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Analogy Between SCS-CN and Muskingum Methods

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Abstract
Soil Conservation Service Curve Number (SCS-CN) method is one of the most widely used, popular, stable, reliable, and attractive rainfall-runoff methods, initially designed for direct surface runoff estimation in small and medium agricultural watersheds. It, in various forms, is now being employed to several areas other than the intended one, such as infiltration, sediment yield, pollutant transport and so on. In this study, the proportionality concept of the SCS-CN method is further extended to the field of flood routing and is shown to either parallel or be analogous to the Muskingum routing method, which is a simplified variant of St. Venant equations. When employed to various real (typical) flood events of four different river reaches available in literature from different sources, and thus, of varying flow and channel settings, the results of SCS-CN concept compare well with those due to Muskingum method in terms of their evaluation for performance through root mean square error (RMSE) for overall hydrograph, and relative error (RE) for peak discharge (Q_p) and time to peak (T_p) of all four flood events. It thus underscores not only the efficacy but also the versatility of the SCS-CN concept in application to one more field of flood/flow routing, which forms to be an element of paramount importance in distributed hydrologic modeling.

Keywords Curve number, Flow/flood routing · Muskingum method · Peak discharge · Rainfall-runoff method · SCS-CN method · St. Venant equation

Abbreviations

A	Surface area of the system (watershed, canal or reservoir) [L^2]
B	Channel width [L]

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CN	Curve number (nondimensional)
C_0, C_1, C_2	Dimensionless parameters of SCS-CN and Muskingum routing procedures (nondimensional)
D_A	Actual detention storage [L]
D_P	Potential maximum detention storage [L]
F	Actual retention storage [L]
F_0	Cumulative dynamic retention [L]
f_0	Initial infiltration capacity [LT^{-1}]
f_c	Constant minimum infiltration capacity [LT^{-1}]
I	Rate of inflow to the system [L^3T^{-1}]
I_p	Peak rate of inflow to the system [L^3T^{-1}]
I_a	Initial abstraction or rainfall losses [L]
I_s	Initial storage depth or condition in routing [L]
i_0	Uniform rainfall intensity [LT^{-1}]
i_e	Uniform effective rainfall intensity [LT^{-1}]
k	Horton's decay coefficient [T^{-1}]
K	Storage coefficient [T]
m	A nondimensional parameter defined as: $m = \theta K/\Delta t$
N	Number of ordinates of outflow hydrograph (nondimensional)
O_{ai}	Ordinates of actual outflow [L^3T^{-1}]
O_{ri}	Ordinates of routed outflow [L^3T^{-1}]
O_{pa}	Actual peak outflow [L^3T^{-1}]
O_{pr}	Routed peak outflow [L^3T^{-1}]
O	Rate of outflow from the system [L^3T^{-1}]
O_b	Base flow [L^3T^{-1}]
P	Total rainfall [L]
P_e	Effective rainfall [L]
Q	Direct surface runoff [L]
S_a	Actual detention storage of a canal reach excluding initial storage (S_i) [L^3]
S_b	Channel bed slope [LL^{-1}]
S_i	Initial storage in canal reach [L^3]
S_{SCS}	Potential maximum retention storage [L]
S_0	Potential storage space prior to rainfall [L]
S_p	Potential detention storage of a canal reach excluding initial storage (S_i) [L^3]
t_p	Time to ponding [T]
T_{pa}	Actual time to peak [T]
T_{pr}	Routed time to peak [T]
X	Outflow depth [L]
Y	Inflow depth [L]
ΔF_0	Change in retention storage of the system [L]
ΔS_{Musk}	Change in Muskingum storage in channel routing [L^3]
Δt	Time interval [T]
β	Initial abstraction coefficient (nondimensional)
λ	Initial abstraction coefficient (nondimensional)
θ	Weighting factor (nondimensional)

1 Introduction

Flood/flow routing in open channels is of paramount importance to water resources engineers (Choudhury et al. 2002), largely for predicting outflow hydrograph corresponding to a specific inflow hydrograph in a channel or reservoir reach. The attendant flood may cause a huge damage in terms of loss of life, property, and economic loss due to disruption of various socio-economic activities. Such information is vital in decision making in many hydrological applications/studies, as for example, flood forecasting; design of flood-control structures (such as bridges, dam spillways, culverts, waterways, scour estimation); canal/reservoir operation; environmental flows for aquatic-habitat needs and so on.

There exist several routing methods in literature (Barati 2014). The first category of distributed models utilizes reach properties to solve both mass conservation and momentum equations. These methods are capable of routing flows/floods both spatially and temporally in the reach under investigation, but require more extensive data and are therefore costlier than others. The second category models utilize a simpler version of the Saint–Venant equations based on both physical concepts and river characteristics. On the other hand, the third category (perhaps the most popular) lumped models exclusively use historical records to calibrate parameters of specific storage function. The calibrated storage function accompanied with continuity equation is utilized to predict the flood events. Ease of execution, simpler concept, and less data requirement are the distinguished features that make them not only attractive but also advantageous in field applications. Their accuracy depends on both the precision of parameter estimation process and the applicability of the selected storage function. The approaches of last category are also further categorized as hydrologic methods, and the others as hydraulic methods (Choudhury 2007). The former methods are based on continuity equation and storage equation, and the latter on continuity and momentum equations, i.e. Saint–Venant equations. The amount of time and effort required to implement, calibrate, and solve the selected model increases with the degree of model sophistication. Although such a model usually provides more accurate results, its use is justified only when there are enough good quality data available.

Thus, a tradeoff is often made in selection of a flood routing model based on the quality of given data, social or economic importance of the project, and safety requirements. In most cases, the field data scarcity prevents the use of Saint–Venant equations (Ponce and Yevjevich 1978; Perumal and Sahoo 2008; Akbari et al. 2012; Akbari and Barati 2012). Therefore, hydrologic routing procedures requiring only a few hydrologic parameters to calibrate based on recorded data are recommended for flood routing by the hydrologic practitioners. Among many models used for flood routing, semi-empirical classical Muskingum method proposed by McCarthy (1938) is one of the most widely used method of hydrologic flood routing procedures. This method was developed by the U.S. Army Corps of Engineers for the Muskingum Conservancy District Flood-Control Project over six decades ago. Perumal (1994) showed that the Muskingum method is an approximate solution of the Saint–Venant equations and proposed a variable parameter Muskingum method directly from these equations. Perumal and Price (2013) derived the fully mass conservative, variable parameter McCarthy Muskingum (VPMM) method directly from the Saint–Venant equations.

According to the linearity of the relation between storage value and weighted inflow & outflow values, various versions of Muskingum models fall in two general categories as linear and nonlinear models. In the first category, the storage value of the reach at a specific time is linearly proportional to inflow and outflow values of the same time while in the latter category, the relation between storage value and weighted inflow and outflow values is nonlinear in most cases. The nonlinear Muskingum model can be a constant-parameter model that considers its parameters to remain unchanged during flood period in contrast to the variable-parameter model, in which parameters change. The most distinguished challenge with the latter model is its calibration (Niazkar and Afzali 2016). The full dynamic wave models, which solve full Saint–Venant equations of mass and momentum conservation, are also prone to numerical issues. To propose a simple SCS-CN-based routing model, this paper first provides an analogy between the Muskingum method of flood routing and the popular Soil Conservation Service Curve Number (SCS-CN) method widely used in rainfall-runoff modelling.

Runoff estimation from rainfall is the most important task in the field of hydrology, specifically in applied water resource sciences and, among myriad rainfall-runoff methods, the SCS-CN method (1956,1964,1969,1971,1972,1985,1993) developed by the United States Department of Agriculture (USDA) is one of the most widely used simple, stable, reliable, and attractive method for estimating runoff from a given rainfall (Williams and LaSeur 1976; Rallison 1980; Hjelmfelt 1980; Bondelid et al. 1982; Garen and Moore 2005; Sahu et al. 2010; Ajmal et al. 2016; Verma et al. 2017; Voda et al. 2019). Though the method was developed originally for direct surface runoff estimation in small and medium agricultural watersheds in the mid-western United States (Ebrahimian et al. 2012), it has been used in several other areas including water resources management, planning of watershed conservation and management practices, reservoir operation, runoff prediction, water balance, irrigation, flood forecasting, rainwater harvesting, flood control, storm water management, design flood estimation, urban hydrology, etc. (Mishra and Singh 1999, 2002; Singh et al. 2010; Durán-Barroso et al. 2019; Walega et al. 2015, 2017; Wang 2018; Baiamonte 2019; Zhang et al. 2019). More specifically, the SCS-CN concept has been applied to areas such as long-term hydrologic simulation, sediment transport, prediction of infiltration and rainfall-excess rates, hydrograph simulation, sediment yield modelling, water quality modelling, transport of pollutant, partitioning of heavy metals (Mishra and Singh 2003; Verma et al. 2021) and so on. Moreover, many research efforts have also investigated utility of the SCS-CN method in quantifying urbanization related land use/cover changes effect on runoff (Hameed 2017; Hu et al. 2020) and effect of forest fires on the hydrological response and the associated hydrological risks (Candela et al. 2005; Nalbantis and Lymperopoulos 2012; Soulis 2018).

Due to its wide application, the SCS-CN method has become an integral part of more complex hydrological and ecological models available commercially. Some of these models are SWMM (Metcalf 1971), CREAMS (Knisel 1980; Smith and Williams1980), HEC-1 (HEC 1981), AGNPS (Young et al. 1989), EPIC (Sharpley and Williams 1990), SWAT (Arnold 1994), and GFMS (Yilmaz et al. 2010), AnnAGNPS (Annualized Agricultural Non-point Source Pollution Model, Baginska and Milne-Home 2003), and EBA4SUB (Event-Based Approach for Small and Ungauged Basins (Petroselli and Grimaldi 2018).

Besides the above extensive applications of the SCS-CN method/concept for various purposes, this concept does not appear to have yet been tested for its applicability to flood routing. Thus, the objective of this study is to first derive its analogy with the Muskingum method, a popular method of flood routing, and then extend its applicability to flood routing using field data.

2 Methodology

2.1 SCS-CN Concept

The SCS-CN method's core proportionality concept equates the ratio of actual runoff to potential runoff with the ratio of actual retention to potential maximum retention in terms of depth units (Mishra and Singh 2003) as:

$$\frac{Q}{P_e} = \frac{F}{S} \tag{1}$$

where Q = direct surface runoff (m), $P_e = (P - I_a)$ = effective rainfall (m), P = total rainfall (m), I_a = initial losses or rainfall losses (m), F = actual retention (m), and S = potential maximum retention (m), which can range (0, ∞), defined as:

$$S = \frac{25400}{CN} - 254 \tag{2}$$

where S is in mm and CN is non-dimensional varying from 0 to 100. The above proportionality is further coupled with the universal water balance Eq. (3), leading to the popular form of the SCS-CN runoff equation, expressed respectively, as follows:

$$P_e = P - I_a = Q + F \tag{3}$$

$$Q = \frac{P_e^2}{P_e + S} \tag{4}$$

To distinguish, the S parameter of the SCS-CN method is defined here-afterwards as S_{SCS}.

2.2 Muskingum Method

The classical Muskingum flood routing method is a combination of continuity and storage functions described, respectively, as below:

$$I - O = \frac{\Delta S_{Musk}}{\Delta t} \tag{5}$$

$$S_{Musk} = K[\theta I + (1 - \theta)O] \tag{6}$$

The variables S_{Musk}, I, and O are the Muskingum channel (detention) storage (m^3), rate of inflow (m^3/sec), and rate of outflow (m^3/sec), respectively, during the passage of a flow through the channel reach within time interval Δt (sec); K = storage coefficient or time constant for the river reach (sec), close to the flow travel time within the reach; and θ = dimensionless weighting factor varying from 0 to 0.5. Coupling the Kalinin-Milyukov concept (Jain 1993) with the popular one-dimensional Saint–Venant equations of flow routing, Perumal (1994) derived the following proportionality:

$$\frac{O}{I} = \frac{S_{Musk} - \theta KI}{KI - \theta KI} \tag{7}$$

from which the Muskingum storage Eq. (6) can be derived. Equation (7) was derived with the following assumptions: Channel has prismatic cross-section of any shape; there is no lateral outflow from or inflow to the reach; friction (energy) slope remains constant at any instant of time in a given routing reach; the magnitude of product of derivatives of flow and section variables (with respect to both time and distance) are negligible; and at any instant of time during unsteady flow, the steady flow relationship is applicable between the stage at the middle of the reach and the discharge passing somewhere downstream of it.

Equation (7) can also be described from Fig. 1 showing different elements of the Muskingum storage concept frequently described using wedge-prism concept in a prismatic channel. In this figure, the areas of various triangles and quadrilaterals represent various detention storages in the channel during the time-period of travel, i.e. K, or time of residence of flow in channel. For example, the product KI represents the potential maximum detention storage in channel when inflow I flows through the whole channel reach during the time of travel K, and similarly, the prism storage KO can be defined. The product θK(I-O) represents the wedge storage that is the part of K(I-O) storage resulting due to (I-O) flowing through the channel reach for time-period K. From this definition, θKI can be defined as the channel storage for the condition when O = 0, implying that there is no outflow. In other words, it represents the initial channel storage (defined as S_i) required to be fulfilled before the outflow O generates at the channel outlet.

Thus, the numerator and denominators of r.h.s. terms of Eq. (7) can be considered as actual (S_a) and potential (S_p) storages (m^3) of a canal reach over and above the initial storage S_i (=θKI), as follows:

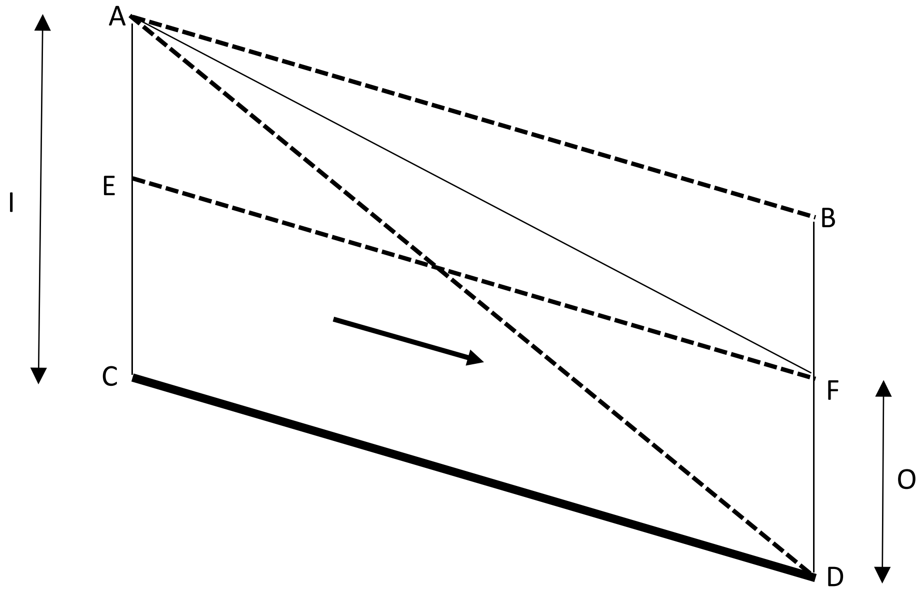

Fig. 1 Elements of Muskingum concept for a prismatic channel: CD = channel bed, AF = water surface, AC = I (inflow) = BD, DF = O (outflow) = CE, AE = BF = I-O, CD = EF = AB, Area AFDC = S_{Musk} (Muskingum storage), Area ABDC = KI (potential maximum storage), Area AFE = θK(I-O) (wedge storage), Area ADC = θKI, Area EFDC = KO (prism storage), Area ABF = (1-θ)K(I-O)

$$\frac{O}{I} = \frac{S_a}{S_p} \tag{8}$$

Equation (8) can be termed as Muskingum proportionality equation.

The validity of definitions of different elements of Eq. (7) as shown in Fig. 1 has also been demonstrated employing elementary trigonometry in Fig. 1, as follows. The numerator of Eq. (7) represents the area of triangle AFD. Similarly, the denominator of this equation represents the area of triangle ABD. From basic principles of trigonometry, it is possible to derive the ratio of area of triangle AFD to the area of triangle ABD to be equal to the ratio of DF (=O, outflow) to DB (=I, inflow).

2.3 Analogy between Muskingum and SCS-CN Proportionalities

To derive a structural analogy between the Muskingum and SCS-CN proportional equalities, the former can also be expressed in terms of depth units considering surface area, A, (m^2) of a prismatic channel during a given time interval (Δt).

$$\frac{Y}{X} = \frac{D_A}{D_p} \tag{9}$$

where Y = O Δt/A = outflow depth (m), X = I Δt/A = inflow depth (m), $D_A = S_a/A = (S_{Musk} - \theta KI)/A$ = actual detention storage depth (m) excluding $I_s = S_i/A = \theta KI/A$ depth (m), and $D_P = S_p/A = (KI - \theta KI)/A$ = potential maximum detention storage depth (m) excluding $I_s = S_i/A = \theta KI/A$ depth (m).

For analogy reasons, the SCS-CN proportional equality (Eq. 1) has been rewritten as:

$$\frac{Q}{P_e} = \frac{F}{S_{SCS}} \tag{10}$$

It is to note that both the terms in numerator and denominator of Eq. (10) exclude I_a. Thus, both Eqs. (9) and (10) are analogous to each other. Table 1 provides an elaborate analogy between various components of the proportional equalities of the two methods.

It can be seen from Table 1 that all the fundamental components of the Muskingum equation are analogous to those of the SCS-CN equation. The input to the Muskingum channel routing i.e. inflow depth (X) analogous to effective rainfall-depth (P_e) of the SCS-CN method in a given time duration/interval (Δt). Likewise, the output of the Muskingum channel routing i.e. outflow depth (Y) is analogous to the direct surface runoff-depth (Q) of SCS-CN method in the given time duration/interval (Δt). The actual (D_A) and potential (D_P) maximum detention storage depths of the Muskingum method are analogous to F and S_{SCS} of the SCS-CN method, respectively. The initial storage depth (I_s) of the Muskingum method is analogous to the initial abstraction depth (I_a) of the SCS-CN method. In both the methods, outflow/runoff generates only after I_s and I_a are exceeded. The weighting parameter (θ) of the Muskingum equation is analogous to initial abstraction coefficient (λ) of the SCS-CN method and both are nondimensional. I_s is also analogous to I_a as cumulative rainfall up to time of ponding ($I_a = i_0 t_p$, where 'i_0' is the rainfall intensity and t_p is time to ponding). Like Eq. (2) describing the runoff curve number CN (described in Table 1 as CN_{SCS}), it is possible to propose a routing curve number (CN_{Musk}) as: $CN_{Musk} = 25400/D_P - 254$, where Dp is in mm and CN_{Musk} is nondimensional varying from 0 to 100.

Table 1 Analogy between various elements of Muskingum and SCS-CN techniques

No.	Component	Muskingum Method	SCS-CN Method
1	Equation	9	10
2	Unit of terms	Depth	Depth
3	Input to the system	$X = I\ \Delta t/A$	P_e
4	Output from the system	$Y = O\ \Delta t/A$	Q
5	Actual detention/retention storage	D_A (detention)	F (retention)
6	Potential maximum detention/retention storage	D_P (detention)	S_{SCS} (retention)
7	Initial storage/abstraction	$I_s = S_i/A = \theta KI/A = (\theta K/\Delta t)X = mX$	$I_a = \lambda S_{SCS}$
8	Initial storage/abstraction coefficient	θ (non-dimensional)	λ (non-dimensional)
9	Treatment of I_s and I_a	Both D_A & D_p exclude I_s	Both F & S_{SCS} exclude I_a
10	Mass conservation	$D_A = X - Y$	$F = P_e - Q$
11	Ratio of output/input	Y/X range (0–1)	Q/P_e range (0–1)
12	Ratio of actual storage/ potential maximum storage (nondimensional)	D_A/D_P range (0–1)	F/S_{SCS} range (0–1)
13	Curve Number (nondimensional)	$CN_{Musk} = 25400/D_P - 254$	$CN_{SCS} = 25400/S_{SCS} - 254$
14	Storage equation	$S_{Musk} = K[\theta I + (1-\theta)O]$	$F_0 = \frac{1}{kA}[\lambda I + (1-\lambda)O]$
15	Storage coefficient (hr)	K	1/k
16	Routing equation	$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$	$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1$
17	C_o	$\left(\frac{-K\theta + \frac{\Delta t}{2}}{K - K\theta + \frac{\Delta t}{2}}\right)$	$\left(\frac{-\frac{\lambda}{k} + \frac{\Delta t}{2}}{\frac{1}{k} - \frac{\lambda}{k} + \frac{\Delta t}{2}}\right)$
18	C_1	$\left(\frac{K\theta + \frac{\Delta t}{2}}{K - K\theta + \frac{\Delta t}{2}}\right)$	$\left(\frac{\frac{\lambda}{k} + \frac{\Delta t}{2}}{\frac{1}{k} - \frac{\lambda}{k} + \frac{\Delta t}{2}}\right)$
19	C_2	$\left(\frac{K - K\theta - \frac{\Delta t}{2}}{K - K\theta + \frac{\Delta t}{2}}\right)$	$\left(\frac{\frac{1}{k} - \frac{\lambda}{k} - \frac{\Delta t}{2}}{\frac{1}{k} - \frac{\lambda}{k} + \frac{\Delta t}{2}}\right)$
20	For no lateral flow condition	$C_0 + C_1 + C_2 = 1$	$C_0 + C_1 + C_2 = 1$

Further, since $I_s = S_i/A = \theta KI/A$, it can be transformed as $I/A = I_s/\theta K$. Substituting $I/A = I_s/\theta K$ into X (=Δt I/A), I_s can be defined as: $I_s = X\ (\theta K/\Delta t)$. Assuming (θK/Δt) as *m* for given event in a channel, I_s can be redefined as: $I_s = mX$, which states that the initial storage is some percentage of inflow-depth and it is analogous to the SCS-CN method's I_a to be equal to βP (Ajmal et al. 2015). Moreover, since $D_A = (S_{Musk} - \theta KI)/A$ and $D_P = (KI - \theta KI)/A$ are actual and potential maximum storage depths, respectively, both excluding I_s. D_A can be transformed as: $D_A = S_{Musk}/A - I_s$ and $D_P = KI/A - I_s$. Here, both D_A and D_P are storage depths that exclude I_s, analogous to the SCS-CN storage/retention depths of F and S_{SCS}, respectively, both excluding I_a.

In terms of mass conservation, the continuity equation ($I - O = \Delta S_{Musk}/\Delta t$) of the Muskingum method is also analogous to the water balance of the SCS-CN method as: $P_e - Q = (i_0 - O/A)\ \Delta t = F$ for $P_e = i_0 \Delta t$ and $Q = O\Delta t/A$.

Furthermore, the ratio of the outflow/inflow depths (Y/X) of the Muskingum method which ranges (0–1) is analogous to the ratio of rainfall/runoff depths (Q/P_e) of the SCS-CN method, which also ranges (0–1). Lastly, the ratio of actual-detention storage/potential maximum-detention storage depths (D_A/D_P) of the Muskingum method also ranges (0–1) and it is analogous to the ratio of actual-retention storage/potential maximum-retention storage depths (F/S_{scs}) of the SCS-CN which ranges (0–1). Furthermore, like CN_{SCS} (Eq. 2), it is also possible to propose a routing curve number (CN_{Musk}) ranging (0–100). Thus, both the Muskingum and SCS-CN proportionalities are parallel/analogous to each other, and therefore, the SCS-CN concept also has the potential for its use in flood routing.

2.4 SCS-CN-based Routing

Based on the above analogy, it is seen that the SCS-CN proportional equality (Eq. 1) is quite similar to that of the Muskingum Eq. (7). Thus, similar to the Muskingum equation, it is possible to derive an expression for routing using SCS-CN concept as well, as follows.

Let us define the cumulative dynamic retention (F_0) and the potential storage space (S_0) available before the onset of rainfall at time t = 0 including I_a as:

$$F_0 = F + I_a \tag{11}$$

$$S_0 = S_{SCS} + I_a \tag{12}$$

For time interval Δt, the outflow and inflow depths are given, respectively, as:

$$Q = O\Delta t/A \tag{13}$$

$$P_e = i_0\Delta t \tag{14}$$

where i_0 is the uniform effective rainfall intensity. Equation (14) assumes rainfall (P) to grow linearly with time (t) (Mishra and Singh 2003). Substituting F, S_{SCS}, Q, and P_e from respective Eqs. (11), (12), (13), and (14) into Eq. (1) yields

$$\frac{(O/A)\Delta t}{i_0\Delta t} = \frac{O/A}{i_0} = \frac{F_0 - I_a}{S_0 - I_a} \tag{15}$$

Here, F_0 which is similar to the detention storage of Muskingum (S_{Musk}) Eq. (7) can be derived as:

$$F_0 = \frac{O/A}{i_0}\left(S_0 - I_a\right) + I_a \tag{16}$$

Substituting $I_a = \lambda S_0$ into Eq. (16) leads to

$$F_0 = \frac{O/A}{i_0}\left(S_0 - \lambda S_0\right) + \lambda S_0 \tag{17}$$

Mishra and Singh (2004) proposed a relationship ($f_0 - f_c = i_0 - f_c = i_e = S_0 k$), which is consistent with the description of Mein and Larson (1971) as well as infiltration data. Here, f_0 is the initial infiltration rate (m/s), k is the decay constant (1/sec) (Horton 1941), f_c is the final infiltration rate (m/s), i_0 is the uniform rainfall intensity (m/s) and i_e is the uniform effective rainfall intensity (m/s). For the condition $f_0 = i_0$ and $f_c = 0$,

$$S_0 = \frac{i_0}{k} \tag{18}$$

On further simplification of Eq. (17) using the proposed S_0, we get

$$F_0 = \frac{O/A}{i_0}\left(\frac{i_0}{k} - \lambda\frac{i_0}{k}\right) + \lambda\frac{i_0}{k} = \left[\frac{1-\lambda}{k}\right]\frac{O}{A} + \frac{\lambda}{k}i_0 \tag{19}$$

If runoff estimated using a rational formula is assumed to be the inflow to the system, i.e. I = C i_0 A. Since runoff coefficient (C) (nondimensional) and surface area (A) (m^2) of the routing reach are constant, the rainfall intensity becomes

$$i_0 = \frac{I}{CA} \tag{20}$$

Hence, F_0 in Eq. (19) can be re-framed as:

$$F_0 = \left[(1-\lambda)\frac{O}{kA}\right] + \left(\frac{\lambda}{kCA}\right) I = \frac{1}{kA}\left[(1-\lambda)O + \frac{\lambda}{C}I\right] \tag{21}$$

The SCS-CN water balance Eq. (3), can be revised using Eqs. (11), (13), (14) and (20) as:

$$F_0 = \left(i_0 - O/A\right)\Delta t = \left(\frac{I}{CA} - \frac{O}{A}\right)\Delta t = \frac{1}{A}\left(\frac{I}{C} - O\right)\Delta t \tag{22}$$

For the beginning and end of Δt, Eq. (22) can be written as:

$$\Delta F_0 = \frac{1}{A}\left[\frac{1}{C}\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{O_1 + O_2}{2}\right)\Delta t\right] \tag{23}$$

Here, subscripts 1 and 2 denote the beginning and end of Δt, respectively. Following Eq. (21) we can write

$$\Delta F_0 = F_{0_2} - F_{0_1} = \frac{1}{kA}\left[(1-\lambda)\left(O_2 - O_1\right) + \frac{\lambda}{C}\left(I_2 - I_1\right)\right] \tag{24}$$

Equating ΔF_0 from Eqs. (23) and (24) leads to the SCS-CN-based routing procedure for determination of outflow (O_2) in terms of coefficients C_0, C_1, and C_2 as follows:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \tag{25}$$

where

$$C_0 = \left(\frac{\frac{\Delta t}{2C} - \frac{\lambda}{kC}}{\frac{(1-\lambda)}{k} + \frac{\Delta t}{2}}\right) \tag{26}$$

$$C_1 = \left(\frac{\frac{\Delta t}{2C} + \frac{\lambda}{kC}}{\frac{(1-\lambda)}{k} + \frac{\Delta t}{2}}\right) \tag{27}$$

$$C_2 = \left(\frac{\frac{(1-\lambda)}{k} - \frac{\Delta t}{2}}{\frac{(1-\lambda)}{k} + \frac{\Delta t}{2}}\right) \tag{28}$$

For no lateral inflow condition, it can be shown that

$$C_0 + C_1 + C_2 = 1 \tag{29}$$

The above similarity is also shown in Table 1.

3 Application

3.1 Data and Study Reaches

To examine the applicability of the SCS-CN method for routing, four different study reaches are selected. The first study reach consists of the observed inflow and outflow hydrographs having double-peak outflow discharges as reported by (Viessman and Lewis 2003). The second one consists of extensively studied data-set of single peak inflow and outflow hydrographs as reported by (Wilson 1974). The third one consists of single peaked inflow-outflow hydrographs data-set as reported by (Wu et al. 1985). The fourth one consists of a data-set of inflow-outflow hydrographs having single peak and it is reported by (O'Donnel 1985). A summary of the characteristics of the four observed flood events used in the study is given in Table 2.

3.2 Performance Evaluation Criteria

The statistics and criteria used for comparing the performance of Muskingum and SCS-CN-based routing methods are root mean square error (RMSE); relative error in peak outflow, RE (O_P); and relative error in time to peak, RE (T_P). The lowest values of these statistics indicate goodness of the method compared to other. These criteria are given respectively as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} \left(O_{ai} - O_{ri}\right)^2}{N}} \tag{30}$$

$$\text{RE}\ (\text{O}_\text{p}) = \frac{O_{pa} - O_{pr}}{O_{pa}} * 100 \tag{31}$$

$$\text{RE}\ (\text{T}_\text{p}) = \frac{T_{pa} - T_{pr}}{T_{pa}} * 100 \tag{32}$$

where O_{ai}, O_{ri}, O_{pa}, O_{pr}, T_{pa}, T_{pr}, and N are ordinates of actual outflow, routed outflow, actual peak outflow, routed peak outflow, actual time to peak, routed time to peak, and number of ordinates of outflow hydrographs, respectively.

3.3 Parameter Estimation

To estimate different parameters of both routing methods, Marquardt (1963) algorithm of constrained least squares was applied using Microsoft Excel software. It performs well in practice and has become a standard of nonlinear optimization techniques in water sector.

Table 2 Characteristics of the four observed flood events used in the study

Event No.	Case studies	Inflow peak (I_p) (m^3/s)	Time to inflow peak (hrs)	Outflow peak (O_{pa}) (m^3/s)	Time to outflow peak (T_{pa}) (hrs)	Time of travel of peak discharge (hrs)	Attenuation (%)
1	2	3	4	5	6	$=6-4$	$=(3-5)\times 100/3$
1	Viessman and Lewis (2003)	1780	8.0	1520	10	2	14.61
2	Wilson (1974)	110	32	84	62	30	23.64
3	Wu et al. (1985)	18.8	17.4	10.4	33.6	16.2	44.68
4	O'Donnel (1985)	1150	82	975	100	18	15.22

Parameters θ and K of the Muskingum method and parameters λ, k and C of the SCS-CN routing methods were optimized using minimum RMSE as the objective function, allowing them to vary in the range (0, ∞). The optimized values of all the parameters are given in Table 3.

4 Results and Discussion

4.1 Characteristics of the Observed Flood Events

The four observed flood events derived from different sources in literature were used in this study. These can be characterized by their peak discharges and time to peak discharges of both inflows and their corresponding outflows (derived graphically), as shown in Table 2. These have been further utilized to derive the (approximate) time of travel of peak discharges (= time to peak outflow – time to peak inflow) and the percent attenuation of peak discharge (= peak inflow – peak outflow)*100/peak inflow. It is seen that time of travel varies from 2 to 30 h (the minimum for Event no. 1 and the maximum for Event no. 2) and the attenuation ranges from 14.61 m^3/s to 44.68 m^3/s (the minimum for Event no. 1 and maximum for Event no. 3). It can be inferred that the channel reach of Event no. 1 has been the shortest in length and/or steepest coupled with highest magnitude of flood peak discharge with highest flow velocity, and of Event no. 2 the maximum/mildest and/or low flood peak magnitude with low flow velocity. Since Event no. 3 showed maximum attenuation, it can be inferred that the reach must have been mildest coupled with lowest flood peak magnitude.

4.2 Parameter Estimation

The parameters of the Muskingum method are K and θ. As seen from Table 3, K values for the four flood events are 2.13, 32.11, 21.28, and 26.96 h, respectively, for case study nos. 1–4. K actually represents the time of travel of flood (= L/c, where L is the reach length and c is the wave celerity = 1.67 times average flow velocity (Ponce 1989)) and it can be approximated as the time difference between the peaks of the inflow and outflow hydrographs, as seen from Table 2.

The weighting parameter θ of the Muskingum method is defined as (Ponce 1989):

$$\theta = 0.5\left(1 - \frac{Ip}{B\,L\,c\,Sb}\right) = 0.5\left(1 - \frac{Ip}{A\,c\,Sb}\right) \tag{33}$$

where Ip is the inflow peak discharge; B and L are the channel top width and length, respectively; and S_b is the channel bed slope. θ ranging (0, 0.5) primarily represents (approximately) the level of attenuation in the peaks of inflow and outflow hydrographs. θ = 0.5 suggests no attenuation or the wave is kinematic in nature whereas θ = 0 indicates highest level of attenuation or the flood water is fully dynamic in nature. As seen from Table 3, its values for the 1–4 flood events are 0.14, 0.15, 0.01, 0.11, respectively, indicating all the flood waves to be fully dynamic in nature but Event no. 3 being the most dynamic with highest attenuation.

On the other hand, the SCS-CN parameters are initial abstraction coefficient (λ), infiltration decay coefficient (k) (1/s), and the runoff coefficient (C). The values of λ for the 1–4

Table 3 Performance evaluation of Muskingum and SCS-CN routing methods

Event No.	Routing methods	RMSE (m^3/s)	RE Q_p (%)	RE T_p (%)	Parameters							
					θ	K (hr)	λ	k (1/hr)	C	C_0	C_1	C_2
1	SCS-CN	68.45	6.70	0.00	-	-	0.13	0.47	0.99	0.09	0.34	0.58
	Muskingum	68.57	7.07	0.00	0.14	2.13	-	-	-	0.09	0.34	0.57
2	SCS-CN	2.73	4.88	10.00	-	-	0.14	0.03	0.98	-0.05	0.25	0.81
	Muskingum	2.74	5.24	10.00	0.15	32.11	-	-	-	-0.06	0.25	0.80
3	SCS-CN	0.44	7.37	0.00	-	-	0.01	0.05	0.99	0.12	0.13	0.75
	Muskingum	0.44	7.59	0.00	0.01	21.28	-	-	-	0.11	0.14	0.75
4	SCS-CN	66.12	15.00	11.76	-	-	0.18	0.03	0.94	-0.09	0.32	0.78
	Muskingum	67.65	15.89	11.76	0.11	26.96	-	-	-	0.00	0.22	0.78

flood events are 0.13, 0.14, 0.01, 0.18, respectively. Except for the third event for which λ is 0.01, λ values for all others range (0.13, 0.18). The values of decay coefficient (k) for these events are 0.47, 0.03, 0.05, and 0.03, respectively. Except for the first event for which k-value is too high, all others range (0.03, 0.05). As expected, the runoff coefficient (C) values are too high ranging (0.94, 0.99), largely due to the inflow in the channel being considered as falling like rainfall over the water surface area (A), in which the losses or abstractions are absolute minimum or nil. It can also be supported by the fact that unlike a watershed in which the rainfall is distributed in space (or the whole watershed), the inflow in the channel is concentrated at the inlet without any loss, and therefore, C can be taken as equal to 1. Such a consideration makes Eq. (21) exactly analogous to that of Muskingum storage Eq. (6) with $K = 1/k$, $\theta = \lambda$, and $I_s = mX$, where $m = f(\theta)$ as shown above. As shown in Table 1 the routing parameters, C_0, C_1, and C_2 of both the Muskingum and SCS-CN methods also match each other. The same can also be inferred from Table 3 for all four flood events, for which all parameters have been optimized using the real inflow/outflow data.

4.3 Performance Evaluation

The performance of both the Muskingum and SCS-CN routing methods is evaluated based on (a) visual closeness of the observed and computed outflow hydrographs and (b) statistical criteria in terms of RMSE, RE in O_p (%), and RE in T_p (%). Figure 2(a–d) indicate the graphical closeness of the observed and computed outflow hydrographs routed by both Muskingum and SCS-CN methods in four different channel reaches. It can be seen from Fig. 2(a) that the SCS-CN method has the capability of capturing the multi-peaked shape of the observed outflow hydrograph quite similar to that of the Muskingum method. Figure 2(b–d) also depict the same.

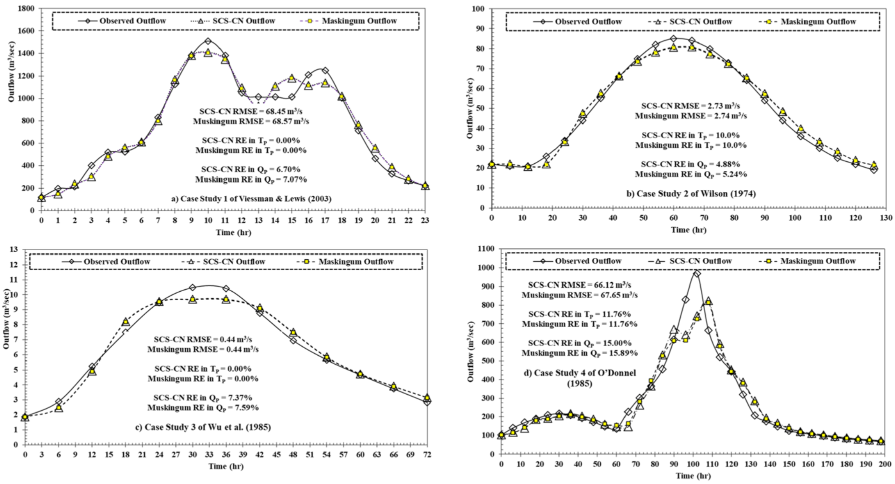

Fig. 2 a-d Graphical comparison of observed and routed outflow hydrographs computed using Muskingum and SCS-CN concept-based methods on four different case studies of Viessman and Lewis (2003); Wilson (1974); Wu et al. (1985); O'Donnel (1985), respectively. These are referred in Table 1 as Case Studies nos. 1 – 4, respectively

The values of RMSE, RE in O_p (%), and RE in T_p (%) are given in Table 3. The RE-values for 1–4 flood events for SCS-CN and Muskingum methods are (68.45, 68.57), (2.73, 2.74), (0.44, 0.44) and (66.12, 67.65) m^3/s, respectively. It can be inferred that RE-values are almost the same for each other and it can also be seen from Fig. 2 that both the methods match the observed outflow shape with almost the same level of accuracy. Similarly, both the methods exhibit the same level of satisfactory performance while fitting the peak and time to peak discharge values of all the four events.

Thus, it can be inferred that, in all study reaches, the SCS-CN concept performs equally well with RMSE, RE in O_p, and RE in T_p being almost equal to or better than those due to the Muskingum method, largely due to both being identical in their mathematical formulation/structure. In peak and time to peak discharge prediction, the SCS-CN method performs equally well. In general, the SCS-CN method is analogous to Muskingum method, not only in its mathematical formulation but also in real-world application and is thus efficacious in flood routing as well, underlining the versatility of the former concept.

5 Conclusion

The SCS-CN concept which is basically used in rainfall-runoff modeling has been shown to be exactly analogous to the popular Muskingum method of flood routing in both mathematical formulation and in application to routing of four (both single- and double-peaked) flood events having occurred in four different routing reaches with the same level of efficacy as the Muskingum method, enhancing the versatility of the SCS-CN concept.

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References

Ajmal M, Khan TA, Kim TW (2016) A CN-based ensemble hydrological model for enhanced watershed runoff prediction. Water 8:20

Ajmal M, Moon GW, Ahn JH, Kim TW (2015) Investigation of SCS-CN and its inspired modified models for runoff estimation in South Korean watersheds. J Hydro-Environ Res 9(4):592–603

Akbari GH, Barati R (2012) Comprehensive analysis of flooding in unmanaged catchments. Proc Inst Civil Eng Water Manag 165:229–238

Akbari G, Nezhad H, A. H., and Barati, R. (2012) Developing a model for analysis of uncertainties in prediction of floods. J Adv Res 3(1):73–79

Arnold J (1994) SWAT (Soil and water assessment tool). Grassland, vol 498. Soil and Water Research Laboratory, USDA, Agricultural Research Service

Baginska B, Milne-Home WA (2003) Parameter sensitivity in calibration and validation of an annualized agricultural non-point source model. Calibr Watershed Models 6:331–345

Baiamonte G (2019) SCS curve number and Green-Ampt infiltration models. J Hydrol Eng 24(10):04019034. https://doi.org/10.1061/(ASCE)HE.1943-5584.0001838

Barati R (2014) Discussion of parameter estimation of the nonlinear Muskingum flood routing model using a Hybrid Harmony Search algorithm by Halil Karahan, Gurhan Gurarslan, and Zong Woo Geem. J Hydrol Eng 19(4):842–845

Bondelid TR, McCuen RH, Jackson TJ (1982) Sensitivity of SCS models to curve number variation. Water Resour Bull Am Water Resour Assoc 18(1):111–116

Candela A, Aronica G, Santoro M (2005) Effects of forest fires on flood frequency curves in a Mediterranean catchment. Hydrol Sci J 50:193–206

Choudhury P (2007) Multiple inflows Muskingum routing model. J Hydrol Eng ASCE 12(5):473–481

Choudhury P, Shrivastava RK, Narulkar SM (2002) Flood routing in river networks using equivalent Muskingum inflow. J Hydrol Eng ASCE 7(6):413–419

Durán-Barroso P, González J, Valdés JB (2019) Improvement of the integration of Soil Moisture Accounting into the NRCS-CN model. J Hydrology 542:809–819. https://doi.org/10.1016/j.jhydrol.2016.09.053

Ebrahimian M, Nuruddin AAB, Soom MABM, Sood AM, Neng LJ (2012) Runoff estimation in steep slope watershed with standard and slope-adjusted curve number methods. Polish J Environ Stud 21(5):1191–1202

Garen DC, Moore DS (2005) Curve number hydrology in water quality modeling: uses, abuses, and future directions. J Am Water Resour Assoc 41(2):377–388

Hameed HM (2017) Estimating the effect of urban growth on annual runoff volume using GIS in the Erbil subbasin of the Kurdistan region of Iraq. Hydrology 4:12

HEC (1981) The new HEC-1 Food hydrograph package. US Army Corps of Engineers. Institute for Water Resources, Hydrologic Engineering Centre, 609, Second street, Davis, CA, 95616

Hjelmfelt AT (1980) Empirical investigation of curve number technique. J Hydraul Div ASCE 106:1471–1476

Horton RE (1941) An approach toward a physical interpretation of infiltration-capacity. Soil Sci Soc Am J 5(C):399–417

Hu S, Fan Y, Zhang T (2020) Assessing the effect of land use change on surface runoff in a rapidly urbanized city: a case study of the central area of Beijing. Land 9:17

Jain SK (1993) Kalinin-Milyukov method of flood routing. National Institute of Hydrology, Roorkee, India

Knisel WG (1980) CREAMS, a field scale model for chemicals, runoff, and erosion from agricultural management systems. USDA ARS, Washington DC, USA (Conservation Research Report 26)

Marquardt DW (1963) An algorithm for least-squares estimation of nonlinear parameters. J Soc Lndust Appl Math 11(2):431–441

McCarthy GT (1938) The unit hydrograph and flood routing. Conf. North Atlantic Division, U.S. Army Corps of Engineers, New London, Conn

Mein RG, Larson CL (1971) Modeling the infiltration component of the rainfall-runoff process. WRRC, Minneapolis, Minnesota

Metcalf E (1971) Storm water management model, vol 1. University of Florida and Water Resources Engineers Inc. Final Report, 11024DOC07/71 (NTIS PB-203289), U.S. EPA, Washington, DC, 20460

Mishra SK, Singh VP (1999) Hysteresis-based flood wave analysis. J Hydrol Engrg ASCE 4(4):358–365

Mishra SK, Singh VP (2002) Chapter 13: SCS-CN-based hydrologic simulation package. In: Singh VP, Frevert DK (eds) Mathematical models in small watershed hydrology. Water Resources Publications, P.O. Box 2841, Littleton, Colorado 80161

Mishra SK, Singh VP (2003) Soil Conservation Service curve number (SCS-CN) methodology. Springer Science & Business Media

Mishra SK, Singh VP (2004) Validity and extension of the SCS-CN method for computing infiltration and rainfall-excess rates. Hydrol Process 18(17):3323–3345

Nalbantis I, Lymperopoulos S (2012) Assessment of flood frequency after forest fires in small ungauged basins based on uncertain measurements. Hydrol Sci J 57:52–72

Niazkar M, Afzali SH (2016) Application of new hybrid optimization technique for parameter estimation of new improved version of Muskingum model. Water Resour Manag 30:4713–4730. https://doi.org/10.1007/s11269-016-1449-9

O'Donnel T (1985) A direct three-parameter Muskingum procedure incorporating lateral inflow. Hydrol Sci J 30(4):479–496

Perumal M (1994) Hydrodynamic derivation of a variable parameter Muskingum method: 1. Theory Solut Proced Hydrol Sci J 39(5):431–442. https://doi.org/10.1080/02626669409492766

Perumal M, Price RK (2013) A fully mass conservative variable parameter McCarthy–Muskingum method: theory and verification. J Hydrol 502:89–102

Perumal M, Sahoo B (2008) Volume conservation controversy of the variable parameter Muskingum-Cunge method. J Hydraul Eng ASCE 134(4):475–485. https://doi.org/10.1061/(ASCE)07339429(2008)134:4(475)

Petroselli A, Grimaldi S (2018) Design hydrograph estimation in small and fully ungauged basins: a preliminary assessment of the EBA4SUB framework. J Flood Risk Manag 11:S197–S210

Ponce VM (1989) Engineering hydrology: principles and practice. Prentice Hall, Englewood Cliffs, New Jersey, p 07632

Ponce VM, Yevjevich V (1978) Muskingum-Cunge method with variable parameters. J Hydraul Div ASCE 104(HY12):1663–1667

Rallison RE (1980) Origin and evolution of the SCS runoff equation. ASCE

Sahu RK, Mishra SK, Eldho TI (2010) An improved AMC-coupled runoff curve number model. Hydrol Process 24:2834–2839. https://doi.org/10.1002/hyp.7695

SCS (1956,1964,1969,1971,1972,1985,1993) Hydrology: national engineering handbook, Supplement A, Section 4, Chapter 10. Soil Conservation Service, USDA, Washington, D.C.

Sharpley AN, Williams JR (1990) EPIC-erosion/productivity impact calculator. I: Model Documentation. II: User Manual. Technical Bulletin-United States Department of Agriculture, (1768)

Singh PK, Gaur ML, Mishra SK, Rawat SS (2010) An updated hydrological review on recent advancements in Soil Conservation Service-curve number technique. J Water Clim Change 01(2):118–134

Smith RE, Williams JR (1980) Simulation of the surface hydrology. In: Knisel W (ed) CREAMS, a field scale model for chemicals, runoff, and erosion from agricultural management systems. USDA ARS, Washington DC, USA (Conservation Research Report 26)

Soulis KX (2018) Estimation of SCS curve number variation following forest fires. Hydrol Sci J 63:1332–1346

Verma S, Mishra SK, Singh A, Singh PK, Verma RK (2017) An enhanced SMA based SCS-CN inspired model for watershed runoff prediction. Environ Earth Sci 76(736):1–20

Verma RK, Verma S, Mishra P, A. (2021) SCS-CN-Based improved models for direct surface runoff estimation from large rainfall events. Water Resour Manag. https://doi.org/10.1007/s11269-021-02831-5

Viessman W Jr, Lewis GL (2003) Introduction to hydrology. Pearson Education Inc, Upper Saddle River, New Jersey, USA

Voda M, Sarpe CA, Voda AI (2019) Romanian river basins lag time analysis. The SCS-CN versus RNS comparative approach developed for small watersheds. Water Resour Manag 33:245–259. https://doi.org/10.1007/s11269-018-2100-8

Walega A, Cupak A, Amatya DM, Drożdżal E (2017) Comparison of direct outflow calculated by modified SCS-CN methods for mountainous and highland catchments in upper Vistula Basin, Poland and lowland catchment in South Carolina, U.S.A. Acta Sci Pol Form Circum 16(1):187–207

Walega A, Michalec B, Cupak A, Grzebinoga M (2015) Comparison of SCS-CN determination methodologies in a heterogeneous catchment. J Mt Sci 12(5):1084–1094

Wang D (2018) A new probability density function for spatial distribution of soil water storage capacity leads to the SCS curve number method. Hydrol Earth Syst Sci 22:6567–6578. https://doi.org/10.5194/hess-22-6567-2018

Williams JR, LaSeur WV (1976) Water yield model using SCS curve numbers. J Hydr Div ASCE 102:1241–1253

Wilson EM (1974) Engineering hydrology. MacMillan Education Ltd., Hampshire, U.K.

Wu Jy, S., King, E. L. and Wang, M. (1985) Optimal identification of Muskingum routing coefficients. Water Resour Bull 21(3):417–421

Yilmaz KK, Adler RF, Tian Y, Hong Y, Pierce HF (2010) Evaluation of a satellite-based global flood monitoring system. Int J Remote Sens 31:3763e3782
Young RA, Onstad CA, Bosch DD (1989) Chapter 26: AGNPS: an agricultural nonpoint source model. In: Singh VP (ed) Computer models of watershed hydrology. Water Resources Publications, Littleton CO
Zhang D, Lin Q, Chen X, Chai T (2019) (2019) Improved curve number estimation in SWAT by reflecting the effect of rainfall intensity on runoff generation. Water 11:163. https://doi.org/10.3390/w11010163