Wireless Pers Commun (2012) 62:633–653
DOI 10.1007/s11277-010-0086-5

Performance Analysis of Space-Time Coded CDMA System Over Nakagami Fading Channels with Perfect and Imperfect CSI

Xiangbin Yu · **Dazhuan Xu**

Published online: 21 July 2010

Abstract In this paper, the performance of multiuser CDMA systems with different space time code schemes is investigated over Nakagami fading channel. Low-complexity multiuser receiver schemes are developed for space-time coded CDMA systems with perfect and imperfect channel state information (CSI). The schemes can make full use of the complex orthogonality of space-time coding to obtain the linear decoding complexity, and thus simplify the exponential decoding complexity of the existing scheme greatly. Moreover, it can achieve almost the same performance as the existing scheme. Based on the bit error rate (BER) analysis of the systems, the theoretical calculation expressions of average BER are derived in detail for both perfect CSI and imperfect CSI, respectively. As a result, tight closed-form BER expressions are obtained for space-time coded CDMA with orthogonal spreading code, and approximate closed-form BER expressions are attained for space-time coded CDMA with quasi-orthogonal spreading code. Computer simulation for BER shows that the theoretical analysis and simulation are in good agreement. The results show that the space-time coded CDMA systems have BER performance degradation for imperfect CSI.

Keywords Multiuser receiver · Code division multiple access (CDMA) · Space-time block code · Nakagami fading channel · Imperfect channel state information

1 Introduction

The ever increasing demand of information has stimulated much interest in MIMO communication system for the development of reliable high data rate transmission over fading channels [1,2]. Especially, space-time coding scheme in MIMO systems can provide

X. Yu (✉) · D. Xu
College of Information Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing, China
e-mail: yxbxwy@nuaa.edu.cn

X. Yu
Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China

effective diversity for combating fading effect [3–10], and has received much interest. But in [4], it is proved that for space-time block code (STBC), a complex orthogonal design which provides full diversity and full rate is not possible for more than two antennae. Considering that full rate transmission is very important to implement high data rate service in wireless environment, and also important for low signal to noise ratios [5], and that low-complexity space-time block decoding algorithm is necessary due to the restriction of receiver size and power, low-complexity and full-rate space-time coding (STC) schemes for three transmit antenna (3Tx) and 4Tx are developed in our previous paper [6]. These full-rate schemes can form efficient spatial interleaving. So concatenated with channel coding, their performance will be improved greatly and outperform the full-diversity STC schemes. Moreover, their complexity for encoding and decoding is also low.

However, the above space-time coding schemes are designed for single-user environment, thus they will be not suitable for multiuser scenario in practice. Hence, it is necessary to extend the above schemes into multiuser CDMA scenario for practical purposes. For superior space-time coded CDMA (STC-CDMA) system, it not only possesses the performance of good STC schemes, but also effectively suppresses the multiuser interference (MUI). Based on different multiuser space-time coded system models, Refs. [11–13] give the corresponding receiver schemes. In [12], the bit error rate (BER) analysis is provided for STC-CDMA system with the conventional matched filter receiver, but the analysis method and system model are applicable only to the BPSK modulation and downlink, and the BER analysis is restricted to the integer m for Nakagami-m fading channel. In [13], a minimum variance linear receiver scheme for multiuser MIMO system is proposed, but the system does not consider the advantage offered by CDMA technique, and needs to design and optimize the weighted matrix to suppress MUI. As a result, the computational complexity will be improved greatly. The performance of space-time coded multicarrier CDMA system is analyzed in Nakagami fading channel [14], but the analysis is limited in Alamouti's space-time code and BPSK modulation. Reference [11] gives the effective combination of CDMA system and different STCs, and the developed decorrelative receiver scheme can decouple the detection of different users, but the decoding complexity is exponential for each user, which will not benefit practical application. Moreover, [11,13] do not provide the BER performance analysis, and the schemes are limited in Rayleigh fading channel. Furthermore, the above references all assumed that the perfect channel state information (CSI) can be available at the receiver. In practice, however, the CSI will be imperfect due to the channel estimation errors. For this reason, we will give the performance analysis of different multiuser STC-CDMA systems in Nakgami fading channel with imperfect CSI. By utilizing the maximum ratio combining (MRC) method and complex orthogonality of STC, low-complexity multiuser receiver schemes are developed for both perfect and imperfect CSI in Nakagami fading channel. With the developed schemes, each user has linear rather than exponential decoding complexity after decorrelating. Based on the orthogonal/quasi-orthogonal spreading codes and the mathematical calculation, the theoretical average BER expressions of the systems are derived in detail for both perfect and imperfect CSI. With these expressions, the impact of imperfect CSI on the BER performance can be effectively analyzed and assessed. Simulation results show that the performance of the proposed scheme is very close to that of the existing scheme [11] with low computation complexity. Moreover, the derived theoretical expressions can match the simulated values well.

The notations we use throughout this paper are as follows. Bold upper case and lower case letters denote matrices and column vectors, respectively. The superscripts $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ represent the Hermitian transposition, transposition, and complex conjugation, respectively.

$E\{.\}$ denotes the statistical expectation, $\|.\|_F$ denotes the matrix Frobenius norm, and we denote the $M \times M$ identity matrix as $\mathbf{I}_M$ and all-zero matrix as $\mathbf{0}_M$.

2 System Model

In this section, we consider a synchronous CDMA communication system with M transmit antennae and K receive antennae and U active users that operates in a MIMO Nakagami flat-fading environment. The multiuser CDMA system adopts the space-time coding scheme (such as full-diversity G_2, G_3, H_3, G_4, H_4 code [3], full-rate X code [6]) to transmit the data. For each user u, the complex element $h_{u,ki} = \alpha_{u,ki}\exp(j\theta_{u,ki})$ denotes the channel gain from the i-th transmit antenna to the k-th receive antenna, which is assumed to be constant over a frame but varied from one frame to another. It has unit power. For Nakagami fading channel, the probability density function (pdf) of $\alpha_{u,ik}$ is given by [15,16]

$$f(\alpha) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \alpha^{2m-1}\exp(-m\alpha^2/\Omega) \tag{1}$$

where $\Gamma(.)$ is the Gamma function [17], $\Omega = E(\alpha_{u,ki}^2) = 1$ is the average fading power and $m \geq 1/2$ [15]. The phase $\theta_{u,ki}$ is uniformly distributed over $[0, 2\pi]$. The Rayleigh distribution, which corresponds to $m = 1$, is a special case of Nakagami-m distribution.

A complex orthogonal space-time coding, which is represented by an $M \times T$ transmission matrix $\mathbf{D}$, is used to encode L input symbols into an M-dimensional vector sequence of T time slots. The matrix $\mathbf{D}$ is a linear combination of L symbols satisfying the complex orthogonality: $\mathbf{D}\mathbf{D}^H = \varepsilon(|d_1|^2 + \cdots + |d_L|^2)\mathbf{I}_M$, where $\{d_l\}_{l=1,\ldots,L}$ are the L input symbols from the constellation Υ, and ε is a constant which depends on space-time coding matrix [10]. For example, $\varepsilon = 1$ if G_2, H_3 and H_4 are employed; and $\varepsilon = 2$ if G_3 and G_4 are used. Accordingly, the rate of the STBC is $R = L/T$. Considering that H_4 and H_3 code have high rate (3/4), but their code matrices have lots of complex multiplication and complex addition. So in this paper, simple 3/4-rate STBC schemes [7,8], denoted by H_4' for 4Tx and H_3' for 3Tx, are employed for space-time coded CDMA system and corresponding performance analysis. The code matrices for H_4' and H_3' are

$$\mathcal{H}_4' = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \\ s_3^* & -s_2 & 0 & s_1^* \end{bmatrix} \quad \text{and} \quad \mathcal{H}_3' = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \end{bmatrix} \tag{2}$$

respectively. For a fair comparison, the system throughput is defined as Throughput $= R \times R_c \times \eta$ [18], where R_c is code rate of the channel coding, η denotes the bandwidth efficiency of modulation scheme.

3 Multiuser Receiver for STC-CDMA Systems with Perfect and Imperfect CSI

In this section, we will give multiuser receiver design for space-time coded CDMA system for both perfect and imperfect CSI, and low-complexity decoding schemes are developed. For space-time coded CDMA system, the block length of space-time code is set equal to P chip periods. Then, according to [11], the transmitted signal matrix of user u at pth ($p = 1, 2, \ldots, P$) chip period is written as

$$\mathbf{G}_u(p) = \mathbf{D}_u \mathbf{S}_u(p) \tag{3}$$

where $\mathbf{D}_u$ is $M \times T$ space-time block coding matrix of user k, $\mathbf{S}_u(p)$ is $T \times 1$ spreading code, and $\mathbf{S}_u = [\mathbf{S}_u(1), \ldots, \mathbf{S}_u(P)]$ corresponds to T normalized spreading codes of length P used to spread $\mathbf{D}_u$ for user $u(u = 1, 2, \ldots, U)$, here conventional orthogonal Walsh–Hadamard (W–H) code and quasi-orthogonal Gold code in CDMA system are considered. We will employ these different spreading codes for different users as well. Based on the analytical method in [11], at the receiver, we can obtain the baseband received signal at pth ($p = 1, 2, \ldots, P$) chip period as follows

$$\mathbf{R}(p) = \sum_{u=1}^{U} \sqrt{\rho_u} \mathbf{H}_u \mathbf{G}_u(p) + \mathbf{Z}(p), \quad p = 1, \ldots, P \tag{4}$$

where $\mathbf{H}_u = [h_u]_{k,i}$ is $K \times M$ channel matrix of user u. $\mathbf{Z}(p)$, $p = 1, 2, \ldots, P$ is $K \times 1$ noise vector, whose elements $\{z_k(p), k = 1, \ldots, K, p = 1, \ldots, P\}$ are independent, identically distributed (i. i. d) complex Gaussian random variables with zero-mean and unit-variance. $M\rho_u$ denotes the average signal-to-noise ratio (SNR) per receive antenna for user u at the receiver during the transmission of space-time coding matrix $\mathbf{D}_u$ (which corresponds to P chip periods), this SNR adopts the definition similar to [11] for comparison consistency.

Substituting (3) into (4) yields

$$\mathbf{R}(p) = \sum_{u=1}^{U} \sqrt{\rho_u}\, \mathbf{H}_u \mathbf{D}_u \mathbf{S}_u(p) + \mathbf{Z}(p), \quad p = 1, \ldots, P \tag{5}$$

In order to express (5) more compactly, we define the following matrices

$\mathbf{Y}_u = \sqrt{\rho_u} \mathbf{H}_u \mathbf{D}_u$, corresponds to $K \times T$ matrix; $\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_U]$, corresponds to $K \times UT$ matrix;
$\mathbf{S}_u = [\mathbf{S}_u(1), \ldots, \mathbf{S}_u(P)]$, corresponds to $T \times P$ matrix; $\mathbf{S} = [\mathbf{S}_1^T, \ldots, \mathbf{S}_U^T]^T$, corresponds to $UT \times P$ matrix;
$\mathbf{R} = [\mathbf{R}(1), \ldots, \mathbf{R}(P)]$, corresponds to $K \times P$ matrix; $\mathbf{Z} = [\mathbf{Z}(1), \ldots, \mathbf{Z}(P)]$, corresponds to $K \times P$ matrix.

Thus (5) is changed into

$$\mathbf{R} = \sum_{u=1}^{U} \mathbf{Y}_u \mathbf{S}_u + \mathbf{Z} = \mathbf{Y}\mathbf{S} + \mathbf{Z} \tag{6}$$

Then according to Ref. [19], the maximum likelihood (ML) estimate of $\mathbf{Y}$ conditioned on $\{\mathbf{H}_u, u = 1, \ldots, U\}$ and $\{\mathbf{D}_u, u = 1, \ldots, U\}$ is obtained by

$$\hat{\mathbf{Y}} = \mathbf{R}\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1} = \mathbf{Y} + \mathbf{Z}\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1} \tag{7}$$

Here, the Moore–Penrose inverse matrix $\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1}$ can be expressed as a multiuser decorrelator [11,19], and thus the ML estimate $\hat{\mathbf{Y}}$ is effective output of the decorrelator with the input being the received data $\mathbf{R}$. By this decorrelator, the multiuser interference is cancelled, and the detection of different users is decoupled. Based on the block structure of $\mathbf{Y}$, the ML estimate $\hat{\mathbf{Y}}_u$ of $\mathbf{Y}_u$ can be easily achieved. While for user u, all data information on the transmitted code matrix $\mathbf{D}_u$ is contained in $\mathbf{Y}_u$. Hence, we can evaluate the code matrix and corresponding information symbols in terms of the achieved $\hat{\mathbf{Y}}_u$.

According to the definition of $\mathbf{Y}_u$, we can assume that $\hat{\mathbf{Y}}_u = [\hat{\mathbf{y}}_{u,1}^T, \hat{\mathbf{y}}_{u,2}^T, \ldots, \hat{\mathbf{y}}_{u,K}^T]^T$, where $\hat{\mathbf{y}}_{u,k}$ is a $1 \times T$ row vector. Thus when G_4 code scheme is employed, we have: $T = 8$ and

$$\hat{\mathbf{Y}}_u = \begin{bmatrix} \hat{\mathbf{y}}_{u,1} \\ \cdots \\ \hat{\mathbf{y}}_{u,K} \end{bmatrix} = \begin{bmatrix} \hat{y}_{u,11} & \hat{y}_{u,12} & \hat{y}_{u,13} & \hat{y}_{u,14} & \hat{y}_{u,15} & \hat{y}_{u,16} & \hat{y}_{u,17} & \hat{y}_{u,18} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hat{y}_{u,K1} & \hat{y}_{u,K2} & \hat{y}_{u,K3} & \hat{y}_{u,K4} & \hat{y}_{u,K5} & \hat{y}_{u,K6} & \hat{y}_{u,K7} & \hat{y}_{u,K8} \end{bmatrix} \tag{8}$$

Since the transmitted information symbols constitute the code matrix $\mathbf{D}_u$, and $\mathbf{D}_u$ is contained in matrix $\mathbf{Y}_u$, the symbol decision values can be achieved by calculating $\hat{\mathbf{Y}}_u$. Moreover, we find that $\mathbf{Y}_u$ has the same receiver signal form as conventional space-time codes in single user scenario [6,10]. Considering that the receiver only knows the estimation $\hat{\mathbf{H}}_u$ of channel $\mathbf{H}_u$. Thus, we may utilize the MRC method and complex orthogonality of space-time coding, and obtain simple decoding scheme for the space-time coded CDMA system with G_4 code and imperfect CSI after performing multiuser decorrelation, i.e.,

$$\begin{cases} \hat{d}_{u,1} = \underset{d_{u,1}\in\Upsilon}{\arg\min} \Bigg\| \sum_{k=1}^{K} \Big(\hat{y}_{u,k1}\hat{h}_{u,k1}^* + \hat{y}_{u,k2}\hat{h}_{u,k2}^* + \hat{y}_{u,k3}\hat{h}_{u,k3}^* + \hat{y}_{u,k4}\hat{h}_{u,k4}^* + \hat{y}_{u,k5}^*\hat{h}_{u,k1} \\ \qquad + \hat{y}_{u,k6}^*\hat{h}_{u,k2} + \hat{y}_{u,k7}^*\hat{h}_{u,k3} + \hat{y}_{u,k8}^*\hat{h}_{u,k4} \Big) - 2\sqrt{\rho_u} \sum_{k=1}^{K}\sum_{i=1}^{4} (|\hat{h}_{u,ki}|^2) d_{u,1} \Bigg\|^2 \\ \hat{d}_{u,2} = \underset{d_{u,2}\in\Upsilon}{\arg\min} \Bigg\| \sum_{k=1}^{K} \Big(\hat{y}_{u,k1}\hat{h}_{u,k2}^* - \hat{y}_{u,k2}\hat{h}_{u,k1}^* - \hat{y}_{u,k3}\hat{h}_{u,k4}^* + \hat{y}_{u,k4}\hat{h}_{u,k3}^* + \hat{y}_{u,k5}^*\hat{h}_{u,k2} \\ \qquad - \hat{y}_{u,k6}^*\hat{h}_{u,k1} - \hat{y}_{u,k7}^*\hat{h}_{u,k4} + \hat{y}_{u,k8}^*\hat{h}_{u,k3} \Big) - 2\sqrt{\rho_u} \sum_{k=1}^{K}\sum_{i=1}^{4} (|\hat{h}_{u,ki}|^2) d_{u,2} \Bigg\|^2 \\ \hat{d}_{u,3} = \underset{d_{u,3}\in\Upsilon}{\arg\min} \Bigg\| \sum_{k=1}^{K} \Big(\hat{y}_{u,k1}\hat{h}_{u,k3}^* + \hat{y}_{u,k2}\hat{h}_{u,k4}^* - \hat{y}_{u,k3}\hat{h}_{u,k1}^* - \hat{y}_{u,k4}\hat{h}_{u,k2}^* + \hat{y}_{u,k5}^*\hat{h}_{u,k3} \\ \qquad + \hat{y}_{u,k6}^*\hat{h}_{u,k4} - \hat{y}_{u,k7}^*\hat{h}_{u,k1} - \hat{y}_{u,k8}^*\hat{h}_{u,k2} \Big) - 2\sqrt{\rho_u} \sum_{k=1}^{K}\sum_{i=1}^{4} (|\hat{h}_{u,ki}|^2) d_{u,3} \Bigg\|^2 \\ \hat{d}_{u,4} = \underset{d_{u,4}\in\Upsilon}{\arg\min} \Bigg\| \sum_{k=1}^{K} \Big(\hat{y}_{u,k3}\hat{h}_{u,k2}^* - \hat{y}_{u,k1}\hat{h}_{u,k4}^* - \hat{y}_{u,k2}\hat{h}_{u,k3}^* - \hat{y}_{u,k4}\hat{h}_{u,k1}^* - \hat{y}_{u,k5}^*\hat{h}_{u,k4} \\ \qquad + \hat{y}_{u,k7}^*\hat{h}_{u,k2} - \hat{y}_{u,k6}^*\hat{h}_{u,k3} - \hat{y}_{u,k8}^*\hat{h}_{u,k1} \Big) - 2\sqrt{\rho_u} \sum_{k=1}^{K}\sum_{i=1}^{4} (|\hat{h}_{u,ki}|^2) d_{u,4} \Bigg\|^2 \end{cases} \tag{9}$$

where $\{\hat{h}_{u,ki}\}$ are the elements of $\hat{\mathbf{H}}_u$, and they are the estimated values of channel coefficients $\{h_{u,ki}\}$. Due to the estimation errors, the available CSI will be imperfect, and thus the estimation matrix $\hat{\mathbf{H}}_u$ is different with actual channel matrix $\mathbf{H}_u$. Only with perfect estimation, $\hat{\mathbf{H}}_u$ will be the same as $\mathbf{H}_u$.

Similarly, when another 4-antenna H_4 code in (2) is used for STC-CDMA system (correspondingly, $T = 4$ and $L = 3$), we may obtain the following simple decoding scheme with imperfect CSI:

$$\begin{cases} \hat{d}_{u,1} = \underset{d_{u,1}\in\Upsilon}{\arg\min} \left\| \sum_{k=1}^{K} \left[\hat{y}_{u,k1}\hat{h}^*_{u,k1} + \hat{y}_{u,k2}\hat{h}^*_{u,k2} + \hat{y}^*_{u,k3}\hat{h}_{u,k3} + \hat{y}^*_{u,k4}\hat{h}_{u,k4}\right] - \sqrt{\rho_u}\sum_{k=1}^{K}\sum_{i=1}^{4}(|\hat{h}_{u,ki}|^2)d_{u,1} \right\|^2 \\ \hat{d}_{u,2} = \underset{d_{u,2}\in\Upsilon}{\arg\min} \left\| \sum_{k=1}^{K} \left[\hat{y}_{u,k3}\hat{h}^*_{u,k1} + \hat{y}^*_{u,k4}\hat{h}_{u,k2} - \hat{y}^*_{u,k1}\hat{h}_{u,k3} - \hat{y}_{u,k2}\hat{h}^*_{u,k4}\right] - \sqrt{\rho_u}\sum_{k=1}^{K}\sum_{i=1}^{4}(|\hat{h}_{u,ki}|^2)d_{u,2} \right\|^2 \\ \hat{d}_{u,3} = \underset{d_{u,3}\in\Upsilon}{\arg\min} \left\| \sum_{k=1}^{K} \left[\hat{y}^*_{u,k1}\hat{h}_{u,k4} - \hat{y}_{u,k2}\hat{h}^*_{u,k3} + \hat{y}^*_{u,k3}\hat{h}_{u,k2} - \hat{y}_{u,k4}\hat{h}^*_{u,k1}\right] - \sqrt{\rho_u}\sum_{k=1}^{K}\sum_{i=1}^{4}(|\hat{h}_{u,ki}|^2)d_{u,3} \right\|^2 \end{cases} \tag{10}$$

For perfect CSI case, the estimation matrix $\hat{\mathbf{H}}_u = \mathbf{H}_u$, and thus the above simple decoding schemes for different STC-CDMA systems under perfect CSI can be easily obtained by substituting $\hat{\mathbf{H}}_u$ with $\mathbf{H}_u$ in Eqs. (9) and (10). The above analysis method for receiver scheme design can be applied to the other space-time coding schemes (such as G_2, G_3, H'_3, and X code, etc.) based STC-CDMA systems, and accordingly, the simple decoding schemes are obtained for both imperfect and perfect CSI.

From Eqs. (9) to (10), it is observed that our developed decoding scheme has linear complexity. For Ref. [11], its receiver decoding schemes with coherent detection (i.e. (44) and (45) in [11]) are shown:

1. For general spreading codes:

$$\hat{\mathbf{D}}_u = \underset{\{d_{u,1},\ldots,d_{u,L}\}\in\Upsilon}{\arg\min} \{\mathrm{vec}^H(\hat{\mathbf{Y}}_u - \sqrt{\rho_u}\mathbf{H}_u\mathbf{D}_u)\Phi_k^{-1}\mathrm{vec}(\hat{\mathbf{Y}}_u - \sqrt{\rho_u}\mathbf{H}_u\mathbf{D}_u)\} \tag{11}$$

2. For orthogonal spreading codes:

$$\hat{\mathbf{D}}_u = \underset{\{d_{u,1},\ldots,d_{u,L}\}\in\Upsilon}{\arg\min} \left\|\hat{\mathbf{Y}}_u - \sqrt{\rho_u}\mathbf{H}_u\mathbf{D}_u\right\|_F^2 \tag{12}$$

With the above two equations, we can see that the decoding scheme in [11] has exponential complexity. Namely, if Υ is a constellation consists of Q symbols, the search times to obtain the transmitted L symbols is Q^L. Thus, when Q and L become larger, the complexity will be much higher, which will result in the significant increase of implementation complexity of the system. While for the developed scheme, the needed search times are only LQ. According to the above analysis, we may give the complexity comparison between the developed scheme and existing scheme [11] in Table 1. In Table 1, scheme 1 and scheme 2 represent the existing decoding scheme [11] and our improved scheme, respectively. From this table, we observe that the proposed scheme 2 has lower complexity than the existing scheme 1. Especially, when the constellation size Q and the number of transmitted symbols L are larger, the low-complexity advantage of our scheme 2 becomes more significant.

Table 1 Comparison of complexity

	QPSK $Q = 4$, $L = 2$	8PSK $Q = 8$, $L = 2$	16QAM $Q = 16$, $L = 3$	16QAM $Q = 16$, $L = 4$	64QAM $Q = 64$, $L = 3$
Scheme 1	16	64	4096	65, 536	262, 144
Scheme 2	8	16	48	64	192

4 BER Analysis of STC-CDMA Systems with Perfect and Imperfect CSI

In this section, we will give the BER performance analysis of space-time coded CDMA system in Nakagami-m fading channel. Let $\overline{\mathbf{Z}} = \mathbf{ZS}^{\mathrm{H}}(\mathbf{SS}^{\mathrm{H}})^{-1}$, and $\overline{\mathbf{Z}}_u$ be $K \times T$ submatrix of $\overline{\mathbf{Z}}$ consisting of the columns starting from $(u-1)T+1$ to uT, then $\overline{\mathbf{Z}} = [\overline{\mathbf{Z}}_1, \ldots, \overline{\mathbf{Z}}_U]$. Considering that the elements of $\overline{\mathbf{Z}}$ are linear combing of complex Gaussian noise variables $\{z_k(p)\}$, these elements are complex Gaussian variables with zero mean. Thus the covariance matrix of $\overline{\mathbf{Z}}$ can be written as

$$
\begin{aligned}
E\{\overline{\mathbf{Z}}^H\overline{\mathbf{Z}}\} &= E\{[\mathbf{ZS}^{\mathrm{H}}(\mathbf{SS}^{\mathrm{H}})^{-1}]^H\mathbf{ZS}^{\mathrm{H}}(\mathbf{SS}^{\mathrm{H}})^{-1}\} \\
&= (\mathbf{SS}^{\mathrm{H}})^{-1}\mathbf{S}E\{\mathbf{Z}^H\mathbf{Z}\}\mathbf{S}^{\mathrm{H}}(\mathbf{SS}^{\mathrm{H}})^{-1}
\end{aligned}
\tag{13}
$$

Since $\{n_k(p), k = 1, \ldots, K, p = 1, \ldots, P\}$ are i.i.d. complex Gaussian random variables with zero mean and variance unit, we have $E\{\mathbf{Z}^H\mathbf{Z}\} = \mathbf{I}_P$. Thus (13) becomes

$$
\mathbf{V}_Z = E\{\overline{\mathbf{Z}}^H\overline{\mathbf{Z}}\} = (\mathbf{SS}^H)^{-1} = \tilde{\mathbf{S}} \tag{14}
$$

where $\tilde{\mathbf{S}} = (\mathbf{SS}^{H)-1}$, with (13) and (14), the following equations can be attained.

$$
\mathbf{V}_{Z_u} = E\{\overline{\mathbf{Z}}_u^H\overline{\mathbf{Z}}_u\} = \tilde{\mathbf{S}}_{(u-1)T+1:uT,(u-1)T+1:uT}, \quad u = 1, \ldots, U. \tag{15}
$$

When the orthogonal W–H code is used for spreading code, the matrix $\tilde{\mathbf{S}}$ will be an identity matrix $\mathbf{I}_{UT}$, and accordingly, $\mathbf{V}_{Z_u}$ is also an identity matrix. While for the quasi-orthogonal Gold code, the $\tilde{\mathbf{S}}$ will be a symmetric Toeplitz matrix with first row being $[a, \mu, \mu, \ldots, \mu]$, where $(UT-1)\mu$ is included. When U and T are given, the elements a, μ are only relative to the correlation coefficients of Gold code [20]. Thus $\mathbf{V}_{Z_u}$ is also a symmetric Toeplitz matrix with first row being $[a, \mu, \mu, \ldots, \mu]$, where $(T-1)\mu$ is included.

According to the above analysis, the elements of $\overline{\mathbf{Z}}$ are complex Gaussians variables with zero mean. So the elements $\{z_{u,kt}\}$ of $\overline{\mathbf{Z}}_u (u = 1, \ldots, U)$ are also complex Gaussians variables with zero mean, and variance unit for orthogonal spreading code case, while in the case of quasi-orthogonal spreading code, their covariance is a or μ. Namely, $E\{|z_{u,kt}|^2\} = a$ and $E\{z_{u,kt}z^*_{u,k't'}\} = \mu$, $k \neq k'$ or $t \neq t'$. Using Eq. (7) and the definition of $\mathbf{Y}_u$, the effective output of the decorrelator of user u can be written as:

$$
\hat{\mathbf{Y}}_u = \mathbf{Y}_u + \overline{\mathbf{Z}}_u = \sqrt{\rho_u}\mathbf{H}_u\mathbf{D}_u + \overline{\mathbf{Z}}_u, \quad u = 1, 2, \ldots, U \tag{16}
$$

4.1 Orthogonal W–H Code and Perfect CSI Case

In this subsection, we will analyze the average BER of the system when the orthogonal W–H code is used for spreading code, where perfect CSI is considered. With the orthogonal spreading code, the covariance of $\overline{\mathbf{Z}}_u$ will be an identity matrix, and its elements are i.i.d. complex Gaussian variables with zero mean and variance unit. For this, it can be shown that orthogonal space-time code converts a matrix channel into a scalar channel with a gain of the Frobenius norm of the matrix channel [8–10]. So before ML detection toward the transmitted symbols, the equivalent output signal $\overline{y}_u$ can be described by

$$
\overline{y}_{u,l} = \varepsilon\sqrt{\rho_u}\,\|\mathbf{H}_u\|_F^2\,d_{u,l} + w_{u,l}, \quad l = 1, 2, \ldots, L,\ u = 1, 2, \ldots, U. \tag{17}
$$

where $w_{u,l}$ is an equivalent noise term after MRC combining with a complex Gaussian distribution $CN(0, \varepsilon\,\|\mathbf{H}_u\|_F^2)$. Therefore, the effective SNR at the receiver is given by

$$
\gamma = \varepsilon^2\,\|\mathbf{H}_u\|_F^4\,E\{|d_{u,l}|^2\}\rho_u/(\varepsilon\,\|\mathbf{H}_u\|_F^2) = \rho_u\,\|\mathbf{H}_u\|_F^2\,/R \tag{18}
$$

To ease notation, we denote $x \sim \mathcal{G}(\kappa, \upsilon)$as a Gamma distributed random variable with parameters κ and υ. Then the pdf of x is $f(x) = \upsilon^{\kappa} x^{\kappa-1} e^{-\upsilon x} / \Gamma(\kappa)$ [21]. According to (1), we have: $\alpha_{u,ki}^2 \sim \mathcal{G}(m, m)$. Let $\beta = \|\mathbf{H}_u\|_F^2$, then $\beta = \sum_{k=1}^{K} \sum_{i=1}^{M} \alpha_{u,ki}^2 \sim \mathcal{G}(mKM, m)$. Hence, the pdf of β can be written as

$$f(\beta) = m^{mKM} \beta^{mKM-1} e^{-m\beta} / \Gamma(mKM) \tag{19}$$

Using (18), (19) and transformation of random variable, the pdf of γ can be obtained by

$$f(\gamma) = (mR\gamma/\rho_u)^{mKM} \exp(-mR\gamma/\rho_u)/[\gamma \cdot \Gamma(mKM)], \quad \gamma \geq 0 \tag{20}$$

According to [22] and utilizing (20), we can evaluate the average BER for STC-CDMA system with coherent MPSK modulation as follows

$$\begin{aligned} P_{e,p} &\cong \frac{2}{\max(\log_2 Q, 2)} \sum_{j=1}^{\max(Q/4,1)} \frac{1}{2} \int_0^{+\infty} f(\gamma) erfc\left(\sqrt{\gamma} \sin((2j-1)\pi/Q)\right) \mathrm{d}\gamma \\ &= \frac{\Gamma(mKM)]^{-1}}{\max(\log_2 Q, 2)} \left(\frac{mR}{\rho_u}\right)^{mKM} \\ &\quad \times \sum_{j=1}^{\max(Q/4,1)} \int_0^{+\infty} \gamma^{mKM-1} \exp\left(-\frac{mR}{\rho_u}\gamma\right) erfc\left(\sqrt{\gamma} \sin\left(\frac{(2j-1)\pi}{Q}\right)\right) \mathrm{d}\gamma \end{aligned} \tag{21}$$

where Q is modulation size, $erfc(z) = (2/\sqrt{\pi}) \int_z^{+\infty} \exp(-x^2)\mathrm{d}x$.

When *mKM* is not an integer, utilizing the analytical results of [23], (21) can be rewritten as

$$\begin{aligned} P_{e,p} &\cong \frac{1}{\max(\log_2 Q, 2)} \frac{\Gamma(mKM + 1/2)}{\Gamma(mKM + 1)} \\ &\quad \times \sum_{j=1}^{\max(Q/4,1)} \frac{\sqrt{s/\pi}}{(1+s)^{mKM+1/2}} {}_2F_1\left(1, mKM + \frac{1}{2}; mKM + 1; \frac{1}{1+s}\right), \\ s &= \sin^2((2j-1)\pi/Q)/(mR/\rho_u) \end{aligned} \tag{22}$$

where the equality $\int_0^{+\infty} e^{-ax} x^{b-1} erfc(\sqrt{cx})\mathrm{d}x = \sqrt{\frac{c}{a+c}} \frac{\Gamma(b+1/2)}{b\sqrt{\pi}} (a+c)^{-b} {}_2F_1\left(1, b+\frac{1}{2}; b+1; \frac{a}{a+c}\right)$ [23] is utilized, and ${}_2F_1(u, v; w; z)$ is the Gaussian hypergeometric function [17]. Equation (22) is a tight closed-form expression of average BER of STC-CDMA with MPSK modulation for non-integer *mKM*.

When *mKM* is an integer, let $F(\gamma) = \int_0^{\gamma} f(v)\mathrm{d}v$, then $F(\gamma)$ with $f(\gamma)$ given in (20) is expressed as

$$F(\gamma) = 1 - \sum_{i=0}^{mKM-1} (mR\gamma/\rho_u)^i \exp(-mR\gamma/\rho_u)/\Gamma(i+1) \tag{23}$$

With (23), the following integral can be obtained.

$$\begin{aligned} \mathcal{I} = \int_0^{+\infty} f(\gamma) erfc\left(\sqrt{g\gamma}\right) \mathrm{d}\gamma &= 1 - \sqrt{\frac{g}{mR/\rho_u + g}} \\ &\quad \times \sum_{i=0}^{mKM-1} \left(\frac{mR/\rho_u}{4(mR/\rho_u + g)}\right)^i \binom{2i}{i} \end{aligned} \tag{24}$$

The detailed derivation of (24) can be found in Appendix.

Using (24), (21) can be simplified to

$$P_{e,p} \cong \frac{1}{\max(\log_2 Q, 2)} \sum_{j=1}^{\max(Q/4,1)} \left[1 - \sqrt{\frac{s}{s+1}} \sum_{i=0}^{mMK-1} (4(1+s))^{-i} \binom{2i}{i} \right] \tag{25}$$

where $s = \rho_u \sin^2((2j-1)\pi/Q)/(mR)$. Equation (25) is a tight closed-form expression of average BER of STC-CDMA with MPSK modulation for integer *mKM*. Moreover, (22) and (25) are exact closed-form expressions of average BER of STC-CDMA system with BPSK and QPSK modulation because the BER of BPSK and QPSK modulation have accurate BER expressions [15].

According to [24,25], the average BER of MQAM with Gray coding over an AWGN channel is given by

$$P_{e,q}(\gamma) = \sum_j \zeta_j erfc\left\{\sqrt{\kappa_j \gamma}\right\} \tag{26}$$

where ζ_j and κ_j are constants which depend on the constellation size Q, and the values of the constant sets $\{\zeta_j, \kappa_j\}$ for MQAM can be found in [24,25]. Hence, using (26) and (20), we can evaluate the average BER of the system with MQAM as follows

$$P_{e,q} = \left(\frac{mR}{\rho_u}\right)^{mKM} \sum_j \zeta_j \int_0^{+\infty} \frac{\gamma^{mKM-1}}{\Gamma(mKM)} \exp\left(-\frac{mR}{\rho_u}\gamma\right) erfc\{\sqrt{\kappa_j \gamma}\} \mathrm{d}\gamma \tag{27}$$

When *mKM* is not an integer, (27) can be rewritten as following (28) according to the above analysis for MPSK modulation.

$$\begin{aligned} P_{e,q} = & \frac{\Gamma(mKM + 1/2)}{\Gamma(mKM+1)} \\ & \times \sum_j \zeta_j \frac{\sqrt{s/\pi}}{(1+s)^{mKM+1/2}} {}_2F_1\left(1, mKM + \frac{1}{2}; mKM+1; \frac{1}{1+s}\right) \end{aligned} \tag{28}$$

where $s = \kappa_j \rho_u/(mR)$, Eq. (28) is an exact closed-form expression of average BER of STC-CDMA with MQAM modulation for non-integer *mKM*.

When *mKM* is an integer, using (24), (27) can be further simplified to

$$P_{e,q} = \sum_j \zeta_j \left[1 - \sqrt{\frac{s}{s+1}} \sum_{i=0}^{mMK-1} (4(1+s))^{-i} \binom{2i}{i} \right] \tag{29}$$

where $s = \kappa_j \rho_u/(mR)$, Eq. (29) is an exact closed-form expression of average BER of STC-CDMA with MQAM modulation for integer *mKM*.

4.2 Orthogonal W–H Code and Imperfect CSI Case

In the previous subsection, we analyze the system performance with perfect CSI. In practice, however, the CSI will be imperfect due to channel estimation errors and quantization. So in this subsection, we will investigate the effect of imperfect CSI on BER with channel estimation errors modeled as complex Gaussian random variables (r.v.s) [26,27] for practical purposes.

From [28] and [15], we know that a Nakagami r.v. with integer parameter m can be modeled as the square root of the sum of $2m$ squared independent Gaussian r.v.s. Using this property, Eq. (18) can be expressed as

$$\gamma = \rho_u/(mR)\sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{v=1}^{m}|h_{u,kiv}|^2 = \rho_u/(mR)\sum_{k=1}^{K}\sum_{i=1}^{M}\sum_{v=1}^{m}\alpha_{u,kiv}^2 \tag{30}$$

where $\{h_{u,kiv}\}$ and $\{\alpha_{u,kiv} = |h_{u,kiv}|\}$ are respectively mKM independent complex Gaussian and Rayleigh r.v.s. with $E(\alpha_{u,kiv}^2) = 1$. Utilizing the analytical results in [26] and [27], the pdf of γ with Gaussian channel errors can be evaluated by

$$f(\gamma) = \sum_{n=1}^{mKM} A_n^{mKM}(c^2)(mR\gamma/\rho_u)^n \exp(-mR\gamma/\rho_u)/[\gamma \cdot \Gamma(n)], \quad \gamma \geq 0 \tag{31}$$

where c is the normalized cross correlation between the actual equivalent complex channel gain $h_{u,kiv}$ and its estimate $\hat{h}_{u,kiv}$, and $c^2 \triangleq |E\{\hat{h}_{u,kiv}h_{u,kiv}^*\}|^2.A_n^{mNK}(c^2) = \binom{mNK-1}{n-1}(1-c^2)^{mNK-n}(c^2)^{n-1}$ is a Bernstein polynomial with the following properties: $\sum_{n=0}^{mNK} A_{n+1}^{mNK+1}(c^2) = 1$ and $A_n^{mNK}(1) = \delta(n-mNK)$. When the channel estimation is perfect, i.e., $\hat{h}_{u,ki} = h_{u,ki}, c^2 = 1$. Using $A_n^{mNK}(1) = \delta(n-mNK)$, the pdf in (31) is equal to (20).

Using (31), (21) and (24), we can obtain the average BER for STC-CDMA system with MPSK modulation and imperfect CSI as follows:

$$\begin{aligned} P_{e,p} &\cong \frac{2}{\max(\log_2 Q, 2)}\sum_{j=1}^{\max(Q/4,1)}\frac{1}{2}\int_0^{+\infty} f(\gamma)erfc\left(\sqrt{\gamma}\sin((2j-1)\pi/Q)\right)\mathrm{d}\gamma \\ &= \frac{1}{\max(\log_2 Q, 2)}\sum_{n=1}^{mKM} A_n^{mKM}(c^2) \\ &\quad\times \sum_{j=1}^{\max(Q/4,1)}\left[1-\sqrt{\frac{s}{s+1}}\sum_{i=0}^{n-1}(4(1+s))^{-i}\binom{2i}{i}\right] \end{aligned} \tag{32}$$

where $s = \rho_u \sin^2((2j-1)\pi/Q)/(mR)$.

Similarly, using (31), (26) and (24), we can obtain the average BER for STC-CDMA system with MQAM modulation and imperfect CSI as follows:

$$\begin{aligned} P_{e,q} &= \sum_{n=1}^{mKM} A_n^{mKM}(c^2) \\ &\quad\times \sum_j \zeta_j\left[1-\sqrt{\frac{s}{s+1}}\sum_{i=0}^{n-1}(4(1+s))^{-i}\binom{2i}{i}\right], \quad s = \kappa_j\rho_u/(mR) \end{aligned} \tag{33}$$

With (32) and (33), we can effectively evaluate the performance of STC-CDMA system with orthogonal W–H code when CSI is not perfectly available.

Note: When the CSI is perfectly known, each channel coefficient is completely correlated, i.e., $c^2 = 1$, and correspondingly, (32) and (33) will reduce to (25) and (29) under perfect

CSI, respectively. Namely, the BER expressions under imperfect CSI include perfect CSI as a special case.

4.3 Quasi-Orthogonal Gold Code and Perfect CSI Case

In this subsection, we will discuss the BER performance when quasi-orthogonal Gold code is employed for spreading code, where perfect CSI is first considered. Under this scenario, the covariance of $\overline{\mathbf{Z}}_u$ will be not an identity matrix. Hence, the above analytical method from Sect. 4.1 needs corresponding revision. For the simplicity of analysis, we take the G_4 code as an example to analyze the corresponding STC-CDMA system performance. When G_4 code is adopted, we can evaluate the decision metrics for the detection of the transmitted symbols $\{d_{u,l}, l = 1, \ldots, 4\}$ according to (9). Due to symmetry considerations, the symbols $d_{u,1}$, $d_{u,2}$, $d_{u,3}$, $d_{u,4}$ have the same error probability. So we can analyze just one of decision metrics and corresponding effective SNR. Without loss of generality, symbol $d_{u,1}$ is considered, then with (9) and (16), the corresponding decision metrics is

$$\begin{aligned} Dm &= \sum_{k=1}^{K} \big(\hat{y}_{u,k1}h^*_{u,k1} + \hat{y}_{u,k2}h^*_{u,k2} + \hat{y}_{u,k3}h^*_{u,k3} + \hat{y}_{u,k4}h^*_{u,k4} + \hat{y}^*_{u,k5}h_{u,k1} + \hat{y}^*_{u,k6}h_{u,k2} \\ &\quad + \hat{y}^*_{u,k7}h_{u,k3} + \hat{y}^*_{u,k8}h_{u,k4}\big) \\ &= 2\sqrt{\rho_u} \sum_{k=1}^{K}\sum_{i=1}^{4} \left(|h_{u,ki}|^2\right) d_{u,1} + z' \end{aligned} \tag{34}$$

where $z' = \sum_{k=1}^{K}(z_{u,k1}h^*_{u,k1} + z_{u,k2}h^*_{u,k2} + z_{u,k3}h^*_{u,k3} + z_{u,k4}h^*_{u,k4} + z^*_{u,k5}h_{u,k1} + z^*_{u,k6}h_{u,k2} + z^*_{u,k7}h_{u,k3} + z^*_{u,k8}h_{u,k4})$ is an equivalent noise. According to the previous analysis, it will be a complex Gaussian random variable with zero mean, and the variance is

$$\mathrm{var}\{z'\} \le 2a \sum_{k=1}^{K}\sum_{i=1}^{4} \left(|h_{u,ki}|^2\right) + 2\mu \sum_{k=1}^{K}\sum_{i=1}^{4} \left(|h_{u,ki}|^2\right) = 2(a+\mu) \sum_{k=1}^{K}\sum_{i=1}^{4} \left(|h_{u,ki}|^2\right) \tag{35}$$

where the inequality $h_{u,ki}h^*_{u,kj} + h_{u,kj}h^*_{u,ki} \le |h_{u,ki}|^2 + |h_{u,kj}|^2$ is utilized, it is due to the fact

$$\begin{aligned} |h_{u,ki} - h_{u,kj}|^2 &= |h_{u,ki}|^2 + |h_{u,kj}|^2 - h_{u,ki}h^*_{u,kj} - h_{u,kj}h^*_{u,ki} \\ &\text{and} \quad |h_{u,ki} - h_{u,kj}|^2 \ge 0 \end{aligned} \tag{36}$$

Thus with (35), we can obtain the lower bound of effective SNR γ by

$$\gamma_l = R^{-1}[\rho_u/(a+\mu)]\,\|\mathbf{H}_u\|_F^2 = R^{-1}\rho_{ul}\,\|\mathbf{H}_u\|_F^2\,, \quad \rho_{ul} = \rho_u/(a+\mu) \tag{37}$$

It is well known that the CDMA system performance with quasi-orthogonal spreading code is worse than that with orthogonal spreading code. Namely, the BER of the former is higher than that of the latter under the same SNR. In other words, the effective SNR (denoted by γ_{no}) of the former is lower than γ_o(effective SNR for the orthogonal spreading code case, i.e. γ in (18)) of the latter. Hence, γ_o can be regarded as the upper bound of γ_{no}. Thus, we have: $\gamma_l \le \gamma_{no} < \gamma_o$. Moreover, μ is small and a is slightly larger than 1 in general, thus γ_l in (37) is indeed lower than γ_o in (18), so the upper bound and lower bound of γ_{no} exist.

Considering that the lower bound of SNR corresponds to the upper bound of BER, we can evaluate the upper bound of average BER of the space-time coded CDMA system with

MPSK modulation when quasi-orthogonal gold code is used. Namely, by substituting ρ_u with ρ_{ul} into (22) and (25), the upper-bound expressions of average BER are attained for the system with non-integer *mKM* and integer *mKM*, respectively. Similarly, by substituting ρ_u with ρ_{ul} into (28) and (29), the upper-bound expressions of average BER for the system with MQAM modulation can be obtained under non-integer *mKM* and integer *mKM* cases, respectively. To attain the approximate BER expression, we can take the mean value between the upper bound and lower bound of γ_{no} as its approximate value, that is,

$$\overline{\gamma}_{no} = (\gamma_l + \gamma_o)/2 = R^{-1}\left\|\mathbf{H}_u\right\|_F^2(\rho_{ul} + \rho_u)/2 = R^{-1}\left\|\mathbf{H}_u\right\|_F^2\overline{\rho}_{no} \tag{38}$$

is regarded as an approximate value of γ_{no}, and $\overline{\rho}_{no} = (\rho_{ul} + \rho_u)/2 = [1 + (a + \mu)^{-1}]\rho_u/2$ is an approximate value of effective SNR accordingly. By substituting ρ_u with $\overline{\rho}_{no}$, utilizing (22) and (25), the closed-form approximation of average BER of the system with MPSK modulation can be given by

$$\begin{aligned} P_{e,p} \cong &\frac{1}{\max(\log_2 Q, 2)}\frac{\Gamma(mKM + 1/2)}{\Gamma(mKM + 1)}\sum_{j=1}^{\max(Q/4,1)}\frac{\sqrt{s/\pi}}{(1+s)^{mKM+1/2}} \\ &\times {}_2F_1\left(1, mKM + \frac{1}{2}; mKM + 1; \frac{1}{1+s}\right), \text{ for non-integer } \quad mKM \end{aligned} \tag{39}$$

and

$$\begin{aligned} P_{e,p} \cong &\frac{1}{\max(\log_2 Q, 2)}\sum_{j=1}^{\max(Q/4,1)}\left[1 - \sqrt{\frac{s}{s+1}}\sum_{i=0}^{mMK-1}(4(1+s))^{-i}\binom{2i}{i}\right], \\ &\text{for integer } mKM \end{aligned} \tag{40}$$

where $s = \overline{\rho}_{uo}\sin^2((2j-1)\pi/Q)/(mR)$, $\overline{\rho}_{no} = [1 + (a + \mu)^{-1}]\rho_u/2$.

According to (28) and (29), by substituting ρ_u with $\overline{\rho}_{no}$, the closed-form approximation of average BER of the system with MQAM modulation can be obtained as follows:

$$\begin{aligned} P_{e,q} = &\frac{\Gamma(mKM + 1/2)}{\Gamma(mKM + 1)}\sum_j \zeta_j \frac{\sqrt{s/\pi}}{(1+s)^{mKM+1/2}}{}_2F_1\left(1, mKM + \frac{1}{2}; mKM + 1; \frac{1}{1+s}\right), \\ &\text{for non-integer} mKM \end{aligned} \tag{41}$$

and

$$P_{e,q} = \sum_j \zeta_j\left[1 - \sqrt{\frac{s}{s+1}}\sum_{i=0}^{mMK-1}(4(1+s))^{-i}\binom{2i}{i}\right], \quad \text{for integer } mKM \tag{42}$$

where $s = \kappa_j\overline{\rho}_{no}/(mR)$, $\overline{\rho}_{no} = [1 + (a + \mu)^{-1}]\rho_u/2$.

Based on the above analytical method, we can easily obtain the approximate closed-form expression of average BER of STC-CDMA system with other space-time coding (such as H_4, H_3', G_3 code, etc.). For STC-CDMA system with G_2code (or X code), it is interesting to find that this system BER has tight closed-form expression similar to the orthogonal spreading code case. This is because the effective SNR γ_{no} has exact value as follows by the related calculation

$$\gamma_{no} = R^{-1}(\rho_u/a)\left\|\mathbf{H}_u\right\|_F^2 = R^{-1}\rho_{no}\left\|\mathbf{H}_u\right\|_F^2, \quad \rho_{no} = (\rho_u/a) \tag{43}$$

By substituting ρ_u with $\rho_{no} = \rho_u/a$, using (22), (25), (27) and (28), we can easily obtain the closed-form expression of average BER of G_2 code (or X code) based STC-CDMA system with MPSK and MQAM modulation, respectively.

4.4 Quasi-Orthogonal Gold Code and Imperfect CSI Case

In this subsection, we will analyze the BER performance of the STC-CDMA systems with imperfect CSI when quasi-orthogonal Gold code is employed. According to the theoretical analysis in Sect. 4.3, the effective SNR of the system under quasi-orthogonal Gold code case may be approximated by $\overline{\rho}_{no}$. Based on this, by substituting ρ_u with $\overline{\rho}_{no}$ into (32) and (33), the closed-form approximation of average BER of the STC-CDMA system with MPSK and MQAM modulation can be respectively obtained by

$$P_{e,p} \cong \frac{1}{\max(\log_2 Q, 2)} \sum_{n=1}^{mKM} A_n^{mKM}(c^2) \times \sum_{j=1}^{\max(Q/4,1)} \left[1 - \sqrt{\frac{s}{s+1}} \sum_{i=0}^{n-1} (4(1+s))^{-i} \binom{2i}{i} \right] \tag{44}$$

and

$$P_{e,q} \cong \sum_{n=1}^{mKM} A_n^{mKM}(c^2) \sum_j \zeta_j \left[1 - \sqrt{\frac{s}{s+1}} \sum_{i=0}^{n-1} (4(1+s))^{-i} \binom{2i}{i} \right] \tag{45}$$

where $\overline{\rho}_{no} = [1 + (a+\mu)^{-1}]\rho_u/2$, $s = \overline{\rho}_{uo}\sin^2((2j-1)\pi/Q)/(mR)$, for MPSK, and $s = \kappa_j \overline{\rho}_{no}/(mR)$ for MQAM.

With the above two expressions, the influence of imperfect CSI on the BER performance of systems can be assessed effectively. When channel estimation is perfect, $c^2 = 1$, then using $A_n^{mKM}(1) = \delta(n - mKM)$, (44) and (45) will reduce to (40) and (42) under perfect CSI case accordingly.

5 Simulation Results and Numerical Analysis

In this section, we will give the performance simulation results for different space-time coded CDMA systems in Nakagami fading channel to testify the validity of the developed scheme and the derived BER expressions. The G_2, G_3, H_3', G_4, H_4' code are used for comparison. In simulation, the channel is assumed to be quasi-static flat fading and the receiver has perfect system synchronization. Every data frame includes 480 information bits, and Gray mapping of the bits to symbol is employed. The Monte-Carlo method is used in simulation. For different STCs, we will adopt different modulation modes to maintain the same throughput. 6-user synchronous CDMA systems are considered, and conventional Gold codes ($P = 63$) and W–H code ($P = 64$) are used for spreading code, respectively. The simulation results are shown in Figs. 1, 2, 3, 4, 5 and 6, respectively. In the following figures, 'G_2-CDMA', 'G_3-CDMA', 'H_3'-CDMA', 'G_4-CDMA', 'H_4'-CDMA' denote the CDMA systems with G_2, G_3, H_3', G_4, H_4' code, respectively. '4Q', '8P', '16Q' and '64Q' represent the 4QAM (QPSK), 8PSK, 16QAM and 64QAM modulation, respectively.

In Fig. 1, we plot the average BER of different STC-CDMA systems with one receive antenna (1Rx) for the Nakagami parameter $m = 1$ and 2. For comparison consistency, perfect

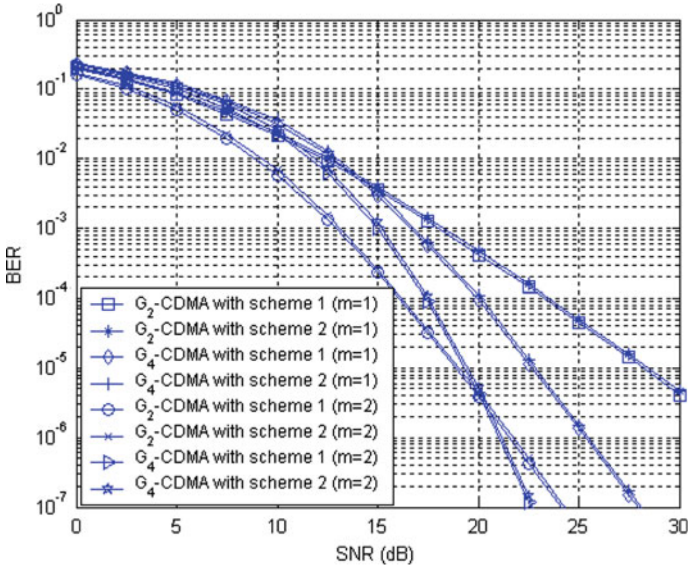

Fig. 1 BER versus SNR for different space-time coded CDMA systems with one receive antenna

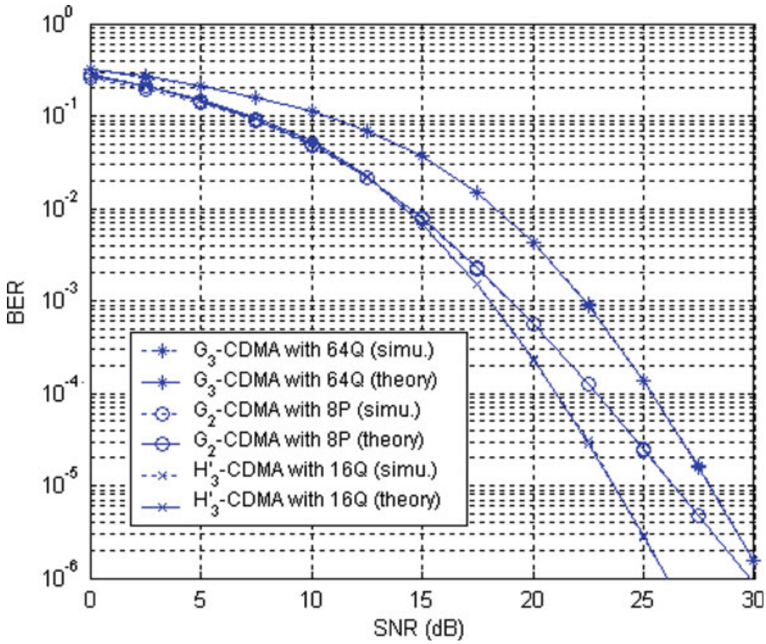

Fig. 2 BER versus SNR for different space-time coded CDMA systems with one receive antenna ($m = 1.5$)

CSI is considered for the developed scheme and existing scheme in simulation. In Fig. 1, 4QAM is employed in conjunction with G_2-CDMA, and 16QAM is applied to G_4-CDMA. Thus the system throughputs all equal 2 bit/s/Hz. Gold code is employed for spreading coding. It is shown in Fig. 1 that the G_4-CDMA performs better than G_2-CDMA because the former has greater diversity than the latter. Moreover, we can see that the bigger the value of m, the smaller the BER is. This is because the fading severity decreases as the Nakagami parameter m increases. These results accord with the existing knowledge as well, which indicates that the scheme 2 (i.e., the developed decoding scheme) is reasonable. Besides, the space-time coded CDMA systems with scheme 2 has almost the same performance as those with scheme 1 (i.e., the existing decoding scheme [11]), but the implement complexity of scheme 2 is much lower than scheme 1. It means that the scheme 2 is effective, and makes a good tradeoff between performance and complexity. In the following simulation, the scheme 2 is used for the performance evaluation because of its simplicity.

In Fig. 2, we plot the theoretical average BER and simulation results of different STC-CDMA systems with 1Rx and Gold code, where $m = 1.5$, $c^2 = 1$ (perfect CSI), the full-diversity G_2, G_3 and H_3' code are considered. For G_2-CDMA, 8PSK modulation is employed, while for H_3'-CDMA and G_3-CDMA, 16QAM and 64QAM modulation are used, respectively. Thus the system throughputs all equal 3 bit/s/Hz. The theoretical BER expression (39) is employed for the systems with MPSK modulation and (41) is used for the system with MQAM modulation. We can see in Fig. 2 that the derived theoretical BER agrees with the simulation results for both H_3'-CDMA and G_3-CDMA, while for G_2-CDMA, the theoretical values from (39) are basically consistent with the corresponding simulated values because (39) is an approximate closed-form expressions for MPSK modulation. Besides, from this figure, it is observed that H_3'-CDMA outperforms G_2-CDMA as expected. Although G_3-CDMA performs worse than G_2-CDMA at low SNR since higher modulation (64QAM) scheme is employed, it will be superior to the latter at very large SNR due to high diversity gain.

In Fig. 3, we give the theoretical average BER and simulation results of different space-time coded CDMA systems with Gold code and 1Rx. In simulation, $m = 2$, perfect CSI ($c^2 = 1$) and imperfect CSI ($c^2 = 0.85$) are considered. G_2, G_3 and H_3' code are used for comparison. For G_2-CDMA, 8PSK modulation is adopted, and for H_3'-CDMA and G_3-CDMA, 16QAM and 64QAM modulation are used, respectively. So the throughputs are 3 bit/s/Hz. For perfect CSI, the theoretical BER expression (40) and (42) are employed for the systems with MPSK and MQAM, respectively, while for imperfect CSI, the theoretical expression (44) and (45) are used for the BER calculation for the systems with MPSK and MQAM, respectively. As shown in Fig. 3, the theoretical BER are basically consistent with the simulated values for different STC-CDMA systems. In Fig. 3, it is found that the multiuser STC-CDMA systems with imperfect CSI perform worse than those with perfect CSI because the former has estimation error and the available CSI is not perfect. Moreover, by comparing the results of Figs. 2 and 3, we can see that the bigger the value of m, the smaller the BER is. It is because the fading severity decreases as m increases.

In Fig. 4, we plot the theoretical average BER and simulation results of different space-time coded CDMA systems with 2Rx and Gold code. In simulation, parameter sets are the same as Fig. 3. Namely, $m = 2$, c^2 is equal to 0.85 or 1, G_2, G_3 and H_3' code are considered. For perfect CSI, Eqs. (40) and (42) are employed for the BER calculation of the systems with MPSK and MQAM, respectively. For imperfect CSI, Eqs. (44) and (45) are used for the BER calculation for the systems with MPSK and MQAM, respectively. From this figure, we can observe similar results as shown in Fig. 3. That is, for perfect CSI, the derived theoretical BER make a good approximation with the actual simulated values. Due to the imperfection of CSI, the system performance with imperfect CSI is still worse than that with perfect CSI for two receive antennae cases. By comparing the results of Figs. 3 and 4, we can see that the STC-CDMA systems with 2Rx perform better than those of using 1Rx because the former has greater diversity than the latter. Moreover, with the number of receive antennae, the

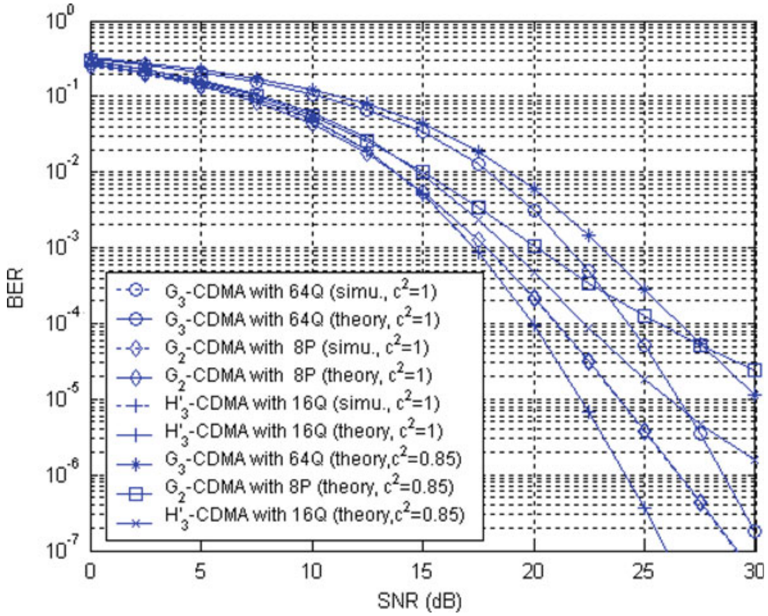

Fig. 3 BER versus SNR for different space-time coded CDMA systems with one receive antenna ($m = 2$)

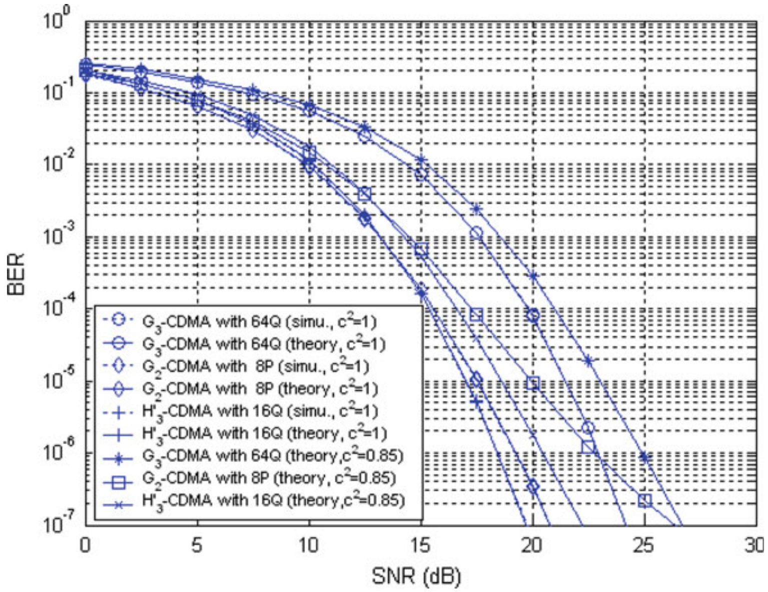

Fig. 4 BER versus SNR for different space-time coded CDMA systems with two receive antennae ($m = 2$)

system performance with imperfect CSI is improved, the performance difference between perfect CSI and imperfect CSI in Fig. 4 is lower than that in Fig. 3. The results from Figs. 2, 3, and 4 show that the derived theoretical formulae for STC-CDMA systems with Gold code are effective and reasonable, and the effect of imperfect CSI on BER performance is also assessed effectively.

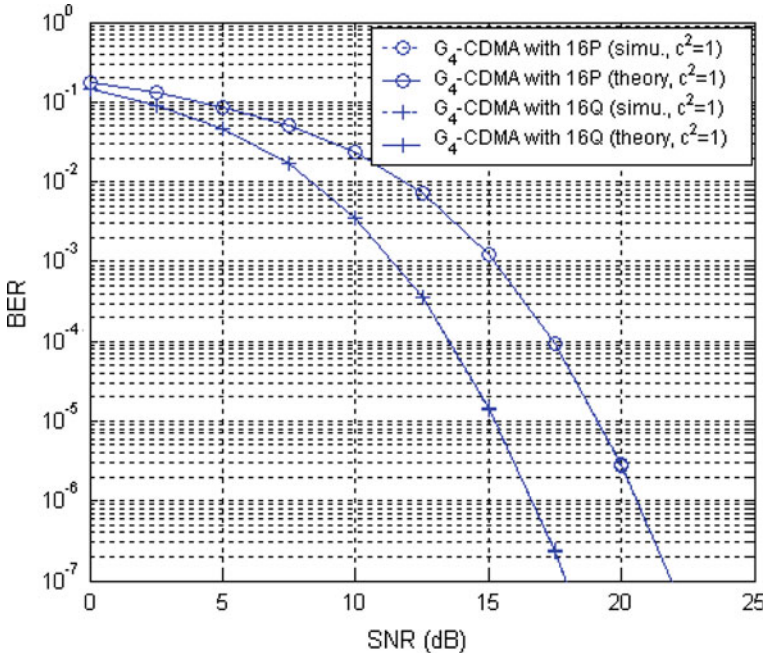

Fig. 5 BER versus SNR for different space-time coded CDMA systems with two receive antennae ($m = 1.5$)

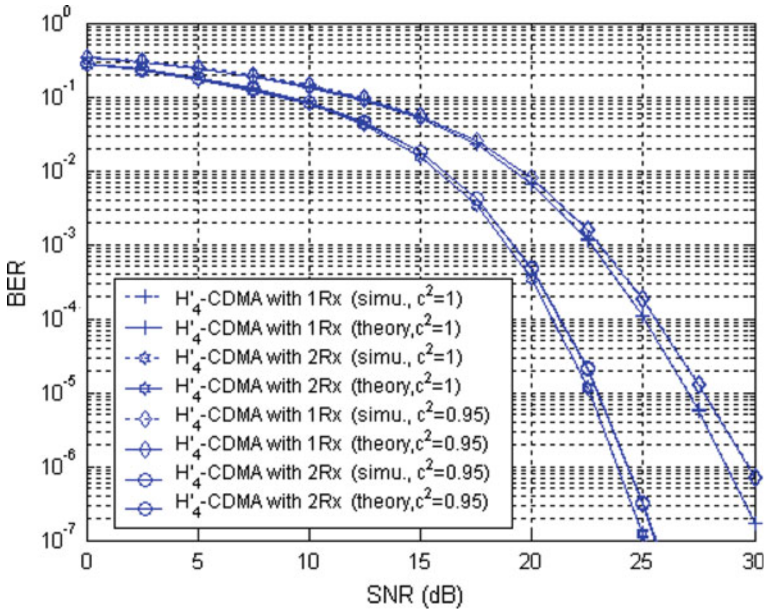

Fig. 6 BER versus SNR for space-time coded CDMA systems with different receive antennae ($m = 2$)

In the following Figs. 5 and 6, we will evaluate the validity of the theoretical BER expressions with orthogonal W–H spreading coding. In Fig. 5, we plot the theoretical average BER and simulation results of different STC-CDMA systems with 4Tx and 2Rx. In simulation, m

is set equal to 1.5, perfect CSI ($c^2 = 1$) and G_4 code are considered. For G_4-CDMA, 16PSK and 16QAM modulation are used, respectively, and thus the throughputs are 2 bit/s/Hz. The theoretical average BER is calculated by (22) for the systems with MPSK modulation, while for MQAM modulation, the theoretical values are obtained by (28). It is shown in Fig. 5 that G_4-CDMA system with 16PSK is worse than that with 16QAM because the minimum distance between 16PSK constellation symbols is smaller than the latter, which accords with the existing knowledge. Moreover, for different STC-CDMA systems, we can see in Fig. 5 that the theoretical average BER accord with the simulation results.

In Fig. 6, we give the theoretical average BER and simulation results of STC-CDMA system with different receive antennae and Nakagami parameter $m = 2$. Imperfect CSI ($c^2 = 0.95$) and perfect CSI ($c^2 = 1$) are considered. In simulation, H_4 code and 64QAM are employed, the number of transmit antenna is 4, the numbers of receive antenna are set equal to 1 and 2, respectively. The theoretical average BER are calculated by (33) and (29) for the system with imperfect and perfect CSI, respectively. It is observed that the system with 2Rx performs better than that with 1Rx because the former has greater diversity than the latter. As shown in Fig. 6, the derived theoretical BER values are in good agreement with the simulation results, that is, the theoretical values and simulated values match well for both perfect and imperfect CSI. Moreover, because of the imperfection of CSI, the system performance with imperfect CSI is worse than that with perfect CSI for both single and multiple receive antenna cases. As shown in this figure, however, the performance difference between imperfect and perfect CSI may be shortened by increasing the number of the receive antennae (which can obtain the high diversity gain) or improving the estimation accuracy (which can obtain high correlation). So the results from Figs. 5 to 6 show that the derived average BER expressions for STC-CDMA system with W–H code are valid and reasonable.

6 Conclusions

Utilizing the existing space-time coding schemes, we have given multiuser space-time coded CDMA systems, and investigated the system performance in Nakagami-m fading channel. Simple and effective receiver schemes are developed for STC-CDMA systems with perfect and imperfect CSI. The schemes can suppress MUI via multiuser detection method, and simplify the high decoding complexity of the existing scheme, obtain the effective tradeoff between the performance and complexity. Based on the perfect and imperfect CSI, by means of the BER analysis and related mathematical calculation, the average BER expressions are respectively derived for the system with MPSK and MQAM, and for perfect CSI, non-integer m and integer m cases are both considered. For orthogonal spread coding, a tight closed-form expression of average BER is attained, and for quasi-orthogonal spread coding, an approximate closed-form expression of average BER is obtained. With these expressions, the system performance under perfect and imperfect CSI can be easily evaluated in theory. Simulation results for BER show that the derived theoretical expressions can match the simulated values. Especially, when the orthogonal spreading code is employed, the simulation results are in good agreement with the theory results. Besides, the imperfection of CSI will affect the performance of STC-CDMA systems, that is, the systems have obvious BER performance degradation.

Acknowledgments This work is supported by the Doctoral Fund of Ministry of Education of China (20093218120021), NUAA Research Funding (NS2010113) and Startup Fund of NUAA (S0855-041). The authors would like to thank the anonymous reviewers for their valuable comments and suggestions which improve the quality of this paper greatly.

Appendix

In this appendix, we give the derivation of Eq.(24). Using (23), the integral. $\mathcal{I}$ is rewritten as

$$\mathcal{I} = \int_0^{+\infty} f(\gamma) erfc\left(\sqrt{g\gamma}\right) \mathrm{d}\gamma = F(\gamma)\ erfc\left(\sqrt{g\gamma}\right)\Big|_0^{+\infty} + \frac{2}{\sqrt{\pi}} \int_0^{+\infty} F(\gamma) e^{-g\gamma} \mathrm{d}\gamma$$

$$= 1 - \frac{1}{\sqrt{\pi}} \sum_{i=0}^{mKM-1} [1/\Gamma(i+1)] \int_0^{+\infty} (mR\gamma/\rho_u)^i \sqrt{g}\gamma^{-1/2} \exp(-mR\gamma/\rho_u - g\gamma) \mathrm{d}\gamma$$

$$= 1 - \frac{1}{\sqrt{\pi}} \sum_{i=0}^{mKM-1} [1/\Gamma(i+1)](mR/\rho_u)^i \frac{\Gamma(i+1/2)\sqrt{g}}{(mR/\rho_u+g)^{i+1/2}}$$

$$= 1 - \sqrt{\frac{g}{mR/\rho_u+g}} \sum_{i=0}^{mKM-1} \frac{(2i-1)!!}{i!2^i} \left(\frac{mR/\rho_u}{mR/\rho_u+g}\right)^i$$

$$= 1 - \sqrt{\frac{g}{mR/\rho_u+g}} \sum_{i=0}^{mKM-1} \left(\frac{mR/\rho_u}{4(mR/\rho_u+g)}\right)^i \binom{2i}{i}$$

where the equality$\Gamma(i+1/2) = \sqrt{\pi}(2i-1)!!/2^i$is utilized, $x!!$ denotes the double factorial of x and is defined recursively by $x!! = x \times (x-2)!!$, if $x \geq 2$; and $x!! = 1$, if $x = 0$ or $x = 1$.

References

1. Paulraj, A. J., Gore, D. A., Nabar, R. U., et al. (2004). An overview of MIMO communications—A key to gigabit wireless. *Proceedings of IEEE, 92*(2), 198–218.
2. Foschini, G. J., & Gans, M. J. (1998). On limits of wireless communication in a fading environment when using multiple antennas. *Wireless Personal Communications, 6*, 311–335.
3. Alamouti, S. M. (1998). A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications, 16*, 1451–1458.
4. Tarokh, V., Jafarkhani, H., & Calderbank, A. R. (1999). Space-time block codes from orthogonal designs. *IEEE Transactions on Information Theory, 45*(5), 1456–1467.
5. Jafarkhani, H. (2001). A quasi-orthogonal space-time block code. *IEEE Transactions on Communications, 49*(1), 1–4.
6. Yu, X., Xu, D., & Bi, G. (2007). Full-rate complex orthogonal space-time block code for multiple antennas. *Wireless Personal Communications, 40*, 81–89.
7. Larsson, E. G., & Stoica, P. (2003). *Space-time block coding for wireless communications*. Cambridge, UK: Cambridge University Press.
8. Ganesan, G., & Stoica, P. (2001). Space-time block codes: A maximum SNR approach. *IEEE Transactions on Information Theory, 47*(6), 2335–2367.
9. Sandhu, S., & Paulraj, A. (2000). Space-time block codes: A capacity perspective. *IEEE Communications Letters, 4*(12), 384–386.
10. Tarokh, V., Jafarkhani, H., & Calderbank, A. R. (1999). Space-time block coding for wireless communications: Performance results. *IEEE Journal on Selected Areas in Communications, 17*(3), 451–460.
11. Li, H., & Li, J. (2002). Differential and coherent decorrelating multiuser receivers for space-time coded CDMA systems. *IEEE Transactions on Signal Processing, 50*(10), 2529–2536.
12. Zhang, H., & Gulliver, T. A. (2005). Capacity and error probability analysis for orthogonal space-time block codes over fading channels. *IEEE Transactions on Wireless Communications, 4*, 808–819.
13. Shabazpanahi, S., Beheshti, M., Gershmann, A. B., Gharavi-Alkhansari, M., & Wong, K. M. (2004). Miniumum variance linear receivers for multiaccess MIMO wireless systems with space-time block coding. *IEEE Transactions on Signal Processing, 52*(12), 3306–3312.

14. Li, B. Z., & Latva-aho, M. (2003). Performance of space-time block coded MC-CDMA in Nakagami fading channels. *IEEE Electronics Letters, 39*(2), 222–224.
15. Proakis, J. G. (2001). *Digital communications* (4th ed.). New York: McGraw-Hill.
16. Alouini, M. S., & Goldsmith, A. J. (2000). Adaptive modulation over Nakagami fading channels. *Wireless Personal Communications, 13*(1–2), 119–143.
17. Gradshteyn, I. S., & Ryzhik, I. M. (2000). *Table of integrals, series, and products* (5th ed.). San Diego, CA: Academic.
18. Liew, T. H., & Hanzo, L. (2002). Space-time codes and concatenated channel codes for wireless communications. *Proceedings of the IEEE, 90*(2), 187–219.
19. Verdu, S. (1998). *Multiuser detection*. Cambridge, UK: Cambridge University Press.
20. Holmes, J. K. (1982). *Coherent spread spectrum systems*. New York: Wiley.
21. Hoel, P. G., Port, S. C., & Stone, C. J. (1971). *Introduction to probability theory*. Boston, MA: Houghton Mifflin.
22. Lu, J., Letaief, K. B., Chuang, J. C.-I., et al. (1999). M-PSK and M-QAM BER computation using signal-space concepts. *IEEE Transactions on Communications, 47*(2), 181–184.
23. Simon, M. K., & Alouini, M.-S. (2000). *Digital communication over fading channels: A unified approach to performance analysis*. New York: Wiley.
24. Simon, M. K., Hinedi, S. M., & Lindsey, W. C. (1995). *Digital communication techniques signal design and detection*. Englewood Cliffs, NJ: Prentice-Hall.
25. Choi, B., & Hanzo, L. (2003). Optimum mode-switching-assisted CP single-and multicarrier adaptive modulation. *IEEE Transactions on Vehicular Technology, 52*(3), 536–559.
26. Gans, M. J. (1971). The effect of Gaussian error in maximal ratio combiners. *IEEE Transactions on Communication Technology, COM-19*(4), 492–500.
27. Maaref, A., & Aissa, S. (2005). Capacity of space-time block codes in MIMO Rayleigh fading channel with adaptive transmission and estimation errors. *IEEE Transactions on Wireless Communicatons, 4*(5), 2568–2578.
28. Lombardo, P., Fedele, G., & Rao, M. M. (1999). MRC performance for binary signals in Nakagami fading with general branch correlation. *IEEE Transactions on Communications, 47*(1), 44–52.

Author Biographies

Xiangbin Yu received his Ph.D. in Communication and Information Systems in 2004 from National Mobile Communications Research Laboratory at Southeast University, China. From 2004 to 2006, he worked as a Postdoctoral Researcher in the Information and Communication Engineering Postdoctoral Research Station at Nanjing University of Aeronautics and Astronautics, Nanjing, China. He has been an Associate Professor with the Nanjing University of Aeronautics and Astronautics since May 2006. Currently, he also works as a Research Fellow in the Department of Electronic Engineering, City University of Hong Kong. Dr. Yu has served as a technical program committee for Globecom 2006 and International conference on communications systems 2008. He has been a member of IEEE ComSoc Radio Communications Committee (RCC) since May 2007. His research interests include Multi-carrier CDMA, space-time coding, adaptive modulation and space-time signal processing.

Dazhuan Xu was graduated from Nanjing Institute of Technology, Nanjing, China, in 1983. He received the M.S. degrees and Ph.D. in Communication and Information Systems from Nanjing University of Aeronautics and Astronautics in 1986 and 2001, respectively. He is now a full professor in College of Information Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing, China. Prof. Xu is a Senior Member of China Institute of Electronics (CIE). His research interests include digital communications, software radio, coding theory, medical signal processing.