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Performance Analysis of Diversity Combining Algorithms in Shadowed Fading Channels

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Abstract. A compound fading model incorporating short term fading and shadowing proposed recently is used to analyze the performance of wireless systems employing microscopic diversity to mitigate the effects of flat fading. This model can account for the presence of different levels of fading and shadowing and provide an analytical solution for the probability density function of the signal-to-noise ratio. Using that model, the performances of MRC and SC diversity combining algorithms were studied. The amount fading (AF) following diversity implementation was calculated and it is seen that the decline in the amount of fading is bound by the level of shadowing present, with the MRC providing a larger decrease in the amount of fading than the SC algorithm. The effect on the error rates was studied using the example of the coherent BPSK modem. Results show that the performances of wireless systems can be analyzed using the compound model for the shadowed fading channels.

Keywords: fading and shadowing, Rayleigh-lognormal (RL) fading, shadowed fading channels, diversity combining, MRC and SC algorithms, bit error rates

Introduction

Wireless channels are subject to random fluctuations in received power arising from the multipath propagation. These short term fading channels generally have been modeled as Rayleigh, Rician and Nakagami [1, 2]. Diversity techniques [1] are routinely employed to mitigate the effects of short term fading. Often, these channels are also subject to long term fading or shadowing resulting from the multiple scattering taking place in the channel. The shadowing is commonly modeled using a lognormal distribution [3–7]. Thus, the channel will be simultaneously subjected to fading and shadowing, making the channel more vulnerable to degradation in performance. Such shadowed fading channels are modeled either as Rayleigh-lognormal (or Suzuki), Rician-lognormal or Nakagami-lognormal [3, 4]. None of these three composite models leads to a closed form solution for the probability density function of the signal-to-noise ratio at the receiver, making the analyses of the diversity approaches very cumbersome when both fading and shadowing are present simultaneously [8–12].

Recently, a composite analytical model was proposed to describe the shadowed fading channels [13]. This model assumes a Nakagami density function for the envelope of the received signal while using a gamma density function to model the average power to account for shadowing. This led to a closed form solution for the power or the signal-to-noise ratio of the signal at the receiver. In this work, this compound pdf which incorporates the short term fading and long term fading (shadowing) is used to obtain the bit error rates in diversity systems. The paper starts with a brief overview of the compound pdf. This is followed by the derivation of the equations for the bit error rate in diversity systems employing both maximal ratio combining and selection combining algorithms. The analytical results are presented along with a discussion on the potential applications of this compound fading and shadowing model in wireless system modeling.

Compound Model of Fading and Shadowing

The multipath fading observed in wireless systems is modeled using the Nakagami distribution. This Nakagami model [1, 2] can account for both Rayleigh and Rician fading conditions, with the former arising when no direct path between the transmitter and receiver is present while the latter arising when there is a direct path between the transmitter and the receiver. It also permits modeling of fading conditions which are far more severe than Rayleigh. Such short term fading conditions modeled using the Nakagami distribution assumes that the average power received is fixed. When multiple scattering conditions are present in addition to multipath, the average power itself becomes a random variable. The randomness of the average power is described as long term fading or shadowing and it is generally modeled in terms of a lognormal distribution for the average power. Since shadowing and fading occur simultaneously, the Rayleigh-lognormal (RL), the Suzuki model [3, 4], and the Nakagami-lognormal models [5–12] have been used to describe shadowed fading channels. All these models are hampered by the absence of closed form solutions for the density function of the received signal power. Availability of closed form solutions for the pdf of the received signal power will provide a simple way to analyze the performance of the wireless systems. The use of K distribution [9] to model shadowing and fading only provided a limited degree of improvement over Rayleigh-lognormal models.

It was shown that the gamma-gamma model used in radar and sonar [9, 13–18] provided a better alternative than the simple K distribution used in modeling clutter in radar and sonar. This approach was used to obtain the density function of the received signal power in presence of fading and shadowing [13]. The probability density function $f_X(x)$ of the received signal power, X, under the Nakagami short term fading becomes

$$f_X(x) = \left(\frac{m}{y}\right)^m \frac{x^{m-1} e^{-\frac{m}{y}x}}{\Gamma(m)}, \quad x > 0 \tag{1}$$

where m is the Nakagami parameter and y is the average power given by $\langle X \rangle$. In Eq. (1), $m = 1$ corresponds to Rayleigh statistics of the envelope and $m > 1$ corresponds to Rician statistics. Even though the lower limit of $m$was taken to be 0.5 in most of the literature, m can take any positive value. Values of m lower than 1 corresponds to severe fading. The severity of fading [2, 9] can be measured as $(\frac{1}{m})$, with negligible fading at high values of m, with the wireless channel becoming a Gaussian channel when $m \to \infty$. Severe fading taking place as $m \to 0$.

When the wireless channel is also subject to shadowing, the local mean power becomes random [1]. This is taken into account by defining y in Eq. (1) to be a random variable with a lognormal pdf. Thus, in presence of shadowing, Eq. (1) can be rewritten by conditioning the power as

$$f_{X|Y}(x \mid y) = \left(\frac{m}{y}\right)^m \frac{x^{m-1} e^{-\frac{m}{y}x}}{\Gamma(m)}, \quad x > 0 \tag{2}$$

The density function of the power in combined fading and shadowing is obtained as

$$f_X(x) = \int_0^{\infty} f_{X|Y}(x \mid y) f_Y(y) dy \tag{3}$$

where $f_Y(y)$ is the pdf of y. When $m = 1$, and $f_Y(y)$ is a lognormal pdf, the resulting fading is described as Rayleigh-lognormal, or Suzuki [1, 4]. With $f_Y(y)$ as a lognormal pdf, Eq. (3) is the Nakagami-lognormal distribution [3, 4]. But, the use of lognormal pdf for y does not lead to a closed form solution for $f_X(x)$ in Eq. (3). Thus, we need to look elsewhere for an appropriate distribution for y.

The problem of local variations in scattering has been studied in radar and sonar [14–16] through the use of a gamma pdf for y in place of the lognormal pdf. If we choose $f_Y(y)$as the gamma pdf given by

$$f_Y(y) = \frac{y^{c-1} e^{-\frac{y}{y_0}}}{\Gamma(c)\, y_0^c}, \quad y > 0, c > 0 \tag{4}$$

where c is the order of the gamma pdf and y_0 is a measure of the mean power [13, 14]. By varying c, it will be possible to generate pdfs ranging from lognormal to Gaussian allowing flexibility. Substituting Eq. (4) in Eq. (3), the density function $f_X(x)$ of the power X becomes [13, 18, 19]

$$f_X(x) = \frac{2}{\Gamma(m)\Gamma(c)} \left(\frac{b}{2}\right)^{c+m} x^{\left(\frac{c+m}{2}\right)-1} K_{c-m}(b\sqrt{x}), \qquad x > 0, m > 0, c > 0 \tag{5}$$

where$K_{c-m}(\ )$ is the modified Bessel function of order $(c - m)$ and $b = 2\sqrt{\frac{m}{y_0}}$. Equation (5) represents the gamma-gamma model or gamma-K model described in radar literature [14, 15] or the McDaniel model in sonar [18]. In wireless systems, Eq. (5) provides a simple way to model all forms of fading including shadowing. By varying c and m, it is possible to create varying levels of fading and shadowing. When $m = 1$, Eq. (5) can adequately represent Rayleigh-lognormal fading as suggested [9]. It is thus clear that Eq. (5) is more versatile than the simple K distribution proposed [9] to model shadowed fading channels. Using the moments [19] of the pdf in Eq. (5), the average received power becomes

$$E[X] = cm\left(\frac{2}{b}\right)^2. \tag{6}$$

The different fading-shadowing conditions can be described by defining AF, the amount of fading in wireless systems [2, 9] as

$$\text{AF} = \frac{\langle X^2 \rangle}{\langle X \rangle^2} - 1. \tag{7}$$

Using the moments of X [13, 16, 19], AF can be expressed as

$$\text{AF} = \left(\frac{1}{m}\right) + \left(\frac{1}{c}\right) + \left(\frac{1}{mc}\right) > 0 \tag{8}$$

with AF $= 0$ corresponding to an ideal Gaussian channel and AF $= \infty$ to severe fading. When $m \to \infty$ and $c \to \infty$, AF $= 0$, we have an ideal channel (Gaussian). When $c \to \infty$, shadowing is absent. When this occurs, we have Rayleigh fading if $m = 1$ and Rician fading if $m > 1$. Low values of m and c correspond to severe fading and shadowing. Values of c in the range of 8 to 10 are more than enough to make the channel depend almost completely on m, making it a Nakagami fading channel with very little shadowing. Equation (5) provides a closed form expression to model fading and shadowing simultaneously offering a significant advantage over the Suzuki [4] or Nakagami-lognormal models [3, 4, 10–12].

Diversity Combining Algorithms

The effects of short term fading are mitigated through microdiversity [20, 21]. The widely used signal processing techniques in diversity systems are the maximal ratio combining (MRC), equal gain combining (EGC) and the selection combining (SC) algorithms [20–23]. MRC provides the best improvement in system performance, followed by the equal gain combining and then by selection combining. We will limit ourselves to MRC and SC algorithms. In MRC systems, the output is the weighted sum of the signals from each branch, with the weights being proportional to the power in each branch. In SC systems, the output is the branch with the highest signal-to-noise ratio or power.

We will start with the MRC diversity. Let M be the order of diversity. If we assume that all the branches are identical, the conditional probability density function of the power or signal-to-noise ratio $X_i \mid Y$ of the ith branch is [22, 23]

$$f_{X_i|Y}(x_i \mid y) = \left(\frac{m}{y}\right)^m \frac{x_i^{m-1} e^{-\frac{m}{y}x_i}}{\Gamma(m)}, \quad x_i > 0, \quad i = 1, 2 \ldots M. \tag{9}$$

If Z is the output of the MRC algorithm, the density function of Z can be expressed as [1]

$$f_{Z\mid Y}(z \mid y) = \left(\frac{m}{y}\right)^{mM} \frac{z^{mM-1} e^{-\frac{m}{y}z}}{\Gamma(Mm)}, \quad z > 0, m > 0, M \geq 1. \tag{10}$$

Note that in Eq. (10), the density is still conditioned on Y. Using Eqs. (2)–(5) and (10), the density function of the output of the MRC algorithm becomes

$$f_Z(z) = \frac{2\left(\frac{b}{2}\right)^{c+Mm} z^{\left(\frac{c+Mm}{2}\right)-1}}{\Gamma(Mm)\Gamma(c)} K_{Mm-c}(b\sqrt{z}), \quad z > 0, m > 0, c > 0, M \geq 1. \tag{11}$$

For the selection combining (SC), in the presence of fading and shadowing, the density function of the signal-to-noise ratio in any branch X_i is given by Eq. (5). If W is the output of the selection combining algorithm, the pdf of W can be expressed as [1]

$$f_W(w) = M[F_X(w)]^{M-1} f_X(w) \tag{12}$$

where $F_X(w)$ is the cumulative distribution function (CDF) obtained from the pdf in Eq. (5). The CDF can be expressed as [16, 19]

$$\begin{aligned} F_X(w) = \int_0^w f_X(x)dx = \frac{\Gamma(m-c)\left(\frac{wb^2}{4}\right)^c}{\Gamma(m)\Gamma(c+1)}\, {}_1F_2\!\left(c, [1-m+c, 1+c], \frac{wb^2}{4}\right) \\ + \frac{\Gamma(c-m)\left(\frac{wb^2}{4}\right)^m}{\Gamma(m+1)\Gamma(c)}\, {}_1F_2\!\left(m, [1-c+m, 1+m], \frac{wb^2}{4}\right) \end{aligned} \tag{13}$$

where ${}_1F_2(x_1; [x_2, x_3]; x_4)$ is the generalized hypergeometric function [19]. Eq. (12) now becomes

$$f_W(w) = M[F_X(w)]^{M-1}\frac{2}{\Gamma(m)\Gamma(c)}\left(\frac{b}{2}\right)^{c+m} w^{(\frac{c+m}{2})-1}K_{c-m}(b\sqrt{w}). \tag{14}$$

The effect of diversity algorithm on fading and shadowing can be explored by calculating the amount of fading (AF) after the implementation of diversity. The amount of fading [Eq. (7)] is obtained from the first and second moments of the density functions given in Eqs. (11) and (14). For the case of MRC algorithm, the expression for AF will be similar to Eq. (8) because of the similarity of the expressions for the pdfs in Eqs. (5) and (11). The amount of fading after MRC algorithm is

$$AF_{MRC} = \left(\frac{1}{mM}\right) + \left(\frac{1}{c}\right) + \left(\frac{1}{mMc}\right). \tag{15}$$

The amount of fading after SC algorithm can be evaluated numerically by estimating the first and second moments of the pdf in Eq. (14). Figure 1 shows the amount of fading in MRC and SC systems. While the amount of fading is inversely proportional to m in the absence of any shadowing, the presence of shadowing keeps the AF from reaching low levels. From Eq. (15), it is seen that the amount of fading in MRC systems asymptotically reaches a value of

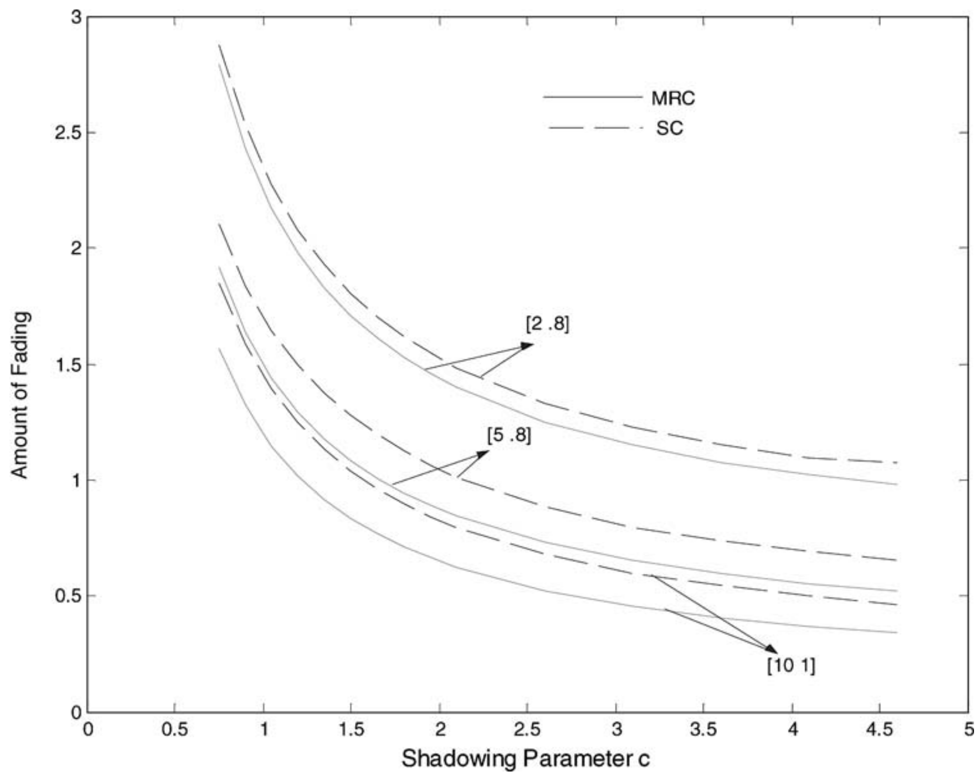

Figure 1. The amount of fading (AF) as a function of the shadowing parameter c is plotted. Three sets of $[M, m]$ curves are shown.

$(\frac{1}{c})$ when $M \to \infty$. In the absence of shadowing, the amount of fading would have been zero in MRC systems as $M \to \infty$. The asymptotic trend of AF in SC systems is similar. As expected, the amount of fading is lower in MRC systems than in SC systems. All the computations were done using Matlab [24].

Average Probability of Error

Before calculating the average probability of error in channels implementing diversity combining algorithms, it is necessary to see and establish the similarity between the compound pdf and the Nakagami-lognormal fading models. This was undertaken by calculating the average probability of error of DPSK using both models. The average probability of error under the Nakagami-lognormal model is given by

$$p_{av}^{\mathrm{NL}} = \int_0^{\infty}\int_0^{\infty}\left[\frac{1}{2}e^{-xy}\right]\left[\left(\frac{m}{y}\right)^m \frac{x^{m-1}e^{-\frac{m}{y}x}}{\Gamma(m)}\right]\left[\frac{K}{y\sqrt{2\pi\sigma^2}}e^{-\frac{[10\log_{10}y-\mu]^2}{2\sigma^2}}\right]dy\,dx \tag{16}$$

where the superscript NL refers to the Nakagami-lognormal fading. In Eq. (16), the quantity in the first bracket is the probability of error of DPSK in an ideal channel, the quantity in the second bracket is the Nakagami pdf of the power and the third bracket contains the lognormal shadowing term. The parameters, μ and σ are in decibels (dB) and K is a dimensionless quantity, $K = \frac{10}{\log_e 10} = 4.3429$ [1]. The severity of shadowing is measured in terms of σ, the higher the value of σ, the higher the shadowing. The relationship and similarities between gamma and the lognormal pdfs were studied by other researchers and their parameters can be related through [25, 26]

$$\sigma = K\sqrt{\psi'(c)}\,\mathrm{dB} \tag{17}$$

and

$$\mu = K[\log_e(y_0) + \psi(c)]\,\mathrm{dB}. \tag{18}$$

where ψ is the digamma function and ψ' is the trigamma function [16]. The average signal-to-noise ratio Z_0 is

$$Z_0 = cy_0. \tag{19}$$

The average probability of error under the compound pdf model is given by

$$p_{av}^{cm} = \int_0^{\infty}\left[\frac{1}{2}e^{-x}\right]\frac{2}{\Gamma(m)\Gamma(c)}\left(\frac{b}{2}\right)^{c+m} x^{(\frac{c+m}{2})-1}K_{c-m}(b\sqrt{x})\,dx \tag{20}$$

where the superscript^{cm} refers to the compound pdf. The average signal-to-noise ratio Z_0 can be expressed as

$$Z_0 = \left(\frac{2}{b}\right)^2 mc. \tag{21}$$

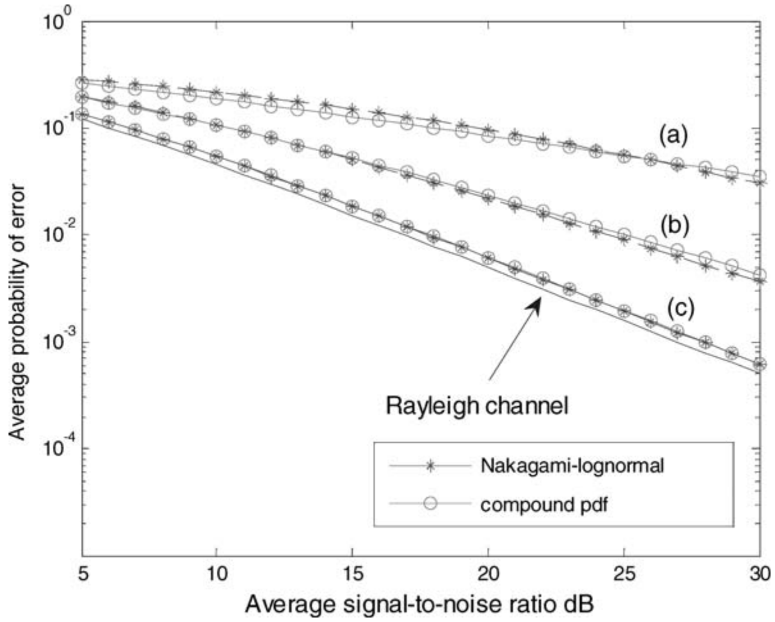

Figure 2. The average probability of error for DPSK is plotted. Curves show excellent agreement between the Nakagami-lognormal and compound pdf based models for the shadowed fading channels. (a) $m = 0.85, c = 0.4$; AF = 6.6; $\sigma^2 = 11.7$ dB (b) $m = 0.8, c = 1.2$; AF = 3.1; $\sigma^2 = 4.9$ dB (c) $m = 1.0, c = 5.0$; AF = 1.4; $\sigma^2 = 2.0$ dB. The results for the pure Rayleigh channel are also shown.

The results are shown in Figure 2. The agreement between the values of the average error probabilities calculated using the Nakagami-lognormal model for the shadowed fading channels and the compound pdf model are excellent. This strongly suggests that the compound pdf of Eq. (5) can model shadowed fading channels more than adequately.

We can now proceed to calculate the average probability of error when the diversity algorithms are implemented. Using the example of BPSK modulation, the average error probabilities can be expressed as

$$\overline{p_{\mathrm{MRC}}} = \int_0^\infty \frac{1}{2} \operatorname{erfc}(\sqrt{z}) f_Z(z)\, dz, \tag{22}$$

and

$$\overline{p_{\mathrm{SC}}} = \int_0^\infty \frac{1}{2} \operatorname{erfc}(\sqrt{w}) f_W(w)\, dw \tag{23}$$

where $f(z)$ and $f(w)$ are given in Eqs. (11) and (14) respectively. The subscripts MRC and SC respectively refer to MRC and SC algorithms. The probability of error in the absence of fading is given by $\frac{1}{2}\operatorname{erfc}(\sqrt{z})$ or $\frac{1}{2}\operatorname{erfc}(\sqrt{w})$. Equations (22) and (23) can be evaluated numerically. The average signal-to-noise ratio per bit in each branch, Z_0, was given in Eq. (21).

Some difficulties are likely to be encountered in the evaluation of the integral in Eq. (23) due to the complexity associated with the generalized hypergeometric function ${}_1F_2\,()$ [16, 19]. Equation (13) contains a gamma function of a negative argument. So long as the argument is not an integer, gamma function of the negative argument is finite. It is safe to assume that m and c can take non-integer values and therefore, the argument of the gamma function is very likely to be a non-integer. These problems can be avoided by expressing Eq. (23) in a double integral form as

$$\overline{P_{\mathrm{SC}}} = \int_0^\infty \int_0^\infty \frac{M}{2}\operatorname{erfc}(\sqrt{w})\left(\frac{m}{y}\right)^m \frac{w^{m-1}e^{-\frac{m}{y}w}}{\Gamma(m)}\left[P\left(\frac{wm}{y}, m\right)\right]^{M-1} \frac{y^{c-1}e^{-\frac{y}{y_0}}}{\Gamma(c)\, y_0^c} dy dw \quad (24)$$

where $P(,)$ is the incomplete gamma function. The parameter y_0 was given in Eq. (19).

The results of the analyses are plotted in Figures 3, 4 and 5. Figure 3 shows the plot of the average probability of error for the MRC algorithm for three sets of values of $[M, m, c]$, the pure Rayleigh channel and the ideal Gaussian channel. Because of the existence of the shadowing component, the drop in error probability with the average signal-to-noise ratio is not as steep as one would expect from diversity combining and processing. Similar trends are seen in Figure 4 which shows the results for the SC algorithm. The performance of MRC algorithm is better than that of the SC algorithm as shown in Figure 5 which compares the average error probabilities for the MRC and SC algorithms side-by-side.

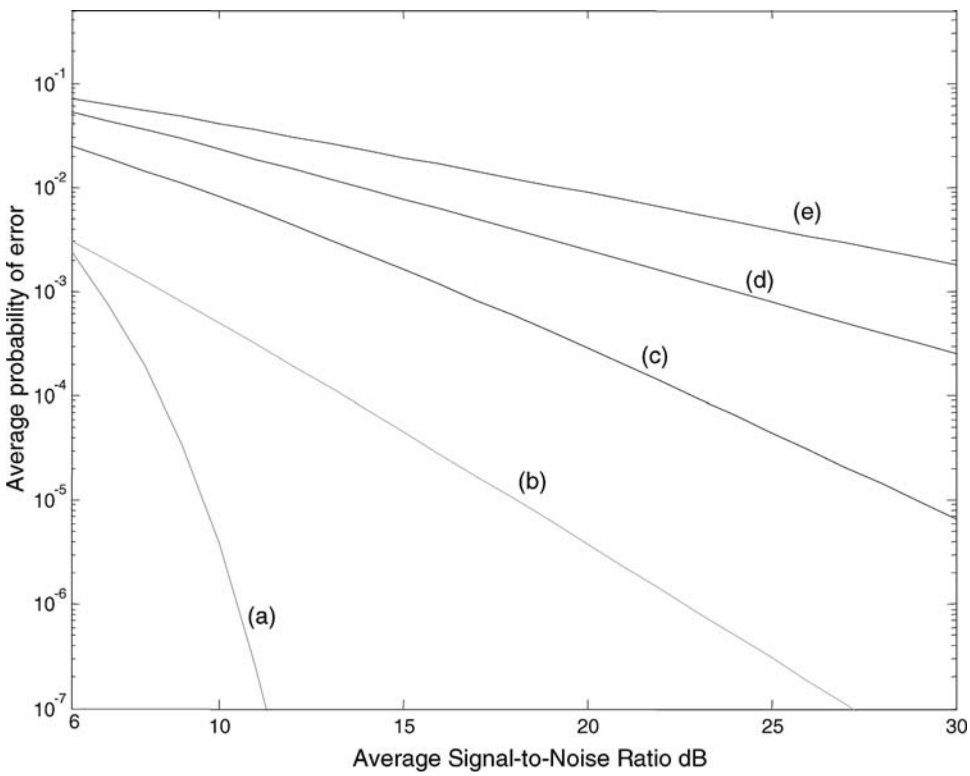

Figure 3. The average probability of error is plotted for the MRC algorithm against the average signal-to-noise ratio for three sets of $[M, m, c]$, the ideal Gaussian channel and the pure Rayleigh channel. (a) Gaussian channel (b) [4, 1.5, 2.2] (c) [2, 1, 1.8] (d) Rayleigh channel (e) [2, .8, .7]. Note that the relationship between the parameters of the compound pdf and the Nakagami-lognormal pdf can be obtained through Eqs. (17) and (18).

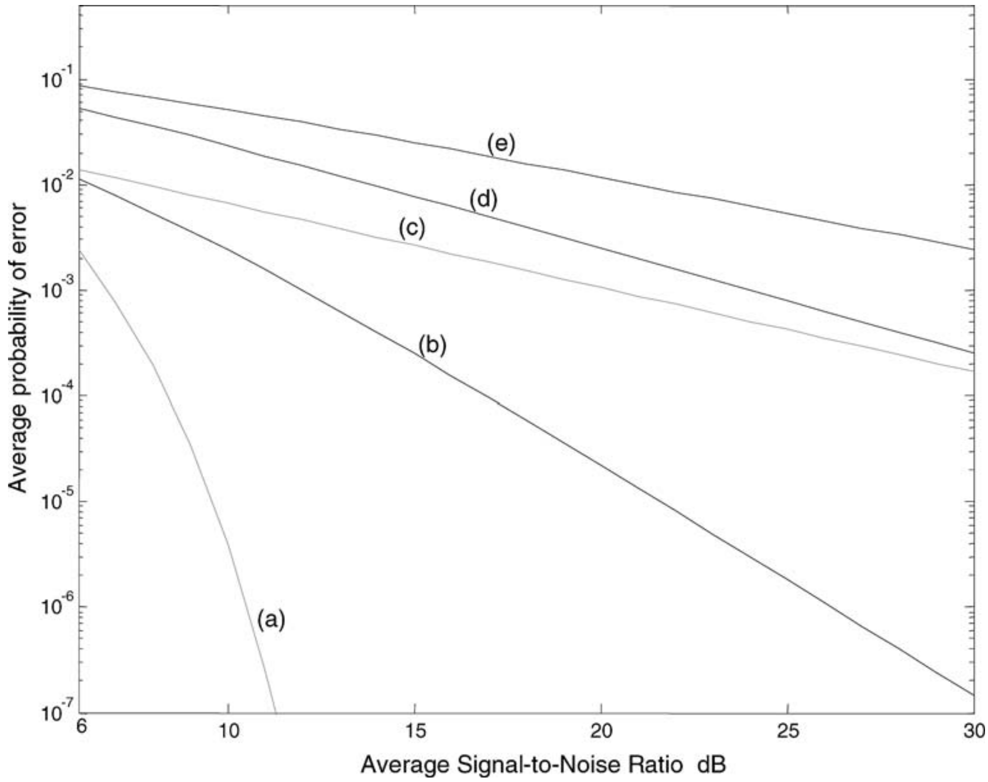

Figure 4. The average probability of error is plotted for the SC algorithm against the average signal-to-noise ratio for three sets of $[M, m, c]$, the ideal Gaussian channel and the pure Rayleigh channel. (a) Gaussian channel (b) [4, 1.5, 2.2] (c) [2, 1, 1.8] (d) Rayleigh channel (e) [2, .7, .7]. Note that the relationship between the parameters of the compound pdf and the Nakagami-lognormal pdf can be obtained through Eqs. (17) and (18).

The general nature of the compound fading model is also seen in Table 1 which contains the values of the average probability of error for two sets of values of M and m. The results shown are for the MRC algorithm. The values of shadow parameter c are taken to be 6.8, 10.8, 14.6 and 15.7. The value of $c = \infty$ corresponds to a pure Nakagami channel. As the value of c increases, the error rates start approaching those of the pure Nakagami channel. These results show that the model permits the flexibility to have variable values of the amount of fading.

Table 1. The values of the average probability of error are tabulated for two sets of M and m for three values of the average-signal-to-noise ratio Z_0 (dB).

Z_0	$M = 4; m = 0.75$					$M = 3; m = 0.95$				
(dB)	$c = 6.8$	$c = 10.8$	$c = 14.6$	$c = 15.7$	$c = \infty$	$c = 6.8$	$c = 10.8$	$c = 14.6$	$c = 15.7$	$c = \infty$
10	5.50E-06	2.50E-06	1.86E-06	1.76E-06	8.50E-07	3.35E-04	2.48E-04	2.18E-04	2.14E-04	1.56E-04
14	1.40E-07	5.50E-08	3.80E-08	3.60E-08	1.55E-08	3.11E-05	2.18E-05	1.88E-05	1.83E-05	1.29E-05
18	2.90E-09	1.00E-09	6.80E-10	6.35E-10	2.68E-10	2.53E-06	1.71E-06	1.47E-06	1.42E-06	9.87E-07

Five values of $c = 6.8, 10.8, 14.6, 15.7, \infty$ were used. The value of $c = \infty$ corresponds to a pure short term fading channel with no shadowing. The results are shown for the MRC algorithm.

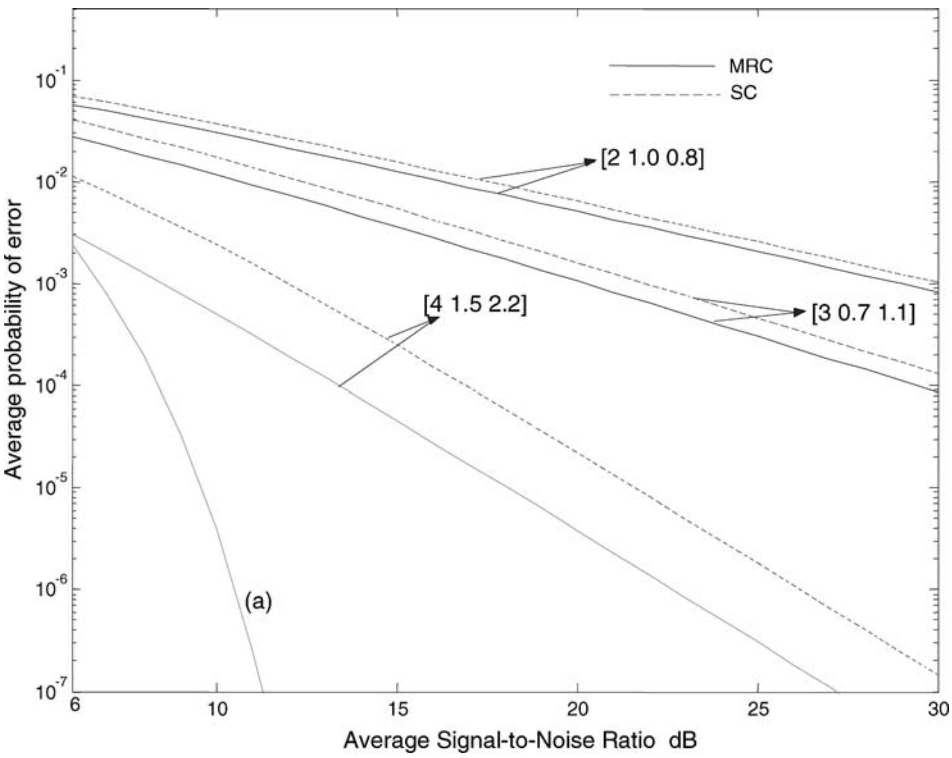

Figure 5. The average probability of error is plotted for the MRC and SC algorithms against the average signal-to-noise ratio for 3 sets of $[M, m, c]$ and the Gaussian channel. (a) Gaussian channel. Note that the relationship between the parameters of the compound pdf and the Nakagami-lognormal pdf can be obtained through Eqs. (17) and (18).

Discussion and Conclusions

Using a general model for shadowed fading channel, the performances of the diversity algorithms have been evaluated. The model is general enough to include Rayleigh, Rician and Nakagami models for short term fading and long term fading (shadowing) including the lognormal. The reduction in the amount of fading in systems employing diversity combining algorithms is limited due to the existence of the shadowing component. Furthermore, the limiting value of the amount of fading is determined by the shadowing factor in these microscopic diversity systems. The mitigation of fading expected from diversity combining algorithms is also limited in shadowed fading channels, with MRC performing better than SC as seen in conventional fading analyses.

The results reported here show that by varying the parameters of the shadowed fading channel, namely m and c, it is possible to simulate different levels of fading and shadowing. The availability of a closed form solution for the pdf of the signal-to-noise ratio in shadowed fading channels is expected to make the study of performance of wireless systems more manageable.

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