

Energy-efficient power allocation in cognitive sensor networks: a coupled constraint game approach

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Abstract In this paper, we investigate power allocation in cognitive sensor networks, where cognitive users (cognitive enabled sensor nodes) opportunistically share the common spectrum with primary users (licensed devices). Consider that sensor nodes are self-interested to maximize their own utilities, we formulate the energy-efficient power allocation problem as a non-cooperative coupled constraint game, by taking the interference temperature into account. An energy efficiency-oriented utility function is defined as a new metric to evaluate the performance of power allocation. Firstly, we prove that there exist Nash equilibriums in the proposed game. Then, we prove that the power allocation game is a super-modular game under some conditions. Finally, we design centralized and distributed Game-based Efficiency-oriented Power Allocation algorithms (i.e., centralized GEPA and distributed GEPA) to obtain the Nash equilibriums. Extensive simulations are conducted to demonstrate that the proposed power allocation algorithms can achieve satisfactory performance in terms of energy efficiency, convergence speed and fairness in cognitive sensor networks.

Keywords Energy efficiency · Cognitive radio · Game theory · Super-modular game · Best response approach

1 Introduction

Wireless sensor networks (WSNs) are composed of a large number of sensor nodes, which are densely deployed in the monitoring area. WSNs have been widely applied to environment monitoring, target tracking, health care, and etc. One of the most important characteristics of WSNs is that each sensor node carries limited, normally irreplaceable batteries as power sources. Therefore, energy efficiency emerges as a critical issue in WSNs [2–4].

Most WSNs operate on the available unlicensed spectrum, i.e., the 900 MHz band and the 2.4 GHz band, which has been becoming more and more crowded due to the rapid development of wireless facilities. Moreover, the licensed spectrum usage is temporally and geographically inefficient. Cognitive radios are adaptive radios that are aware of spectrum environment. They have the capability of detecting the vacant spectrum, known as spectrum holes [5].

Sensor nodes equipped with cognitive radio constitute cognitive sensor networks (CSNs) [6]. Sensor nodes equipped with cognitive radio can change their transmission frequency and make use of vacant spectrum in an opportunistic manner, so as to improve the quality of service (QoS) [7].

In the licensed spectrum, the licensed device, which is called primary user (PU), has the priority to access the allocated channel. Cognitive users should not cause any harmful interference to PUs, by either switching to an available band or limiting its interference with PUs under an acceptable level [5, 8]. The second category, i.e.,

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limiting cognitive users' interference within the PUs' tolerable level, is more flexible and leads to more efficient spectrum utilization, which is also the focus of this paper.

It is noteworthy that the transmission power of each cognitive user can be translated into the interference noise to PUs. Interference temperature model is widely used to evaluate such interference to PUs from cognitive users. Power allocation or power control has been a fundamental problem in the research area of CSNs. Especially, it is a huge challenge to allocate the transmission power of cognitive users, by taking account of both the interference to PUs and energy efficiency.

Different from previous works in power allocation, we formulate a non-cooperative coupled constraint game among cognitive users in terms of energy efficiency. We assume that each cognitive user behaves in a rational and selfish manner, i.e., each sensor node only concerns its own utility rather than that of others. Moreover, it is possible to design a distributed algorithm at each cognitive user, by taking advantage of game tools. Especially, the interference temperature model introduces a coupled constraint for the non-cooperative game, and thus brings the complexity to its solution.

In this paper, we focus on energy efficiency of the power allocation problem in CSNs and introduce the game-based efficiency-oriented power allocation algorithms for cognitive users. The main contributions are summarized as follows:

1. We investigate power allocation among cognitive users based on the interference temperature model. Such a problem is formulated as a non-cooperative coupled constraint game. Moreover, we prove that this game is a super-modular game, and use the best response approach to reach the Nash equilibriums.
2. We adopt an energy efficiency-oriented utility function as a metric to evaluate energy efficiency, which should be taken into account for CSN system due to the limited energy capacity.
3. We design centralized and distributed Game-based Energy-efficient Power Allocation (GEPA) algorithms to derive the Nash equilibriums. Especially, the distributed GEPA-combines the Lagrangian multiplier, gradient projection and the best response algorithm to tackle the non-cooper-ative coupled constraint game. Extensive simulations have been conducted to evaluate the proposed algorithm in terms of energy efficiency, convergence speed and fairness.

The rest of this paper is organized as follows. Section 2 discusses the related works. In Sect. 3, we present the system model and formulate the problem as a non-cooperative couple-constraint game. Then we proceed in Sect. 4 to introduce the centralized GEPA. We firstly demonstrate

the existence of Nash equilibrium, then prove that the game is a super-modular game under some certain conditions, and finally design the algorithm to achieve Nash equilibrium based on best response approach. A distributed GEPA is presented in Sect. 5. We give the numerical results in Sect. 6, and draw a conclusion in Sect. 7.

2 Related work

Power allocation in cognitive radio network has long been recognized as a difficult and significant problem [9–11]. Most of the efforts have been devoted to providing solutions to power control in cognitive radio networks, among which one significant branch focuses on the optimization of QoS. For example, in [12], the authors propose a joint algorithm for sensing adaptation and opportunistic resource allocation, which aims to minimize the cost of sensing and transmission while guaranteeing the QoS constraint. In [13], a robust distributed power allocation algorithm is proposed to maximize the social utility of secondary users with consideration of interference uncertainties. In [14], resource allocation in OFDM-based cognitive radio networks is studied under a number of practical limitations. In [15], power allocation schemes are investigated while taking the spectrum sensing error into account.

Moreover, game theory has been widely used to analyze the competitive interaction of the power allocation problem in CSNs [16, 17]. For example, in [18], the authors study spectrum access for cognitive users under the interference temperature constraint. In [19], the authors introduce a potential game approach by approximating a non-cooperative game to a “close” potential game.

Most of the previous works on the power control game in CSNs pay lots of attention to QoS rather than energy efficiency. However, energy issue is vital for CSNs and has attracted more focus in recent works. The authors in [20] define a different objective function taking energy efficiency into account in a multi-channel environment. However, they assume that each user can access the multiple channel simultaneously, which is not suitable for CSNs since currently one sensor node is only equipped with one transceiver. In [21], energy-efficient power allocation for OFDM-based cognitive radio networks is solved by convex optimization in a centralized way, which is not suitable for CRNs due to the large information exchange overhead. In [22], the authors study the energy efficiency aspect of spectrum sharing and power allocation in heterogeneous cognitive radio networks with femtocells, which is different from the scenario in this paper.

Some studies focus on energy-efficient power allocation in wireless communication with game theory, e.g., relay-assisted interference wireless networks in [23], code-

division multiple access (CDMA) networks in [24], multicarrier CDMA systems in [25, 26]. Furthermore, energy efficiency is also considered in MIMO cognitive radio system in [27, 28]. A game theoretical energy efficient power control for CSN is different with the previous work, since the interference constraints should be taken into account. Also, MIMO cognitive radio is not widely adopted in CSN.

3 System model and problem formulation

3.1 Preliminaries of game theory

Compared with centralized protocols and algorithms, distributed ones, with less information exchange, computational complexity and energy consumption, are more suitable for WSNs. It is sensible to make the assumption that sensor nodes behave in a selfish and rational manner. Game theory is a study of selfish and rational players, which is a formal model of an interactive decision-making situation.

A non-cooperative game $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ consists of the following three components [29, 30]:

1. Player set $\mathcal{N} = \{1, \dots, i, \dots, n\}$: where i is the identification of a sensor node and n is the number of players in the game.
2. Strategy space \mathcal{S} : Each player i in a game selects a strategy s_i from its strategy set S_i . $\mathcal{S} = \times_i S_i$ represents the strategy space of the game. It is noteworthy that the strategy space of the game is a Cartesian product of the strategy space of each cognitive user. Usually let $s = (s_i, s_{-i})$ denote the strategy vector, where s_i is the player i 's strategy, and s_{-i} is the strategies of all other players.
3. Utility set $\mathcal{U} = \{u_1, \dots, u_i, \dots, u_n\}$. Note that player i 's utility is determined by strategy vector s . Each player wants to select the appropriate strategy, according to all other players' strategies, to maximize its own utility.

Nash equilibrium is such a static stable strategy vector that no player has any incentive to unilaterally change its strategy from Nash equilibrium. Moreover, Nash equilibrium is the most important concept of the equilibrium condition in game theory.

Definition 1 An strategy vector $s^* = (s_i^*, s_{-i}^*)$ is called Nash equilibrium if and only if

$$u_i(s^*) \geq u_i(s_i, s_{-i}^*) \quad \forall i \in \mathcal{N}, \forall s_i \in S_i \tag{1}$$

Definition 2 If the strategy space is over \mathbf{R} , super-modular game restricts the utility function such that

$$\frac{\partial^2 u_i(s)}{\partial s_i \partial s_j} \geq 0 \quad \forall i \in \mathcal{N}, \forall j \neq i \in \mathcal{N} \tag{2}$$

We can use best response approach to achieve pure Nash equilibrium under some certain conditions (maximal-value condition or minimum-value condition) in super-modular game analysis [29].

3.2 System model

Consider a CSN with multiple cognitive users sharing the same licensed spectrum with a unique PU. The CSN is deployed for a long period environment monitoring. Each cognitive user is equipped with a transmitter and a dedicated receiver. The cognitive user (also known as the secondary user) set $\mathcal{N} = \{1, 2, \dots, n\}$ allocates the transmission power $p = (p_1, p_2, \dots, p_n)$. Let p_i denote the allocated transmission power of user i . Since sensor nodes are densely deployed in the geographic area, the number of cognitive users is much greater than that of PUs.

The obtained data rate of a cognitive user is directly determined by its receiver's Signal to Interference plus Noise Ratio (SINR). Note that other cognitive users' transmission power will be translated into the interference noise. The SINR of user i , denoted by S_i , is given by

$$S_i = \frac{h_{ii} \times p_i}{\sum_{j \neq i, j \in \mathcal{N}} h_{ij} \times p_j + I} \tag{3}$$

where h_{ij} is the link gain between user j 's transmitter and user i 's receiver, which is assumed to be fixed, and $\alpha_c^2 = I_e + I_p$ in which I_e is the environmental noise and I_p is the interference noise from PU to cognitive users. Let r_i denote the obtained data rate of user i , which depends on the power allocation vector $p = (p_1, p_2, \dots, p_n)$, i.e.,

$$r_i = B \log(1 + S_i) \tag{4}$$

where B is wireless spectrum bandwidth. We assume that the cognitive user i 's available transmission power p_i ranges from p_{\min} to p_{\max} , i.e.,

$$0 \leq p_{\min} \leq p_i \leq p_{\max} \tag{5}$$

Since PU has the priority to access the licensed spectrum, we should protect PU from the interference of cognitive users' transmission. A number of interference models have been proposed to measure the interference, among which the interference temperature model is the most widely used to analyze the spectrum sharing problem. The interference temperature limit represents the maximal amount of the tolerable interference for the specific licensed spectrum in a given geographic area [18]. Let M denote the interference

temperature limit, which is described as the threshold of the total received power at the PU's receiver, i.e.,

$$\sum_{i \in \mathcal{N}} h_{i0} \times p_i \leq M \quad (6)$$

where h_{i0} is the link gain between the cognitive user i 's transmitter and the PU's receiver, and is assumed to be fixed.

3.3 Energy efficiency metric

In the previous works on CSNs power allocation, the cognitive users who share the licensed spectrum with licensed users want to achieve high QoS. The high QoS is referred to as a high level of performance or achieved service quality, e.g., high obtained data rate, low latency or low bit error probability.

When determining the transmission power of cognitive users, we should consider that the power allocation achieves high energy efficiency. Some research works use a specific class of games with the cost structure to balance between transmission rate and energy consumption. We need a more exact and clear definition of energy efficiency. Specifically, we want to maximize the obtained data rate per power unit. Note that we also incorporate the sensing and computational energy consumption into the energy consumption model. In such a case, each cognitive user faces the power allocation problem as follows,

$$\begin{aligned} \max_{p_i} & \frac{B \log(1 + S_i)}{p_i + \alpha} \\ \text{s.t.} & \sum_{i \in \mathcal{N}} h_{i0} \times p_i \leq M \end{aligned} \quad (7)$$

where α means power that is required not only by the transmitter electronic circuitry to operate the device, but also by sensing and computation [28].

Figure 1 depicts the energy efficiency of the cognitive user over the transmission power level under different α . It can be found that with the increase of α , energy efficiency drops straightly under the same transmission power. Meanwhile, the optimal transmission power which reaches the maximal energy efficiency becomes larger.

4 Energy-efficient power allocation game

A basic modeling assumption in our work is that each cognitive user behaves rationally in a self-interested manner. Each user wants to maximize its utility function under the interference temperature constraint. Let $\mathcal{G} = \{\mathcal{N}, \mathcal{S}, \mathcal{U}\}$ denote the energy-efficient power allocation game. Cognitive users are the players in the game. Cognitive user i 's

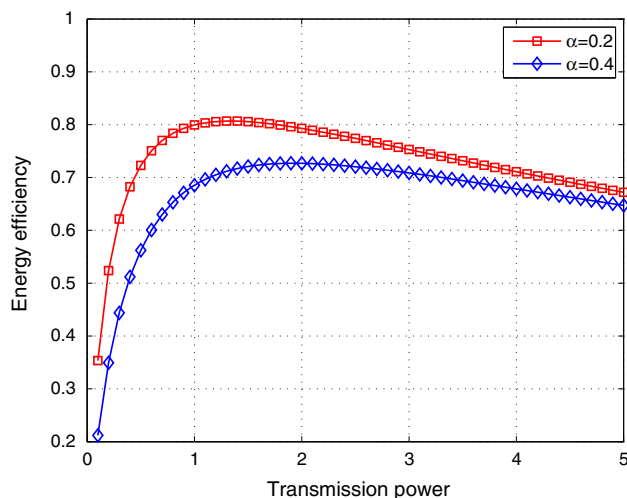


Fig. 1 Energy efficiency of cognitive user under different α

available transmission power p_i is the strategy set of player i . The cartesian product of individual strategy sets of n players is the joint strategy space of this game. Each player receives the payoff $u_i(p)$ which is defined as follows,

$$u_i(p_i, p_{-i}) = \frac{B \log(1 + S_i)}{p_i + \alpha} \quad (8)$$

As some previous work, we assume that the channel gain information is a prior knowledge for each cognitive user [18].

4.1 Existence and uniqueness of Nash equilibrium

A game can be shown to have a pure Nash equilibrium if the following conditions are satisfied [29]:

1. The player set is finite.
2. The strategy sets are closed, bounded, and convex.
3. The utility functions are continuous and quasi-concave in the strategy space.

Theorem 1 *There exist Nash equilibriums in the energy-efficient power allocation game \mathcal{G} .*

We show in Appendices that the above three conditions are satisfied. Thus, we can obtain the conclusion that there exist Nash equilibriums in the energy-efficient power allocation game \mathcal{G} .

For the uniqueness of Nash equilibrium, it has been demonstrated in [24].

4.2 Super-modular game

Consider our utility function and derive its differential function,

$$\frac{\partial^2 u_i}{\partial p_i \partial p_j} = B \log(e) \left[\frac{-h_{ii} h_{ij}}{(p_i + \alpha) \left(\sum_{j \in \mathcal{N}} h_{ij} p_j + I \right)^2} + \frac{h_{ij} h_{ii} p_i}{(p_i + \alpha)^2 \left(\sum_{j \in \mathcal{N}} h_{ij} p_j + I \right) \left(\sum_{j \neq i \in \mathcal{N}} h_{ij} p_j + I \right)} \right] \quad (9)$$

Note that if the following condition is satisfied, the equation (2) can be satisfied. Thus, we can draw a conclusion that this energy-efficient power allocation game is a super-modular game.

$$\alpha \leq S_i p_i, \forall i \in \mathcal{N} \quad (10)$$

As we all know, compared to the transmission power consumption, the sensing and computational power consumption is quite limited and small-scale. For most sensor nodes, SINR is large enough to satisfy the above condition. For successful transmission, SINR at the receiver side should be large enough to guarantee that [25]. That is to say, a lower bound should be given for transmission power of each cognitive user. Meanwhile, we assume even at the lower bound point, the condition (10) is satisfied.

Thus it is reasonable to make an assumption that this energy-efficient power allocation game is a super-modular game.

4.3 Centralized game-based energy-efficient power allocation algorithm

Without the coupled constraint, the Nash equilibrium can be directly obtained through the best response approach [31, 32]. In other word, tackling the coupled constraint enhances the complexity of the problem, which should be combined with the best response approach to fully solve the problem. Two algorithms are proposed to tackle the coupled constraint, and the first one, i.e., the centralized GEPA complies the interference constraint from the perspective of system.

It is noteworthy that the goal is to maximize the energy efficiency of each user, and guarantee that the interference temperature to PU does not exceed the threshold. To tackle the coupled constraint, we punish cognitive users in terms of reduced utility to each user when the threshold value gets exceeded. If the interference temperature exceeds the threshold, we get

$$m = \frac{\sum_{i \in \mathcal{N}} h_{i0} p_i^*}{M} > 1 \quad (11)$$

where p_i^* denotes user i 's optimal transmission power through the best response approach. Multiplying the transmission power of each user by $\frac{1}{m}$, we get the new transmission power of each user finally. We summarize the approach as Algorithm 1.

It is noteworthy that the operating point is the best choice for each user in that condition. That is to say, we can obtain the Nash equilibrium finally.

Theorem 2 *The final result of the centralized GEPA is a Nash equilibrium for the cognitive users.*

Proof Note that the optimal transmission power of each user through the best response approach satisfies the following condition,

$$B \log(e) \frac{\frac{h_{ii}(p_i^* + \alpha)}{I + \sum_{j \in \mathcal{N}} h_{ij} p_j^*} - \ln \left(1 + \frac{h_{ii} p_i^*}{I + \sum_{j \neq i \in \mathcal{N}} h_{ij} p_j^*} \right)}{(p_i^* + \alpha)^2} = 0 \quad (12)$$

When the threshold value is violated, the transmission power of each user is reduced by multiplying $\frac{1}{m}$. Observe the following function with m ,

$$f(m) = B \log(e) \frac{\frac{h_{ii} \left(\frac{p_i^*}{m} + \alpha \right)}{I + \sum_{j \in \mathcal{N}} h_{ij} \frac{p_j^*}{m}} - \ln \left(1 + \frac{h_{ii} \frac{p_i^*}{m}}{I + \sum_{j \neq i \in \mathcal{N}} h_{ij} \frac{p_j^*}{m}} \right)}{\left(\frac{p_i^*}{m} + \alpha \right)^2} \quad (13)$$

where m is a positive constant. For ease of presentation, we set $k = \frac{1}{m}$ and do not consider $\frac{B \log(e)}{(p_i^* + \alpha)^2}$ in Eq. (13). We have

$$g(k) = \frac{h_{ii}(k p_i^* + \alpha)}{I + \sum_{j \in \mathcal{N}} h_{ij} k p_j^*} - \ln \left(1 + \frac{h_{ii} k p_i^*}{I + \sum_{j \neq i \in \mathcal{N}} h_{ij} k p_j^*} \right), \text{ and get}$$

$g(1) = \frac{h_{ii}(p_i^* + \alpha)}{I + \sum_{j \in \mathcal{N}} h_{ij} p_j^*} - \ln \left(1 + \frac{h_{ii} p_i^*}{I + \sum_{j \neq i \in \mathcal{N}} h_{ij} p_j^*} \right)$, which corresponds to equation (27), i.e., the equilibrium condition. Moreover this function decreases with k . So the Eq. (13) is positive, and the utility function of user i increases with p_i within the interval $[0, \frac{p_i^*}{m}]$. The transmission power allocated now is the best choice for each user and the Nash equilibrium is derived. \square

Algorithm 1: The Centralized GEPA: Executed by the cognitive users

- 1: Initial condition: Each user i transmits at its maximum power level p_{\max} to achieve large data rate.
- 2: Adaption condition: User i , who is selected from a sequential order, chooses an exact transmission power to maximize its utility according to the following equation:

$$p_i^* = \arg \max_{p_i \in \mathcal{S}} u_i(p_i, p_{-i})$$

- In each iteration, each user updates its transmission power while other users keep their power levels fixed.
- 3: Repeat 2 until no user revises its transmission power.
- 4: Receive m , which is calculated by the central controller according to (11).

- 5: **if** $m > 1$ **then**
 - 6: Multiply the transmission power p_i^* by $\frac{1}{m}$.
 - 7: **end if**
-

One coordinator or a control node is needed to implement the three steps. After receiving the information of all cognitive users, the control node computes m defined in Eq. (11), and broadcast the information to all cognitive users. Then the cognitive users update the transmission power accordingly.

5 Game with coupled constraint

In this section, the former problem (7) is considered as a coupled constraint game. Dual decomposition is utilized to decouple the coupled constraint. Finally, a decoupled game-based energy efficiency-oriented power allocation algorithm, i.e., distributed GEPA is proposed. It is noticed that the distributed GEPA complies the interference constraint from the perspective of individual.

5.1 Dual decomposition

Note the interference temperature limit is a coupled constraint, which makes it difficult to design a distributed algorithm for cognitive users. To decouple the constraint, (6) is taken into the objective function by Lagrangian multiplier λ_i ,

$$\mathcal{L}_i(p_i, \lambda_i) = u_i(p_i, p_{-i}) + \lambda_i \left(M - \sum_{j \in \mathcal{N}} h_{j0} p_j \right) \quad (15)$$

Note that the interference temperature limit (6) is the same for each cognitive user, i.e., $\lambda_i = \lambda_j, \forall i \neq j$. Thus we only need a unique Lagrangian multiplier λ to relax the coupled constraint (6).

The Lagrangian dual function is the maximum value of (15) over the transmission power p_i ,

$$\mathcal{D}_i(\lambda) = \sup_{p_i \in \mathcal{S}} \mathcal{L}_i(p_i, \lambda) \quad (16)$$

Notice that the Lagrangian dual function can be expressed as follows,

$$\mathcal{D}_i(\lambda) = f_i(\lambda) + \lambda M \quad (17)$$

where

$$f_i(\lambda) = \max_{p_i \in \mathcal{S}} \left[u_i(p_i, p_{-i}) - \lambda \sum_{j \in \mathcal{N}} h_{j0} p_j \right] \quad (18)$$

denotes the Lagrangian to be maximized. The dual problem is

$$\min_{\lambda \geq 0} \mathcal{D}_i(\lambda) \quad (19)$$

Through the dual decomposition approach, the formal coupled constraint game is now decoupled. According to

duality theory, there exists a dual optimal Lagrangian multiplier λ^* such that $p_i(\lambda^*)$ will be globally optimal [33]. Given an arbitrary Lagrangian multiplier λ , the cognitive user will converge to Nash equilibrium with the previous algorithm. However, it may be suboptimal, but not globally optimal. In such a sense, the Lagrangian multiplier λ^* can be regarded as a coordination signal with aligns local optimality of Eq. (18) with global optimality of Eq. (7).

Notice that the second partial differential function of the utility function keeps unchanged after the decoupling approach. Thus the decoupled game is still a super-modular game, which can be solved by the best response approach.

5.2 Two-level iteration

According to [33], the dual problem (19) can be efficiently solved by using a gradient projection method in an iterative manner. The Lagrangian multiplier is adjusted as follows,

$$\lambda^{m+1} = \left[\lambda^m - \varepsilon \frac{\partial \mathcal{D}_i(\lambda^m)}{\partial \lambda^m} \right]^+ \quad (20)$$

where $\varepsilon > 0$ is the step size, $[z]^+ = \max\{z, 0\}$, and m denotes the iteration number. Notice that $p_i(\lambda)$ denotes the unique maximizer in Eq. (18), and thus the Lagrangian multiplier adjustment can be expressed as follows,

$$\lambda^{m+1} = \left[\lambda^m - \varepsilon \left(M - \sum_{j \in \mathcal{N}} h_{j0} p_j \right) \right]^+ \quad (21)$$

It is noteworthy that $\frac{\partial \mathcal{D}_i(\lambda^m)}{\partial \lambda^m} = M - \sum_{j \in \mathcal{N}} h_{j0} p_j, \forall i \in \mathcal{N}$. Thus, λ^{m+1} is equivalent for all cognitive users even they have different \mathcal{D}_i . In another word, λ corresponds to the constraint, i.e., Eq. (6), which is a constraint for all cognitive users. Thus, λ should be equal to all cognitive users.

Remark 1 If the interference temperature $\sum_{j \in \mathcal{N}} h_{j0} p_j$ is larger than the threshold M , the Lagrangian multiplier λ will increase (see Eq. (21)). This results in each cognitive user reducing its transmission power p_i (see Eq. (18)). In such a way the interference temperature will gradually decrease to the threshold.

Combined with the best response approach, a two-level iteration algorithm is proposed to solve the decoupled game. As shown in Algorithm 2, Lagrangian multiplier is adjusted in the upper-level iteration, meanwhile the transmission power of each cognitive user is adjusted in the lower-level iteration. Finally, the cognitive users' transmission power will converge to the Nash equilibrium. Moreover, we show in Appendices that the Lagrangian multipliers will converge in the distributed GEPA, i.e., Algorithm 2.

Algorithm 2: The distributed GEPA: Executed by the cognitive users

- 1: Each user i begins with an initial Lagrangian multiplier $\lambda^1 = 0$. For (18), run **Algorithm 1** and converge to the Nash equilibrium, which is taken as the initial transmission power value, i.e., $p_i(\lambda^1)$.
- 2: Each user i updates its Lagrangian multiplier λ^{m+1} for next iteration according to equation (21).
- 3: With the new Lagrangian multiplier λ^{m+1} , for (18), run **Algorithm 1** and converge to the Nash equilibrium, i.e., $p_i(\lambda^{m+1})$. In the adaption step, each user chooses the best response value as follows,

$$p_i^* = \arg \max_{p_i \in P_i} \left[u_i(p_i, p_{-i}) - \lambda^{m+1} \sum_{j \in \mathcal{N}} h_{j0} p_j \right]$$

where the Lagrangian multiplier λ^{m+1} acts as an adaption state which incorporates the coupled constraint.

- 4: Repeat 2-3 until the Lagrangian multiplier of each cognitive user does not change.

5.3 Discussion on proposed algorithms

Algorithm 1 is different from Algorithm 2 since it is not a fully distributed algorithm. For implementing Algorithm 1, a central controller is needed to calculate the interference temperature and send the control message.

The interference temperature limit defined in (6) influences the performances in such a way: larger constraint will result in better performance, i.e., higher energy efficiency and higher data rate, and vice versa. The reason can be explained as that larger interference temperature means larger available set for each cognitive user.

Moreover, the influence of α on energy efficiency has been presented as Fig. 1. With the increasing of α , the final transmission power of each cognitive will increase. Thus the data rate will increase. However, the energy efficiency will decrease.

6 Numerical results

In this section, we conduct several simulations to evaluate the game-based approach. We compare our results with that of [18], where the game-based QoS-oriented power allocation algorithm (GQPA) is proposed. The bandwidth of the shared licensed spectrum is 10 MHz with Gaussian noise 1. The cross-gain coefficients h_{ij} and the self-gain coefficient h_{ii} are randomly selected from intervals [0.05, 0.15] and [0.8, 1.2], respectively. The cross-gains from cognitive user to PU are randomly selected from interval [0.3, 0.6]. Note that available transmission power of each

cognitive user is $[p_{\min}, p_{\max}]$. The lower bound p_{\min} is set at the value of 0.1, the upper bound p_{\max} is set at the value of 5. Cognitive users are chosen in a random order to update their transmission power.

6.1 Without exceeding interference temperature limit

Figure 2 compares the initial transmission power level with the final transmission power level for both GQPA and GEPA. It depicts that the final transmission power of GEPA is much smaller than that of GQPA.

Figure 3 compares cognitive users’ data rate under two algorithms. Some users obtain higher data rate under GEPA than that under GQPA. However, its transmission power under GEPA is lower than that under GQPA. The simulation shows that GEPA is effective in enhancing the energy efficiency.

Figure 4 compares cognitive users’ initial utility and final utility under two algorithms. It shows that through GEPA approach, the utility of each cognitive user increases, and GEPA has a better performance in terms of energy efficiency than GQPA.

6.2 With exceeding interference temperature limit

The previous simulation results correspond to the situation where the interference of cognitive users to PU is under the threshold. Figure 5 shows the results in the case that the initial interference of cognitive users exceeds the threshold. The final results of GEPA can be guaranteed to be Nash equilibrium, which are different from that of GQPA. In GQPA, some cognitive users transmit at a certain high power, which is not their best choice for maximizing utility and will result in more noise to other cognitive users.

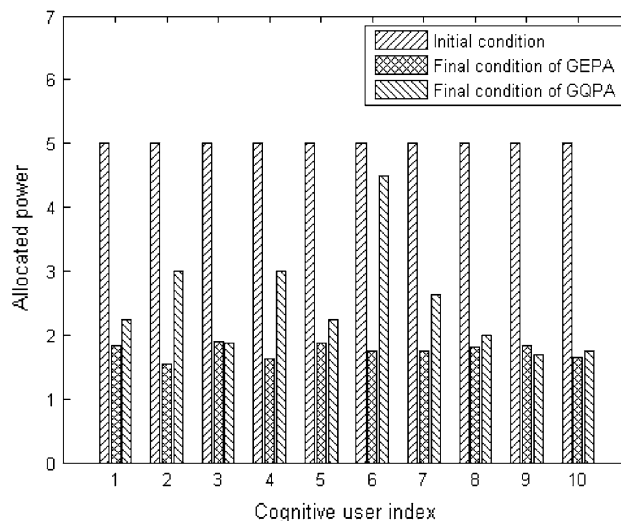


Fig. 2 Initial power and final power under GEPA and GQPA

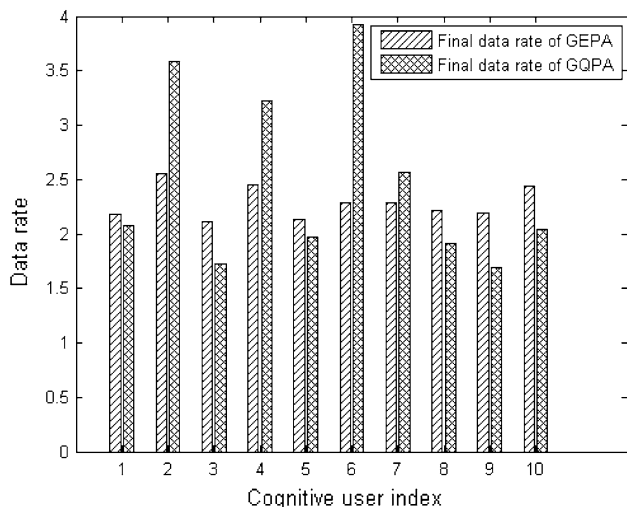


Fig. 3 Data rate under GEPA and GQPA

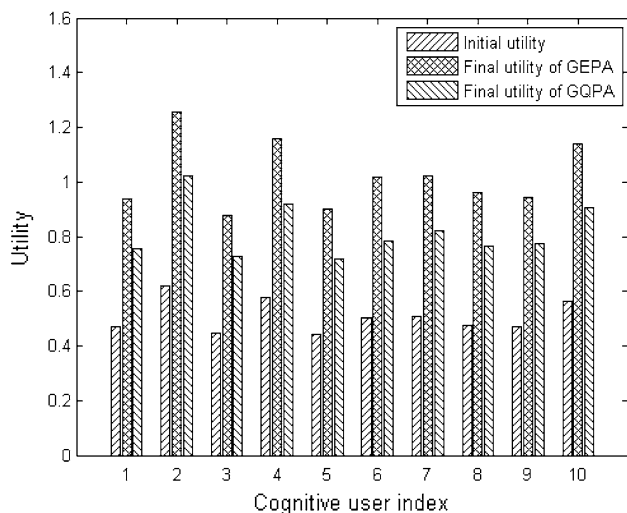


Fig. 4 Final utility under GEPA and GQPA

Hence both its energy efficiency-oriented utility and others' utilities will be degraded in GQPA.

6.3 Decoupled approach to coupled constraint game

To evaluate the performance of Algorithm 2, we implement another group of simulations. Figure 6 depicts the convergence of the Lagrangian multiplier, which is updated in the upper level of the Algorithm 2. Figure 7 depicts the convergence of the allocated power of each cognitive user. The observations are obtained as follows:

1. Both the Lagrangian multiplier and the allocated power of each cognitive user converge in a few step, which means the algorithm achieves a good performance of convergence speed.

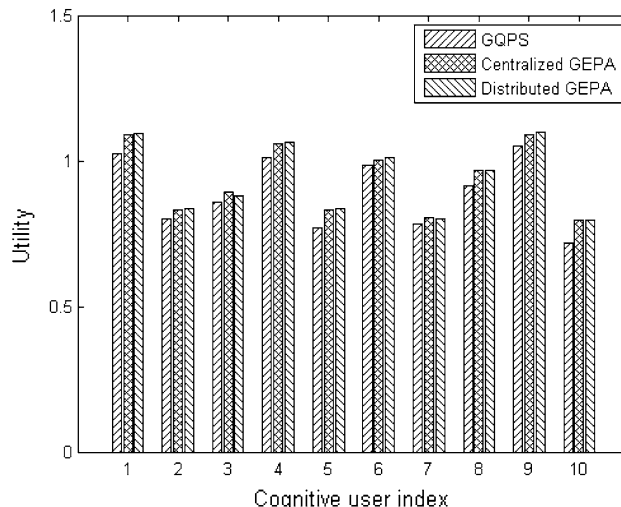


Fig. 5 Final utility under GQPA, centralized GEPA and distributed GEPA

2. The Lagrangian multiplier is not equal to 0, which means that the interference temperature introduced by cognitive users is equal to the threshold.
3. Each cognitive user finally achieves the Nash equilibrium while satisfying the interference temperature limit.

6.4 Performance evaluation of proposed algorithms

Now we concern about the convergence process of cognitive users. One of the cognitive users is chosen and the transmission power is set at the different level as the initial condition. After running best response algorithm, the transmission power converges to the same fixed value finally. Figure 8 depicts the convergence process of one user from different initial conditions.

Cognitive users' utilities are corresponding to the obtained data rate per power unit. Here we define the system utility as the average value of cognitive users' utilities. A higher system utility means more efficient power usage in the whole CSN.

$$\frac{\sum_{i \in \mathcal{N}} \frac{B \log(1+S_i)}{p_i + \alpha}}{n} \tag{23}$$

Besides, consider that the equal distribution of energy resources is also extremely vital for CSNs. A low fairness will result in the situation that some cognitive users will run out of energy and meanwhile other users still have a high energy capacity. In addition, the fairness index can be expressed as [34],

$$f(p_1, p_2, \dots, p_n) = \frac{(\sum_{i \in \mathcal{N}} p_i)^2}{n \sum_{i \in \mathcal{N}} p_i^2} \tag{24}$$

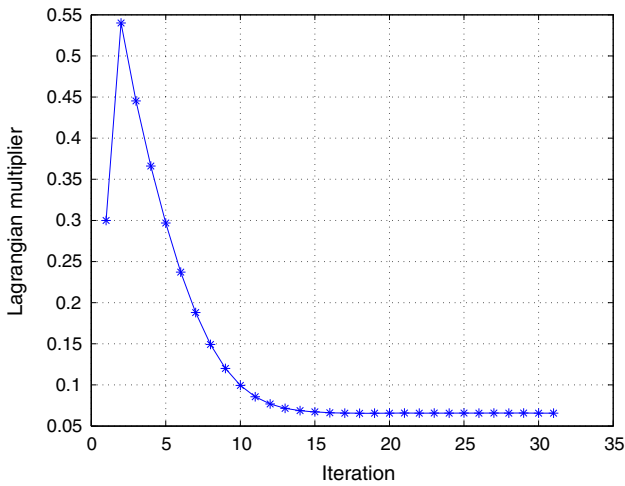


Fig. 6 Convergence of lagrangian multiplier under distributed GEPA

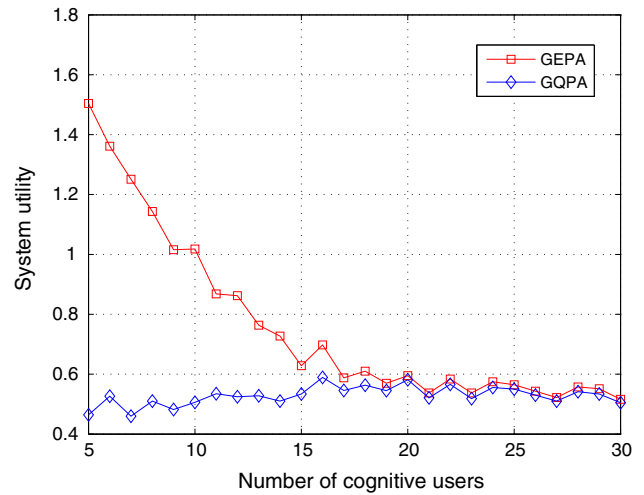


Fig. 9 System utility with different numbers of cognitive users

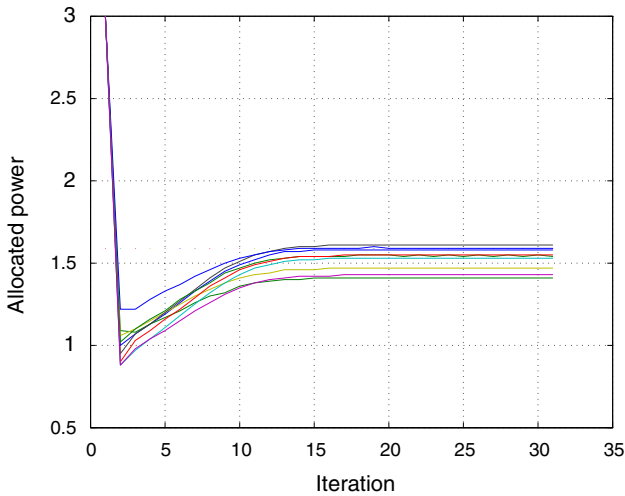


Fig. 7 Convergence of allocated power under distributed GEPA

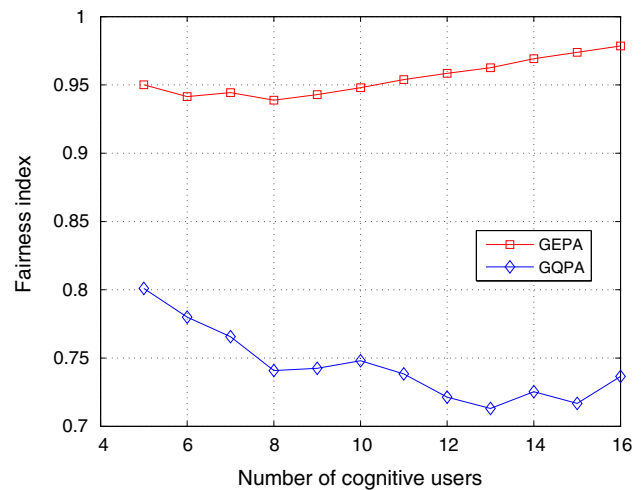


Fig. 10 Fairness index under GEPA and GQPA

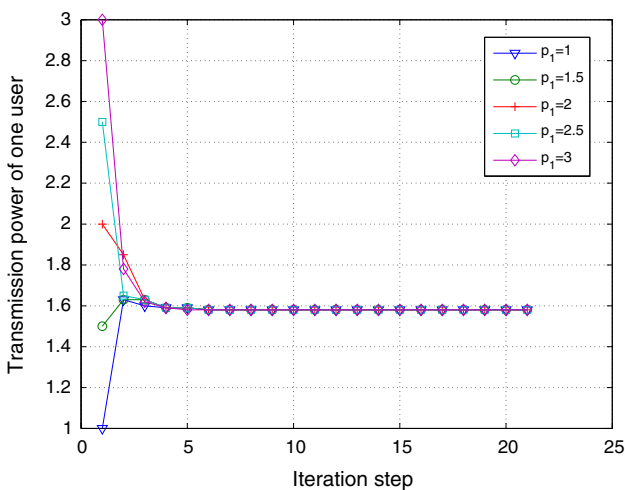


Fig. 8 Convergence process from different initial conditions

The results of the proposed GEPA solutions are compared with those of GQPA solutions. It replies that GEPA achieves better performance in terms of energy efficiency and fairness than GQPA. Figure 9 compares the system utility under two power allocation solutions. It replies that as the user number increases, the system utility of GEPA decreases dramatically, but that of GQPA nearly keeps stable. Interference to cognitive users increases due to the increasing number of users, so each user has an incentive to allocate more power to achieve the most efficient operation point and then the system utility drops dramatically. Moreover, GEPA solutions achieve a higher system utility than GQPA solutions. In addition, it also clarified that even in the case that the threshold is exceeded, GEPA is effective to tackle the coupled constraint and performs better than GQPA.

We have implemented 100 Monte Carlo trails to compare the fairness index under two algorithms. Figure 10 compares the performance of these two algorithms on the fairness index. From the picture, it is clear that with the increase of the user number, the fairness index of GEPA is higher and more stable than that of GQPA.

7 Conclusion

In this paper, we investigate power allocation in CSNs to achieve high energy efficiency. Different from the existing power control schemes, we take priority of energy efficiency to prolong the network lifetime, and propose a game-based energy efficiency-oriented power allocation (GEPA) algorithm. We have proven the existence of Nash equilibriums. With the best response approach, we can identify the unique Nash equilibrium. Moreover, GEPA makes the fair transmission power allocation available. Simulation results show that GEPA achieves better network performance than GQPA in terms of energy efficiency and fairness index. As part of future work, we will investigate the design of more practical distributed protocols where cognitive users choose their power according to the knowledge of only incomplete and partial network state including PUs' interference. Moreover, a more comprehensive comparison with other latest power allocation algorithms will be considered in the future work.

Appendices

Existence of Nash equilibriums

Proof There are n cognitive users involved in the energy-efficient power allocation game. Obviously, the first condition is satisfied. Note that the available transmission power of each user ranges from p_{\min} to p_{\max} , which means that the strategy set of each cognitive user is an interval. Intervals and cartesian products of intervals are closed, bounded and convex. Therefore the second condition is satisfied.

As long as the third condition is satisfied, we can get the result that there exist Nash equilibriums. Now we analyze properties of the continuous utility function,

$$u_i(p_i, p_{-i}) = \frac{B \log(1 + S_i)}{p_i + \alpha} \quad (25)$$

Then we get the differential function of the utility function,

$$\frac{\partial u_i(p)}{\partial p_i} = B \log(e) \frac{\frac{h_{ij}(p_i + \alpha)}{1 + \sum_{j \in \mathcal{N}} h_{ij} p_j} - \ln \left(1 + \frac{h_{ij} p_i}{1 + \sum_{j \neq i \in \mathcal{N}} h_{ij} p_j} \right)}{(p_i + \alpha)^2} \quad (26)$$

It is obvious that $u(0, p_{-i}) = 0$, $\left. \frac{\partial u_i(p)}{\partial p_i} \right|_{p_i=0} > 0$, and there exists only one constant p_c such that $\left. \frac{\partial u_i(p)}{\partial p_i} \right|_{p_i=p_c} = 0$. The utility function increases with p_i within the interval $[p_{\min}, p_c]$ and decreases within the interval $[p_c, p_{\max}]$.

Definition 3 A function $f: S \rightarrow R$ defined on a convex subset S to a real vector space is quasi-concave if for $\forall x, y \in S, \forall \lambda \in [0, 1]$,

$$f(\lambda x + (1 - \lambda)y) \geq \min(f(x), f(y)) \quad (27)$$

Given a random interval $[a, b] \in [p_{\min}, p_{\max}]$, if $a \geq p_c$, then the utility function is decreases within the interval $[a, b]$, and $u(b, p_{-i}) = \min(u(a, p_{-i}), u(b, p_{-i}))$, so it is obvious that $\lambda a + (1 - \lambda)b \leq b$, and then $u_i(\lambda a + (1 - \lambda)b, p_{-i}) \geq u_i(b, p_{-i})$. Similarly, if $b \leq p_c$, then the utility function is still quasi-concave due to the same reason. Otherwise, $\min u_i(p_i, p_{-i}) = u_i(a, p_{-i})$ or $\min u_i(p_i, p_{-i}) = u_i(b, p_{-i})$ over the internal $[a, b]$. We get the result that the utility function is a quasi-concave function over the available set $[p_{\min}, p_{\max}]$.

From the above, we conclude that the player set is finite, the strategy sets are closed, bounded, and convex, and the utility functions are continuous and quasi-concave in the strategy space. This completes the proof of Theorem 1, such that there exist Nash equilibriums in the energy-efficient power allocation game \mathcal{G} . \square

Convergence of lagrangian multiplier

Proof According to Eq. (25), we can obtain the differential function,

$$\frac{\partial^2 u_i}{\partial p_i^2} = B \log(e) \left[\frac{-A^2(p_i + \alpha)^2}{(p_i + \alpha)^3(1 + A p_i)^2} - 2 \frac{A \frac{(p_i + \alpha)}{(1 + A p_i)} - \ln(1 + A p_i)}{(p_i + \alpha)^3} \right] \quad (28)$$

in which $A = \frac{h_{ij}}{1 + \sum_{j \neq i \in \mathcal{N}} h_{ij} p_j}$ and A can be regarded as a constant since the transmission power of other cognitive users keeps unchanged. Notice the first part is always negative, i.e., $\frac{-A^2(p_i + \alpha)^2}{(p_i + \alpha)^3(1 + A p_i)^2} < 0$. According to the analysis in Appendices, the second part satisfies

$$-2 \frac{A \frac{(p_i + \alpha)}{(1 + A p_i)} - \ln(1 + A p_i)}{(p_i + \alpha)^3} \begin{cases} < 0, & p_i < p_c \\ = 0, & p_i = p_c \\ > 0, & p_i > p_c \end{cases} \quad (29)$$

There exists one point p_t make $\frac{\partial^2 u_i}{\partial p_i^2} \Big|_{p_i=p_t} = 0$. Moreover, $p_t > p_c$.

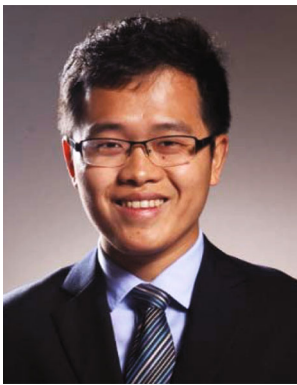
Within the interval $[0, p_c]$, the utility function is increasing and concave. Since the relationship between the Lagrangian multiplier λ and the transmission power p_i is linear and the utility function is strictly concave, there exists a sufficiently small step size ε that guarantee p_i and λ to converge to the optimal solution [35].

Within the interval $[p_c, p_t]$, the utility function is decreasing and concave. Within the interval $[p_t, \infty]$, the utility function is decreasing and convex. With the similar reason, both the transmission power p_i and λ will converge to the optimal solution. \square

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Co-Chair, IEEE DCOSS 2011 Publicity Co-Chair, IEEE ICDCS 2012 Publicity Co-Chair, IEEE ICC 2012 Communications QoS and Reliability Symposium Co-Chair, IEEE SmartGridComm The Whole Picture Symposium Co-Chair, IEEE MASS 2013 Local Chair, Wireless Networking and Applications Symposium Co-chair, IEEE ICC 2013, Ad hoc and Sensor Network Symposium Co-chair, IEEE ICC 2014 and TPC member for IEEE ICDCS'10,'12,'13,'14, IEEE MASS'10,'11,'13, IEEE SECON'11,'12, IEEE INFOCOM'11,'12,'13,'14 *etc.*