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Parity between prescription and visitation rates in healthcare

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Abstract
In the refinement of healthcare practices, interrelated visitation and excessive medications rates are vital. This article extracts pertinent data evidence in a novel manner using a new bivariate probability model and probes their balancing nature. For this purpose, we introduce a *parameter,* $0<\xi<1$ and investigate its role. All expressions in our new methodology are explained with big healthcare data. Some comments are made for future work to refine more healthcare practices.

Keywords Australian Health Survey data for 1977–1978 · Bivariate Poisson · Number of prescriptions · Patient's and physician's reaction

1 Introduction

The healthcare practices currently undergo a thorough refinement, largely due to heterogeneity among patients and also diverse practicing styles by the physicians. However, a common concern is felt across many nations, and it is the excessive medications that are consumed by the patients. When the incidence of excessive medications occurs repeatedly, it unnecessarily inflates the healthcare cost.

Consequently, the healthcare efficiency is weakened. Why does healthcare rather than any other service sector emerge to the top concern in the list? In many nations, the federal and state governments subsidize the healthcare sectors (larger segment of the economy) creating a budget deficit year after year, increasing the national debt. The healthcare sector constitutes approximately 10% of every country's GDP. The Center for Medicare and Medicaid Services in USA confirmed that 30% of the healthcare budget is waste. Major reason for the waste is cited to be excessive and unnecessary medications consumed by the patients. This pattern of waste needs a scrutiny for the sake of creating cost effective healthcare operations. A comprehension of this is targeted in this article via a new bivariate probability model and the data.

First, let us capture what has been done so far in this topic of excessive medications. This research topic needs much more attention of the scholars. An existence of excessive use of antibiotics was pointed out by Li et al. [4]. An alert made by He [3] was that the excessive medication was intended by the physicians as defence against the medical malpractice lawsuits. The China's health reforms of 2009 (see Ding et al. [2] for details) identified that the medical insurance system changed the pattern of prescribing medications. A warning was made by Sacarny et al. [6] that unnecessary medication threatens the continuation of Medicare. Based on 230,800 medications prescribed during 2007 through 2009 in 784 community health clinics covering 28 cities across China. It was concluded by Parveen et al. [5] that the most probable number of medications prescribed by a physician is three. At times, the style of prescribing excessive medications inflated the patient's critical drug resistance level. Hence, the healthcare reforms require a proper realization of the consequences of excessive medications.

To be exact, let a random variable $X\geq 0$ indicates *number of visitations to physician* (with $\lambda>0$ meaning *visitation rate*). Likewise, let a related random variable $Y\geq 0$ be the *number of prescribed medications to a patient by a physician* (with $\theta>0$ indicating the *medication rate*). This article provides probabilistic reasons for $X\leq Y$. How much this scenario undercuts the healthcare efficiency? A

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discussion would help to formulate a better healthcare policy. Do excessive medications induce more reimbursement from the insurance industries?

With appropriate probability concepts, this article explains the consequences of excessive medications and how to extract evidence from healthcare data for the existence of excessive medications. The literature does not have a clear approach on this issue. Hence, why not we develop a new innovative approach to discuss the pattern of excessive medications?

For this purpose, we introduce a *parameter*, $0<\xi<1$ and explore its assistance. In Sect. 2, we derive a model with this parameter and its statistical properties. The model's parameters are estimated and interpreted with a big healthcare data in Sect. 3. The article concludes with recommendations to create strategic policies for better healthcare practices in the last section. We point out that this methodology is applicable to issues in electronic and wireless communications as well.

2 A new bivariate Poisson model

Let the correlated bivariate Poisson random variables X and Y denote *the number of visits to doctor and the number of written prescriptions*. How much might be their correlation? To answer this question, an expression is derived later in the article.

In the past, bivariate probability models have been utilized in studies of healthcare type. However, none of them is appropriate in discussion of excessive medications. Because, our *scenario* depicts an imbalance in favor of more y than x. It dictates a modification in bivariate probability model before its adaptation in our set up.

What are out there in the literature so far as probability models. The incidences of rapes are explained by a bumped-up Poisson model. The infrastructures within menopause types is illustrated by a bivariate model. A bivariate model describes hospital's management efficiency. A specific bivariate model estimates the level of xenophobia in a survey sampling. A pattern of non-adherence by the patients to prescribed medications is explained by a bivariate model. See Shanmugam and Chattamvelli [7] for details.

Risk for unpleasant adverse reactions is likely to increase under the existence of excessive medications. Do excessive medications occur because of aggressively advertised *medicines*? Medications of antibiotics, narcotic pain killers are some examples of advertised medicines. More than 25% of the patients receive antibiotics or pain killers.

First, let us understand an ideal situation. It is one in which the patient receives one medication per visit to the physician. In such an ideal scenario, the number of prescriptions equals the number of visits. On the contrary, a situation might arise favoring more y than x, due to writing excessive medications by the physicians. Let the probability for a deviation from the ideal scenario is $0\le\xi\le1$. Then the *odds* for prescribing an additional medication by the physician is $\frac{\xi\lambda}{\theta}$. The odds diminish when the medication rate, θ increases. The odds increase along with the increasing visitation rate, λ as the chance, ξ for the imbalance to increase. The bivariate model to describe excessive medications is

$$\begin{aligned}&\Pr[Y=y,X=x]\\&\quad=e^{-(\theta+\xi\lambda)}\theta^{y}\left(\frac{\xi\lambda}{\theta}\right)^{x}/x!(y-x)!;\\&\qquad y\ge x,x,y=0,1,2,\ldots,\infty;\quad\lambda,\theta,0\le\xi\le1.\end{aligned}\tag{1}$$

Both X and Y in (1) are dependent. Shanmugam and Chattamvelli [7] enlists ways to verify independence among variables. In the model (1), the probability $\pi^{00}=\Pr(Y=0,X=0)=e^{-\theta}\{e^{-\lambda\xi}\}$ denotes the likelihood of receiving no medication from the physician times an exponentiated likelihood of no visitation to the physician, where the moderation is done by the parameter ξ (see Fig. 1). When $\xi\to0$, note that

$$\pi^{00}=e^{-\theta}\lim_{\substack{\xi\to0\\x\to0\\y\to0}}\binom{x}{y}\left(\frac{\xi\lambda}{\theta}\right)^{y}=e^{-\theta},$$

referring just the proportion receiving no medication from the physician. When $\xi\to1$, the model (1) changes to

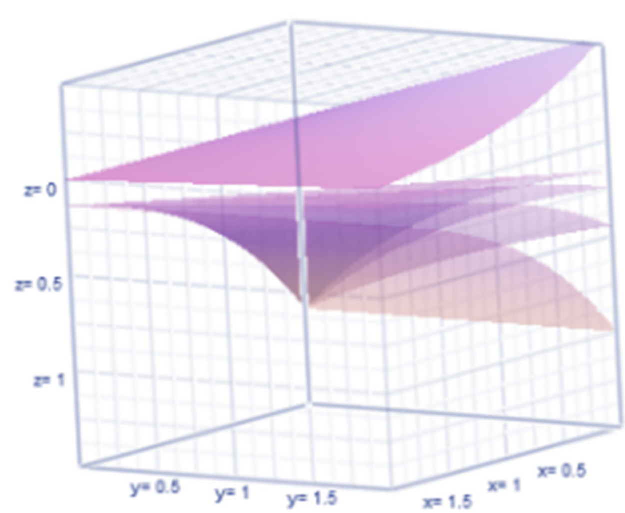

Fig. 1 The nonlinear π^{00} for values $\xi=0,0.1,0.4,0.7,$ and 0.9

$$\Pr[Y = y, X = x] \rightarrow \left[\frac{e^{-\lambda}\lambda^{x}}{x!}\right]\left[\frac{e^{-\theta}\theta^{(y-x)}}{(y-x)!}\right]; y \geq x; \quad y = 0, 1, 2, \ldots, \infty; \lambda > 0, \theta > 0, \xi \to 1,$$

meaning that x and y are sample space dependent but probabilistically independent. Incidentally, an implication is $-\ln \pi^{00} - \lambda < \theta < -\ln \pi^{00}$ due to $0 < \xi < 1$ (see Fig. 2).

The number of visits follows marginally a probability pattern

$$\Pr[X = x] = e^{-\xi\lambda}(\xi\lambda)^{x}/x!; x = 0, 1, 2\ldots, \infty; \quad 0 \leq \xi \leq 1; \lambda > 0. \tag{2}$$

The unrestrictive expected visits $E[X = x|\lambda, \theta, 0 \leq \xi \leq 1] = \xi\lambda$ is lesser than the restrictive (that is, $\xi = 1$) expected visits $E[X = x|\lambda, \theta] = \lambda$. The unrestrictive volatility $Var[X = x|\lambda, \theta, 0 \leq \xi \leq 1] = \xi\lambda$ is smaller than the restrictive (that is, $\xi = 1$) volatility $Var[X = x|\lambda, \theta] = \lambda$. The *jump rate* from no visit to one visit is then $\xi\lambda$. The jump rate is moderated by the parameter ξ, but is delinked from medication rate θ. The excessive visitation tendency is explicated in Theorem 1.

Theorem 1 *A patient has an excessive visitation tendency if s/he makes more than their most probable number* $v = [\xi\lambda]$ *with chance* $Pa_{phobia} = \Pr(\chi^2_{2vdf} \leq 2\xi\lambda)$, *because of cumulative Poisson and Chi squared probabilities, where df denotes the degrees of freedom.*

Proof Let $v = [\xi\lambda]$ be the most probable number of visits. The probability that a patient visit more than the most probable number, v is

$$Pa_{phobia} = \Pr[X > v|\xi, \lambda] = \sum_{x=v+1}^{\infty} e^{-(\xi\lambda)}(\xi\lambda)^{x}/x!$$

because the random variable X follows Poisson distribution, according to (2). Consider an incomplete gamma integral $\int_{\xi\lambda}^{\infty} e^{-x}x^{v}dx$. Put $u = x - \xi\lambda$ so that the range of the integral changes to 0 to ∞. Then,

$$\int_{\xi\lambda}^{\infty} e^{-x}x^{v}dx = \int_{0}^{\infty} e^{-(u+\xi\lambda)}(u + \xi\lambda)^{v}du = e^{-\xi\lambda}\int_{0}^{\infty} e^{-u}\sum_{j=0}^{v}\binom{v}{j}u^{j}(\xi\lambda)^{v-j}du.$$

By pulling the coefficients independent of u outside the integral, we notice that

$$e^{-\xi\lambda}\sum_{j=0}^{v}\binom{v}{j}(\xi\lambda)^{v-j}\int_{0}^{\infty} e^{-u}u^{j}du.$$

Realizing that $\int_{0}^{\infty} e^{-u}u^{j}du = \Gamma(j + 1) = j!$ and substituting in the above expression and expanding $\binom{v}{j} = \frac{v!}{j!(v-j)!}$, we notice that j! gets cancelled out yielding $\int_{\xi\lambda}^{\infty} e^{-x}x^{v}dx = e^{-\xi\lambda}\sum_{j=0}^{v} v!(\xi\lambda)^{v-j}/(v - j)!$. Let us divide by v! on both sides and replace v! by $\Gamma(v + 1)$. Note that$\sum_{j=0}^{v} e^{-\xi\lambda}(\xi\lambda)^{v-j}/(v - j)! = \frac{1}{\Gamma(v+1)}\int_{\xi\lambda}^{\infty} e^{-x}x^{v}dx$.Put $v - j = k$ in the sum. When j = 0, k = v and when j = v, k = 0. Then, the sum is equivalent to $\sum_{j=0}^{v} e^{-\xi\lambda}(\xi\lambda)^{k}/k!$. Subtract both sides from 1. Then, the left side is $\sum_{k=v+1}^{\infty} e^{-\xi\lambda}(\xi\lambda)^{k}/k!$, which is the Pa_{phobia}. But the right side is $\frac{1}{\Gamma(v+1)}\int_{0}^{\xi\lambda} e^{-x}x^{v}dx$, which is the cumulative function of the gamma probability density function. By change of variable $x = \frac{1}{2}\chi^2$ with $v = \frac{2v}{2}$, where χ^2 is the Chi squared random variable, the right side changes to

$$\frac{1}{\Gamma\left(\frac{2v}{2} + 1\right)}\int_{0}^{2\xi\lambda} e^{-\frac{\chi^2}{2}}\left(\frac{\chi^2}{2}\right)^{\left(\frac{2v}{2}+1\right)-1} d\left(\frac{\chi^2}{2}\right),$$

which is the cumulative function of the Chi squared distribution, $\Pr(\chi^2_{2vdf} \leq 2\xi\lambda)$ with $2v$ degrees of freedom. Hence, the theorem 1 is proved. □

The marginal prescription probability pattern is

$$\Pr[Y = y|\lambda, \theta, 0 \leq \xi \leq 1] = \frac{(\theta + \xi\lambda)^{y}e^{-(\theta+\xi\lambda)}}{y!}; \quad y = 0, 1, 2\ldots, \infty; \theta > 0; \lambda > 0; 0 \leq \xi \leq 1. \tag{3}$$

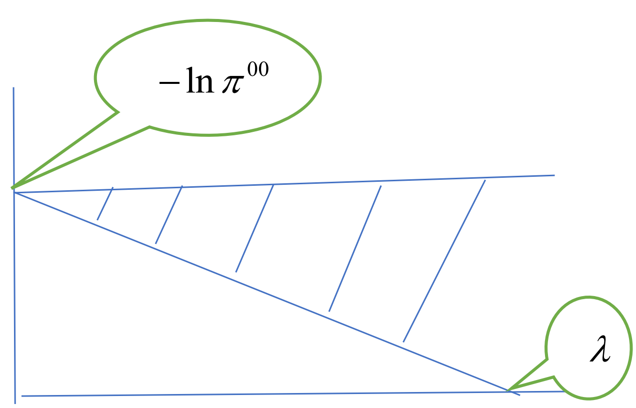

Fig. 2 π^{00} versus visitation rate

The chance for no prescription is $\Pr[Y=0]=e^{-(\theta+\xi\lambda)}$, which decreases as the parameter ξ and/or the visitation rate λ increases, according to (3). The likelihood for a patient to receive one medication is $(\theta+\xi\lambda)\Pr[Y=0|\lambda,\theta,0<\xi<1]$. The *jump rate* in prescription from none to one is $(\theta+\xi\lambda)$, suggesting that it is influenced by the parameter ξ in addition to visitation and prescription rates. The unrestrictive expected prescriptions is $E[Y=y|\lambda,\theta,0\le\xi\le 1]=\theta+\xi\lambda$, where the prescription increases when the visitation rate λ or the parameter ξ increases. Also, it is more from its counterpart $E[Y=y|\lambda,\theta]=\theta$ under the ideal Scenario ($\xi=0$). The unrestrictive volatility $Var[Y=y|\lambda,\theta,0\le\xi\le 1]=\theta+\xi\lambda$ is more than the restrictive volatility $Var[Y=y|\lambda,\theta]=\theta$ of the ideal Scenario. The excess volatility increases when the visitation rate λ and/or the parameter ξ increases. The excessive prescription tendency is stated in Theorem 2.

Theorem 2 *A physician prescribes excessively with a probability* $Ph_{phobia}=\Pr(\chi^2_{2mdf}\le 2m)$, *where* $m=[\theta+\xi\lambda]$.

Proof The line of argument runs parallel to that for Theorem 1 with a replacement of v by $m=[\theta+\xi\lambda]$ which is the most probable number of prescriptions. The probability that a physician writes more than the most probable number, m is

$$Ph_{phobia}=\Pr[Y>m|\theta,\xi,\lambda]$$
$$=\sum_{x=m+1}^{\infty}e^{-(\theta+\xi\lambda)}(\theta+\xi\lambda)^{y}/y!$$

because the random variable Y follows Poisson distribution, according to (3). Consider an incomplete gamma integral $\int_{\theta+\xi\lambda}^{\infty}e^{-y}y^{y}dy$. Put $w=y-(\theta+\xi\lambda)$ so that the range of the integral changes to 0 to ∞. Then,

$$\int_{\theta+\xi\lambda}^{\infty}e^{-y}y^{m}dx=\int_{0}^{\infty}e^{-(w+\theta+\xi\lambda)}(w+\theta+\xi\lambda)^{m}dw$$
$$=e^{-(\theta+\xi\lambda)}\int_{0}^{\infty}e^{-w}\sum_{j=0}^{m}\binom{m}{j}w^{j}(\theta+\xi\lambda)^{w-j}dw.$$

By pulling the coefficients independent of w outside the integral, we notice that

$$e^{-(\theta+\xi\lambda)}\sum_{j=0}^{m}\binom{m}{j}(\theta+\xi\lambda)^{m-j}\int_{0}^{\infty}e^{-w}w^{j}dw.$$

Realizing that $\int_{0}^{\infty}e^{-w}w^{j}dw=\Gamma(j+1)=j!$ and substituting in the above expression and expanding $\binom{m}{j}=\frac{m!}{j!(m-j)!}$, we notice that j! gets cancelled out yielding

$$\int_{\theta+\xi\lambda}^{\infty}e^{-y}y^{m}dy=e^{-(\theta+\xi\lambda)}\sum_{j=0}^{m}m!(\theta+\xi\lambda)^{m-j}/(m-j)!.$$

Let us divide by m! on both sides and replace m! by $\Gamma(m+1)$. Note that

$$\sum_{j=0}^{m}e^{-(\theta+\xi\lambda)}(\theta+\xi\lambda)^{m-j}/(m-j)!$$
$$=\frac{1}{\Gamma(m+1)}\int_{\theta+\xi\lambda}^{\infty}e^{-y}y^{m}dy.$$

Put $w-j=k$ in the sum. When j = 0, k = m and when j = m, k = 0. Then, the sum is equivalent to $\sum_{k=0}^{m}e^{-(\theta+\xi\lambda)}(\theta+\xi\lambda)^{k}/k!$. Subtract both sides from 1. Then, the left side is $\sum_{k=m+1}^{\infty}e^{-(\theta+\xi\lambda)}(\theta+\xi\lambda)^{k}/k!$, which is the Ph_{phobia}. But the right side is $\frac{1}{\Gamma(m+1)}\int_{0}^{\theta+\xi\lambda}e^{-y}y^{m}dy$, which is the cumulative function of the gamma probability density function. By change of variable $y=\frac{1}{2}\chi^2$ with $m=\frac{2m}{2}$, where χ^2 is the Chi squared random variable, the right side changes to

$$\frac{1}{\Gamma\left(\frac{2m}{2}+1\right)}\int_{0}^{2(\theta+\xi\lambda)}e^{-\frac{\chi^2}{2}}\left(\frac{\chi^2}{2}\right)^{\left(\frac{2m}{2}+1\right)-1}d\left(\frac{\chi^2}{2}\right),$$

which is the cumulative function of the Chi squared distribution, $\Pr(\chi^2_{2mdf}\le 2(\theta+\xi\lambda))$ with $2m$ degrees of freedom. Hence, the theorem 2 is proved. □

When a patient makes excessive visits, a physician may react ($\Re_{physician}$) as in (4). How likely it is? Its probability is

$$\Pr[\Re_{physician}]=\frac{[1-\Pr(\chi^2_{2vdf}>2(\theta+\xi\lambda))\Pr(\chi^2_{2vdf}>2\xi\lambda)]}{\Pr(\chi^2_{2vdf}\le 2(\theta+\xi\lambda))+\Pr(\chi^2_{2vdf}\le 2\xi\lambda)},\quad(4)$$

because of the DeMorgan's probability laws.

How probable for a patient to reciprocally react ($\Re_{patient}$)? Its chance is as in (5).

$$\Pr[\Re_{patient}]=\frac{[1-\Pr(\chi^2_{2mdf}>2(\theta+\xi\lambda))\Pr(\chi^2_{2mdf}>2\xi\lambda)]}{\Pr(\chi^2_{2mdf}\le 2(\theta+\xi\lambda))+\Pr(\chi^2_{2mdf}\le 2\xi\lambda)}\quad(5)$$

The number of visits, x for a given y prescriptions follows a binomial probability pattern is stated in (6). Note that

$$\Pr[X = x|Y = y] \\ = y!\theta^{y-x}(\xi\lambda)^x/(y-x)!x!(\theta+\xi\lambda)^y; \\ x \le y; x, y = 0, 1, 2, \ldots, \infty. \quad (6)$$

It means that the expected visits to the physician at a given prescriptions y are

$$E[X = x|y, \lambda, \theta, 0 \le \xi \le 1] = \left[\frac{\xi\lambda}{\theta+\xi\lambda}\right]y,$$

which is only a proportion $\left[\frac{\xi\lambda}{\theta+\xi\lambda}\right]$ of the number of prescriptions. The unrestrictive expected visits are $E[X = x|\lambda, \theta, 0 \le \xi \le 1] = \xi\lambda$. The adjustment factor is $\left[\frac{y}{\theta+\xi\lambda}\right]$. The unrestrictive volatility, $Var[X = x|y, \lambda, \theta, 0 \le \xi \le 1]$ is only a fraction $\frac{\theta}{\theta+\xi\lambda}$ of its expected visits $E[X = x|y, \lambda, \theta, 0 \le \xi \le 1]$. When $\xi \to 0$, the unrestrictive expected visits and its volatility become negligible.

The number, y of prescriptions for a given x visits has a probability pattern is displayed in (7). Note that

$$\Pr[Y = y|X = x] \\ = e^{-\theta}\theta^{y-x}/(y-x)!; y \ge x, x = 0, 1, 2, \ldots, \infty; \\ \lambda > 0, \theta > 0, 0 \le \xi \le 1. \quad (7)$$

The expected number of prescriptions $E[Y = y|x, \lambda, \theta, 0 \le \xi \le 1] = x + \theta$ is restrictive and different from the expected number of prescriptions $E[Y = y|\lambda, \theta, 0 \le \xi \le 1]. = \theta + \xi\lambda$. However, the restrictive volatility $Var[Y = y|x, \lambda, \theta, 0 \le \xi \le 1] = \theta$ is smaller than the unrestrictive volatility

$$Var[Y = x|\lambda, \theta, 0 \le \xi \le 1] = \theta + \xi\lambda.$$

Interestingly, both the restricted expected number and its volatility are immune to the parameter, ξ.

How much correlated are x and y? For it, we obtain first the product moment in (8). Note that

$$E[YX|y, \lambda, \theta, 0 \le \xi \le 1] \\ = E(E[X|Y = y, \lambda, \theta, 0 \le \xi \le 1]) \\ = \frac{\lambda\xi}{(\theta+\xi\lambda)}E[Y|\lambda, \theta, 0 \le \xi \le 1] = \xi\lambda. \quad (8)$$

Their covariance is $Cov(Y, X) \approx \xi\lambda$ and their correlation (see Fig. 3) is

$$\rho_{xy} = Corr(Y, X) \approx \left(1 + \frac{\theta}{\xi\lambda}\right)^{-1/2} \to 0, \quad (9)$$

when the parameter $\xi \to 0$. The *bivariate survival function* (*BSF*) is displayed in (10). Note that

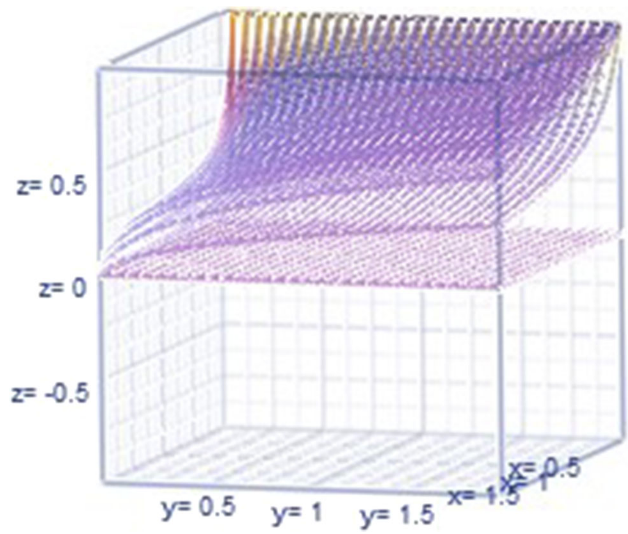

Fig. 3 Shift of expression (11) for $\xi = 0, 0.1, 0.4, 0.7,$ and 0.9

$$SF(X \ge r, Y \ge s|\lambda, \theta, 0 \le \xi \le 1] \\ \approx \Pr\left\{F_{2rdf,2(s-r+1)df} \le \frac{(s-r+1)}{r}\right\}e^{-(\theta+\xi\lambda)}\theta^s/s!, \quad (10)$$

where $\Pr\{F_{v_1df,v_2df} \le a\}$ denotes the cumulative F-probability. The *bivariate hazard rate* (*BHR*) is

$$HR(X = r, Y = s|\lambda, \theta, \xi] \\ \approx (Shift)HR(X = r|\lambda, \xi]HR(Y = s|\theta],$$

where

$$Shift \approx \frac{\Pr(\chi^2_{2[r+1]df} \le 2\xi\lambda)\Pr(\chi^2_{2[s+1]df} \le 2(\xi\lambda+\theta))}{\Pr\left\{F_{2[r+1]df,2(s-r+1)df} \le \frac{(s-r+1)}{(r+1)}\right\}}. \quad (11)$$

Let us discuss this scenario. At a threshold $\tau > 0$, the *excessive visitation to physician* is expected to be the amount in (12). Note that

$$EEVisit_x = E[X = x - \tau|\tau, \lambda, \theta, \xi] \\ = \frac{\sum_{i=\tau+1}^{\infty} SF(X \ge i|\lambda, \xi)}{SF(X \ge \tau|\lambda, \xi)} \\ = \left|\frac{1 - \sum_{x=1}^{\tau}\Pr(\chi^2_{2xdf} \le 2\xi\lambda)}{\Pr(\chi^2_{2\tau df} \le 2\xi\lambda)} - 1\right|. \quad (12)$$

The *risk* for making more visits is

$$TVaR_\tau(X) = E[X|X \ge \tau, \lambda, \theta, \xi] \\ = \tau - 1 + \frac{\lambda\xi\Pr(\chi^2_{2[\tau-1]df} \le 2\xi\lambda)}{\Pr(\chi^2_{2\tau df} \le 2\xi\lambda)}. \quad (13)$$

For a given minimum number of prescriptions $\kappa > 0$, the *excessive prescriptions* ($EE\,Prescriptions_y$) is shown in (14). We obtain

$$EE\,Prescriptions_y = E[Y = y - \kappa|\kappa, \lambda, \theta, \xi] = \left| \frac{1 - \sum_{y=1}^{\kappa} \Pr(\chi^2_{2ydf} \leq 2(\theta + \xi\lambda))}{\Pr(\chi^2_{2\kappa df} \leq 2(\theta + \xi\lambda))} - 1 \right|. \tag{14}$$

The *risk* for more receiving extra prescriptions is then

$$TVaR_\kappa(Y) = E[Y|Y \geq \kappa, \lambda, \theta] = \kappa - 1 + \frac{(\theta + \xi\lambda)\Pr(\chi^2_{2[\kappa-1]df} \leq 2(\theta + \xi\lambda))}{\Pr(\chi^2_{2\kappa df} \leq 2(\theta + \xi\lambda))}. \tag{15}$$

In some situations, patient *inclines* to visit more. A stable situation is one in which a patient makes only the necessary visits and the stability requires $\Pr(X > m + v) = \Pr(X > m)\Pr(X > v)$. In other words, the parameter

$$\delta = \frac{\Pr(\chi^2_{2[r+t]df} \leq 2\xi\lambda)}{\Pr(\chi^2_{2rdf} \leq 2\xi\lambda)\Pr(\chi^2_{2tdf} \leq 2\xi\lambda)}$$

Portrays patient's more, less, necessary visits (depends on whether $\delta < 1$, $\delta > 1$ or $\delta = 1$ respectively). Similarity occurs with respect to prescriptions. The parameter

$$\gamma = \frac{\Pr(\chi^2_{2[r+t]df} \leq 2(\theta + \xi\lambda))}{\Pr(\chi^2_{2rdf} \leq 2(\theta + \xi\lambda))\Pr(\chi^2_{2tdf} \leq 2(\theta + \xi\lambda))}$$

captures prescribing tendency. The physician prescribes more, less, or just needed medications based on whether $\gamma < 1$, $\gamma > 1$ or $\gamma = 1$ respectively.

Consider a sample $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ of size $n \geq 3$ from the bivariate Poisson population governed by the model (1). The maximum likelihood estimates (mle) of the parameters λ, θ, and ξ are preferable due to their invariance property. That is, the *mle* of a function of the parameters is the function of their *mle*. Let $\bar{x}, \bar{y}$ and r_{xy} denote sample average visits, average prescriptions, and their correlation respectively. Proportionally, log likelihood function is $\ln L \propto n(\bar{x} - \bar{y})\ln\theta - n(\theta + \xi\lambda) + n\bar{y}(\ln\lambda + \ln\xi)$.

Simultaneous solution of $\partial_\xi \ln L = 0$. $\partial_\lambda \ln L = 0, \partial_\theta \ln L = 0$, yield the *mle*. This redundancy does not cause computational problems. That is, $\hat{\theta}_{mle} = \bar{y} - \bar{x}$, $\hat{\xi}\hat{\lambda} = \bar{x}$ and $\hat{\rho}_{xy,mle} = \frac{\bar{x}}{\bar{y}}$.

Is the estimate $\hat{\rho}_{xy}$ significant with a confidence level $1 - \alpha \in (0, 1)$? Note that the projected number of prescriptions is

$$\hat{Y} = \bar{y} + \left(1 + \sqrt{\frac{\bar{x}}{\bar{y}}}\right)(x - \bar{x})$$

at given $X = x$ with a sampling error $(1 - \hat{\rho}^2_{xy})V\hat{a}r(Y)$. Similarly, the projected number of visits is

$$\hat{X} = \bar{x} + \frac{\bar{x}}{\bar{y}}(y - \bar{y})$$

at given $Y = y$ with a sampling error $(1 - \hat{\rho}^2_{xy})V\hat{a}r(X)$. The estimate $\hat{\rho}_{xy}$ is significant when the bracket $\frac{1}{2}\ln\left(\frac{1+\hat{\rho}_{xy,mle}}{1-\hat{\rho}_{xy,mle}}\right) \pm \frac{z_{\alpha/2}}{\sqrt{n-3}}$ does not contain zero, where $z_{\alpha/2}$ is the $(1 - \frac{\alpha}{2})^{th}$ standard normal percentile. All results of this section are illustrated in the next section.

3 Illustration using Australian health survey data

Cameron et al.'s [1] data are displayed in Table 1 of n = 4704 cases of *Australian Health Survey*. The visitation rate and the prescriptions rate are respectively 0.99 and 1.37. Based on (2), the proportion never visited physician and not received a prescription is $\pi^{00} = 0.25$. The *most likely* visits are $v = 2$. The chance for any *patient* to visit *more than the most likely number* v is the probability $Pa_{phobia} = \Pr(\chi^2_{4df} \leq 1.98) = 0.26$, based on Theorem 1. There is at 26% chance for any patient to make unnecessary visit. Per Theorem 2, the *most likely number of prescriptions* is $m = 3$. The *chance for over-prescribing* is $Ph_{phobia} = \Pr(\chi^2_{6df} \leq 2.74) = 0.16$.

The chance for physician's reaction is $\Pr[\Re_{physician}] = 0.84$. In other words, for every 84 reacting physicians, there are 16 non-reacting physicians. Likewise, the probability that a patient reacts is $\Pr[\Re_{patient}] = 0.94$. For every 94 patients reacting, there are only 6 non-reacting patients. See the conditional mean and conditional variance of visits and prescriptions are sketched in Figs. 4 and 5 respectively. Using (9) and Table 1, the estimate of the correlation between the number of prescriptions and the number of visits is $\hat{\rho}_{xy} = 0.85$. Based on (11), an estimate of the *bivariate hazard rate* (*BHR*) is 3.16 with r = 2 and s = 3.

With $\kappa = 4$, the *expected excessive prescriptions* are $EEprescriptions_y = 0.37$. Based on (15), the *tail value at risk* of more prescriptions is

$$TVaR_\kappa(Y) = 3 + \frac{1.37\Pr(\chi^2_{6df} \leq 2.74)}{\Pr(\chi^2_{8df} \leq 2.74)} = 7.33.$$

Analogously, with $\tau = 3$, the patient's *expected excessive visits* are $EEVisit_x = 0.58$. Using (13), the *risk* for making more visits is

Table 1 Data on visits and prescriptions in Australian Healthcare during 1977–1978

$\frac{Y\rightarrow}{X\downarrow}$	0	1	2	3	4	5	Sum	$\bar{y}\|X=x$	$s^2_{y\|X=x}$
0	907	67	55	44	20	9	1102	0.39	0.96
1		2566	135	108	77	35	2921	1.24	0.56
2			134	124	86	50	394	3.13	1.05
3				84	68	54	206	3.85	0.65
4					40	28	68	4.41	0.24
5						13	13	5	0
Sum	907	2633	324	360	291	189	4704		
$\bar{x}\|Y=y$	0	0.97	1.24	1.69	2.11	2.50			
$s^2_{x\|Y=y}$	0	0.02	0.53	0.93	1.32	1.62			
$\hat{\theta}_{mle}$	0.38								
$(\hat{\lambda}\hat{\xi})_{mle}$	0.99								
$\hat{\rho}_{xy}$	0.73								

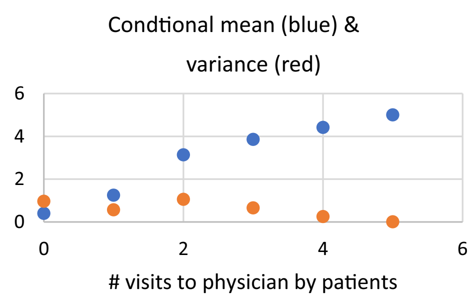

Fig. 4 Mean and variance of # visits

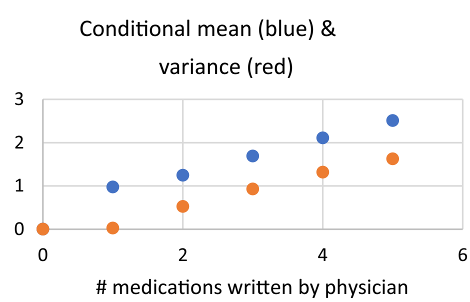

Fig. 5 Mean and variance of # prescriptions

$$TVaR_\tau(X) = 2 + \frac{0.99\,\Pr(\chi^2_{4df} \le 1.98)}{\Pr(\chi^2_{6df} \le 1.98)} = 5.28.$$

The parameter

$$\delta = \frac{\Pr(\chi^2_{10df} \le 1.98)}{\Pr(\chi^2_{4df} \le 1.98)\Pr(\chi^2_{6df} \le 1.98)} = 0.17$$

and hence, patients have visited more. Similarly, the parameter

$$\gamma = \frac{\Pr(\chi^2_{10df} \le 2.74)}{\Pr(\chi^2_{6df} \le 2.74)\Pr(\chi^2_{4df} \le 2.74)} = 0.21$$

and the number of prescriptions is excessive. Based on (9). the *mle* is $\hat{\rho}_{xy,mle} = 0.84$ and it is significant because the $1-\alpha = 0.95$ confidence interval

$$\frac{1}{2}\ln\left(\frac{1+\hat{\rho}_{xy,mle}}{1-\hat{\rho}_{xy,mle}}\right) \pm \frac{z_{\alpha/2}}{\sqrt{n-3}} = (1.22, 1.28)$$

does not enclose zero. The number of prescriptions is estimated to be $\hat{Y} = 7$, with a sampling error $(1-\hat{\rho}^2_{xy})V\hat{a}r(Y) = 0.38$ for a given $x = 4$ visits. Likewise, the number of visits is expected to be $\hat{X} = 2$ with a standard error $(1-\hat{\rho}^2_{xy})V\hat{a}r(X) = 0.27$ for a given $y = 3$ prescriptions.

4 Comments and conclusions

Not only this article provides a new bivariate probability model to illustrate the pattern in the incidences of the number of visits to the physician by the patients and the number of prescriptions written by the physician but also an expression to compute the correlation between the number of visits and the number of prescriptions.

Our model (1) of this article constructs a probabilistic interpretation and secure data evidence about the excessive medications prescribed by the physicians. The statistical significance of the correlation between visits and prescriptions can be assessed. The chance for the patient's reaction to visit to the physician is captured, estimated and interpreted. The probability of the existence of the physician's tendency to prescribe more medications is established, estimated using an analytic expression and is interpreted. It is worthwhile to extend this breakthrough methodology to find reasons and circumstances in which the physicians possess such an excessive prescription tendency.

For excessive prescriptions to happen, data on related covariates about the prescribing physicians need to be collected and examined. The healthcare professionals ought to pay extra attention to collect such big data. The analytic minded statisticians ought to build a multivariate regression methodology to make a projection on how many more excessive medications are possible in a situation.

A discovery of reasons for such an imbalanced situation (quite different from an ideal situation in which one medication per single visit of the patient occurs) is a necessity. Consequently, the goal for this twenty-first century ought to be putting together intensive efforts to reform the healthcare practices towards cost effectiveness could be attained. This article is a pioneering first step in the direction to attain such goal in big data analysis. The conceptual framework is constructed in this article for further advancements to do big data analysis.

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