in terms of MSE performance. In Fig. 2(a), λ is chosen based on the derived MSE expression which provides the theoretical minimum MSE in the steady state. As revealed in the simulation results, the performance of the proposed scheme improves as the time index increases. The MSEs at the time indexes 4 and 100 are getting closer and it demonstrates the convergence property of the proposed algorithm. It also verifies that the derived MSE expression at the time index $n = 100$ is approximately matched with the simulation result. Similarly, Fig. 2(b) presents the MSE performance comparison for other simulation parameters and similar results are observed. In Fig. 3, the simulation result verifies that the theoretical selection of the coefficient is appropriate. The minimum MSE is achieved approximately at the location of the selected parameter λ as indicated in the simulation results by varying the parameter λ, where the MSE performance of the selected λ is marked with 'x'.

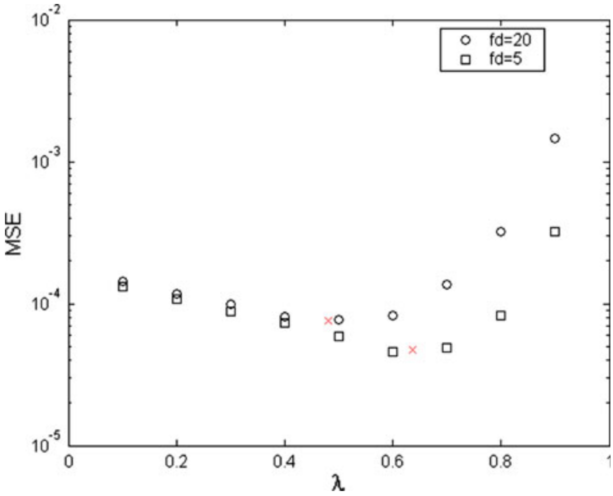


Fig. 3 MSE versus the parameter λ at SNR = 20 dB, with $f_d = 20$ Hz, $\tau_{\max} = 20$ μs and $f_d = 5$ Hz, $\tau_{\max} = 5$

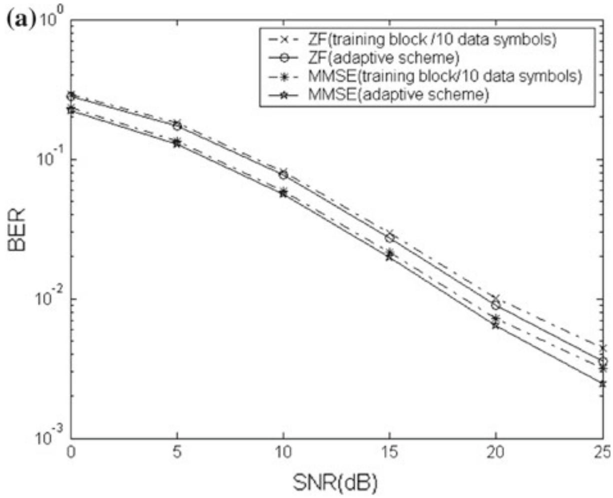
In the simulation results of Fig. 4, we transmit ten data symbols after a training symbol is transmitted. The statistics are calculated at the tenth symbol period. To demonstrate the superior performance of the proposed algorithm, we provide the bit error rate (BER) performance versus SNR for the zero-forcing (ZF) and the minimum mean-squared error (MMSE) equalizers based on the estimated channel. Denote $\tilde{\mathbf{t}}[n,k]$, $\mathbf{r}[n,k]$ and $\tilde{\mathbf{H}}[n,k]$ the estimated signal vector, the received signal vector, and the estimated channel parameter matrix, respectively. The estimated frequency response of the channel is obtained through (3) with the proposed adaptive channel estimation scheme. By taking the two antenna case as an example, we have

$$\tilde{\mathbf{t}}[n,k] = \begin{pmatrix} \tilde{t}_1 & [n,k] \\ \tilde{t}_2 & [n,k] \end{pmatrix}, \quad \mathbf{r}[n,k] = \begin{pmatrix} r_1 & [n,k] \\ r_2 & [n,k] \end{pmatrix}, \quad \text{and } \tilde{\mathbf{H}}[n,k] = \begin{pmatrix} \tilde{H}_{11} & [n,k] & \tilde{H}_{21} & [n,k] \\ \tilde{H}_{12} & [n,k] & \tilde{H}_{22} & [n,k] \end{pmatrix}$$

Referring to (2), the ZF equalization for detection is expressed as $\tilde{\mathbf{t}}_{ZF}[n,k] = (\tilde{\mathbf{H}}^H[n,k]\tilde{\mathbf{H}}[n,k])^{-1}\tilde{\mathbf{H}}^H[n,k]\mathbf{r}[n,k]$, where $\tilde{\mathbf{t}}_{ZF}[n,k]$ is the two element column vector of ZF equalized data symbols. Similarly, the minimum mean-square error (MMSE) equalization is given by $\tilde{\mathbf{t}}_{\text{MMSE}}[n,k] = (\tilde{\mathbf{H}}^H[n,k]\tilde{\mathbf{H}}[n,k] + \sigma_n^2/\sigma_t^2\mathbf{I})^{-1}\tilde{\mathbf{H}}^H[n,k]\mathbf{r}[n,k]$, where σ_t^2 is the variance of the transmitted data symbols. Note that the equalization for the signal detection is performed in the frequency domain. As indicated in the results, if the channel variation is fast, the proposed adaptive scheme can tremendously improve performance of the training-block based scheme due to the tracking capability and providing the updated channel information.

6 Conclusion

In this paper, a simple adaptive channel estimation algorithm for multiple antenna OFDM systems is developed to provide competitive performance and tracking capability. It shows promising for the real-time implementation. The simulation results show that the simple adaptive channel scheme offers better performance in terms of MSE and BER performance



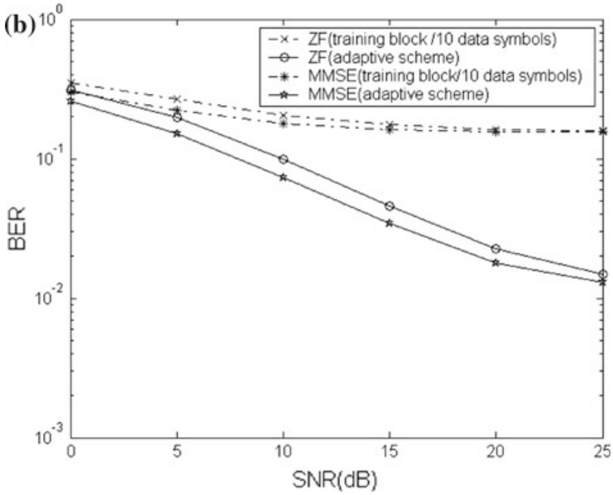


Fig. 4 BER versus SNR with **a** $f_d = 5$ Hz and $\boldsymbol{\tau}_{\max} = 5\,\mu$s. **b** $f_d = 100$ Hz and $\boldsymbol{\tau}_{\max} = 20\,\mu$s

measure than the existing scheme. The derived MSE expression is tight and matched to the simulation result. λ can be properly selected to achieve the optimal performance associated with a Doppler frequency detection device under an operation environment.

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