

A Computationally Efficient Tree-PTS Technique for PAPR Reduction of OFDM Signals

Byung Moo Lee · Rui J. P. de Figueiredo · Youngok Kim

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Abstract The high peak-to-average power ratio (PAPR) of time domain signals has been a major problem in orthogonal frequency division multiplexing (OFDM) systems, and thus various PAPR reduction algorithms have been introduced. Partial transmit sequence (PTS) is one of the most attractive solutions because of its good performance without distortion. However, it is considered as an impractical solution for the realization of high-speed data transmission systems due to its high computational complexity. In this paper, a novel PAPR reduction algorithm based on a tree-structured searching technique is proposed to reduce the PAPR with low complexity. In the proposed scheme, the computational complexity of searching process is decreased by adjusting the size of tree with two parameters, width and depth, while preserving good performance. The simulation results show that proposed scheme provides similar performance with optimum case with remarkably reduced computational complexity.

Keywords Peak-to-average-power ratio (PAPR) · Partial transmit sequence (PTS) · OFDM

1 Introduction

Since orthogonal frequency division multiplexing (OFDM) has high spectral efficiency and is robust against inter-symbol interference and frequency-selective fading channel, it is

B. M. Lee
Central R&D Laboratory, Korea Telecom (KT), Seoul 137-792, Korea
e-mail: blee@kt.com

R. J. P. de Figueiredo
Laboratory for Intelligent Signal Processing and Communications, The Henry Samueli School of Engineering, University of California, Irvine, CA 92697-2625, USA
e-mail: rui@uci.edu

Y. Kim (✉)
Department of Electronic Engineering, Kwangwoon University, Seoul 139-701, Korea
e-mail: kimyoungok@kw.ac.kr

widely chosen for European digital audio/video broadcasting (DAV/DVB), wireless local metropolitan area network standards (WLAN/WMAN), and now it is a strong candidate for future broadband wireless communication systems.

However, one of major problems of OFDM technology is its high peak-to-average-power ratio (PAPR), which causes a distortion of signal at the nonlinear high power amplifier (HPA) of transmitter. Thus, the power efficiency of HPA is seriously limited to avoid nonlinear distortion, otherwise, high PAPR results in significant performance degradation. To reduce the high PAPR, a number of algorithms have been introduced, such as companding [1,2], block coding [3], active constellation extension [4,5], partial transmit sequence (PTS) [6,7] and selected mapping (SLM) [8,9]. Among various schemes, PTS is considered as one of the most attractive solutions because of its good performance without distortion. However, it is considered as an impractical solution for the realization of high-speed data transmission systems due to its high computational complexity. Therefore, many extensions of PTS scheme with reduced computational complexity, such as adaptive PTS [10] and iterative flipping algorithm [11], have been proposed although there is a significant performance gap between ordinary PTS and computationally efficient schemes.

In this paper, a novel PAPR reduction algorithm based on a tree-structured searching technique is proposed to reduce the PAPR with low complexity. In the proposed scheme, the computational complexity of searching process is decreased by adjusting the size of tree with two parameters, width and depth, while preserving good performance. It is also shown that the proposed scheme is a generalized version with adjustable complexity between ordinary PTS and iterative flipping schemes. The simulation results show that the proposed scheme provides similar performance with optimum case with remarkably reduced computational complexity.

The rest of this paper is organized as follows. In Sect. 2, the system model is described and previous approaches are briefly discussed. The novel PAPR reduction algorithm based on a tree-structured searching technique is proposed in Sect. 3, and the computational complexity and the performance of proposed scheme are analyzed and compared with those of previous approaches in Sect. 4. Finally, conclusions are given in Sect. 5.

2 System Model and Previous Approaches

Since the PTS is firstly introduced to reduce the PAPR of OFDM signal, various effective schemes in terms of performance and/or complexity have been proposed. In this section, two representative techniques, the original PTS [6] and iterative flipping technique [10,11] are briefly reviewed.

2.1 Ordinary PTS Technique

An OFDM signal of N subcarriers is represented as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi f_k t}, \quad 0 \leq t \leq T_s, \quad (1)$$

where T_s is the duration of the OFDM signal and $f_k = \frac{k}{T_s}$, and the high PAPR of the OFDM signal arises from the summation in the inverse discrete Fourier transform (IDFT) process.

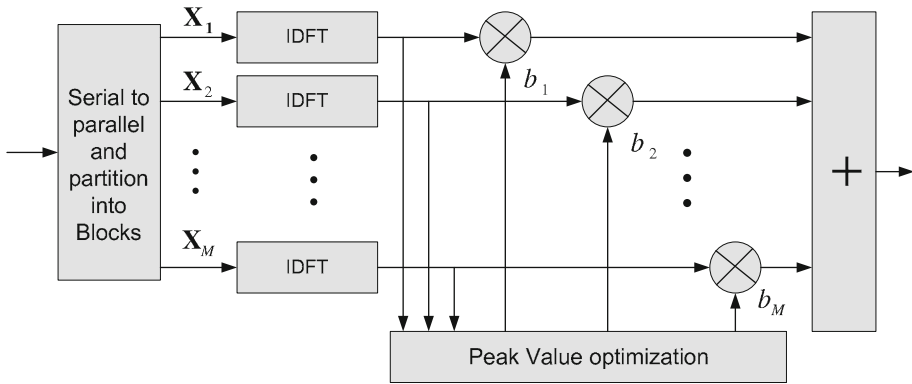


Fig. 1 Block diagram of the PTS scheme

A block diagram of the PTS technique is shown in Fig. 1. As shown in the figure, the input signal vector is partitioned into M disjoint subblocks, which can be represented as

$$\mathbf{X}_m = [X_0^m, X_1^m, \dots, X_{N-1}^m]^T, \quad m = 1, \dots, M, \tag{2}$$

where $X_k^m = X_k$ or 0, and thus, the input signal vector is expressed as follows:

$$\mathbf{X} = \sum_{m=1}^M \mathbf{X}_m. \tag{3}$$

The IDFT is then applied to each subblock and the oversampled time-domain PTS \mathbf{x}_m is expressed as

$$\mathbf{x}_m = [x_0^m, x_1^m, \dots, x_{NL-1}^m]^T, \quad m = 1, \dots, M, \tag{4}$$

where L is the oversampling factor. It is known that $L = 4$ is sufficient to approximate the PAPR in the analog domain. To reduce the PAPR, different phase factors $b_m = e^{j2\pi w/W}$, where $w = 0, 1, \dots, W - 1$ and W is the number of phases, are multiplied with the M time-domain PTS, \mathbf{x}_m . Due to the high computational complexity of the PTS technique, only a few phase factors $b_m \in \{\pm 1, \pm j\}$ are used to rotate the phase without multiplication operation [12]. Finally, the peak value optimization block iteratively searches the optimal phase sequence with minimum PAPR, which can be expressed as follows:

$$\text{PAPR}_{\text{optimal}} = \frac{\min_{b_1, \dots, b_M} \left(\max_{0 \leq n \leq LN} \left| \sum_{m=1}^M b_m x_n^m \right|^2 \right)}{E(|x(n)|^2)}. \tag{5}$$

After finding the optimal phase sequence, all the subblocks are summed and transmitted.

2.2 Iterative Flipping Technique

The iterative flipping algorithm [11] is briefly described as follows. After dividing the input signal vector into M subblocks, the PAPR of the OFDM signal is calculated with assumption of $b_m = 1$ for all of subblocks. Then the sign of the first phase factor is changed from 1 to -1 ($b_1 = -1$), and the PAPR of OFDM signal is re-calculated. If the PAPR of previously

calculated signal is bigger than that of current signal, keep $b_1 = -1$. Otherwise, revert to the previous phase factor, $b_1 = 1$. This procedure is performed until reaching to the end of subblocks. Similar adaptive PTS scheme was proposed by Jayalath and Tellambura [10]. In the adaptive PTS scheme, the flipping procedure does not necessarily reach to the end of subblocks. To reduce complexity, flipping procedure can be stopped in the middle of procedure if the desired PAPR OFDM signal is obtained during the procedure.

3 Proposed Approach

We propose a novel PAPR reduction algorithm based on a tree-structured searching technique. In the iterative flipping algorithm, the performance is relatively degraded compared with that of ordinary PTS, because it is based on PAPR comparison over the subset of combinations of phase factors. For example, the phase factors for the previous subblocks do not necessarily preserve the minimum PAPR property as the flipping procedure goes on for the following subblocks. That is, the choice of the phase factor for current subblock can affect the best decision for the previous subblocks. Therefore, it is natural that the enhanced performance can be achieved if the decision is made with the information of phase factors as many as possible. When all the combinations of phase factors are considered, optimal performance can be achieved through comparison over all the possible combinations and it would be ordinary PTS.

Since the computational complexity also should be considered, however, a tree-structured searching technique with two parameters S and T , which are the width and the depth of tree respectively, is proposed.

Figure 2 shows the tree structure of proposed tree-PTS (T-PTS) scheme. When S and T are set, as shown in the figure, there are S^T nodes at the T th subblock of tree and each node corresponds to a different sub-path going down the tree or an alternative representation for an OFDM symbol. Note that each node $\mathbf{x}(\mathbf{b}_{t,s})$ of tree has its own phase information $\mathbf{b}_{t,s}$, which is the phase factor of s th node at t th subblock, and its corresponding PAPR. Therefore, the node with minimum PAPR is determined at T th subblock and then the parent node is decided as the final decision of phase sequence while the unselected nodes are discarded. After that, we set the parent node of the node with minimum PAPR at the T th subblock as a new root node, and the procedure is continued for deciding the phase sequence with minimum PAPR until reaching to the node at the M th subblock. Depending on the parameters, S , T , and M , the proposed algorithm could be double or more roof.

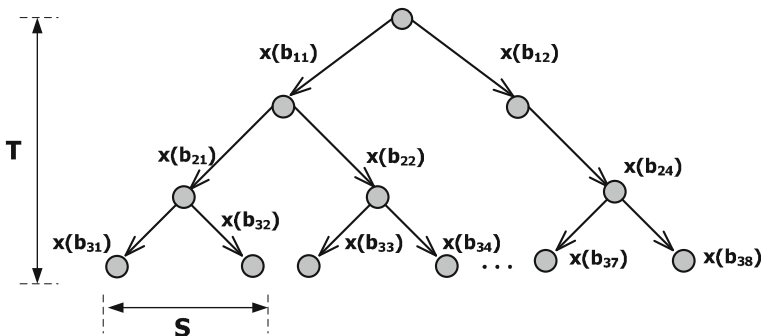


Fig. 2 Tree structure of the proposed T-PTS scheme with $S = 2$ and $T = 3$ as an example

The steps of proposed algorithm are described in details as follows:

1. Partition the input signal vector into M disjoint subblocks.
2. Set S and T , where $1 \leq S \leq W$ and $1 \leq T \leq M$.
3. Perform zero padding with a factor of L and IDFT.
4. Set $\mathbf{x}(\mathbf{b}_{00})$ as a root node and initialize all the phase factors as $\{1, 1, \dots, 1\}$.
5. Multiply different W phase factors to the first/current data subblocks and measure PAPR of each signal.
6. Choose S nodes, which show the lowest PAPR among W nodes, and discard the rests.
7. From each chosen node, repeat steps 5 and 6 until reaching T th subblock.
8. Set the parent node of the node with minimum PAPR at the T th subblock as a new root node and repeat the steps 5–8 until reaching to the M th subblock.

Table 1 Example of T-PTS algorithm, $S = 2, T = 2$

Figures	Descriptions
	<p>In the first subblock, calculate PAPRs of the OFDM signals after rotating phase of the first subblock using W phases factors</p> <p>Keep only $S = 2$ phase factors which show minimum PAPRs among W phase factors</p>
	<p>From each node (in this case from $\mathbf{x}(\mathbf{b}_{11})$ and $\mathbf{x}(\mathbf{b}_{12})$), calculate PAPRs of the OFDM signals after rotating phase using W phase factors at the second subblock.</p> <p>Find the minimum S PAPRs in the second subblock at each node, in this case $S = 2$.</p> <p>Since $T = 2$, stop the algorithm in the second subblock and among S^T candidates, find the one which shows minimum PAPR.</p> <p>The $(T - 1)$th = 1th parent node of the minimum PAPR node is a final decision for the first subblock.</p> <p>Since $T = 2$, we did a final decision for the $(T - 1)$th = 1th parent node at the second subblock.</p>
	<p>Assume the minimum PAPR signal in the second subblock is $\mathbf{x}(\mathbf{b}_{23})$</p> <p>Then the $(T - 1)$th = 1th parent node of $\mathbf{x}(\mathbf{b}_{23})$, $\mathbf{x}(\mathbf{b}_{12})$ is a final decision for the first subblock.</p> <p>Discard other unrelated information to reduce complexity.</p> <p>Keep the parent node and children nodes which are necessary for next step.</p> <p>Set $\mathbf{x}(\mathbf{b}_{12})$ as a new root node.</p> <p>Do this procedure iteratively until the end of subblock (Mth subblock).</p>

- The final phase factors are ones at the path from the root node $\mathbf{x}(\mathbf{b}_{00})$ to the node with minimum PAPR at the M th subblock.

When the final decision is performed with $S = W$ and $T = M$, it would be ordinary PTS. On the other hand, if $S = 1$, it would be Cimini and Sollenberger’s iterative flipping algorithm. By adjusting the two parameters, therefore, performance can be enhanced at the cost of increased computational complexity.

Examples of Table 1 may help to understand the details of the proposed algorithm.

We also provide an example ($S = 2, T = 2$) of the selected phases for PAPR reduction and corresponding PAPRs of each subblock in Fig. 3 and Table 2 based on a randomly generated OFDM signal. In Fig. 3, since $S = 2$, we select two nodes in each subblock. The calculated PAPR for each subblock is shown in Table 2. The vector in each row indicates PAPR (dB) of OFDM signal when the signal is multiplied by $[1 - 1j - j]$ in order. The first subblock requires 4 calculation, and from the second to forth subblock, it requires 8 calculation, since we choose $T = 2$. According to the Table 2, the original PAPR of OFDM signal in this example is 8.3467 dB. This PAPR is reduced to 7.0476 by selecting $-j$ in the first subblock, and reduced to 6.8635 dB by selecting j in the second subblock. The PAPR is maintained in the third subblock obviously by selecting 1 and finally it is reduced to 6.4869 dB by selecting j in the fourth subblock. Thus the final decision of phase factors for minimum PAPR is $[-jj1j]$.

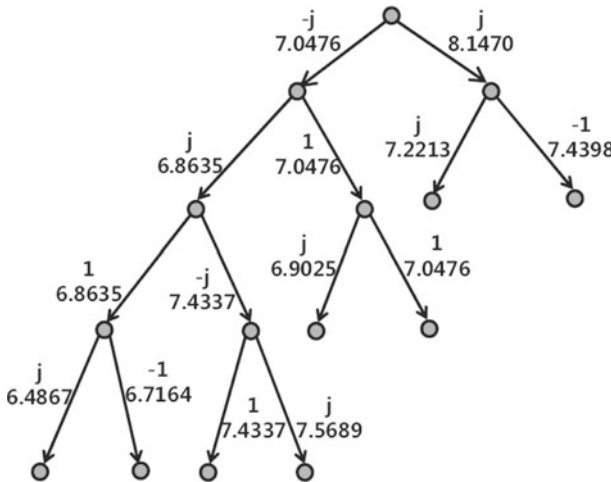


Fig. 3 An example of selected phase factors and corresponding PAPRs (dB) in each subblock ($S = 2, T = 2$)

Table 2 A PAPR example of each block when $S = 2, T = 2$

Subblock	PAPR(dB), Phase factors = $[1 - 1j - j]$
1st subblock	[8.3467 8.2115 8.1470 7.0476]
2nd subblock	[7.0476 8.1935 6.8635 8.1149] [8.1470 7.4398 7.2213 8.6964]
3rd subblock	[6.8635 7.8444 7.9597 7.4337] [7.0476 7.8484 6.9025 7.4211]
4th subblock	[6.8635 6.7164 6.4869 7.7607] [7.4337 8.4966 7.5689 8.3877]

4 Numerical Results

In this section, the computational complexity and the performance of proposed scheme are analyzed and compared with those of previous approaches.

4.1 Computational Complexity Analysis

The computational complexity comparison for the ordinary PTS, the iterative flipping, and the proposed T-PTS schemes is performed in terms of the number of iterations. Since the phase factor of the first subblock can be considered as 1 ($b_1 = 1$) without loss of generality, the number of iterations, C_{PTS} , for ordinary PTS can be derived as follows [10]:

$$C_{PTS} = W^{(M-1)}, \tag{6}$$

where W and M represent the numbers of phases and subblocks respectively. As shown in (6), the number of iterations for ordinary PTS is exponentially proportional to the number of subblocks, M .

The computational complexity of the proposed T-PTS scheme can be similarly analyzed with ordinary PTS, but the number of iterations is a function of two parameters, S and T . In the proposed scheme, after performing the initial tree search procedure with T subblocks, the same procedure with the initial tree search is just repeated until reaching to the end subblock. Therefore, the number of iterations, $C_{T-PTS}(S, T)$, for the proposed scheme can be derived as follows:

$$C_{T-PTS}(S, T) = 1 + \left(\sum_{t=1}^T (W - 1) \cdot S^{(t-1)} \right) + (W - 1)S^{(T-1)}(M - T), \quad 1 \leq T \leq M. \tag{7}$$

The first term of (7) is from the initialization process of step 4. The second term is from the initial tree search procedure until reaching to the T th subblock (step 5 and 7). The last term is from the repetition of tree search procedure until reaching to the end subblock (step 10). Note that $(W - 1)$ instead of W is used not to include the initialization step repeatedly in the following tree search procedure. As shown in (7), if S and T are preset as constants, the number of iterations for the proposed scheme is linearly proportional to the number of subblocks, M .

Figure 4 shows the number of iterations over the number of subblocks, M , for the ordinary PTS, the iterative flipping, and the proposed T-PTS schemes when $W = 4$, $S = 2, 3, 4$ and $T = 2$ are used. Figure 5 shows the number of iterations over the number of phase factors, W , for the ordinary PTS, the iterative flipping, and the proposed T-PTS schemes when $M = 4$, $S = 2, 3, 4$ and $T = 2$ are used. As shown in the figures, the computational cost of proposed scheme is remarkably reduced as M or W increases, compared to that of ordinary PTS, while it is slightly increased compared to that of the iterative flipping. In the following section, we will discuss the PAPR reduction performance associated with the computational complexity.

By adjusting parameters in (7), meanwhile, the proposed algorithm can be represented as the ordinary PTS and the iterative flipping schemes. From (7), the number of iterations for $S = 1$ and $T = 1$ can be derived as follows:

$$C_{T-PTS}(1, 1) = 1 + (W - 1) + (W - 1)(M - 1) = W + (W - 1)(M - 1), \tag{8}$$

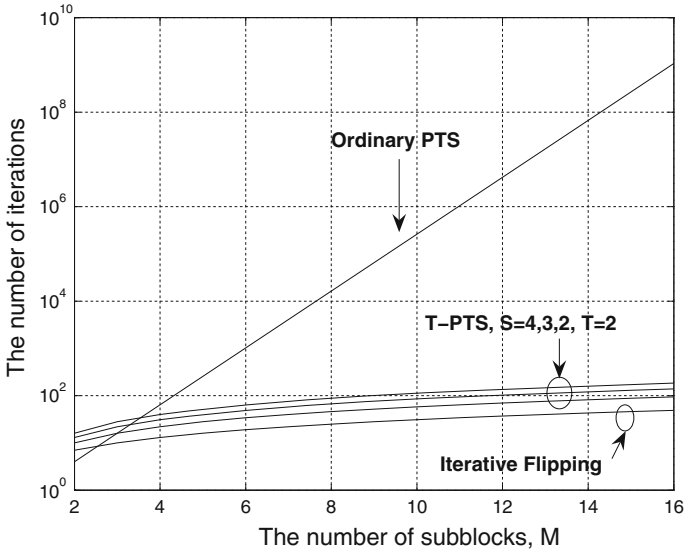


Fig. 4 Number of iterations vs. number of subblocks, M , for the ordinary PTS, the iterative flipping, and the proposed T-PTS schemes with $W = 4, S = 2, 3, 4$ and $T = 2$

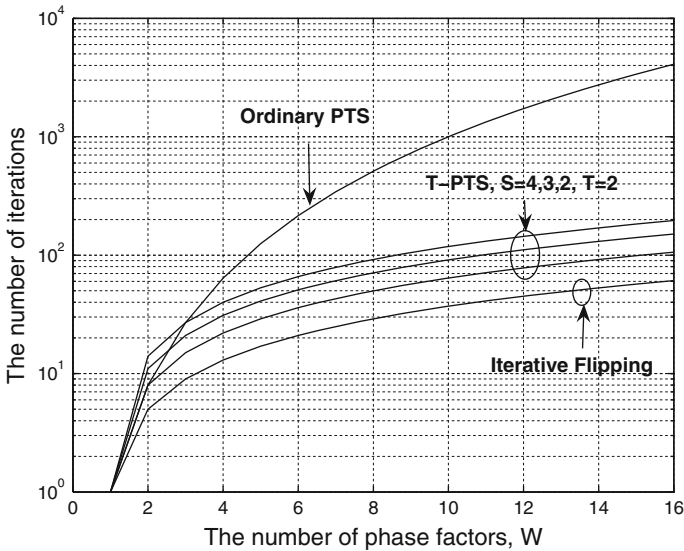


Fig. 5 Number of iterations vs. number of phase factors, W , for the ordinary PTS, the iterative flipping, and the proposed T-PTS schemes with $M = 4, S = 2, 3, 4$ and $T = 2$

and, (8) is equivalent to the complexity of the iterative flipping algorithm. The number of iterations for $S = W$ and $T = M$ also can be derived as follows:

$$\begin{aligned}
 C_{T\text{-PTS}}(W, M) &= 1 + [(W - 1) + (W - 1)W + (W - 1)W^2 + \dots + (W - 1)W^{M-1}] \\
 &= W^M,
 \end{aligned}
 \tag{9}$$

and, (9) is equivalent to the complexity of the ordinary PTS when the phase factor of the first subblock is not fixed.

4.2 Performance Analysis

The performance of proposed scheme is analyzed through the simulations. In the simulations, 10^5 OFDM symbols were randomly generated with 16-QAM, $N = 64$ subcarriers, $W = 4$ ($b_m \in \{\pm 1, \pm j\}$), and $M = 4, 8$. An input signal vector is divided into M subblocks, where each subblock consists of adjacent N/M subcarriers for simplicity. The signal is oversampled with a factor of $L = 4$ to estimate the PAPR of analog domain.

Figure 6 shows the PAPR reduction performance of proposed algorithm with various S and $T = 2$. As shown in the figure, the proposed scheme can reduce the PAPR around 3 dB for $M = 4$ and 4 dB for $M = 8$, respectively, at the 0.1% of complementary cumulative distribution function (CCDF, $\Pr(\text{PAPR} > \text{PAPR}_0)$), while the PAPR of original OFDM signal exceeds 10.5 dB. Note that the difference of PAPR between the ordinary PTS and the proposed scheme with $S = 4$ and $T = 2$ is only 0.15 dB for $M = 4$ and 0.9 dB for $M = 8$, respectively, at the 0.1% of CCDF.

The computational complexity, which is derived from (7) and also verified by simulations, is provided in Table 3. As shown in the table, the enhanced performance can be achieved with only 62% for $M = 4$ and 0.54% for $M = 8$ of the computational complexity of ordinary PTS.

Figure 7 shows the PAPR reduction performance of proposed algorithm as the T is increased. As shown in the figure, the performance difference between the ordinary PTS and the proposed scheme with $S = 2$ and $T = 8$ is only around 0.25 dB at 0.1% of CCDF. According to the Table 3, moreover, the enhanced performance can be achieved with only 4.74% of the computational complexity of ordinary PTS for $M = 8$. Note that the case of increasing T provides more enhanced performance at the increased computational complexity.

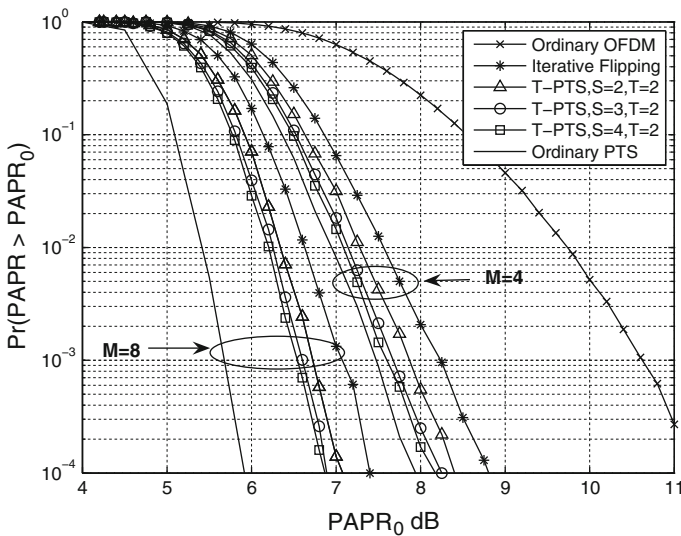


Fig. 6 CCDFs comparison between the previous approaches and the proposed T-PTS scheme with various S and $T = 2$ when $N = 64$, 16-QAM, $W = 4$, and $M = 4, 8$

Table 3 Number of iterations for ordinary PTS, iterative flipping, and proposed T-PTS with $W = 4$

	Number of subblocks, M	Ordinary PTS	Iterative flipping	Proposed T-PTS (S,T)								
				(2,2)	(3,2)	(4,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)
Number of iterations	4	64	13	22	31	40	34	46	–	–	–	–
	8	1,6384	25	46	67	88	82	142	238	382	574	766

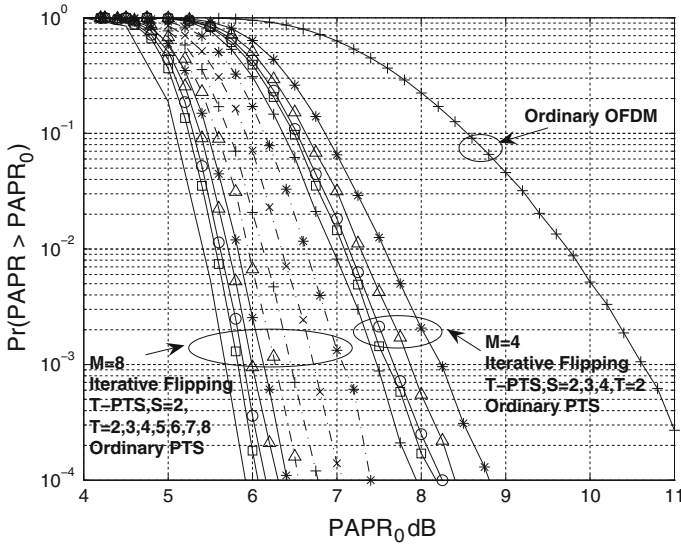


Fig. 7 CCDFs comparison between the previous approaches and the proposed T-PTS scheme with various T when $N = 64, 16$ -QAM, $W = 4$, and $M = 4, 8$

5 Conclusion

A novel PAPR reduction algorithm based on a tree-structured searching technique is proposed to reduce the PAPR with low complexity. Since the complexity of proposed scheme is adjustable while preserving good performance, the proposed scheme can be considered as a practical solution for high-speed data transmission systems. Simulation results demonstrated the effectiveness of proposed scheme in terms of computational complexity and PAPR reduction performance.

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Author Biographies



Byung Moo Lee received his Ph.D. degree in Electrical and Computer Engineering from the University of California, Irvine, CA, U.S.A. in 2006. From 2006 to 2007, he was a research specialist at the same University. Since 2007, he has been with the Central R&D Laboratory of Korea Telecom (KT), Seoul, Korea. His research interests include wireless communications, nonlinear signal processing, and optimization.



Rui J. P. de Figueiredo received the B.S. and M.S. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, and the Ph.D. degree in applied mathematics from Harvard University, Cambridge. After several years on the faculty of Rice University, Houston, TX, he joined the University of California, Irvine, in 1990, where he holds the position of Professor of electrical engineering and computer science, bio-medical engineering, and mathematics. He has published more than 370 papers and chaired or co-chaired 12 national or international conferences. Dr. de Figueiredo was the President of the IEEE Circuits and Systems Society in 1998 and served on several national and international committees and panels. For all of these contributions, he received a number of awards, including the IEEE Fellow award; the 1994 IEEE Circuits and Systems Society Technical Achievement Award; this society's 2002 M. E. Van Valkenburg Society Award and its 1999 Golden Jubilee Medal; the IEEE Third Millennium Medal; the Gh. Asachi Medal from the Technical University of

Iasi, Romania; the 2000 IEEE Neural Networks Transactions Outstanding Paper Award; and the 2003 IEEE Circuits and Systems Transactions Guillemin-Cauer Best Paper Award.



Youngok Kim received B.S. degree in mechanical engineering from Yonsei University, Seoul, Korea in 1999, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Texas at Austin, Austin, in 2002 and 2006, respectively. From 2006 to 2008, he was a senior researcher at Infra Laboratory of Korea Telecom (KT), Seoul, Korea. In March 2008, he joined the Department of Electronic Engineering of Kwangwoon University, Seoul, Korea, as an Assistant Professor. His research interests include ultra-wide band wireless communication systems, OFDM-based systems, precise ranging and location systems, PAPR reduction techniques, diversity techniques for wireless systems, and multiple-access schemes in multi-carrier systems.