

Diversity Analysis of Relay Selection Schemes for Two-Way Wireless Relay Networks

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Abstract One-way wireless relay networks have recently received a lot of attention due to their ability to provide spatial diversity in fading wireless environment. Moreover, performing single-relay selection is a very attractive method due to its cost effective implementation and superior performance. However, one-way relay networks with the half-duplex signalling suffer from a spectral efficiency loss. To overcome such a drawback, two-way wireless relay networks have been proposed and these are also the networks considered in this paper. The paper analyzes the diversity orders of various relay selection schemes, including the best-relay selection, best-worse-channel selection, and maximum-harmonic-mean selection. The analysis is done for the amplify-and-forward protocol and under the two-step and three-step transmission procedures. In particular, it is shown that full diversity orders of R and $R + 1$ can be achieved in a R -relay wireless network with the two-step and three-step procedures, respectively. Numerical and simulation results are provided to verify our analysis.

Keywords Cooperative diversity · Two-way relay network · Coding gain · Relay selection · Diversity order

1 Introduction

Using multiple antennas has been widely accepted as one of the effective techniques to increase the capacity and reliability of wireless communications in the presence of fading [1,2]. Moreover, it can be combined with other diversity techniques, e.g., time and frequency, to further increase the diversity order. However, in some wireless applications, implementing

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multiple transmit and/or receive antennas might not be possible due to the size and cost limitations. For example, with a small mobile device, to make the propagation paths between transmit and receive antennas statistically independent is a difficult task due to space limitation. Cooperative (or relay) diversity has been proposed to overcome the above limitations [3–6]. The basic idea is that the source cooperates with other mobile nodes (or relays) in the network to form a virtual multiple antenna system [7].

In a relay network, it is common to consider either the amplify-and-forward (AF) or the decode-and-forward (DF) protocol at the relays. In AF, relays receive noisy versions of the source's messages, amplify and re-transmit to the destination. With DF, relays decode the source's messages, re-encode and re-transmit to the destination [8]. In the conventional cooperation, all the relays participate in relaying the signals. Accordingly, it increases the cost and complexity of the network. Recently, a large amount of research work related to single-relay-selection schemes for one-way relay networks has been presented in the literature. In such an approach, only one *best* relay is selected to cooperate with the source to transmit the messages to the destination. Since only one relay participates in the cooperation with the source, the overall signal processing in the network is greatly simplified. In addition, the approach can also lead to a higher spectral efficiency than the conventional relaying approach in which all the relays in the network cooperate. This is because signal transmission in the relay selection approach can be completed in two time slots, regardless of the number of relays R employed in the network. In contrast, $R + 1$ time slots are needed to complete the transmission of the conventional relaying approach. Nevertheless, a central processing unit is required to decide which relay is selected to cooperate. In terms of AF, based on some criterion, such as the best-relay selection [9, 10], the best-worse-channel selection [11], the maximum-harmonic-mean selection [11], a relay from the set of R relays is selected. Performance improvement with these single-relay selection schemes over the conventional relaying approach is clearly demonstrated in [8, 10].

The above-mentioned relay selection schemes, however, were only considered for one-way relay networks. In other words, only half-duplex systems, i.e., all the nodes cannot transmit and receive signals simultaneously, are considered. The drawback of a one-way relay network is a loss in spectral efficiency due to half-duplex signalling. Recently, two-way wireless relay networks were proposed to overcome the low spectral efficiency of the one-way networks [12–15]. In a two-way relay network, two terminal nodes communicate with each other via intermediate relay(s). AF and DF are also the two main signalling options at the relays. Two-step and three-step procedures have been proposed for each protocol in [16–18], where all the relays in the network cooperate to transmit the signals between two terminal nodes.

Motivated by the effectiveness of single-relaying selection schemes in a one-way relay network, this paper studies single-relaying selection schemes for a two-way relay network in order to further improve its spectral efficiency. The main objective is to analyze the diversity orders of various relay selection schemes so that their relative performance can be fully understood. This in turns gives a useful guideline in selecting a cooperation and transmission protocol for a two-way wireless network. It is assumed that all the nodes, including R relays and two terminal nodes, know all the channel coefficients through training. The network is symmetrical (or reciprocal) in the sense that all the channel coefficients are the same for both backward and forward transmissions. The results show that full diversity orders are achieved by various single relay selection schemes. Moreover, the two-step and three-step procedures provide a superior performance compared to the four-step procedure.

The remainder of this paper is organized as follows. Section 2 describes a two-way wireless relay network with the two-step transmission procedure. The single-relay-selection schemes

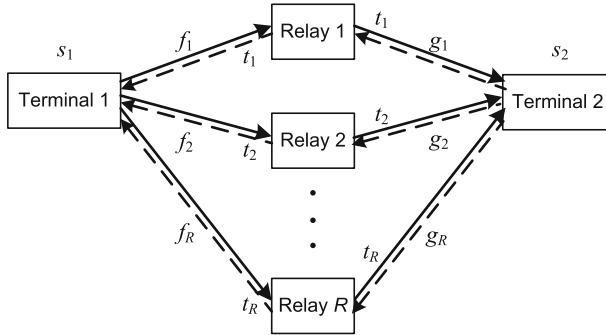


Fig. 1 A two-way wireless relay network

and their diversity orders are also analyzed in this section. Section 3 examines the three-step procedure. Numerical and simulation results are presented in Section 4. Finally, Section 5 concludes the paper.

Notations: $E\{x\}$ is the expectation of x and $\mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric Gaussian random variable with variance σ^2 .

2 Two-Way Wireless Relay Networks with Two-Step Procedure

Consider a wireless network illustrated in Fig. 1. Two terminal nodes, $\mathbf{T}_1, \mathbf{T}_2$, exchange their messages to each other with the assistance of R relay nodes. Each of the $R + 2$ nodes has only one antenna and operates in a half-duplex mode, i.e., a node cannot transmit and receive simultaneously. The transmission is carried out in two steps. In the first step, information bits are encoded at both terminal nodes to produce information symbols $s_m, m = 1, 2$. The information symbols are normalized such that $E\{|s_m|^2\} = 1$. The two terminals send $\sqrt{2P_m}s_m$ to R relays, where P_m is the average transmitted power at terminal $\mathbf{T}_m, m = 1, 2$. Denote the received signal at the i th relay as r_i , which is affected by either the fading coefficient f_i or g_i , and the noise v_i . Here, $f_i \in \mathcal{CN}(0, \sigma_{f_i}^2)$ and $g_i \in \mathcal{CN}(0, \sigma_{g_i}^2)$ are the gains of the fading channels from terminals \mathbf{T}_1 and \mathbf{T}_2 to the i th relay, respectively, while $v_i \in \mathcal{CN}(0, N_0)$. The received signal at the i th relay is then written as

$$r_i = \sqrt{2P_1}f_i s_1 + \sqrt{2P_2}g_i s_2 + v_i, \quad i = 1, \dots, R \tag{1}$$

Since AF relaying is considered, in the second step the i th relay scales its received signal by $\epsilon_i \alpha_i e^{j\theta_i}$ and forwards the scaled signal to the two terminals. The transmitted signal at the i th relay is:

$$t_i = \epsilon_i \alpha_i e^{j\theta_i} r_i \tag{2}$$

where α_i is the normalization factor to maintain the average power of $P_{R,i}$ at the i th relay and it is given by

$$\alpha_i = \sqrt{\frac{2P_{R,i}}{2P_1|f_i|^2 + 2P_2|g_i|^2 + N_0}} \tag{3}$$

The parameter $\epsilon_i \in \{0, 1\}$ is to decide whether the i th relay is active (or selected), and $\theta_i = -(\angle f_i + \angle g_i)$ is used to correct the phases of the received signals at the i th relay in the first step.

As mentioned before, the channel coefficients are assumed to be the same for both backward and forward transmissions. Denote the received signals at two terminal nodes by $y_m, m = 1, 2$. They can be written as follows:

$$y_1 = \sum_{i=1}^R f_i t_i + w_1 = \sqrt{2P_1} \sum_{i=1}^R \epsilon_i \alpha_i e^{j(\angle f_i - \angle g_i)} |f_i|^2 s_1 + \sqrt{2P_2} \sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| s_2 + \sum_{i=1}^R \epsilon_i \alpha_i e^{-j\angle g_i} |f_i| v_i + w_1 \tag{4}$$

$$y_2 = \sum_{i=1}^R g_i t_i + w_1 = \sqrt{2P_1} \sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| s_1 + \sqrt{2P_2} \sum_{i=1}^R \epsilon_i \alpha_i e^{j(\angle g_i - \angle f_i)} |g_i|^2 s_2 + \sum_{i=1}^R \epsilon_i \alpha_i e^{-j\angle f_i} |g_i| v_i + w_2 \tag{5}$$

where w_1 and w_2 are the noise components at terminal nodes \mathbf{T}_1 and \mathbf{T}_2 , respectively.

With the assumption of perfect channel state information, coherent detection can be implemented at both terminals $\mathbf{T}_1, \mathbf{T}_2$. Furthermore, since s_1 is known at \mathbf{T}_1 , the received signal-to-noise ratio (SNR) for terminal node \mathbf{T}_1 , denoted by $\overline{\text{SNR}}_{\mathbf{T}_1}$, is found to be

$$\overline{\text{SNR}}_{\mathbf{T}_1} = \frac{2P_2 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |f_i|^2 \right)} \tag{6}$$

Similarly, the received SNR for terminal node \mathbf{T}_2 , denoted by $\overline{\text{SNR}}_{\mathbf{T}_2}$, is

$$\overline{\text{SNR}}_{\mathbf{T}_2} = \frac{2P_1 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |g_i|^2 \right)} \tag{7}$$

Therefore, the average received SNR for both terminal nodes is

$$\begin{aligned} \overline{\text{SNR}} &= \frac{\overline{\text{SNR}}_{\mathbf{T}_1} + \overline{\text{SNR}}_{\mathbf{T}_2}}{2} \\ &= \frac{P_2 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |f_i|^2 \right)} + \frac{P_1 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |g_i|^2 \right)} \end{aligned} \tag{8}$$

Based on (8), the most general relay selection scheme is as follows:

$$\max_{\epsilon_1, \dots, \epsilon_R} \overline{\text{SNR}} \quad \text{subject to } \epsilon_i \in \{0, 1\} \tag{9}$$

However, due to their simplicity and effectiveness this work only considers single-relay selection schemes, i.e., (9) with the constraint that $\sum_{i=1}^R \epsilon_i = 1$.

If the i th relay is selected, the average received SNR can be written as

$$\gamma_r = \overline{\text{SNR}}_i = \frac{P_2 (\alpha_i |f_i g_i|)^2}{N_0 (1 + \alpha_i^2 |f_i|^2)} + \frac{P_1 (\alpha_i |f_i g_i|)^2}{N_0 (1 + \alpha_i^2 |g_i|^2)} \tag{10}$$

To simplify our derivation, we assume $P_1 = P_2 = P_{R,i} = \gamma/2$ and $N_0 = 1$. The parameter γ shall be generally referred to as the channel signal-to-noise ratio (CSNR). Then the average received SNR of the network can be rewritten as

$$\gamma_r = \frac{1}{2} \left(\frac{\gamma^2 F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma^2 F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) \tag{11}$$

where $F_i = |f_i|^2$, $G_i = |g_i|^2$. In the following subsections, three selection criteria are considered. They are the best-relay selection, the best-worse-channel selection, and the maximum-harmonic-mean selection. The diversity order of each selection criterion is analyzed.

2.1 Best-Relay Selection

For a one-way relay network, the best-relay selection has been proposed in [8–10, 19], where the relay is selected so that the received SNR is maximized. With a two-way relay network, the best relay shall be determined to be the one that maximizes the average received SNR. The achievable diversity order of this selection scheme is stated in the following theorem.

Theorem 1 *The asymptotic diversity order of the best-relay selection scheme in a two-way R-relay wireless network with the two-step procedure is R.*

Proof The proof is outlined in ‘‘Appendix 2’’. Note that Proposition 3 in ‘‘Appendix 2’’ also provides upper and lower bounds on the symbol error rate (SER) for the best-relay selection scheme. □

2.2 Best-Worse-Channel Selection

In the best-worse-channel selection scheme, the relay whose worse channel, $\min\{|f_i|^2, |g_i|^2\}$, is the best is selected. The diversity order achieved by this selection scheme is stated in the following theorem.

Theorem 2 *The asymptotic diversity order of the best-worse-channel selection for a two-way R-relay wireless network with the two-step procedure is R, i.e., a full diversity order.*

Proof If $x_i = \min\{|f_i|^2, |g_i|^2\} \geq 1/\gamma$, then $1 \leq \gamma|f_i|^2$ and $1 \leq \gamma|g_i|^2$. Therefore

$$\begin{aligned} \gamma_r &= \frac{1}{2} \left(\frac{\gamma^2 |f_i g_i|^2}{1 + 2\gamma |f_i|^2 + \gamma |g_i|^2} + \frac{\gamma^2 |f_i g_i|^2}{1 + \gamma |f_i|^2 + 2\gamma |g_i|^2} \right) \\ &\geq \frac{|f_i g_i|^2 \gamma}{2(|f_i|^2 + |g_i|^2)} \geq \frac{1}{4} \min\{|f_i|^2, |g_i|^2\} \geq \frac{|f_i g_i|^2 \gamma}{2(|f_i|^2 + |g_i|^2)} \\ &\geq \frac{1}{4} \min\{|f_i|^2, |g_i|^2\} \gamma = \frac{x_i \gamma}{4} \end{aligned} \tag{12}$$

Based on (12), the proof can be carried out in the same manner as in [20]. □

2.3 Maximum-Harmonic-Mean Selection

In the maximum-harmonic-mean selection scheme [7, 11], the relay that has the maximum harmonic mean of the two channel gain magnitudes, i.e., $(|f_i|^{-2} + |g_i|^{-2})^{-1}$, is selected. The following theorem establishes the achievable diversity order of this selection criterion.

Theorem 3 *The asymptotic diversity order of the maximum-harmonic-mean selection for a two-way R-relay wireless network with the two-step procedure is R, i.e., a full diversity order.*

Proof As before, if $x_i = \min\{|f_i|^2, |g_i|^2\} \geq 1/\gamma$, then $1 \leq \gamma|f_i|^2$ and $1 \leq \gamma|g_i|^2$. Therefore, one has

$$\gamma_r \geq \frac{|f_i g_i|^2 \gamma}{2(|f_i|^2 + |g_i|^2)} \tag{13}$$

Let $y_i = \frac{1}{|f_i|^{-2} + |g_i|^{-2}}$. If $x_i \geq 1/\gamma$, then

$$\gamma_r \geq \frac{|f_i g_i|^2 \gamma}{2(|f_i|^2 + |g_i|^2)} \geq \frac{y_i \gamma}{2} \geq \frac{y_i \gamma}{2}. \tag{14}$$

Again, based on the above expression, the proof of Theorem 3 can be carried out as in [20]. □

To summarize, this section has shown that all the three considered single-relay selection schemes achieve the maximum diversity order of R in a two-way R -relay wireless network with the two-step procedure. However, with the two-step procedure, the direct transmission between the two terminal nodes is not exploited. The next section considers the three-step procedure, where direct transmissions between the two terminals are also taken into account. As expected, the analysis confirms a higher diversity order achieved with the three-step procedure at the expense of a lower bandwidth efficiency (for the same modulation format).

3 Two-Way Relay Networks with Three-Step Procedure

Different from the two-step procedure, the transmission is completed in three steps as follows. In the first step, terminal node \mathbf{T}_1 transmits s_1 to R relays and terminal node \mathbf{T}_2 . In the second step, terminal node \mathbf{T}_2 transmits s_2 to R relays and terminal node \mathbf{T}_1 . The received signals at the i th relay in the first and second steps are as follows:

$$r_i^{(1)} = \sqrt{3P_1} f_i s_1 + v_i^{(1)}, \tag{15}$$

$$r_i^{(2)} = \sqrt{3P_2} g_i s_2 + v_i^{(2)}, \tag{16}$$

where $v_i^{(1)}$ and $v_i^{(2)}$ are $\mathcal{CN}(0, N_0)$ random variables that represents AWGN at the i th relay in the first and second steps. In the third step, the i th relay first superimposes its received signals in the previous two steps. The superimposed signal at the i th relay is

$$r_i^{(3)} = r_i^{(1)} + r_i^{(2)} = \sqrt{3P_1} f_i s_1 + \sqrt{3P_2} g_i s_2 + v_i^{(1)} + v_i^{(2)}. \tag{17}$$

Then it scales $r_i^{(3)}$ with $\epsilon_i \alpha_i e^{j\theta_i}$ and forwards the scaled signal to both terminals. The transmitted signal at the i th relay is

$$t_i = \epsilon_i \alpha_i e^{j\theta_i} r_i^{(3)}, \tag{18}$$

where $\delta_i \in \{0, 1\}$, $\theta_i = -(\angle f_i + \angle g_i)$, and

$$\alpha_i = \sqrt{\frac{3P_{R,i}}{3P_1|f_i|^2 + 3P_2|g_i|^2 + 2N_0}} \tag{19}$$

is the normalization factor employed by the i th relay.

Clearly, the received signals at two terminal nodes are:

$$\begin{aligned} y_1^{(2)} &= \sqrt{3P_2}h_0s_2 + w_1^{(2)} \\ y_1^{(3)} &= \sqrt{3P_1} \sum_{i=1}^R \epsilon_i \alpha_i e^{j(\angle f_i - \angle g_i)} |f_i|^2 s_1 + \sqrt{3P_2} \sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| s_2 \\ &\quad + \sum_{i=1}^R \epsilon_i \alpha_i e^{-j\angle g_i} |f_i| (v_i^1 + v_i^2) + w_1^{(3)} \\ y_2^{(1)} &= \sqrt{3P_1}h_0s_1 + w_2^{(1)} \\ y_2^{(3)} &= \sqrt{3P_1} \sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| s_1 + \sqrt{3P_2} \sum_{i=1}^R \epsilon_i \alpha_i e^{j(\angle g_i - \angle f_i)} |g_i|^2 s_2 \\ &\quad + \sum_{i=1}^R \epsilon_i \alpha_i e^{-j\angle f_i} |g_i| (v_i^1 + v_i^2) + w_2^{(3)} \end{aligned}$$

where $y_m^{(n)}$ and $w_m^{(n)}$ denote the received signal and noise components at terminal \mathbf{T}_m in the n th step, respectively.

Since s_1 is known at \mathbf{T}_1 , s_2 is known at \mathbf{T}_2 , both terminals have knowledge of all the channel gains f_i, g_i and h_0 through training, using the maximum-ratio-combining (MRC) method, the average received SNR for both terminal nodes is found to be:

$$\begin{aligned} \overline{\text{SNR}} &= \frac{1}{2} \left(\frac{3P_1|h_0|^2}{N_0} + \frac{3P_2 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + 2 \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |f_i|^2 \right)} \right. \\ &\quad \left. + \frac{3P_2|h_0|^2}{N_0} + \frac{3P_1 \left(\sum_{i=1}^R \epsilon_i \alpha_i |f_i g_i| \right)^2}{N_0 \left(1 + 2 \sum_{i=1}^R \epsilon_i^2 \alpha_i^2 |g_i|^2 \right)} \right) \tag{20} \end{aligned}$$

Again, to simplify our derivation, it is assumed that $P_1 = P_2 = P_{R,i} = \frac{\gamma}{3}$ and $N_0 = 1$. If the i th relay is selected in a single-relay selection scheme according to some criterion, the corresponding average received SNR can be rewritten as

$$\gamma_r = \overline{\text{SNR}}_i = \gamma \beta_0 + \frac{1}{2} \left(\frac{\gamma^2 F_i G_i}{2 + 3\gamma F_i + \gamma G_i} + \frac{\gamma^2 F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) \tag{21}$$

where $\beta_0 = |h_0|^2$, $F_i = |f_i|^2$, $G_i = |g_i|^2$.

Based on (21), the following theorem establishes the diversity order achieved with the three-step procedure when the best-relay selection is implemented. The proof of the theorem is outlined in ‘‘Appendix 3’’.

Theorem 4 *The asymptotic diversity order of the best-relay selection in a two-way R -relay wireless network with the three-step procedure is $R + 1$.*

Theorem 4 applies for the best-relay selection scheme. The same result in terms of the achievable diversity order of $R + 1$ can also be obtained for the best-worse-channel and maximum-harmonic-mean selection schemes with the three-step procedure in a two-way relay network. The proofs are rather straightforward and this omitted.

4 Simulation Results

This section presents numerical and simulation results to confirm the diversity orders achieved by various single-relay selection schemes for both the two-step and three-step procedures. In all simulations, the channels between any two nodes are assumed to be Rayleigh flat fading and reciprocal. In particular, all the channel coefficients are modeled and generated as $\mathcal{CN}(0, 1)$ random variables. The noise components at both terminals and the relays are also modeled and generated as i.i.d. $\mathcal{CN}(0, 1)$ random variables. For each average SNR value, 10^8 information bits were tested in the simulation to compute the average SER of each scheme.

The SER comparison between the two-step, three-step, and the four-step procedures are also made in this section. Here the four-step procedure simply applies a one-way relaying protocol separately for each direction, hence it also achieves the full diversity order of R for all the single-relay selection schemes considered. The SER comparison is performed under the same bandwidth and power constraints, which means that different modulation formats shall be chosen for the three transmission procedures under comparison.

Figure 2 shows the bit-error-rate¹ (BER) performance of the best-relay selection scheme with the two-step procedure in two-way networks that have two and three relays, respectively. Here binary-phase shift-keying (BPSK) is used and therefore k in (36) is set to 2. Also shown in the figure are the upper and lower bounds given in (35). It can be seen that the bounds are tight at high CSNR region. Moreover, the diversity orders 2 and 3 can be verified for the 2-relay and 3-relay networks, respectively.

Figure 3 then plots the BER performance of the three single-relay selection schemes considered in this paper and for the 2-relay, 3-relay and 4-relay networks with the two-step procedure. Observe that, consistent with our analysis, all three single-relay selection schemes achieve the same diversity order of R in a R -relay network. Moreover, it is interesting to see that there is virtually no difference in the BER performance of all three selection methods.

With the three-step procedure in a two-way network, Fig. 4 shows the bounds and simulation results of the BER obtained with the best-relay selection scheme and BPSK modulation. Consistent with our analysis, it can be verified that a diversity order of $R + 1$ can be achieved in a R -relay network with the three-step procedure.

Finally, three transmission procedures in a two-way wireless relay network are now compared. In order to maintain the same bandwidth efficiency, 4-PSK, 8-PSK, and 16-PSK are employed in the two-step, three-step, and four-step procedures, respectively. A network with 3 relays is considered. Figure 5 shows the SER performance of each procedure. First, it can be clearly seen that the two-step and three-step procedures significantly outperform the four-step procedure. Second, the performance of the two-step procedure is slightly better

¹ For BPSK, the SER and BER are the same.

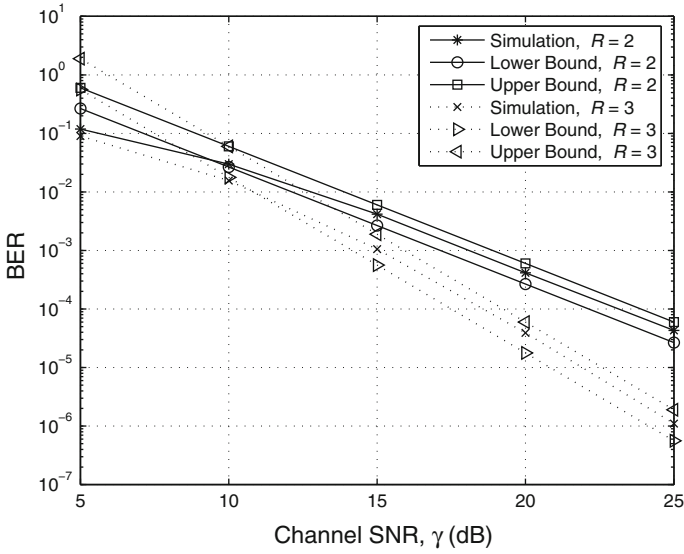


Fig. 2 BER and its bounds obtained with the best-relay selection scheme with the two-step procedure, BPSK modulation

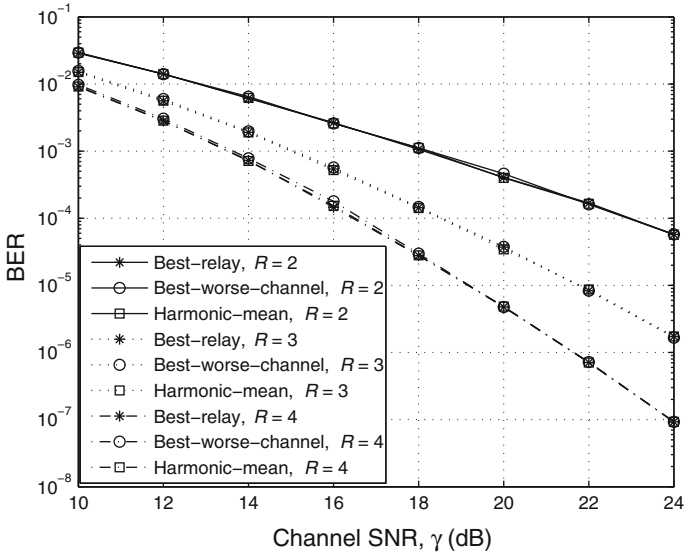


Fig. 3 BER of various single-relay selection schemes with BPSK modulation

than that of the three-step procedure in the low and medium CSNR regions. However, at the high CSNR region the SER of the three-step procedure starts to outperform the SER of the two-step procedure. This is expected from the higher diversity order achieved with the three-step procedure.

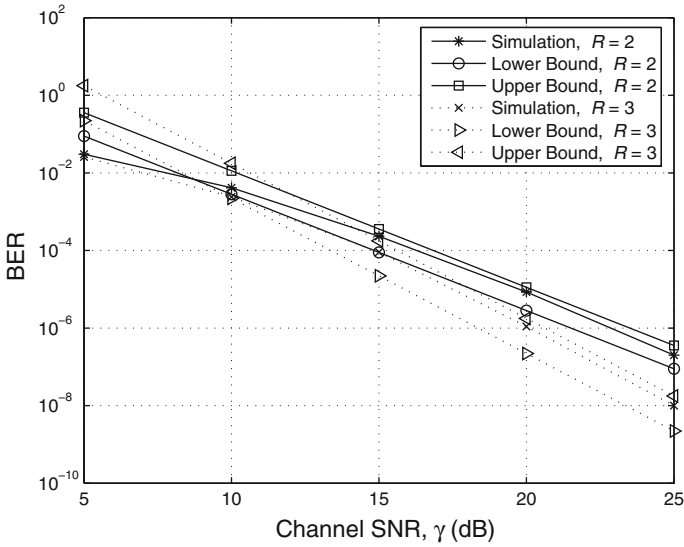


Fig. 4 BER and its bounds obtained with the best-relay selection scheme with the three-step procedure, BPSK modulation

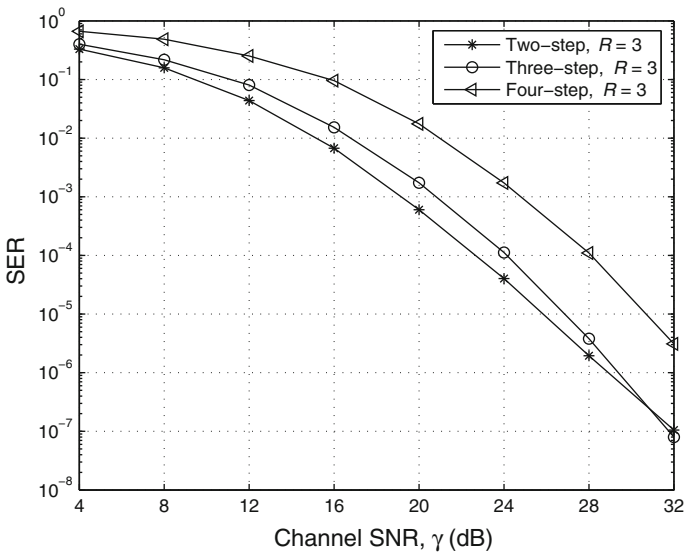


Fig. 5 SER comparison for the best-relay selection scheme with the two-step, three-step, and four-step procedures

5 Conclusion

In this paper, we have investigated the single-relay selection schemes in a two-way wireless relay network in which each node is equipped with a single antenna and the channels are Rayleigh fading. In particular, the paper analyzed the diversity orders of the best-relay selection, best-worse-channel selection, and maximum-harmonic-mean selection schemes with

two-step and three-step transmission procedures. It was shown that, in a R -relay network the diversity orders are R and $R + 1$ for the two-step and three-step procedures, respectively. Simulation results were presented to corroborate the analytical results. Performance comparison of different transmission procedures reveals that the two-step procedure is the best choice in the low and medium CSNR regions, while the three-step procedure outperforms the other two procedures at high CSNR region due to its higher achievable diversity order.

Appendix 1

Theorem 1 and Propositions 1, 2 and 3 are based on Fact 1 and Lemma 1 in [8]. They are summarized here for completeness. The interested reader is referred to [8] for the complete proofs.

Fact 1 [8]: Let u be an exponential random variable with parameter λ_u . Then, for a function $g(t)$ continuous about $t = t_0$ and satisfying $g(t) \rightarrow 0$ as $t \rightarrow t_0$

$$\lim_{t \rightarrow t_0} \frac{1}{g(t)} \Pr_u(g(t)) = \lambda_u \tag{22}$$

where $\Pr_u(g(t))$ is the cumulative distribution function (cdf) of the random variable u .

Lemma 1 [8]: Let u, v be independent exponential random variables with parameters λ_u, λ_v , respectively. Let $f(x, y) = xy/(x + y + 1)$. Let δ be positive, and let $r_\delta = \delta f(u/\delta, v/\delta)$. Let $h(\delta) > 0$ be continuous with $h(\delta) \rightarrow 0$ and $\delta/h(\delta) \rightarrow d < \infty$ when $\delta \rightarrow d$. Then

$$\lim_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) = \lambda_u + \lambda_v. \tag{23}$$

Appendix 2

Theorem 1 can be proved by the following three propositions.

Proposition 1 Let u, v be exponential random variables with parameters λ_u, λ_v , respectively. Let $f(x, y) = \frac{1}{2} \left(\frac{xy}{1+2x+y} + \frac{xy}{1+x+2y} \right)$. Let δ be positive, and let $r_\delta = \delta f(u/\delta, v/\delta)$. Let $h(\delta) > 0$ be continuous with $h(\delta) \rightarrow 0$ and $\delta/h(\delta) \rightarrow d < \infty$ when $\delta \rightarrow 0$. Then

$$\frac{4}{3}(\lambda_u + \lambda_v) \leq \lim_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) \leq 2(\lambda_u + \lambda_v). \tag{24}$$

Proof To prove the lower bound, start with

$$\begin{aligned} \Pr(r_\delta < h(\delta)) &= \Pr\left(\frac{1}{r_\delta} > \frac{1}{h(\delta)}\right) = \Pr\left[\frac{2}{\frac{uv}{\delta+2u+v} + \frac{uv}{\delta+v+2w}} > 1/h(\delta)\right] \\ &\geq \Pr\left[\max\left(\frac{4}{3u}, \frac{4}{3v}\right) > 1/h(\delta)\right] = 1 - \Pr\left[\frac{3u}{4} > h(\delta)\right] \Pr\left[\frac{3v}{4} > h(\delta)\right] \\ &= 1 - \exp\left[-\frac{4}{3}(\lambda_u + \lambda_v)h(\delta)\right]. \end{aligned} \tag{25}$$

□

Then using Fact 1 (see ‘‘Appendix 1’’) yields²

$$\liminf_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) \geq \frac{4}{3} (\lambda_u + \lambda_v). \tag{26}$$

To prove the upper bound, first note that

$$f(x, y) \geq \frac{xy}{1 + 2x + 2y}. \tag{27}$$

Then by applying Lemma 1 (see ‘‘Appendix 1’’) with $l(x, y) = \frac{4xy}{1+2x+2y}$, one has

$$\lim_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r'_\delta < h(\delta)) = \frac{1}{2} (\lambda_u + \lambda_v) \tag{28}$$

where $r'_\delta = \delta l(u/\delta, v/\delta)$. Moreover, $r_\delta > \frac{1}{4} r'_\delta$. Therefore

$$\limsup_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) \leq \limsup_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr\left(\frac{1}{4} r'_\delta < h(\delta)\right) = 2(\lambda_u + \lambda_v). \tag{29}$$

Combining (26) with (29) completes the proof of Proposition 1.

Proposition 2 *For high γ , the cumulative distribution function (cdf) of the average received SNR of the best-relay selection scheme under Rayleigh fading in a two-way R-relay wireless network is bounded as follows:*

$$\left(\frac{4}{3}\right)^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{\gamma_r}{\gamma}\right)^R \leq F(\gamma_r) \leq 2^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{\gamma_r}{\gamma}\right)^R \tag{30}$$

where λ_{F_i} and λ_{G_i} are the parameters of the exponential random variables F_i and G_i , respectively, i.e., $f_{F_i}(x) = \lambda_{F_i} \exp(-\lambda_{F_i}x)$, $f_{G_i}(x) = \lambda_{G_i} \exp(-\lambda_{G_i}x)$.

Proof Write $F(\gamma_r)$ as follows:

$$\begin{aligned} F(\gamma_r) &= P \left[\max_{i=1, \dots, R} \overline{\text{SNR}}_i < \gamma_r \right] \\ &= P \left[\max_{i=1, \dots, R} \frac{1}{2} \left(\frac{\gamma^2 F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma^2 F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) < \gamma_r \right] \\ &= P \left[\max_{i=1, \dots, R} \frac{1}{2} \left(\frac{\gamma F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) < \frac{\gamma_r}{\gamma} \right] \\ &= \prod_{i=1}^R P \left[\frac{1}{2} \left(\frac{\gamma F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) < \frac{\gamma_r}{\gamma} \right]. \end{aligned} \tag{31}$$

□

Let $\delta = \frac{\gamma_r}{\gamma}$ and $h(\delta) = \delta$. It follows from Proposition 1 that

$$\begin{aligned} \frac{4}{3} (\lambda_{F_i} + \lambda_{G_i}) &\leq \lim_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) \\ &= \lim_{\gamma \rightarrow \infty} P \left[\frac{1}{2} \left(\frac{\gamma F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) < \frac{\gamma_r}{\gamma} \right] \\ &\leq 2(\lambda_{F_i} + \lambda_{G_i}) \end{aligned} \tag{32}$$

² The notation ‘‘inf’’ (‘‘sup’’) denote an infimum (supremum) value in a set when all elements of the set are at least as large (small) as that value.

Thus

$$\begin{aligned}
 & \left(\frac{4}{3}\right)^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \leq \lim_{\gamma \rightarrow \infty} \frac{1}{h(\delta)^R} F(\gamma_r) \\
 & = \prod_{i=1}^R \lim_{\gamma \rightarrow \infty} \frac{1}{h(\delta)} P \left[\frac{1}{2} \left(\frac{\gamma F_i G_i}{1 + 2\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{1 + \gamma F_i + 2\gamma G_i} \right) < \frac{\gamma_r}{\gamma} \right] \\
 & \leq 2^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \tag{33}
 \end{aligned}$$

Clearly

$$\left(\frac{4}{3}\right)^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{\gamma_r}{\gamma}\right)^R \leq F(\gamma_r) \leq 2^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{\gamma_r}{\gamma}\right)^R \tag{34}$$

which proves Proposition 2.

Proposition 3 *With a linear modulation and under AWGN, the symbol error rate (SER) at high CSNR of the best-relay selection scheme in a two-way R-relay network with the two-step procedure is bounded as follows:*

$$\frac{4^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) (2R - 1)!!}{(3k\gamma)^R} \frac{1}{2} \leq P_e \leq \frac{2^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) (2R - 1)!!}{(k\gamma)^R} \frac{1}{2} \tag{35}$$

Proof Conditioned on the instantaneous received SNR, the SER of a linear modulation format under AWGN can be approximately by $Q(\sqrt{k\gamma_r})$, where k is a constant related to the modulation format and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt$. The unconditional SER is obtained by averaging $Q(\sqrt{k\gamma_r})$ over the pdf of γ_r . That is

$$P_e \approx E_{\gamma_r} \{ Q(\sqrt{k\gamma_r}) \}. \tag{36}$$

□

Let Y be the standard Gaussian random variable, i.e., $Y \sim \mathcal{N}(0, 1)$. Then

$$\begin{aligned}
 P_e & \approx E_{\gamma_r} \left\{ P(Y > \sqrt{k\gamma_r}) \right\} = E_{\gamma_r} \left\{ \int_{\sqrt{k\gamma_r}}^{+\infty} f_Y(Y) dY \right\} \\
 & = \int_0^{+\infty} \left(\int_{\sqrt{k\gamma_r}}^{+\infty} f_Y(Y) dY \right) f_{\gamma_r}(\gamma_r) d\gamma_r \\
 & = \int_0^{+\infty} \left(\int_0^{Y^2/k} f_{\gamma_r}(\gamma_r) d\gamma_r \right) f_Y(Y) dY \\
 & = \int_0^{+\infty} F(Y^2/k) f_Y(Y) dY \tag{37}
 \end{aligned}$$

where $F(\cdot)$ is the cdf of the average received SNR, i.e., γ_r .

From Proposition 2, one has

$$\int_0^\infty \left(\frac{4}{3}\right)^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{y^2}{k\gamma}\right)^R \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy \leq P_e \leq \int_0^\infty 2^R \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i}) \left(\frac{y^2}{k\gamma}\right)^R \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy \tag{38}$$

It then follows from [21, Eq. 3.461.2] that

$$\int_0^\infty \left(\frac{y^{2R}}{\sqrt{2\pi}}\right) \exp\left(-\frac{y^2}{2}\right) dy = \frac{(2R - 1)!!}{2}. \tag{39}$$

Proposition 3 is thus proved.

Appendix 3

Theorem 4 can be proved by the following three propositions.

Proposition 4 *Let u, v be an exponential random variables with parameters λ_u, λ_v , respectively. Let $f(x, y) = \frac{1}{2} \left(\frac{xy}{2+3x+y} + \frac{xy}{2+x+3y}\right)$. Let δ be positive, and let $r_\delta = \delta f(u/\delta, v/\delta)$. Let $h(\delta) > 0$ be continuous with $h(\delta) \rightarrow 0$ and $\delta/h(\delta) \rightarrow d < \infty$ when $\delta \rightarrow 0$. Then*

$$\frac{3}{2}(\lambda_u + \lambda_v) \leq \lim_{\delta \rightarrow 0} \frac{1}{h(\delta)} \Pr(r_\delta < h(\delta)) \leq 3(\lambda_u + \lambda_v) \tag{40}$$

Proof The proof is similar to that of Proposition 1 and thus omitted. □

Proposition 5 *For high CSNR, the cdf of the average received SNR of the best-relay selection scheme under Rayleigh fading in a two-way R-relay wireless network with the three-step procedure is bounded as follows:*

$$\begin{aligned} & \frac{3^R \lambda_{\beta_0} \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i})}{2^R (R + 1)} \left(\frac{\gamma_r}{\gamma}\right)^{R+1} \leq F(\gamma_r) \\ & \leq \frac{3^R \lambda_{\beta_0} \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i})}{R + 1} \left(\frac{\gamma_r}{\gamma}\right)^{R+1} \end{aligned} \tag{41}$$

where $\lambda_{\beta_0}, \lambda_{F_i}$ and λ_{G_i} are the parameters of the exponential random variables β_0, F_i and G_i , respectively.

Proof $F(\gamma_r)$ can be written as follows:

$$\begin{aligned}
 F(\gamma_r) &= P \left[\max_{i=1, \dots, R} \overline{\text{SNR}}_i < \gamma_r \right] \\
 &= P \left[\max_{i=1, \dots, R} \gamma\beta_0 + \frac{1}{2} \left(\frac{\gamma^2 F_i G_i}{2 + 3\gamma\beta_{1_i} + \gamma\beta_{2_i}} + \frac{\gamma^2 F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) < \gamma_r \right] \\
 &= P \left[\max_{i=1, \dots, R} \frac{1}{2} \left(\frac{\gamma F_i G_i}{2 + 3\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) < \frac{\gamma_r}{\gamma} - \beta_0 \right] \\
 &= \int_0^\delta P \left[\max_{i=1, \dots, R} \frac{1}{2} \left(\frac{\gamma F_i G_i}{2 + 3\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) < \delta - x \right] \lambda_{\beta_0} e^{-\lambda_{\beta_0} x} dx \\
 &= \int_0^\delta \prod_{i=1}^R P \left[\frac{1}{2} \left(\frac{\gamma F_i G_i}{2 + 3\gamma F_i + \gamma G_i} + \frac{\gamma F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) < \delta - x \right] \lambda_{\beta_0} e^{-\lambda_{\beta_0} x} dx
 \end{aligned} \tag{42}$$

where $\delta = \frac{\gamma_r}{\gamma}$. Applying Proposition 4 with $x' = 1 - \frac{x}{\delta}$ (note that $0 \leq x \leq \delta$, so $0 \leq \delta x' \leq \delta$), one has

$$\begin{aligned}
 \frac{3}{2}(\lambda_{F_i} + \lambda_{G_i}) &\leq \\
 \lim_{\delta \rightarrow 0} \frac{1}{\delta x'} \Pr \left(\frac{1}{2} \left(\frac{\gamma^2 F_i G_i}{2 + 3\gamma F_i + \gamma G_i} + \frac{\gamma^2 F_i G_i}{2 + \gamma F_i + 3\gamma G_i} \right) < \delta x' \right) & \\
 &\leq 3(\lambda_{F_i} + \lambda_{G_i})
 \end{aligned} \tag{43}$$

From (42, 43), [19] and after some straightforward calculations, (41) can be verified. Thus, Proposition 5 is proved. □

Proposition 6 *Under the linear modulation and AWGN, the symbol error rate (SER) at high CSNR of the best-relay selection in a two-way R-relay network with the three-step procedure is bounded as follows:*

$$\begin{aligned}
 \frac{3^R \lambda_{\beta_0} \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i})}{2^R (k\gamma)^{R+1}} \frac{(2R + 1)!!}{2} &\leq P_e \\
 &\leq \frac{3^R \lambda_{\beta_0} \prod_{i=1}^R (\lambda_{F_i} + \lambda_{G_i})}{(k\gamma)^{R+1}} \frac{(2R + 1)!!}{2}
 \end{aligned} \tag{44}$$

Proof The proof is similar to that of Proposition 3 and thus omitted. □

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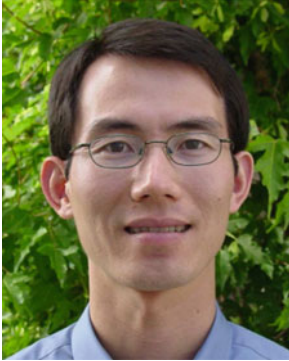
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