

A robust energy-efficient power control algorithm for cognitive radio networks

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Abstract

To achieve energy efficiency is very important for future cognitive radio networks since we need less power consumption and much more transmitted information. In this paper, we propose a power allocation scheme with robust energy efficiency consideration for appropriately guaranteeing target signal to interference plus noise ratio (SINR) requirement for cognitive users and the received interferences at primary receivers below a threshold. A time-varying interference threshold protection factor and a protection margin to the SINR targets are introduced for the above purpose. This problem is formulated as a fraction programming problem solved by an iterative algorithm based on Lagrange dual approach by convex transformation. Simulation results show the validity of the proposed algorithm on both energy efficiency and robustness under channel gain disturbance.

Keywords Cognitive radio · Power control · Energy efficiency

1 Introduction

The explosive growth of real time wireless communication applications requires unprecedented demand on the radio spectrum [1]. However, wireless spectrum is scarce resource and the fixed spectrum allocation approach has seriously affected the development and the applications of wireless networks [2]. Therefore, to improve spectral efficiency becomes main goal for wireless communication networks and a challenging task for researchers worldwide. Cognitive radio (CR) has been proposed as a promising technology to upgrade the efficiency by using "spectrum holes" without affecting the spectrum used by primary users [3].

Power control plays an important role in CR networks for the improvement of the spectrum efficiency in wireless communications. For the past few years, some scientists have been working on enhancing network throughput [4–7] where the cognitive users adjust their transmit power to maximize their network rate under different constraints, including interference power limit from primary users, transmit power budget or SINR requirements for cognitive users.

High speed data transmission consumes more energy to guarantee quality of service (QOS) in wireless communications [8]. To overcome this problem, energy efficiency for cognitive radio networks has become increasingly crucial and attracted more attention from academic field recently. In [9], the authors consider a joint subchannel allocation and power control strategy to maximize energy efficiency of each cognitive transmitter for orthogonal frequency-division multiple access (OFDMA) CRNs with multiple cognitive transmitters. In [10], the authors consider power allocation schemes that adopt spectrum sharing combined with soft-sensing information, adaptive sensing thresholds, and adaptive power to design an energy efficient system. A power allocation scheme, basing on maximizing energy efficiency for spectrum sharing cognitive radio systems, is proposed in [11] with constraints in peak or average of power. In [12], the authors specifically deal with the problem of energy efficiency power control for single user OFDM-CR system. In [13], user association and

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power allocation in mm Wave-based ultra dense networks is considered with attention to load balance constraints, energy harvesting by base stations, user quality of service requirements, energy efficiency, and cross-tier interference limits. Zhang et al. [14] proposes a distributed radio resource allocation scheme to maximize the energy-efficiency in uplink OFDMA dense femtocell networks. They model the subchannel allocation and power control problem into a non-cooperation game. In [15], the studied scheme maximizes energy efficiency by allocating both transmit power of each small cell base station to users and bandwidth for backhauling, according to the channel state information and circuit power consumption. However, these existing works normally assume perfect knowledge about channel gain information, which is difficult or even infeasible to obtain perfectly in a practical CR network. In particular, the minimum acceptable SINR for all cognitive users can not be guaranteed when there exist perturbations in channel gains.

As an important issue for consideration in CRNs, robust energy efficiency power allocation has been studied in [16]. The authors investigate a robust energy efficiency maximization problem in underlay CRNs with multiple cognitive users and primary users. They consider that all channels lie in some bounded uncertainty regions. Under the worst-case conditions, a scheme to handle power allocation problem via fractional programming is proposed. An energy-efficient resource management with channel uncertainty is proposed in [17]. The authors aim to maximize the energy efficiency of a CR network while considering practical restrictions, including the power budget of the system, the interference thresholds of primary users, the rate requirements of cognitive users, and the fairness among them. The interference constraint is introduced as chance-constrained form and treated by the Bernstein approximation. These works consider the channel uncertainty with norm-bounded parameter perturbations. In [18], the authors propose a power control and sensing time optimization problem in a cognitive small cell network, where cognitive radio enabled small cell architecture is designed to opportunistically access the spectrum via cognitive small basestation. And there are no base stations or communication infrastructures in this paper.

The SINR of an active cognitive radio link may dip below the target requirement in the continuously changing environment. Different from the robustness considered in [16, 17], a protection algorithm is provided where robustness is captured in the constraints and the energy efficiency of communication links in the objective function in this paper. Our target is to maximize the energy efficiency of the cognitive system while having interference threshold protection factor and SINR requirement protection margin. The main contributions of this paper are summarized as follow:

- (1) An interference protection factor to provide a protection margin for the interference threshold of primary users is provided to keep the interference from cognitive-user transmitters below a permissible threshold that the primary user can tolerate. Thus the protection scheme can guarantee quality of service (QOS) of the primary users when there are disturbances in the network.
- (2) Based on protection factor, by using constrained fractional programming optimization that takes protection factors as the uncertain variables, a robust energy efficiency power allocation scheme is proposed with the consideration of total SINR of dB per joule of energy metric as an energy efficient objective function. Additionally, this scheme is also robust because of the introduction of interference constraint and target SINR requirement constraint.

The remainder of the paper is organized as follows. The system model for cognitive radio networks is introduced in Sect. 2. In Sect. 3, we present our robust energy efficiency power allocation algorithm. In Sect. 4, performance analysis for our proposed algorithm is performed through simulation results. Finally, the conclusion of this work is given in Sect. 5.

2 System model

We consider a cognitive radio network where a secondary system coexists with a primary system and each cognitive user has single transmitter and single receiver. We assume that there are M active cognitive radio transmitter–receiver pairs and only one primary user in the vicinity of interest, as shown in Fig. 1.

In this system cognitive users share the radio spectrum with primary user via the underlay mode. For the underlay based CR systems, cognitive users can use same frequency



Fig. 1 System model: underlay cognitive radio networks

spectrum with primary user as long as the interference power provided by cognitive users is less than a predefined tolerable threshold [19]

$$\sum_{i} p_{i} h_{i} \le IT \tag{1}$$

where p_i denotes the transmit power of cognitive user *i*. h_i denotes the channel gain between the primary receiver and cognitive transmitter *i*. *IT* is defined in a way to make sure that the tolerant interference power threshold does not violate the threshold at the primary receivers [20]. This constraint protects the communication links of primary users.

Let g_{ij} be the channel gain from the cognitive transmitter *j* to the cognitive receiver *i* and it includes all path loss and fading effects. Let H_{i1} denote the channel from the primary transmitter to the cognitive receiver *i*. T_1 represents the transmit power for the primary user. The SINR at the receiver on the active link *i* can be written as

$$r_{i} = \frac{g_{ii}p_{i}}{\sum_{j \neq i} g_{ij}p_{j} + H_{i1}T_{1} + n_{i}}$$
(2)

where n_i is the background noise at the cognitive receiver *i*.

The SINR of cognitive user constitutes the QOS of the user. Since the cognitive user adjusts it's transmit power to achieve the target SINR and maintain the desired QOS, we must make sure the SINR at each cognitive receiver achieves the value of its own target requirement. The achieved SINR satisfies

$$r_i \ge r_i^{fixed} \tag{3}$$

This constraint is the requirement that the *i*th received SINR is above a given SINR target r_i^{fixed} for all *i*. In this work, we want to provide a flexible power control scheme, since it is addressed to QOS-flexible services [21], in such a way that the fixed target SINR can be adjusted according to a given performance criterion, which the fixed target SINR modified to support the time-varying target SINR

$$r_i \ge r_i^t \tag{4}$$

where r_i^t can be regarded as the minimum time-varying SINR determined from the QOS constraints.

The constraints (4) can be equivalently represented as

$$(R-G)P \ge N \tag{5}$$

where *R* is a diagonal matrix with $\frac{1}{r'_1}, \frac{1}{r'_2}, \dots, \frac{1}{r'_M}$, matrix $N = \left[\frac{H_{11}T_1 + n_1}{g_{11}}, \frac{H_{21}T_1 + n_2}{g_{22}}, \dots, \frac{H_{M1}T_1 + n_M}{g_{MM}}\right], P = [p_1, p_2, \dots, p_M]^T$, and the element of matrix *G* is

$$G_{ij} = \begin{cases} \frac{g_{ij}}{g_{ii}}, & i \neq j \\ 0, & i = j \end{cases}$$
(6)

As we know that the performance of the green radio systems are usually measured by energy efficiency metric which presents the total SINR delivered to the destination per joule of consumed energy, i.e.,

$$\eta_{EE} = \frac{\sum_{i} r_i}{\sum_{i} p_i + P_C} \tag{7}$$

r

where η_{EE} denotes the energy efficiency. P_C presents the circuit power consumption of the source in the transmit mode.

Our goal is to maximize the energy efficiency of cognitive user with the constraints on that the total interference of cognitive users on primary user is below a given threshold and each cognitive user achieves a predefined SINR. This scheme is an energy efficiency algorithm (EEA). To be specific, we can figure out this problem of maximizing energy efficiency as the following fraction programming

$$\max \frac{\sum_{i} r_{i}}{\sum_{i} p_{i} + P_{C}}$$

s.t.
$$C1 : \sum_{i} p_{i} h_{i} \leq IT$$

$$C2 : r_{i} \geq r_{i}^{t}$$
 (8)

In our proposed power allocation algorithm, we assume that the instantaneous channel gains are perfectly estimated at the receiver. In particular, since this problem is not convex optimization, fractional programming approach is proposed in [22].

3 Robust energy efficiency power allocation algorithm

The channel gains in cognitive radio systems are imperfect owing to many practical factors. The energy efficiency power allocation algorithms without considering channel uncertainty are not appropriate when we concern this fact, since the allocated power by the traditional energy efficiency power allocation algorithms will break the optimization rules that assure the interference to primary user and the SINR requirement of the cognitive users under certain predefined threshold.

In addition, when new users enter the cell in the system or the channel gains perturbations exist, the optimal transmit power must be adjusted since the interference generated by the cognitive users may be violated the interference power threshold. When the interference power at a primary receiver is greater than the interference threshold, an outage event happens. The QOS of primary user will not be guaranteed. An active primary link protection scheme introduces a protection factor ε to the interference power threshold. The introduction of ε can cushion the perturbation of the channel gains, which is modified the interference constraint C1 in (8) as

$$\sum_{i} p_{i} h_{i} \leq IT(1-\varepsilon) \tag{9}$$

where $\varepsilon > 0$ and it is a simple percentage. This margin must be large enough such that the interference from cognitive users does not larger than the threshold. In particular, the larger ε , the better we can protect primary users. Furthermore, ε should be a time-varying variable according to time-varying target SINR requirement. That is

$$\sum_{i} p_{i} h_{i} \le IT(1 - \varepsilon(t)) \tag{10}$$

For our power allocation scheme, we call $\varepsilon(t)$ the interference protection factor.

The SINR requirements for cognitive users should remain feasible and near-optimal under the perturbation of parameters in the nominal optimization problem. To guarantee that active cognitive users continue to have acceptable SINR when there are disturbances in the CRN, a protection margin $\xi_i(t)$ is provided [20], i.e.,

$$r_i \ge r_i^{\scriptscriptstyle I} (1 + \zeta_i(t)) \tag{11}$$

where $\xi_i(t)$ is the protection margin for each user's SINR. $\xi_i(t)$ must large enough so that the SINR of each cognitive user do not drop below the SINR target and cause SINR outage. We use the term "robust" in the sense of safety margin against SINR outage.

The proposed energy efficiency power control algorithm is modified to a robust version which solves the following power control problem subject to the time-varying robustness settings

$$\max \frac{f_r(p_i)}{f_c(p_i)}$$
s.t.
$$C3: \sum_i p_i h_i \le IT(1 - \varepsilon(t))$$

$$C4: r_i \ge r_i^t (1 + \xi_i(t))$$
(12)

where $f_r(p_i) = \sum_i r_i$, and $f_c(p_i) = \sum_i p_i + P_c$. The robust energy efficiency algorithm (REEA) is formulated.

In the following section, we present robust power allocation formulation in cognitive radio network by using protection margins. We will enhance the interference constraint and the target SINR constraint to deal with this problem.

Robust optimization techniques based on the worst case analysis are more appropriate. In our proposed scheme, we take robustness into consideration through the introduction of the protection factors $\varepsilon(t)$ and $\xi_i(t)$ to the constraints C3 and C4 respectively. Larger protection factors guarantee that the interference power limits will not be violated and the SINR for each user will achieve the predefined value for larger perturbations. Constraint C4 in problem (12) can be equivalently written as

$$C4': \frac{r_i^t(1+\xi_i(t))}{p_i} \le \frac{1}{\sum_{j \ne i} G_{ij} p_j + \frac{H_{il}T_1 + n_i}{g_{ii}}}$$
(13)

We note that the problem (12) is nondeterministic polynomial (NP)-hard. Therefore, we must have its transformed forms that can provide an optimal solution. The original maximization optimize problem can be reformulated as a minimization problem so as to (12) can be written as

$$\min \frac{\sum_{i} p_{i} + P_{C}}{\sum_{i} r_{i}}$$
s.t.
$$C3 : \sum_{i} p_{i}h_{i} \leq IT(1 - \varepsilon(t))$$

$$C4' : \frac{r_{i}^{t}(1 + \xi_{i}(t))}{p_{i}} \leq \frac{1}{\sum_{j \neq i} G_{ij}p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}}$$

$$(14)$$

The global optimal solution is difficult to obtain since (14) defines a non-linear fractional programming problem. Fortunately, (14) can be transformed into an equivalent parametric problem using fractional programming [23]. We introduce a time-varying parameter $\alpha(t)$ by convex programming, and then we obtain a new optimization problem as

$$\min \sum_{i} p_{i} + P_{C} - \alpha(t) \left(\sum_{i} \frac{p_{i}}{\sum_{j \neq i} G_{ij} p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}} \right)$$

s.t. C3 : $\sum_{i} p_{i} h_{i} \leq IT(1 - \varepsilon(t))$
C4' : $\frac{r_{i}^{t}(1 + \xi_{i}(t))}{p_{i}} \leq \frac{1}{\sum_{j \neq i} G_{ij} p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}}$
(15)

where $\alpha(t)$ is for the target SINR requirement. If $p^* = [p_1^*, \ldots, p_I^*]^T$ is Nash equilibrium of the robust game (15) according to the particular value of $\alpha(t)$ given by $\alpha^*(t) = \frac{f_r^*(p_i)}{f_c^*(p_i)}$, p^* is also the optimal solution of (14) [17]. That is, we can analyze energy efficiency over (15), subject to two constraints, which is equivalent to directly solve (12).

We use the Lagrange dual algorithm to get the optimal solution. The Lagrange function of the optimization problem in (15) for cognitive user *i* is

$$L_{i}(p_{1},...,p_{I}) = \sum_{i} p_{i} + P_{C} - \alpha(t) \left(\sum_{i} \frac{p_{i}}{\sum_{j \neq i} G_{ij}p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}} \right)$$
$$+ \lambda_{i} \left(\sum_{i} p_{i}h_{i} - IT(1 - \varepsilon(t)) \right)$$
$$+ \sum_{i} v_{i} \left(\frac{r_{i}^{t}(1 + \xi_{i}(t))}{p_{i}} - \frac{1}{\sum_{j \neq i} G_{ij}p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}} \right)$$
(16)

where λ_i and v_i are the Lagrange multipliers for two constraints in (15), respectively.

The points to meet the Karush–Kuhn–Tucker (KKT) conditions are the optimal solutions since (15) is a convex optimization problem. The KKT conditions [24, 25] for the cognitive user *i* are as follows

$$0 \leq p_{i} \perp 1 - \frac{\alpha(t)}{\sum_{j \neq i} G_{ij}p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}} + \lambda_{i}h_{i} - \frac{\upsilon_{i}r_{i}^{t}}{p_{i}^{2}} \geq 0$$

$$0 \leq \lambda_{i} \perp \sum_{i} p_{i}h_{i} - IT(1 - \varepsilon(t)) \geq 0$$

$$0 \leq \upsilon_{i} \perp \frac{r_{i}^{t}(1 + \xi_{i}(t))}{p_{i}} - \frac{1}{\sum_{j \neq i} G_{ij}p_{j} + \frac{H_{i1}T_{1} + n_{i}}{g_{ii}}} \geq 0$$
(17)

where \perp signifies orthogonal of the corresponding variables.

From the KKT conditions of (17), the maximum energy efficiency occurs at a power level for which the partial derivation of $L_i(p_1, ..., p_I)$ with respect to p_i is zero, i.e.,

$$p_{i}^{*} = \sqrt{\frac{r_{i}^{t}(1+\xi_{i}(t))v_{i}}{1-\frac{\alpha(t)}{\sum_{j\neq i}G_{ij}p_{j}+\frac{H_{i1}T_{1}+n_{j}}{S_{ii}}} + \lambda_{i}h_{i}}}$$
(18)

where λ_i and v_i are the Lagrange multipliers for the constraints (14). These Lagrange multipliers can be updated by

$$\lambda_{i} = \left\{ \lambda_{i} + \beta_{1} \left(\sum_{i} p_{i} h_{i} - IT(1 - \varepsilon(t)) \right) \right\}^{+}$$
(19)

$$v_{i} = \left\{ v_{i} + \beta_{2} \left(\frac{r_{i}^{t}(1 + \xi_{i}(t))}{p_{i}} - \frac{1}{\sum_{j \neq i} G_{ij} p_{j} + \frac{H_{i}(T_{1} + n_{i})}{g_{ii}}} \right) \right\}^{+}$$
(20)

where $[x]^+ = \max(x, 0)$. β_1 and β_2 are iteration steps.

We shall further discuss the property of the protection factor $\varepsilon(t)$ and $\zeta_i(t)$ about the interference power and SINR respectively. Our robust power allocation scheme is different from the robust model in Ref. [26]. We use the term robust in the sense of safety margin against the outage.

In [26], let $(h_i + \Delta h_i)$ denote the channel gain between the cognitive transmitter *i* and the primary receiver, where h_i and Δh_i are the nominal value and the corresponding deviation part respectively. The first constraint in (8) can be written as

$$\sum p_i(h_i + \Delta h_i) \le IT \tag{21}$$

To be convenient for explanation, let $\Delta h_i = \kappa(t)h_i.\kappa(t)$ is percentage value. Re-writing (21) in percentage form

$$\sum p_i h_i (1 + \kappa(t)) \le IT \tag{22}$$

The interference power constraint at the primary receiver is given by (10), it is also expressed as follows:

$$\sum p_i \frac{h_i}{(1 - \varepsilon(t))} \le IT \tag{23}$$

The inequalities (23) and (24) indicate that $\varepsilon(t)$ has relationship with $\kappa(t)$, i.e.,

$$1 + \kappa(t) = \frac{1}{1 - \varepsilon(t)} \tag{24}$$

According to (24), it is necessary that the protection factor be at a value $\varepsilon(t)$ related to $\kappa(t)$ via

$$\varepsilon(t) = 1 - \frac{1}{1 + \kappa(t)} \tag{25}$$

If the value $\varepsilon(t)$ is roughly equivalent to the $\left(1 - \frac{1}{1+\kappa(t)}\right)$ for practical cognitive systems, our robust framework is equivalent to the robust algorithm [26] when other constraints unchanged.

The constraint C4 of problem (12) can be converted into the following form,

$$\frac{p_i}{\sum_{j \neq i} G_{ij}(1 + \xi_i(t))p_j + \frac{H_{i1}T_1 + n_i}{g_{ii}}(1 + \xi_i(t))} \ge r_i^t$$
(26)

The SINR constraint has the same part as the robust model [26] except $(1 + \xi_i(t))$. This indicates that these algorithms have the same effect if the denominator of the SINR constraint [26] times $(1 + \xi_i(t))$.

4 Simulation results

In this section, we provide simulation results to show the impact of protection factor $\varepsilon(t)$ and protection margin $\xi_i(t)$ on the energy efficiency attained with our optimal power control. We also present the performance analysis of the proposed algorithm for guaranteeing both interference power and target SINR requirements to each cognitive user. An ad hoc cognitive network is considered where a cognitive system coexists with a primary system.

We assume that there are three cognitive links and one primary link in the network. The simulation parameters here are similar with those given in [27], the channel gain h_i is chosen randomly from the interval [0.16, 0.20], with uniform distributions. The transmit power for primary user



Fig. 2 The total transmit power of REEA with different interference protection factor

 T_1 and the interference power threshold *IT* are 0.8 and 0.315 mW respectively. The circuitry power consumption is $P_C = 0.2$ mW, and the link gains **g** from the cognitive transmitter *j* to the cognitive receiver *i* is

$$\mathbf{g} = \begin{bmatrix} 0.9531 & 0.0454 & 0.0318\\ 0.0494 & 0.9889 & 0.0547\\ 0.0233 & 0.0498 & 0.9540 \end{bmatrix}$$
(27)

In the first scenario, the target SINR requirement is $r_i^t = 3.52$ dB for all users. Figure 2 depicts the convergence of the total transmit power by the REEA algorithm with different $\varepsilon(t)$ and $\xi_i(t) = 0$. From Fig. 2, the total transmit power of cognitive users increases with the increasing parameter $\xi_i(t)$ under a fixed SINR protection factor, which means that more transmit power is required to give the cognitive users more protection under channel uncertainty. Larger $\varepsilon(t)$ means higher channel uncertainty. Each cognitive user requires higher transmit power to overcome the impact of channel uncertainty.

The total transmit power for REEA with different SINR protection factor are shown in Fig. 3. The $\varepsilon(t)$ is zero, and other parameters are identical to previous scenario. It is obvious that the REEA algorithm with the higher SINR protection factor has higher total transmit power. The higher total transmit power can ensure that the SINR of each cognitive user is greater than the target SINR. In addition, higher transmit power is required to avoid the SINR at cognitive receiver below the target SINR under the imperfect channel state information (CSI).

In the second scenario, the disturbance parameters margin are $\varepsilon(t) = 10\%$ and $\zeta_i(t) = 5\%$. The power allocation for the cognitive users for EEA and REEA based on utility function is presented in Fig. 4. We find that similar results are obtained when users update their transmit power



Fig. 3 The total transmit power of REEA with different SINR protection factor



Fig. 4 Transmit power allocated by two algorithms for $r_i^t = 3.52$ dB

with different algorithm. The REEA algorithm and EEA algorithm can converge to the equilibrium points quickly. Additionally, the powers allocated by REEA algorithm are less than those obtained by EEA algorithm, since there is a trade-off between power consumption and robustness consideration. As a result, each cognitive user requires lower transmit power to guarantee the interference power threshold from primary user, i.e., Eq. (10), and an acceptable level of performance under worst case interference conditions. Large $\varepsilon(t)$ offers more protection to the cognitive users. From Eq. (18), the optimal transmit power of each cognitive user relates to $\xi_i(t)$ by considering the uncertainty of channel gain. $\xi_i(t)$ will increase under large perturbations. Larger $\xi_i(t)$ means higher channel uncertainty. Each cognitive user requires higher power to overcome the impact of channel uncertainty. $\varepsilon(t)$ has greater influence than $\xi_i(t)$. While EEA algorithm does not



Fig. 5 Transmit power and energy efficiency of REEA and EEA schemes for different target SINR requirement. **a** Transmit power for REEA and EEA algorithms. **b** Energy efficiency for REEA and EEA algorithms

consider the disturbance of the channel gains, hence the transmit power is constant. When the channel uncertainty happens, the unchanged powers of EEA algorithm may violate primary users' interference power threshold.

In this scenario, the target SINR has changed to $r_i^t = 5.11$ dB at the twenty-first time-step. All other parameters are the same as in the second scenario. The transmit power for all users are shown in Fig. 5(a) which shows the superiority of our robust REEA scheme because of lower power consumption. When we give perturbed parameters as $G_{\Delta} = \sum_{j \neq i} |\Delta G_{ij}|^2 \le 0.015^2$ and $h_{\Delta} = \sum_i |\Delta h_i|^2 \le 0.05^2$ for the consideration of practical situation where ΔG_{ij} and Δh_i denote the perturbation part to the channel gains of G_{ij} and h_i respectively. The energy efficiency for EEA and REEA algorithms is provided Fig. 5(b). REEA scheme has higher energy efficiency since more number of bits is delivered to the destination per joule of energy consumed.



Fig. 6 SINR of REEA and EEA algorithms for different target SINR requirement



Fig. 7 Interference power for REEA and EEA algorithms

In addition, as the target SINR increases, the optimal transmit power by two algorithms for all users' increases, however, the energy efficiency of two schemes decreases. And with the protection margin are captured in the constraints, the energy efficiency increases.

We compare the performance of the two schemes for guaranteeing the target SINR requirement. Figure 6 shows the evolution of received SINRs for all cognitive users when target SINRs change at time slot 20. Solid line shows the requirement imposed by the target SINR. REEA algorithm is closer to the SINR requirements. Due to the disturbance of the channel gains, the SINRs for REEA scheme are lower than those of EEA algorithm, but they satisfy the users' needs. REEA offers the same disturbance scene as EEA but with much less energy expenditure. And from the perspective of energy efficiency, REEA algorithm is superior to EEA algorithm.



Fig. 8 Transmit power of REEA and EEA schemes when new user enters the network



Fig. 9 SINR and interference power of REEA and EEA schemes when new user enters the network. **a** SINR of each user. **b** Interference power of REEA and EEA schemes

As mentioned previously, the interference power generated by cognitive users should be lower than the permissible interference power threshold. Figure 7 shows the interference power of two schemes for different target



Fig. 10 Energy efficiency of REEA and EEA schemes when new user enters the network

SINR. We find that REEA algorithm demonstrates good robustness since the requirement on the interference power is always met. However, EEA algorithm breaks the limit when the target SINR increases, which implies that this scheme is not appropriate in cognitive radio networks since the interest of primary users is ruined. In fact, EEA scheme stops iterating when the interference threshold is violated.

When new users enter the network, the interference power threshold of the primary users or the SINR of cognitive users may be violated by this new interference. In this scenario, we set the target SINR is $r_i^t = 4.08$ dB, the interference protection factor is $\varepsilon(t) = 10\%$, and the SINR protection margin is $\xi_i(t) = 7\%$. We suppose a new cognitive user joins the network at twentieth time-slot. The protection against the disturbance from the new user is shown in Fig. 8. We find that the transmit powers of each cognitive user for EEA and REEA algorithms are both able to reach equilibrium in several time-slots. In order to guarantee the target SINR requirement, each user must adjust the transmit power when the new user comes for both schemes. The optimal power of each user for REEA is lower than that of the corresponding allocated power for EEA scheme, since the proposed REEA scheme is a conservative algorithm which considers the worst situations.

Both the channel uncertainty between cognitive users and primary users and the channel uncertainty between cognitive users are considered in the proposed CRN. We set the channel uncertainty parameters $G_{\Delta} = 0.02^2$ and $h_{\Delta} = 0.02^2$ in the simulation, and the SINR of each user and the interference power for two schemes are shown in Fig. 9. We note that the transmit power of EEA scheme is larger than that of REEA scheme from Fig. 9(a). Once we get the optimal power from these two schemes, we compare the actual interference generated by the schemes to the primary network, respectively. In this way we will see that EEA scheme cannot be used, since the interference to the primary user is larger than the threshold in this case. Moreover, more interference will impact the cognitive users with the increasing number of SU. But in Fig. 9(b), we find that REEA scheme performs well, since the interference from cognitive users to primary user produced by REEA is lower than the threshold.

Figure 10 shows the energy efficiency of the two schemes. We know that the energy efficiency of REEA scheme is superior to that of EEA scheme. When a new user joins the networks, the power allocated to each user is adjusted with rapid convergence speed, then the energy efficiency of the two schemes will remain stable and constant.

5 Conclusion

In this paper, we investigate the problem of energy efficiency power allocation in underlay CRNs with protection factors to enhance the interference power constraint and the target SINR requirement constraint. The advantage of this model is that it not only takes into account robustness for power allocation but also the energy efficiency maximization. And we formulate the energy efficiency maximization problem as a fractional programming problem with convex constraints. It was shown in our simulations that our proposed algorithm can strictly guarantee the target SINR requirement for all users and provide satisfied protection for primary user.

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