Wireless Pers Commun (2012) 62:893–906
DOI 10.1007/s11277-010-0099-0

Multi-cell Optimal Downlink Beamforming Algorithm with Per-base Station Power Constraints

Fan Huang · Yafeng Wang · Jian Geng · Dacheng Yang

Published online: 12 August 2010

Abstract We consider a problem of optimizing multi-cell downlink throughput in multiple-input single-output (MISO) beamforming with single user per sub-channel in the wireless communication system. Previous work based on the generalization of uplink-downlink duality has already reformulated the maximum achievable downlink throughput into dual uplink throughput maximization problem. Since the dual uplink problem is nonconvex, it is difficult to find its optimal solution. The main contribution of this paper is a novel practical algorithm based on heuristic to find the solution of beamforer design satisfying the necessary optimality conditions of the dual uplink problem. Meanwhile the converged beamforming vectors can in turn improve the system sum rate significantly. As the dual problem is a mixed optimization, we also provide algorithms for its two sub-optimal problems. Simulation results validate the convergence and the efficiency of proposed algorithms.

Keywords Multi-cell · Downlink beamforming · Throughput maximization · Power constraint · Numerical algorithm

1 Introduction

The cellular mobile communication systems nowadays are mostly based on orthogonal frequency-division multiplexing (OFDM) technology so that intra-cell interference can be neglected. In order to further improve the system throughput, to lower inter-cell interference, which is inherent to the cellular systems, is one of the main issues for research. Traditional approaches mostly mitigate interferences from neighboring sites by partitioning resources, e.g. frequency resources, for different sites [1–3]. However, full spectral reuse can allow the system to achieve higher capacity due to the high resource utilization efficiency. By providing each base station (BS) with the whole frequency resources, coordinated scheduling [4–6]

F. Huang (✉) · Y. Wang · J. Geng · D. Yang
Wireless Theories and Technologies Lab (WT&T), Beijing University of Posts and Telecommunications, Beijing, People's Republic of China
e-mail: buptfan@gmail.com; gj.garry@gmail.com

and joint transmit beamforming (or joint processing) [7–9] are the new and hot topics for inter-cell interference mitigation and strengthening received signal, which requires sharing information between cooperative BSs. Compared with the joint processing, where both data and channel information of all users need to be shared, coordinated scheduling requires only the channel information, which may be practicable in some scenarios that do not have enough bandwidth.

The objective of the problem is usually multi-cell downlink throughput maximization, which is a constrained nonconvex optimization problem and it needs some skills to solve this problem. Efficient sub-optimal algorithms for coordinated scheduling in the single-input single-output (SISO) system are proposed in [4]. Its main idea is to solve the frequency and power resources allocation separately, which includes the combinatorial optimization problem of scheduling and the numerical optimization problem of power allocation. To be specific, it fixes the frequency resource allocation while finding the optimal power allocation. The lower bound of the power allocation turns to be a convex problem and it can use the iteration method to increase the lower bound gradually until convergence. Then it fixes power allocation to solve the optimal frequency resource allocation problem. Its strategy proved to be nondecreasing and converge to the first order optimal conditions. Though the complexities of the algorithms are determined by the times of iteration which is uncertain, it offers a better way to find the near-optimal solution than exhaustive search algorithm. Some greedy schemes are proposed in [5,6] for coordinated scheduling and power allocation with single antenna transmission. In addition, a general introduction to convex optimization for communications and signal processing is given in [10] including linear and nonlinear problems with constraints.

So far, there have been plenty of works on the downlink beamforming for multi-antenna wireless systems in the past. A main tool to solve these problems is uplink-downlink duality which was also discussed in [10]. Authors in [11] used Lagrangian duality to prove that for both the beamforming problem and the capacity region problem the duality of a multi-antenna downlink channel with per-antenna power constraints is an uplink channel with an uncertain and diagonally constrained noise. Under the signal to interference and noise ratio (SINR) constraints, Ref. [12] provided an algorithm to find the global optimal downlink beamforming vector to minimize transmit power.

One can see that there are not many works on coordinated multi-cell downlink beamforming problem, which aims to maximize the throughput of the systems. Some greedy algorithms were proposed in [13], which considered this problem under a graphic framework and solved the problem by separating interference reduction and resource allocation intuitively. In [14], the uplink-downlink beamforming duality with SINR constraints in the multi-cell environment was considered. In that work, it was shown that the optimal downlink beamforming in MISO system with single user per sub-channel can be the minimum mean squared error (MMSE) beamforming in the dual uplink. It also proved that the dual uplink problem and the primal downlink problem achieve the same optimal throughput.

Our work is mainly based on the theoretical analysis of [14]. The dual uplink problem considered in this paper is a min-max mixed optimal problem. Since the dual problem is a nonconvex optimization, it is not easy to compute the globally optimal solution. The main contribution of this paper is a novel efficient algorithm for improving the sum rate of the system through finding the solution satisfying the necessary optimality conditions of dual problem. Numerical results show the convergence of the proposed algorithm.

The remainder of this paper is organized as follows. Section 2 describes the problem under consideration. I-MC-DB algorithm is presented in Sect. 3. In Sect. 4, numerical examples are provided. Conclusions are shown in Sect. 5.

Notation $\mathbb{C}^{i\times j}$ represents the complex space with $i \times j$ dimensions. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and the transpose conjugate and $\|\cdot\|_F$ to be the Frobenius norm.

2 Problem Formulation

We consider a multi-cell downlink cellular network with B base stations (BSs). Each BS is equipped with M transmitting antennas and each mobile station (MS) with single receiving antenna. As we mainly focus on designing the beamforming vectors, the scheduling procedure and power allocation are assumed to be performed before. Therefore, for simplicity, we also assume that only one MS is assigned to a sub-channel in each cell. Perfect channel state information of all the channels in the system is available in every BS. For a specific sub-channel, the downlink throughput maximization beamforming problem with constraints can be formulated as

$$\max \left\{ \sum_b \log_2 \left(1 + \frac{p_b \left|(\mathbf{h}_{b.b})^H \mathbf{u}_b\right|^2}{\sigma_n^2 + \sum_{b'\neq b} p_{b'} \left|\left(\mathbf{h}_{b',b}\right)^H \mathbf{u}_{b'}\right|^2} \right) \middle| \mathbf{p} = \mathbf{P}_T, \mathbf{u}_b \in U_1, \forall b \right\}, \quad (1)$$

where p_b is the transmit power from the bth BS, where $b \in \{1, 2, \ldots, B\}$ and B is the number of BSs, $\mathbf{h}_{b',b} \in \mathbb{C}^{M\times 1}$ is the channel response vector from the b'th BS to the bth MS, $\mathbf{u}_b \in \mathbb{C}^{M\times 1}$ is the transmit beamforming vector with unit norm at the bth BS, σ_n^2 is the power of additive white Gaussian noise (AWGN), $\mathbf{p} = [p_1, p_2, \ldots, p_B]$ is the BS transmit power vector, $\mathbf{P}_T = [P_{T,1}, P_{T,2}, \ldots, P_{T,B}]$ is the BS target transmit power vector, and $U_1 = \left\{\mathbf{x}\,|\,\|\mathbf{x}\| = 1, \mathbf{x} \in \mathbb{C}^{M\times 1}\right\}$. The primal downlink problem of (1) proved to achieve the same optimum value as a dual uplink problem in [14]:

$$\begin{aligned} &\min_{\bar{q}_b} \max_{\mathbf{u}_b \in U_1, \bar{g}_b} \left\{ \sum_b \log_2 \left(1 + \frac{\bar{g}_b \left|(\mathbf{u}_b)^H \mathbf{h}_{bb}\right|^2}{\sum_{b'\neq b} \bar{g}_{b'} \left|(\mathbf{u}_b)^H \mathbf{h}_{bb'}\right|^2 + \bar{q}_b} \right) \right\}, \\ &s.t. \quad \begin{cases} \mathbf{1}^T \bar{\mathbf{g}} \le \mathbf{1}^T \mathbf{P}_T, & \bar{\mathbf{q}}^T \mathbf{P}_T \le \sigma_n^2 \mathbf{1}^T \mathbf{P}_T, \\ \bar{q}_b \ge 0, \forall b, & \bar{g}_b \ge 0, \forall b, \end{cases} \end{aligned} \quad (2)$$

where $\bar{\mathbf{g}} = [\bar{g}_1, \ldots, \bar{g}_B]$ and $\bar{\mathbf{q}} = [\bar{q}_1, \ldots, \bar{q}_B]$ are the vectors of dual variables, and $\mathbf{1}$ is a $B \times 1$ vector where all elements are 1.

This dual problem can be seen as an uplink beamforming problem in which $\bar{\mathbf{g}}$ will be regarded as the transmit power vector of the MSs and $\bar{\mathbf{q}}$ the thermal noise power vector. The optimum point satisfies the following expression:

$$\mathbf{u}_b^* = \zeta_b \left(\sum_{b'} \bar{g}_{b'}^* \mathbf{h}_{bb'} (\mathbf{h}_{bb'})^H + \bar{q}_{b'}^* \mathbf{I}_{M\times M} \right)^{-1} \mathbf{h}_{bb}, \quad (3)$$

where $\mathbf{u}_b^*$, $\bar{g}_{b'}^*$ and $\bar{q}_{b'}^*$ are the optimal values of $\mathbf{u}_b$, $\bar{g}_{b'}$ and $\bar{q}_{b'}$, respectively and ζ_b is to insure that $\left|\mathbf{u}_b^*\right| = 1$. Let $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_B]$. To simplify (2), we denote $\overline{h}_{bb'} = \left|(\mathbf{u}_b)^H \mathbf{h}_{bb'}\right|^2$ and $\overline{h}_{bb} = \left|(\mathbf{u}_b)^H \mathbf{h}_{bb}\right|^2$. Thus, one can get:

$$\begin{aligned}&\min_{\bar{q}_b}\ \max_{\mathbf{u}_b\in U_1,\bar{g}_b}\left\{\sum_b \log_2\left(1+\frac{\bar{g}_b\bar{h}_{bb}}{\sum_{b'\neq b}\bar{g}_{b'}\bar{h}_{bb'}+\bar{q}_b}\right)\right\},\\ &s.t.\quad \begin{cases}\mathbf{1}^T\bar{\mathbf{g}}\leq \mathbf{1}^T\mathbf{P}_T, & \bar{\mathbf{q}}^T\mathbf{P}_T\leq \sigma_n^2\mathbf{1}^T\mathbf{P}_T,\\ \bar{q}_b\geq 0,\forall b, & \bar{g}_b\geq 0,\forall b.\end{cases}\end{aligned}\tag{4}$$

One of the most difficulties of these problems is the non-convexity of both the primal and the dual problems. Therefore, it may be not feasible to find its global optimal solution. In this work, we provide a greedy iterative algorithm to improve the objective value of the primal problem by obtaining the near-optimal solution of the dual problem, which satisfies the necessary optimality conditions.

3 Numerical Algorithm

In this section, we propose a numerical algorithm to solve (4), which we call the iterative multi-cell downlink beamforming (I-MC-DB) algorithm, which includes three sub-algorithms of which two are also iterative algorithms. When the I-MC-DB converges, $\bar{\mathbf{g}}$ and $\bar{\mathbf{q}}$ must satisfy their necessary conditions respectively.

3.1 Transmit power optimization

We fix $\mathbf{u}_b$ and $\bar{q}_b$, then (4) can be expressed as

$$\begin{aligned}&\max_{\bar{\mathbf{g}}\geq 0}\underbrace{\sum_b \log_2\left(1+\frac{\bar{g}_b\bar{h}_{bb}}{\sum_{b'\neq b}\bar{g}_{b'}\bar{h}_{bb'}+\bar{q}_b}\right)}_{y(\bar{\mathbf{g}})},\\ &s.t.\quad \mathbf{1}^T\bar{\mathbf{g}}\leq \mathbf{1}^T\mathbf{P}_T.\end{aligned}\tag{5}$$

By this, the dual problem is transformed into optimization of power allocation for single antenna scenario. Similar with the spectrum balancing problem in multi-user digital subscriber lines (DSL) networks [15] or coordinated scheduling and power allocation problem in downlink OFDMA networks [4], we provide an algorithm to solve this.

Problem (5) is nonconvex due to the difference of concave functions in $\bar{\mathbf{g}}$. We introduce the following lower bound derived in [15]

$$\alpha\log_2 z+\beta\leq \log_2(1+z)\qquad \begin{cases}\alpha=\frac{z_0}{1+z_0},\\ \beta=\log_2(1+z_0)-\frac{z_0}{1+z_0}\log_2 z_0,\end{cases}\tag{6}$$

which is tight at $z=z_0$. With (6) the objective function in (5) can be relaxed as

$$y\left(\bar{\mathbf{g}}\right)\geq \sum_b\left(\alpha_b\log_2\left(\frac{\bar{\mathbf{g}}_b\bar{\mathbf{h}}_{bb}}{\sum_{b'\neq b}\bar{\mathbf{g}}_{b'}\bar{\mathbf{h}}_{bb'}+\bar{\mathbf{q}}_b}\right)+\beta_b\right),\tag{7}$$

where α_b and β_b are calculated according to (6). In order to reformulate (7) into a concave optimization problem, a transformation $\tilde{g}_b = \ln \tilde{g}_b, \forall b$ is applied, therefore we have

$$\max_{\tilde{\mathbf{g}}} \underbrace{\sum_b \left(\alpha_b \left(\log_2 \bar{h}_{bb} + \tilde{g}_b - \log_2 \left(\sum_{b' \neq b} e^{\tilde{g}_{b'}} \bar{h}_{bb'} + \bar{q}_b\right)\right) + \beta_b\right)}_{\tilde{y}(\tilde{\mathbf{g}})}, \quad (8)$$
$$s.t. \quad \sum_b e^{\tilde{g}_b} \leq \sum_b P_{T,b}.$$

Note that log-sum-exp function is convex [16], so each term in (8) is concave, and the constraint is convex since it is a sum of convex exponentials. Therefore, (8) is a standard concave maximization problem.

The Lagrangian function of (8) is

$$L(\tilde{\mathbf{g}}, \lambda_g) = \tilde{y}(\tilde{\mathbf{g}}) - \lambda_g \left(\sum_b e^{\tilde{g}_b} - \sum_b \mathbf{P}_{T,b}\right), \quad (9)$$

where λ_g is the Lagrangian multiplier. So the dual problem is $\min_{\lambda_g} \max_{\tilde{\mathbf{g}}} L(\tilde{\mathbf{g}}, \lambda_g)$. The inner dual maximization can be solved by finding the stationary point with respect to $\tilde{\mathbf{g}}$:

$$\frac{\partial L(\tilde{\mathbf{g}}, \lambda_g)}{\partial \tilde{g}_b} = 0 = \frac{1}{\ln 2}\alpha_b - e^{\tilde{g}_b}\left(\lambda_g + \frac{1}{\ln 2}\sum_{b' \neq b} \frac{\alpha_{b'}\bar{h}_{b'b}}{\sum_{j \neq b'} e^{\tilde{g}_j}\bar{h}_{b'j} + \bar{q}_{b'}}\right).$$

Transform the partial derivative with $\tilde{\mathbf{g}} = \ln \bar{\mathbf{g}}$ and we can get

$$\bar{g}_b = \frac{\alpha_b}{\ln 2\lambda_g + \sum_{b' \neq b} \frac{\alpha_{b'}\bar{\mathbf{h}}_{b'b}}{\sum_{j \neq b'} \bar{\mathbf{g}}_j \bar{\mathbf{h}}_{b'j} + \bar{\mathbf{q}}_{b'}}}, \quad (10)$$

which is a fixed-point equation.

Proposition 1 *The right hand side of equation* (10) *is a standard interference function.*

Proof Let

$$I_b(\bar{\mathbf{g}}) = \frac{\alpha_b}{\ln 2\lambda_g + \sum_{b' \neq b} \frac{\alpha_{b'}\bar{\mathbf{h}}_{b'b}}{\sum_{j \neq b'} \bar{\mathbf{g}}_j \bar{\mathbf{h}}_{b'j} + \bar{\mathbf{q}}_{b'}}}. \quad (11)$$

According to the definition in [17], if (11) is a standard interference function, then when $\bar{\mathbf{g}} \geq 0$, we should have

1) Positivity ($I_b(\bar{\mathbf{g}}) > 0$):
   Positivity is obvious.
2) Monotonicity (if $\bar{\mathbf{g}} \geq \bar{\mathbf{g}}'$, then $I_b(\bar{\mathbf{g}}) \geq I_b(\bar{\mathbf{g}}')$):

$$I_b(\bar{\mathbf{g}}) = \frac{\alpha_b}{\ln 2\lambda_g + \sum_{b' \neq b} \frac{\alpha_{b'}\bar{h}_{b'b}}{\sum_{j \neq b'} \bar{g}_j \bar{h}_{b'j} + \bar{q}_{b'}}},$$

   if $\bar{\mathbf{g}} \geq \bar{\mathbf{g}}'$, then

$$\sum_{b' \neq b} \frac{\alpha_{b'}\bar{\mathbf{h}}_{b'b}}{\sum_{j \neq b'} \bar{\mathbf{g}}_j \bar{\mathbf{h}}_{b'j} + \bar{\mathbf{q}}_{b'}} \leq \sum_{b' \neq b} \frac{\alpha_{b'}\bar{\mathbf{h}}_{b'b}}{\sum_{j \neq b'} \bar{\mathbf{g}}'_j \bar{h}_{b'j} + \bar{\mathbf{q}}_{b'}},$$

Table 1 The sub-algorithm 1

1:	Initializest the maximum iteration number $s_{\max}$ and set $s = 0$
2:	Initialize $\bar{\mathbf{g}}_0$ and $\tilde{\mathbf{g}}_0$
3:	Calculate the initial $\boldsymbol{\alpha}_0$ and $\boldsymbol{\beta}_0$ using (6)
4:	**Repeat:**
4.1	update $\tilde{\mathbf{g}}_{s+1}$ using (10) and $\bar{\mathbf{g}}_{s+1}$
4.2	$d = \left\|\bar{\mathbf{g}}_{s+1} - \bar{\mathbf{g}}_s\right\|$
4.3	update $\boldsymbol{\alpha}_{s+1}$ and $\boldsymbol{\beta}_{s+1}$ using (6)
4.4	$s = s + 1$
5:	**Until** convergence ($d \le \Delta_1$) or $s = s_{\max}$

So

$$I_b(\bar{\mathbf{g}}) \ge I_b(\bar{\mathbf{g}}').$$

3) Scalability ($\gamma I_b(\bar{\mathbf{g}}) \ge I_b(\gamma\bar{\mathbf{g}}), \forall\gamma > 1$):

$$\gamma I_b(\bar{\mathbf{g}}) = \frac{\alpha_b}{\ln 2 \frac{\lambda_g}{\gamma} + \sum_{b' \ne b} \frac{\alpha_{b'} \bar{h}_{b'b}}{\sum_{j \ne b'} \gamma \bar{g}_j \bar{h}_{b'j} + \gamma \bar{q}_{b'}}} > \frac{\alpha_b}{\ln 2\lambda_g + \sum_{b' \ne b} \frac{\alpha_{b'} \bar{h}_{b'b}}{\sum_{j \ne b'} \gamma \bar{g}_j \bar{h}_{b'j} + \bar{q}_{b'}}} = I_b(\gamma\bar{\mathbf{g}}).$$

According to the property of standard interference function, $\bar{g}_b$ can be iteratively updated by this fixed-point equation and it can have a global convergence.

We use a gradient descent to update λ_g for fixed $\bar{\mathbf{g}}$

$$\lambda_g^{(s+1)} = \left[\lambda_g^{(s)} + \varepsilon_1\left(\sum_b \bar{g}_b - \sum_b P_{T,b}\right)\right]^+, \tag{12}$$

where s is the iteration number with ε_1 a scalar step size which is small enough and $[\cdot]^+$ is defined as $[\cdot]^+ = \max(0, \cdot)$. When the power constraint is violated, λ_g would go up and vice-versa. An equilibrium value would be reached and solves the problem where both λ_g and $\bar{\mathbf{g}}$ are converged.

After the searching procedure for $\bar{g}_b$ and λ_g, α and β should be updated according to (6) to tighten the relaxation, and thereafter a new iteration starts.

To sum up, the procedure for finding the optimal $\bar{g}_b$ is show in Table 1.

Proposition 2 *The iterative procedure above improves the value of objective function at each iteration and converges to the solution satisfying the Karush-Kuhn-Tucker (KKT) conditions* [16].

Proof We can see that the objective function value would be improved due to the fact that

$$\begin{aligned} \cdots &\le f_{\boldsymbol{\alpha}_s,\beta_s}(\bar{\mathbf{g}}_s) = f_{\boldsymbol{\alpha}_s,\beta_s}(\tilde{\mathbf{g}}_s) \\ &\le f_{\boldsymbol{\alpha}_s,\beta_s}(\bar{\mathbf{g}}_{s+1}) = f_{\boldsymbol{\alpha}_{s+1},\beta_{s+1}}(\bar{\mathbf{g}}_{s+1}) \le \cdots \end{aligned}$$

where s is the iteration number.

The finite transmit power and channel gain lead to an upper bound to the sum rate, so the procedure must converge. And the transmit powers must satisfy the KKT conditions when convergent [4].

3.2 Optimal thermal noise power

In this case we fix $\mathbf{u}_b$ and $\bar{g}_b$ to derive the following:

$$\begin{aligned} &\min_{\bar{\mathbf{q}}} \sum_b \log\left(1+\frac{\bar{g}_b\bar{h}_{bb}}{\sum_{b'\neq b}\bar{g}_{b'}\bar{h}_{bb'}+\bar{q}_b}\right),\\ &s.t.\begin{cases}\bar{\mathbf{q}}^T\mathbf{P}_T\leq\sigma_n^2\mathbf{1}^T\mathbf{P}_T\\ \bar{q}_b\geq 0,\forall b.\end{cases}\end{aligned} \tag{13}$$

Proposition 3 (13) *is a convex optimization problem and its solution must be global optimal.*

Proof One can obtain the Hesse matrix of the objective function of (13), which is a diagonal matrix. The diagonal elements can be expressed as follow:

$$a_{ii}=\frac{\left(2\left(\sum_{j\neq i}\bar{g}_j\bar{h}_{ij}+\bar{q}_i\right)+\bar{g}_i\bar{h}_{ii}\right)\bar{g}_i\bar{h}_{ii}}{\ln 2\left(\left(\sum_{j\neq i}\bar{g}_j\bar{h}_{ij}+\bar{q}_i\right)^2+\bar{g}_i\bar{h}_{ii}\sum_{j\neq i}\bar{g}_j\bar{h}_{ij}+\bar{q}_i\right)^2}\geq 0,\quad i\in\{1,2,\ldots,B\},$$

where a_{ii} represents the element in the i-th row and the i-th column of the Hesse matrix. With $\bar{\mathbf{q}}>0$ and all other elements nonnegative, the Hesse matrix is semi-positive definite, which indicate the convexity of the objective function. With linear constraints, (13) is a convex optimization problem. Thus its KKT point is global optimal.

The Lagrangian dual problem of (13) can be denoted as: $\max\limits_{\lambda_q}\min\limits_{\bar{q}\geq 0} L(\bar{\mathbf{q}},\lambda_q)$, $\lambda_q\geq 0$, where

$$L(\bar{\mathbf{q}},\lambda_q)=\sum_b\log_2\left(1+\frac{\bar{\mathbf{g}}_b\bar{\mathbf{h}}_{bb}}{\sum_{b'\neq b}\bar{\mathbf{g}}_{b'}\bar{\mathbf{h}}_{bb'}+\bar{\mathbf{q}}_b}\right)-\lambda_q\left(\sigma_n^2\sum_b P_{T,b}-\sum_b\bar{\mathbf{q}}_b\mathbf{P}_{T,b}\right), \tag{14}$$

with λ_q the Lagrangian multiplier.

Using KKT condition and considering the constraint of $\bar{\mathbf{q}}\geq 0$, we can update $\bar{\mathbf{q}}$ as follow:

$$\begin{aligned} \bar{q}_b&=\max\left\{\frac{-\bar{\mathbf{g}}_b\bar{\mathbf{h}}_{bb}+t}{2}-\sum_{b'\neq b}\bar{\mathbf{g}}_{b'}\bar{\mathbf{h}}_{bb'},0\right\},\\ t&=\sqrt{\left(\bar{\mathbf{g}}_b\bar{\mathbf{h}}_{bb}\right)^2+\frac{\bar{\mathbf{g}}_b\bar{\mathbf{h}}_{bb}}{\ln 2\left(\lambda_q\mathbf{P}_{T,b}\right)}}.\end{aligned} \tag{15}$$

And λ_q is updated by

$$\lambda_q^{(s+1)}=\left[\lambda_q^{(s)}+\varepsilon_2\left(\sum_b\bar{q}_b P_{T,b}-\sigma_n^2\sum_b P_{T,b}\right)\right]^+. \tag{16}$$

The procedure of its solution is very similar to part A (Transmit power optimization) which is denoted in Table 2.

Convergence follows from convexity. It should be noted that the changes between adjoining two iterations may be very little as discussed in the next section since the updating step needs to be small enough. One can easily show that the convergence must occur on the boundary where the constraint is satisfied with equality since increasing the value of arbitrary $\bar{q}_b$ can lower the value of the objective function as long as $\bar{\mathbf{q}}^T\mathbf{P}_T<\sigma_n^2\mathbf{1}^T\mathbf{P}_T$. So in practice, to speedup convergence, we can add another convergence condition to increase the updating step appropriately, which is that the constraint of the weighed sum of $\bar{\mathbf{q}}$ is satisfied with equality, while still avoid advance convergence.

Table 2 The sub-algorithm 2

1:	Initialize $\lambda_{q,0}$ and S_{max}, set $s = 0$
2:	**Repeat**
2.1	update $\bar{\mathbf{q}}_{b,s+1}$ by (14), $d_q = \left|\bar{\mathbf{q}}_{b,s+1} - \bar{\mathbf{q}}_{b,s}\right|$
2.2	update $\lambda_{q,s+1}$ by (15), $d_\lambda = \left|\lambda_{q,s+1} - \lambda_{q,s}\right|$
2.3	$s = s + 1$
3:	**Until** convergence ($d_q \leq \Delta_2 andd_\lambda \leq \Delta_3$) or $s = s_{max}$

3.3 Optimal Beamforming Vector

Assume that $\left|\mathbf{u}_b^*\right| = 1$. Here we use the optimal MMSE receive criterion to update the uplink beamforming:

$$\mathbf{u}_b = \zeta_b \left(\sum_{b'} \bar{g}_{b'} \mathbf{h}_{bb'} \left(\mathbf{h}_{bb'}\right)^H + \bar{q}_{b'} \mathbf{I}_{M\times M} \right)^{-1} \mathbf{h}_{bb}. \tag{17}$$

Proof The received signal is

$$y_b = \left(\mathbf{u}_b\right)^H \left(\sum_{b'} \sqrt{\bar{g}_{b'}} \mathbf{h}_{bb'} x_{b'} + \bar{q}_b \right),$$

where $x_{b'}$ indicates the signal from the b'th user, and

$$E\left(x_{b'}\right) = 0,\ E\left(x_i x_j^*\right) = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}.$$

According to the MMSE criterion:

$$\mathbf{u}_b = \arg\min_{\mathbf{u}_b} E\left(\left|x_b - y_b\right|^2\right),$$

we can get

$$\begin{aligned} E\left(\left|x_b - y_b\right|^2\right) &= E\left(x_b x_b^*\right) - \sqrt{g_b} \mathrm{Re}\left\{\left(\mathbf{u}_b\right)^H \mathbf{h}_{bb}\right\} E\left(x_b x_b^*\right) \\ &\quad + \sum_{b'} g_{b'} \left|\left(\mathbf{u}_b\right)^H \mathbf{h}_{bb'}\right| E\left(x_{b'} x_{b'}^*\right) + q_b \left|\left(\mathbf{u}_b\right)^H\right|. \end{aligned}$$

Let $\nabla_{\mathbf{u}_b} E\left(\left|x_b - y_b\right|^2\right) = 0$, we have

$$-\sqrt{\bar{g}_b}\mathbf{h}_{bb} + \sum_{b'} \bar{g}_{b'} \mathbf{h}_{bb'} \left(\mathbf{h}_{bb'}\right)^H \mathbf{u}_b + \bar{q}_b \mathbf{I} \mathbf{u}_b = 0.$$

Since $q_b^> 0$ in actual system, $\sum_{b'} g_{b'} \mathbf{h}_{bb'} \left(\mathbf{h}_{bb'}\right)^H + q_b \mathbf{I}$ is positive definite matrix and it can be inversed. After some manipulation, the proposition can be proved.

ζ_b is calculated by

$$\zeta_b = \frac{1}{\left|\left(\sum_{b'} \bar{g}_{b'}^* \mathbf{h}_{bb'} \left(\mathbf{h}_{bb'}\right)^H + \bar{q}_{b'}^* \mathbf{I}\right)^{-1} \mathbf{h}_{bb}\right|}, \forall b. \tag{18}$$

Table 3 The I-MC-DB algorithm

1:	Initialize the max iteration number s_{max} and set $s = 0$
2:	Initialize $\bar{\mathbf{g}}_0$ and $\bar{\mathbf{q}}_0$
3:	**Repeat**
3.1	update beamforming matrix $\mathbf{U}_{s+1}$ according to (17), $d_u = \left\| \mathbf{U}_{s+1} - \mathbf{U}_s \right\|_F$
3.2	solve the sub-problem (5) and update $\bar{\mathbf{g}}_{s+1}$
3.3	solve (12) to obtain $\bar{\mathbf{q}}_{s+1}$
3.4	$s = s + 1$
4:	**Until** convergence ($d_u \leq \Delta_4$) or $s = s_{max}$

3.4 I- MC-DB Algorithm

Since the update of $\mathbf{u}_b^*$ in part C may be different from the previous one, it needs to implement the algorithms in part A and part B again to update the corresponding values of optimal transmission power and thermal noise power. Motivated by the idea of solving the problems in Sects. 3.1 and 3.2, which are mixed optimizations, we solve the inner maximization and the outer minimization in (4) separately. This yields the I-MC-DB algorithm which is described in Table 3. We apply the converged beamforming vectors to the primal problem. The convergence of this iteration algorithm has not been proved in this work. However, simulations in the next section show that the algorithm can always converge and the converged beamforming vectors can always improve the system sum rate. It is worth noting that the iteration algorithm (10) works for asynchronous situation, To be specific, the updates of some $\bar{g}_b$ can use the outdated information of $\bar{g}_{b'}$ where $b' \neq b$. In other words, it allows updates of some $\bar{g}_b$be faster than others but still achieve the unique convergence [17]. In addition when parts of the channel information are outdated due to some kinds of delay, performance of the proposed algorithm will be affected.

In fact, the sequence of the three sub-optimal processes can be exchanged and, as a result, different starting points may lead to different converged points. Here we discuss three staring points:

Starting point case 1: each BS calculates the beamforming vector to the corresponding MS independently aiming at maximize SNR in the MS side and can be expressed as

$$\mathbf{u}_b = \frac{(\mathbf{h}_{bb})^H}{|\mathbf{h}_{bb}|} \tag{19}$$

The algorithm starts with these beamforming vectors. The sequence of the three sub-optimal processes is first solving (5) and then solving (12), which is followed by the update of beamforming matrix.

Starting point case 2: it starts with the initialized $\bar{\mathbf{g}}$ and $\bar{\mathbf{q}}$ instead of beamforming vectors and is the same as that depicted in Table 3.

Starting point case 3: the starting beamforming vectors are set randomly. The sequence of the sub-optimal processes is the same as Starting point case 1.

4 Simulation

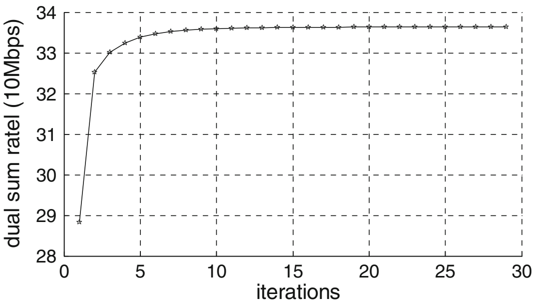
Our simulation considers 7 cells and 4 antennas in each BS. Note that scenario considered in this work can include multiple MSs with each sub-channel of each cell assigned just one MS. Thus, to evaluate the performance of the proposed algorithm, it is equivalent to assume that only a single MS is allocated to each cell. We evaluate the performance of the cell instead of the performance of a specific sub-channel. To simplify the model, we also assume each BS has the same transmit power value. The main parameters of simulation are expressed as follows:

- Transmission power of BS: 46 dBm
- System frequency bandwidth: 10 MHz
- Power spectral density of AWGN: -174 dBm/Hz
- Antenna configuration: 4 * 1, Omni antenna
- Path loss: $L = 128.1 + 37.6 \log_{10}(R)$, R in kilometers
- Penetration Loss: 20 dB
- Shannon capacity $\log_2(1 + SINR)$ is used to calculate the rate of each MS.

In addition, Δ_1, Δ_2, Δ_3, Δ_4 are set as $0.01, 0.01, 0.01 * 10^{-8}$ and 0.01 respectively. It should also be note that the update step and the tolerable error for sub-problem (8) need to be small enough to avoid premature convergence.

We first show the results of the sub-algorithms of Sects. 3.1 and 3.2. Convergence of the sub-algorithm 1 and sub-algorithm 2 in Sect. 3 are described in Figs. 1 and 2 respectively. In Fig. 1, where x-axis denotes the times of updating the lower bound of the objective func-

Fig. 1 Convergence of transmit power optimization



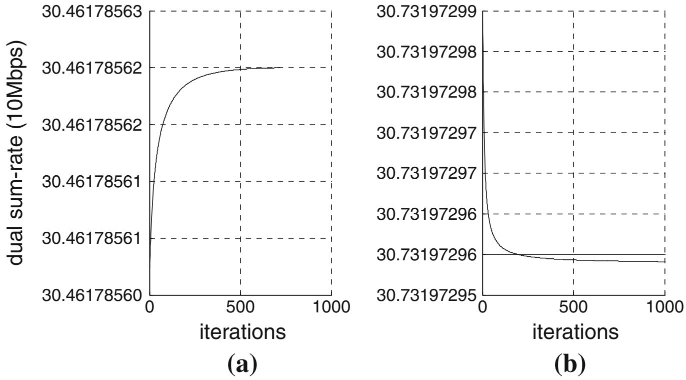


Fig. 2 Convergence of optimal thermal noise power

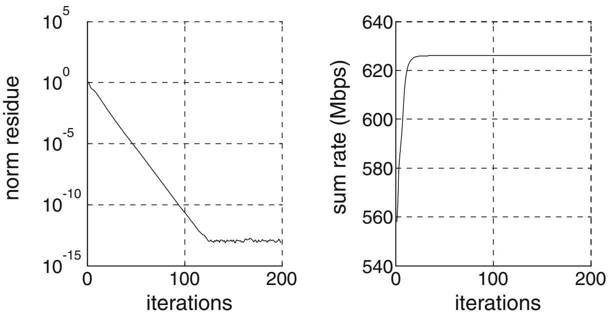


Fig. 3 Convergence of I-MC-DB algorithm

tion, it can be confirmed that the value of objective function of sub-maximization problem is monotonically increased at each operation and the sub-algorithm 1 turns to be efficient. Interestingly, we found that when the I-MC-DB converges, the values of transmit power usually lie near the boundary, which is coincident with the conclusion in [18] about the optimal power values. It is observed from Fig. 2 that it may need several hundred times of updating $\bar{\mathbf{q}}$ to find the optimal solution of sub-minimization. This is due to the fact that the step for updating Lagrangian dual variable λ_q needs to be small enough since it is in the denominator of (14) and a small change of λ_q may make a large difference of certain q_b, especially when λ_q is near zero. There are two different convergent traces for sub-algorithm 2 which can be found in Fig. 2. Fig. 2a describes the initial values of $\bar{\mathbf{q}}$ by (14) are out of feasible domain and Fig. 2b shows the situation of those initial values are in the feasible domain. This can be explained by the convexity of the problem since the objective function of (12) is monotonically decreasing when $\bar{\mathbf{q}} \geq 0$. And if the weighted sum of initial values of $\bar{\mathbf{q}}$ exceeds the sum power constraint, the value of the objective function computed by $\bar{\mathbf{q}}$ will be less than the optimal value of (12) and the convergent trace will be monotonically increasing, which applies to Fig. 2a, else the convergent trace goes as Fig. 2b. So it is a global convergence for all starting points. In the simulation, values of $\bar{\mathbf{q}}$ are much less than $\bar{g}_{b'}\bar{h}_{bb'}$, thus, the optimization of $\bar{\mathbf{q}}$ will have very little affection on the dual sum rate in each iteration.

In Fig. 3, the norm residue (Euclidean norm distance between the optimal beamforming matrix and the beamforming matrix at the sth iteration) is illustrated against the number of iterations, which is defined as [4]

$$R_s = \left\| u_s - u^* \right\|_F ,$$

where $\mathbf{u}_s$ is the beamforming matrix at the sth iteration and $\mathbf{u}^*$ indicates the optimal beamforming matrix. In Fig. 3, the x-axis presents the iteration times of computing (3), (5) and (12) together. Convergences of I-MC-DB algorithm are always found during plenty times of simulations and efficient since the numbers of iteration are always less than 30 times. This is an encouraging result for further study of the convergence of this algorithm. It should be noted that the value of norm residue of beamforming matrix over iteration may not be monotonic since every time updating beamforming matrix by MMSE will change the value of $\bar{\mathbf{g}}$ and $\bar{\mathbf{q}}$ and they would in turn affect the

optimal beamformers through MMSE method (3). However, when it is near the convergent beamforming matrix, it turns to be monotonic. It is observed that the convergence performance of our algorithms showed in the figures is in accordance with the theoretical derivation.

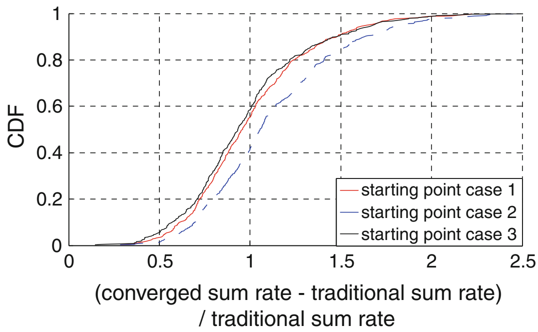


Fig. 4 CDF curves of the relative improvement on sum rate

Compared with the traditional way of beamforming, where each BS schedules independently and computes the beamforming vectors based on (19), the relative improvements on sum rate of the proposed algorithm is depicted in Fig. 4. Even though the optimum solution of the dual problem may not be reached, the converged beamforming vectors can always contribute significantly to the improvement of system sum rate, with the average gain being about 100%, as shown in the figure. It also shows that starting point case 2 performs comparatively better than the other two cases. Therefore, the proposed algorithm shows to be efficient.

5 Conclusion

In this work, we have investigated the strategy for multi-cell optimal downlink beamforming problem in MISO system with per-base station power constraints and single user per cell. This paper deals with beamformer design for multi-cell transmission, which is still a largely unresolved problem. An iterative algorithm to obtain sub-optimal solutions satisfying the necessary optimality conditions of the dual uplink problem is provided. Moreover, the converged beamforming vectors of the proposed algorithm can be employed to improve system sum rate. Numerical results validate the convergence and efficiency of the provided I-MC-DB algorithm. However, numbers of open problems still remain. Indeed, convergence of I-MC-DB algorithm is not rigorously proven. Also, solutions of the optimum value of the dual uplink problem and the starting points of transmit power and noise power of I-MC-BD need to be further studied.

Acknowledgments This work was supported by Key project (2009ZX03003-008-01) of China, National Natural Science Foundation of China (60811120097) and Research Institute of China Mobile Communications Corporation.

References

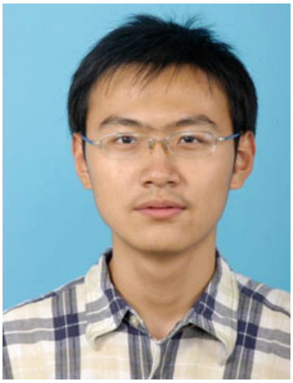
1. Jorgušeski, L., Le, T. M. H., Fledderus, E. R., & Prasad, R. (2008). *Downlink resource allocation for evolved UTRAN and WiMAX cellular systems*. In 19th IEEE international symposium on personal, indoor and mobile radio communications (PIMRC 2008).
2. Zhang, X., He, C., Jiang, L., & Xu, J. (2008). Inter-cell interference coordination based on softer frequency reuse in OFDMA cellular systems. *IEEE Neural Networks and Signal Processing*, 270–275.
3. Assaad, M. (2008). Optimal fractional frequency reuse (FFR) in multicellular OFDMA system. *IEEE Vehicular Technology Conference* 1–5.

4. Venturino, L., Prasad, N., & Wang, X. (2009). Coordinated scheduling and power allocation in downlink multicell OFDMA networks. *IEEE Transactions on Vehicular Technology, 58*(6), 2835–2848.
5. Gjendemsjø, A., Gesbert, D., Øien, G. E., & Kiani, S.G. (2008). Binary power control for sum rate maximization over multiple interfering links. *IEEE Transactions on Wireless Communications, 7*(8), 3164–3173.
6. Zhong, C. X., Li, C. G., Yang, L. X., & He, Z. Y. (2008). *Dynamic resource allocation for the downlink of multi-cell systems with full spectral reuse*. In IEEE neural networks and signal processing, 2008 international conference (pp. 173–177).
7. Karakayali, M., Foschini, G., & Valenzuela, R. (2006). Network coordination for spectrally efficient communications in cellular systems. *IEEE Wireless Communications Magazine, 13*(4), 56–61.
8. Zhang, H., & Dai, H. (2004). Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks. *EURASIP Journal on Applied Signal Processing, 2*, 222–235.
9. Venkatesan, S. (2007). *Coordinating base stations for greater uplink spectral efficiency in a cellular network*. In Proceedings of international symposium on personal, indoor, mobile radio communications (PIMRC) (pp. 1–5).
10. Luo, Z.-Q., & Yu, W. (2006). An introduction to convex optimization for communications and signal processing. *IEEE Journal on Selected Areas in Communications, 24*(8), 1426–1438.
11. Yu, W., & Lan, T. (2007). Transmitter optimization for the multi-antenna downlink with per-antenna power constraints. *IEEE Transactions on Signal Processing, 55*(6), 2646–2660.
12. Dahrouj, H., & Yu, W. (2008). *Coordinated beamforming for the multi-cell multi-antenna wireless system*. In Proceedings of 42nd annual conference on information sciences and systems (pp. 429–434).
13. Chang, Y., Tao, Z. Zhang, J., Kuo, J. “Multi-cell ofdma downlink resource allocation using a graphic framework,” *IEEE Trans. Veh. Technol.,* in press.
14. Janghoon, Y., & Ku Kim, D. (2008). Multi-cell uplink-downlink beamforming throughput duality based on Lagrangian duality with per-base station power constraints. *IEEE Communications Letters, 12*(4), 277–279.
15. Papandriopoulos, J., & Evans, J. (2006). *Low-complexity distributed algorithms for spectrum balancing in multi-user dsl networks*. In Proceedings of 2006 IEEE international conference on communications (Vol. 7, pp. 3270–3275).
16. Boyd, S., & Vanderberghe, L. (2004). *Convex Optimization*. Cambridge: Cambridge University Press.
17. Yates, R. D. (1995). A framework for uplink power control in cellular radio systems. *IEEE Journal on Selected Areas in Communications, 13*(7), 1341–1348.
18. Gjendemsjø, A., Gesbert, D., Øien, G. E., & Kiani, S. G. (2008). Binary power control for sum rate maximization over multiple interfering links. *IEEE Transactions on Wireless Communictions, 7*(8), 3164–3173.

Author Biographies

Fan Huang He received B.E. degree in Communication Engineering from Beijing University of Posts and Telecommunications (BUPT), Beijing, China in 2006. He is currently a Ph.D. candidate in Communications and Information Systems at BUPT. His research interests are in the areas of wireless communications, especially the cooperation communication technology and radio resource management. He is a student member of both IEEE and IEICE.

Yafeng Wang He is an associate professor of Electronic Engineering in the School of Information and Telecommunications at Beijing University of Posts and Telecommunications (BUPT). His research mainly focuses on wireless communications and information theory. He received his Ph.D, M.E. and B.Sc. in 2003, 2000 and 1997 respectively from BUPT, University of Electronic Science and Technology of China and Baoji University of Arts and Science. In 2008, he worked as a visiting scholar in the Faculty of Engineering and Surveying at the University of Southern Queensland, Australia. He is a senior member of IEEE.



Jian Geng He received B.E. degree in Communication Engineering from Beijing University of Posts and Telecommunications (BUPT) in 2009. Now he is pursuing his M.S. degree in Communications and Information Systems at BUPT. His research interests focus on radio resource management and cooperative transmission of wireless communications.

Dacheng Yang He received M.S. (1982) and Ph.D. (1988) degrees in Circuits and Systems from Beijing University of Posts and Telecommunications (BUPT). From 1992 through 1993 he worked at the University of Bristol, United Kingdom, as a senior visiting scholar, where he was engaged in Project Link-CDMA of the RACE program. In 1993 he returned to BUPT as an associate professor. Currently he is a professor and doctoral advisor at BUPT, the director of the Wireless Theories and Technologies (WT&T) Laboratory. He is also the chief Chinese scientist of the BUPT-Qualcomm Joint Research Center. His research interests include communications theories, wireless applications, domestic production and intellectual property of mobile communications system.