



Muskingum Models' Development and their Parameter Estimation: A State-of-the-art Review

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Abstract

River flood routing is an important issue in current water resources management. As a popular hydrological flood routing method, Muskingum model has always been the dominant method of flood routing. This paper reviews the development of Muskingum model and the research status of its parameter estimation. The characteristics and relationships of different types of Muskingum models are compared, and it is found that the combination of mathematical techniques and evolutionary algorithms has shown good results in parameter estimation in recent years. In addition, this paper also gives a brief overview of six accuracy evaluation criteria and nine research case data sets commonly used in the literature. It also introduces some challenges of the Muskingum model and new trends in future research, which should interest researchers and engineers.

Keywords Flood routing · Muskingum model · Parameter estimation · Evolutionary algorithms · Hybrid algorithms

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Nomenclature

LMM	Linear Muskingum model
NLM	Nonlinear Muskingum model
NLM-VEP	Nonlinear Muskingum model with variable exponent parameter
LMM-LF	Linear Muskingum model with lateral flow
NLM-LF	Nonlinear Muskingum model with lateral flow
NLM-VEP-LF	Nonlinear Muskingum model with variable exponent parameter and lateral flow
NLM-LF-GS	Nonlinear Muskingum model with variable exponential parameters and transverse flow in the presence of stable GW-SW interaction process
NLM-GS	Nonlinear Muskingum model considering the nonlinear relationship between lateral and channel inflow
HJ	Hooke Jeeves pattern search
LR	Linear Regression
CG	Conjugate Gradient method
DFP	Davidon-Fletcher-Powell
HJ + DFP	HJ pattern search in conjunction with Davidon-Fletcher-Powell
NONLR	Nonlinear Multivariate Parameter estimation technique
GA	Genetic Algorithm
HS	Harmony Search Algorithm
PSO	Particle Swarm Optimization Algorithm
ICSA	Immune Clonal Selection Algorithm
DE	Differential Evolution Algorithm
SFLA	Shuffled Frog Leaping Algorithm
MHBMOA	Modified Honey Bee Mating Optimization Algorithm
CSA	Cuckoo Search Algorithm
IGSA	Improved Gravitational Search Algorithm
NMS	Nelder-Mead simplex Algorithm
GRG	Generalized Reduced Gradient Algorithm
BFGS	Broyden-Fletcher-Goldfarb-Shanno
GA-NMS	Hybrid GA and Nelder-Mead simplex Algorithm
GA-GRG	Hybrid GA and GRG
HS-BFGS	Hybrid HS and BFGS
SFLA-GRG	Hybrid SFLA and GRG
BA	Bat Algorithm
GGA	Gray-encoded Genetic Algorithm
BGA	Binary-encoded Genetic Algorithm
HCGA	Hybrid Chaotic Genetic Algorithm
SAA	Simulated Annealing Algorithm
SAVCA	Self-Adaptive Vision Correction Algorithm
CSS	Charged System Search
LM	Leven Berg-Marquardt Algorithm
PRP	Polak-Ribière-Polyak method
SA	Shark Algorithm
SSD	Sum of squared deviations
SAD	Sum of absolute value of deviations,
DPO	Absolute value deviations between peaks of observed and routed flows
DPOT	Absolute value of deviations of peak times of observed and routed flows

MARE	Mean absolute relative error
NSC	Nash-Sutcliffe criterion

1 Introduction

Flood is one of the most deadly disasters in the world, and it is also the most common and frequent natural disaster for life, property, economy and environment in the world (Wijayarathne and Coulibaly 2020). Therefore, flood routing has become a hot issue in water resources management (Yuan et al. 2016). River flood routing is an important aspect of water resources management and one of most challenging practical problems. Its essence is to predict reservoir discharge for specific inflow processes. Generally speaking, the flood path method can be divided into two main methods, namely “hydraulic routing” and “hydrologic routing”. The former is based on the concept of storage function, while the latter is based on the principle of mass conservation and momentum conservation.

However, the hydraulic routing method is often not so widely used in practice, mainly due to the unavailability of morphological and hydrological inputs required at small spatial scales (Perumal and Price 2013). Many researchers have conducted research in this topic (Price Roland 2009; Reggiani et al. 2016; Swain and Sahoo 2015; Todini 2007; Yadav et al. 2015). Simplification methods developed from the continuity equations and simplified forms of mass and momentum equations provide many important contributions. Many researchers have provided assessments, reviews, comparisons, and field applications (Fenton 2019; Gaşiorowski and Szymkiewicz 2022; Koussis 2009; Perumal et al. 2001; Perumal and Sahoo 2012, 2007; Wei and Song 2022).

McCarthy (1938) proposed a primitive hydrological computation method, the Muskingum model, which is based on the storage-continuity relationship and involves multiple computation parameters, and all hydraulic and geomorphological characteristics of the river reach are concentrated into multiple model parameters (Todini 2007). As a popular hydrological flood routing method, the classical Muskingum model and its development can efficiently deal with many complex problems and are widely used in flood routing problems.

In practical applications, the accuracy of Muskingum model in predicting river flow depends on the definition of model structure and the determination of model parameters. In the past few decades, there are two main methods to improve the accuracy of the Muskingum model: (1) improving the structure of the Muskingum model and (2) developing optimization techniques for parameter estimation (Kang et al. 2017; Niazkar and Afzali 2016). In recent decades, some researchers have focused on changing the structure of the Muskingum model storage equation to improve the accuracy of fitting measured hydrological data. In general, the available models can be divided into the following categories: (1) Linear Muskingum model; (2) Nonlinear Muskingum model; (3) Muskingum model with variable exponent parameter; (4) Lateral flow Muskingum model; (5) Variable exponent parameters and lateral flow Muskingum model and (6) Muskingum model considering the interaction between groundwater and surface water. With the increase of parameters, parameter optimization becomes more and more complex. Therefore, in practical applications, the parameter estimation of Muskingum model is the key to flood routing (Sheng et al. 2014). Up to now, various optimization algorithms have been developed and used to search the optimal solution of Muskingum model optimization problem. At present, the optimization techniques for

parameter estimation of Muskingum model can be divided into three categories: mathematical techniques, evolutionary algorithms and hybrid algorithms.

Although the research on Muskingum model has been very rich in recent years, it is still necessary to review and analyze the development of Muskingum model and parameter estimation technology. Analyzing the advantages and disadvantages of various improved models, so as to clearly study the challenges and improvements of the Muskingum model, provides better prediction skills for river flood evolution, and provides sufficient data for fitting observed hydrological data. In addition, this paper attempts to summarize the accuracy evaluation criteria used in most literature and the case data sets studied to evaluate various Muskingum models. Therefore, 80 years after the classic Muskingum model was proposed, a recent review article on this will inspire the future.

The rest of this paper is structured as follows: Section 2 introduces the Muskingum modelling development. Section 3 presents main optimization methods and optimization technology for estimating the parameters of Muskingum models. Section 4 is a brief description of the adopted precision evaluation criteria for evaluating the performance of models and optimization techniques. Section 5 gives some studies case data sets. Finally, the work is concluded in Section 6.

2 Model Development

Many researchers focused on modifying the structure of the storage equation with the objective of improving accuracy in fitting observed hydrograph data. Muskingum models can be classified into six groups as follows:

2.1 Linear Muskingum Model

McCarthy (1938) presented continuity and storage equations for flood routing. The continuity equation is written as:

$$\frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} = I - Q \quad (1)$$

where I and Q denote upstream and downstream flow rates, respectively; S is the channel storage volume; t is the time; and $\Delta S/\Delta t$ is the change in storage during a time interval Δt . The linear Muskingum model (LMM) is expressed as:

$$S = K[XI + (1 - X)Q] \quad (\text{LMM}) \quad (2)$$

where K denotes a time storage constant for the river reach, and X is the weighting factor given to the inflow and outflow in a river reach.

2.2 Nonlinear Muskingum Models

The LMM has two parameters (K and X). However, it was observed to have a nonlinear relationship between the weighted flow and storage volume. The use of a nonlinear model might be more appropriate (Haddad et al. 2015). A nonlinear Muskingum model with three parameters was suggested by Chow (1959):

$$S = K[XI^\alpha + (1 - X)Q^\alpha] \text{ (NLM1)} \quad (3)$$

Gill (1978) added an exponent parameter m to Eq. (3):

$$S = K[XI + (1 - X)Q]^m \text{ (NLM2)} \quad (4)$$

Gavilan and Houck (1985) introduced a generalized NLM1 model with four parameters.

$$S = K[XI^{\alpha_1} + (1 - X)Q^{\alpha_2}] \text{ (NLM3)} \quad (5)$$

where α , α_1 and α_2 are flow parameters, and m is an exponent parameter.

Easa (2014b) presented a new nonlinear Muskingum model with four parameters that combined NLM1 and NLM2:

$$S = K[XI^\alpha + (1 - X)Q^\alpha]^m \text{ (NLM4)} \quad (6)$$

Easa et al. (2014) proposed a general form of storage discharge relationship of the Muskingum model:

$$S = [C_1I^{\alpha_1} + C_2Q^{\alpha_2}]^m \text{ (NLM5)} \quad (7)$$

Vatankhah (2014b) proposed a nonlinear Muskingum model with five parameters:

$$S = K[XI^{\alpha_1} + (1 - X)Q^{\alpha_2}]^m \text{ (NLM6)} \quad (8)$$

Easa et al. (2014) proposed a nonlinear Muskingum model with six parameters based on a model with five parameters. It preserved the physical significance of the original Muskingum model parameters (Easa 2013):

$$S = [X(C_1I^{\alpha_1}) + (1 - X)(C_2Q^{\alpha_2})]^m \text{ (NLM7)} \quad (9)$$

Bozorg-Haddad et al. (2015) proposed a nonlinear Muskingum model with seven parameters by adding two constraint coefficients (C_1 and C_2) based on NLM6.

$$S = K[X(C_1I^{\alpha_1}) + (1 - X)(C_2Q^{\alpha_2})]^m \text{ (NLM8)} \quad (10)$$

Niazkar and Afzali (2017) proposed a new Muskingum model by utilizing eight constant parameters in order to relate reach storage to inflow and outflow values:

$$S = K[X_1I^{\alpha_1}(1 + X_2I^{\alpha_2}) + X_3Q^{\alpha_3}]^m \text{ (NLM9)} \quad (11)$$

Bozorg-Haddad et al. (2019) introduced four Muskingum models with improved, generalized, nonlinear storage equations:

$$S_t = K[X_1I_t^\alpha + X_2I_{t+1}^\alpha + (1 - X_1 - X_2)Q_t^\alpha] \text{ (NLM10)} \quad (12)$$

$$S_t = K[X_1I_t + X_2I_{t+1} + (1 - X_1 - X_2)Q_t]^m \text{ (NLM11)} \quad (13)$$

$$S_t = K[X_1I_t^\alpha + X_2I_{t+1}^\alpha + (1 - X_1 - X_2)Q_t^\alpha]^m \text{ (NLM12)} \quad (14)$$

$$S_t = K[X_1(C_1I_t^\alpha) + X_2(C_1I_{t+1}^\alpha) + (1 - X_1 - X_2)C_2Q_t^\alpha]^m \text{ (NLM13)} \quad (15)$$

where X_1 and X_2 are weighting factors reflecting the significance of inflows in t^{th} and $(t+1)^{\text{th}}$ time intervals, respectively.

2.3 Muskingum Models with a Varying Exponent Parameter

Based on the NLM2 model, Easa (2013) proposed an improved Muskingum model with a varying exponent parameter. The exponent parameter, which has no physical meaning, is assumed to vary with the inflow level.

$$S_t = K[XI_t + (1 - X)Q_t]^{m_t} \text{ (NLM-VEP1)} \tag{16}$$

where m_t is related to I_t , m_t denotes exponent parameter at time t , and letting the inflow range be divided into L levels, m_t is defined as

$$m_t \begin{cases} m_1 I_t \leq \mu_1 \\ m_2 \mu_1 \leq I_t \leq \mu_2 \\ \dots \dots \\ m_i \mu_{i-1} \leq I_t \leq \mu_i \\ \dots \dots \\ m_L I_t \geq \mu_{L-1} \end{cases} \tag{17}$$

where μ_i denotes inflow dividing value for level i ; and μ_i is given as

$$\mu_i = r_i \times I_{\max}, \quad i = 1, 2, \dots, (L - 1) \tag{18}$$

where r_i denotes inflow dividing parameters for level i , and I_{\max} denotes the maximum inflow.

Vatankhah (2014a) employed a general functional m_t based on NLM-VEP1:

$$S_t = K[XI_t + (1 - X)Q_t]^{m_t} \text{ (NLM-VEP2)} \tag{19}$$

$$m_t = \alpha_1 + \alpha_2 \sin[a_3 + a_4(t + 1)^{a_5}] \tag{20}$$

where the angle of the sine function is expressed in radian, and constant coefficients $\alpha_i (i = 1, 2, \dots, 5)$ are obtained by using an optimization procedure.

Easa (2015) proposed a general model formulation nonlinear Muskingum model with a varying exponent parameter:

$$S_t = K(\mu_t)[X(\mu_t)I_t^{\alpha(\mu_t)} + (1 - X(\mu_t))Q_t^{\alpha(\mu_t)}]^{m(\mu_t)} \text{ (NLM-VEP3)} \tag{21}$$

where $K(\mu_t)$, $X(\mu_t)$, $\alpha(\mu_t)$, and $m(\mu_t)$ are varying parameters and μ_t is a dimensionless inflow variable (0 to 1) for time interval t , given by

$$\mu_t = I_t / I_{\max} \tag{22}$$

where I_t denotes the inflow for time interval t and I_{\max} denotes the maximum inflow during the routing period.

Easa (2014a) proposed a Muskingum model with an inflow based continuous parameter by assuming a continuous parameter $m(\mu_t)$, which is a function of dimensionless inflow variable μ_t :

$$S_t = K[XI_t + (1 - X)Q_t]^{m(\mu_t)} \text{ (NLM-VEP4)} \tag{23}$$

$$m(\mu_t) = a + be^{-c\mu_t} \quad (24)$$

where $m(\mu_t)$ denotes exponent parameter for time interval t , a , b , c denote coefficients to be determined using optimization, and μ_t denotes dimensionless inflow variable (0 to 1), expressed by Eq. (22).

2.4 Muskingum Models with Lateral Flow

All the above introduced models ignored the reality that lateral flow existed in the river reach in actual flood events (Kang et al. 2017). O'Donnell (1985) proposed the first Muskingum model with lateral flow and assumed that the lateral flow entering the river reach was directly proportional to the inflow with a proportionality factor β :

$$\frac{dS_t}{dt} = (1 + \beta)I_t - Q_t \quad (25)$$

$$S_t = K[X(1 + \beta)I_t + (1 - X)Q_t] \quad (\text{LMM-LF}) \quad (26)$$

The LMM-LF was used for studying flow routing process during river ice thawing-breakup period (Yang et al. 2019).

Karahan et al. (2015) presented a nonlinear Muskingum model with lateral flow by integrating lateral flow assumption with NLM2, and

$$S_t = K[X(1 + \beta)I_t + (1 - X)Q_t]^m \quad (\text{NLM-LF1}) \quad (27)$$

where m is a positive exponent different from 1.

Kang et al. (2017) proposed a nonlinear Muskingum model with lateral flow by introducing a similar lateral flow assumption into NLM4.

$$S_t = K[X(1 + \beta)I_t^\alpha + (1 - X)Q_t^\alpha]^m \quad (\text{NLM-LF2}) \quad (28)$$

2.5 Muskingum Models with Varying Exponent Parameter and Lateral Flow

Zhang et al. (2017) presented a varying exponent parameter Muskingum model with lateral flow by considering that the exponent parameter m_t of NLM-LF1 varied with inflow levels:

$$S_t = K[X(1 + \beta)I_t + (1 - X)Q_t]^{m_t} \quad (\text{NLM-VEP-LF1}) \quad (29)$$

Kang et al. (2017) proposed another nonlinear Muskingum model with lateral flow by considering that the exponent parameter m_t of NLM-LF2 varied with inflow levels.

$$S_t = K[X(1 + \beta)I_t^\alpha + (1 - X)Q_t^\alpha]^{m_t} \quad (\text{NLM-VEP-LF2}) \quad (30)$$

2.6 Muskingum Models Considering Interaction of Groundwater and Surface Water

Lu et al. (2021) presented an improved NLM-LF1 by assuming that a stable GW-SW interaction process existed before a flood event:

$$S_t = K[X(1 + \beta)I_t + (1 - X)Q_t + e]^m \text{ (NLM-LF1-GS)} \quad (31)$$

where e denotes the stable lateral inflow due to the GW-SW interaction.

Considering the nonlinear relationship between lateral inflow and channel inflow, the following equation is given by Lu et al. (2021):

$$S_t = K[XI_t + (1 - X)Q_t + e + I_t^\beta]^m \text{ (NLM2-GS)} \quad (32)$$

where β denotes a power exponent.

2.7 Comparison and Summary of Each Model

In the development of nonlinear Muskingum model, researchers have continued to improve it by increasing the method of considering parameters, so that it has higher degrees of freedom, better fitting ability, and the development tends to be realistic. But to some extent, it will also increase the complexity of the model.

However, only studying the nonlinear Muskingum model is obviously not in line with reality and lacks practical significance, because there are many factors in the actual flood process. Therefore, scholars began to consider the exponential case on the basis of non-linearity, lateral flow, and the later Muskingum models contained both conditions. These models further optimized the ability to predict flood paths. Not only that, some researchers have also considered the exchange of surface water and groundwater in reality, so the Muskingum model including surface water and groundwater is studied, which renders the development of Muskingum model closer the actual situation. The following appendix shows the advantages and disadvantages of each Muskingum model. It is believed that with the development of science and technology, scholars will constantly improve the Muskingum model according to the reality.

3 Parameter Estimation Methods

From literature, it is known that the estimation of model parameters is the most important work for applying Muskingum models. Thus, many research works carried out over recent decades have focused on estimating the parameters of Muskingum models. Based on surveyed literature, the parameter estimation techniques of the Muskingum models can be divided into three groups as follows:

3.1 Mathematical Techniques

S-LSM method was suggested by Gill (1978) to solve a system of nonlinear equations at each time using a trial-and-error procedure. But, if more than a few time steps are included, the S-LSM method could be very time-consuming (Barati 2011). The LSR technique was adapted to estimate Muskingum flood routing model coefficients for multiple tributaries (Khan 1993). An iterative nonlinear multivariate parameter estimation technique (NONLR) was proposed by Yoon and Padmanabhan (1993), which was an iterative procedure including nonlinear least squares regression and required an initial assumption of parameter values. The tedium of piecewise linearization could be avoided by using NONLR, which directly fitted the nonlinear model to the data using nonlinear least-squares regression (Yoon and Padmanabhan 1993). Das (2007) proposed a chance-constrained optimization-based model,

which determined the parameters of the Muskingum model by minimizing the SSD of difference between the observed and routed outflows. However, the developed model by Das (2007) was very complex and required massive computation for parameter estimation of the Muskingum model (Niazkar and Afzali 2015). Barati (2011) recommended Nelder-Mead simplex algorithm (NMS) to find the values of parameters in nonlinear Muskingum model. Although the algorithm was simple for programming and it converged quickly to optimal values, it demanded an initial guess for the parameter estimation (Barati 2011; Niazkar and Afzali 2015). In order to eliminate the limitation of parameter estimation procedures, Barati (2013) developed parameter estimation for nonlinear Muskingum models using Excel Generalized Reduced Gradient (GRG) Solver, which was a deterministic method and needed initial value assumption for parameter estimation. Gasiorowski and Szymkiewicz (2020) used Powell's algorithm for the identification of parameters influencing the accuracy of the solution of the nonlinear Muskingum Equation. Spiliotis et al. (2021) proposed a fuzzy Muskingum model, which treated the parameters as fuzzy symmetric trigonometric numbers to make the fuzzy parameters closer to nature and increase the security of prediction. These mathematical techniques were easy programmed and were quite efficient for finding an optimal solution very quickly, but they lacked global optimality and achieved global optimal solutions contingent on the specification of suitable initial parameter estimates, which was a nontrivial task (Barati 2011; Bozorg-Haddad et al. 2015).

3.2 Evolutionary Algorithms

Genetic algorithm (GA) is an evolutionary iterative search engine, which was first used by Mohan (1997) to find optimal parameters of nonlinear Muskingum model. GA could reach better results without requiring initial guesses close to optimal parameter values (Niazkar and Afzali 2015). Wang et al. (2009) proposed a hybrid chaotic genetic algorithm (HCGA) based on chaotic sequence and GA for parameter estimation of Muskingum model. Chu and Chang (2009) applied PSO to the parameter estimation for nonlinear Muskingum model. PSO did not require any initial guess of each parameter and thus avoided the subjective estimation usually found in traditional estimation methods and reduced the likelihood of finding a local optimum of parameter values (Chu and Chang 2009). Although results achieved by PSO were better than those of GA, they were not as good as those by other algorithms such as HS and BFGS (Niazkar and Afzali 2015). Luo and Xie (2010) proposed an immune clonal selection algorithm (ICSA), which did not demand any initial estimate of values to solve parameters of nonlinear Muskingum model. However, a sensitivity analysis was required for the determination of algorithm parameters such as clonal scale, mutation probability, and crossover probability (Barati 2011). Xu et al. (2012) investigated DE for parameter estimation problem of nonlinear Muskingum model, and the application results demonstrated that using DE to estimate parameters could get the best objective values compared with LSM, HJ+DFP, NONLR, GA, HS, BFGS and PSO. Orouji et al. (2013) applied simulated annealing algorithm (SAA) and SFLA to determine parameters in the Muskingum model, and the application results of two benchmarks and real cases showed that SFLA could give the best parameter estimation compared to SAA. Niazkar and Afzali (2015) used a modified honey bee mating optimization algorithm (MHBMOA) with a modified routing procedure for the estimation of Muskingum parameters. Although MHBMOA did not require initial guesses to initiate parameters, it necessitated that they be selected wisely (Niazkar and Afzali 2015). Karahan et al. (2015) used CSA for the calibration of parameters of a modified nonlinear Muskingum model. Yuan et al. (2016) developed an improved BSA to estimate parameters of nonlinear Muskingum model, and results indicated the improved BSA could obtain better performance of solution

precision and computational efficiency than PSO, GA and DE. Kang and Zhang (2016) applied elitist-mutated PSO and IGSA to estimate parameters of Muskingum flood routing models. They suggested that the two approaches could be confidently employed to estimate parameters of both linear and nonlinear Muskingum models in engineering applications. Farzin et al. (2018) used IBA to optimize estimated values of three parameters associated with the Muskingum model. Farahani et al. (2019) used Shark Algorithm (SA) to estimate parameters of nonlinear Muskingum with four parameters and results indicated SA outperformed several other evolutionary algorithms. In order to improve the accuracy of outflow prediction, Akbari et al. (2020) proposed a nonlinear Muskingum model with four variable parameters. This model was superior to other nonlinear Muskingum models used by other researchers at that time. Lee (2021) used the meta-heuristic optimization algorithm SAVCA to adjust the parameters of a Muskingum flood evolution model. Norouzi and Bazargan (2021) used particle swarm optimization to optimize the parameters of a linear Muskingum method and data from a single elementary flood, and then applied these parameters to compute outflow hydrographs for four floods, which improved the accuracy of the linear Muskingum method for estimating hydrographs. Bozorg-Haddad et al. (2021) used the Teaching-Based Optimization Algorithm (TLBO) to estimate three parameters of the nonlinear Muskingum model. The combination of TLBO and Muskingum bypassed the need for specific algorithms to optimize parameters, which rendered the Muskingum model flood routing more accurate. Sun et al. (2021) used genetic algorithms to optimize uncertain parameters in Muskingum's method and compared them with the measured flow. Results showed that the absolute error of about 85% of the observed data was less than 15%, and the runoff and groundwater level changes were well simulated. Norouzi and Bazargan (2022) used particle swarm optimization algorithm to optimize linear Muskingum parameters through the time variation of Y , and proposed a computation method for downstream flood depth with high accuracy. These algorithms did not need initial values of the designed parameters and search randomly for a near-global optimal solution. But these algorithms required careful attention for algorithm parameters, which would affect the execution of the algorithm.

3.3 Hybrid Algorithms

In the last ten years, hybrid global–local optimization algorithms have become popular solution approaches for solving engineering optimization problems (Ayvaz et al. 2009; Kayhan et al. 2010). Easa (2013) used GA and GRG to obtain the global optimal solution for an improved nonlinear Muskingum model with a varying exponent parameter. Karahan et al. (2013) proposed a hybrid HS and BFGS algorithm for improving the Muskingum parameters by coupling a heuristic method for global search and a gradient-based method for local search. Ouyang et al. (2014) combined PSO with NMS to optimize parameters of the Muskingum model. The hybrid PSO-NMS did not require initial values for each parameter, which helped to decrease the computation for global optimum search of parameter values. Hamedi et al. (2015) proposed a hybrid optimization (SFLA-GRG) method for optimal parameter estimation of nonlinear Muskingum. Results demonstrated that SFLA-GRG method was more effective than Excel solver for parameter optimization of nonlinear Muskingum models. Haddad et al. (2015) proposed a novel hybrid SFLA-NMS method for optimal parameter estimation of two new nonlinear Muskingum models in three case studies. Results showed that the hybrid SFLA-NMS method could be successfully applied to optimize parameter values of nonlinear Muskingum models. Niazkar and Afzali (2016, 2017) proposed a new hybrid technique, which combined MHBMO and GRG algorithms for Muskingum parameter estimation. Kang et al. (2017) combined an improved real-coded

adaptive GA and NMS algorithm, which was utilized for parameter estimation of two improved nonlinear Muskingum models considering lateral flow. Ehteram et al. (2018) reported a hybrid PSO and BA which could prevent trapping in local optima and increase the convergence speed by substituting a weaker BA solution with the best PSO solution. Bozorg-Haddad et al. (2019) successfully used SFLA-NMS method to estimate optimal parameter values with different generalized nonlinear Muskingum models. Okkan and Kirdemir (2020) mixed PSO with the Levenberg–Marquardt (LM) algorithm to estimate the parameters of a nonlinear Muskingum model with three parameters. Compared with other algorithms, the optimal results were obtained faster. Akbari and Hessami-Kermani (2021) used PSO-GA algorithm to optimize Muskingum parameters with faster convergence speed and higher accuracy. Yuan et al. (2021) proposed a modified PRP algorithm based on line search technique and modified PRP formula, which had better performance in estimating the parameters of nonlinear Muskingum model. These hybrid methods overcame the shortcomings of both mathematical optimization techniques and evolutionary algorithms, and thus appeared to be the most efficient in solving parameter estimation with nonlinear Muskingum models (Bozorg-Haddad et al. 2019; Niazkar and Afzali 2016).

3.4 Method Summary

In the above mathematical methods for optimizing the Muskingum model, from the initial S-LSM method to the recent fuzzy parameter method, it has been constantly closer to the uncertainty of the problem. Among them, the fuzzy estimation method is relatively good, which not only makes the fuzzy parameters closer to nature but also improves the safety of prediction. Moreover, the generated blur band can largely include the output of observation data. Global optimization is the key advantage of such methods, but the premise is that the initial guess of the optimal solution should be close to the global optimum.

With the continuous development of computer technology, researchers continue to explore new optimization algorithms and pertinent modified algorithms to improve the accuracy of the Muskingum model in the process of river flood routing. Among the above evolutionary algorithms, PSO and GA algorithm are the fastest, most accepted and most widely used. However, these algorithms need to pay close attention to the parameters of the algorithm, because they are plagued by the slow convergence of random search computation intensive to the global optimum, which will affect the execution of the algorithm.

Using hybrid algorithms to estimate parameters can continuously combine the advantages and disadvantages of different algorithms to complement each other. Recently, Akbari and Hessami-Kermani (2021) used PSO-GA algorithm to estimate for continuous improvement. At present, the combination of PSO-GA is the best. They have fast convergence speed, high precision and are superior to other hybrid algorithms in the Muskingum model. Among these parameter estimation methods, the third type (hybrid algorithm) seems to be the most effective solution to parameter estimation of the nonlinear Muskingum model.

4 Performance Evaluation Criteria

Parameters of different versions of Muskingum models can be estimated by various techniques. To evaluate the performance of models and optimization techniques, six precision evaluation criteria are generally employed, namely sum of squared deviations

(SSD), sum of absolute value of deviations (SAD), absolute value deviations between peaks of observed and routed flows (DPO), absolute value of deviations of peak times of observed and routed flows (DPOT), mean absolute relative error (MARE), and Nash–Sutcliffe criterion (NSC) (Barati 2013; Niazkar and Afzali 2015).

4.1 Outflow Criteria

The accuracy of the outflow can be determined by SSD or SAD (Barati 2011; Mohan 1997; Xu et al. 2012):

$$SSD = \sum_{t=1}^N (O_o^t - O_r^t)^2 \quad (33)$$

$$SAD = \sum_{t=1}^N |O_o^t - O_r^t| \quad (34)$$

where O_o^t and O_r^t are observed and routed outflows respectively at time t , and N is the total number of time intervals.

4.2 Peak Magnitude Criterion

DPO is a measure of the accuracy of the peak magnitude of the outflow (Yoon and Padmanabhan 1993):

$$DPO = |O_o^p - O_r^p| \quad (35)$$

where O_o^p and O_r^p are peak magnitudes of observed and routed outflows, respectively.

4.3 Peak Time Criterion

DPOT is a measure of the accuracy of the peak time of the outflow (Yoon and Padmanabhan 1993):

$$DPOT = |T_o^p - T_r^p| \quad (36)$$

where T_o^p and T_r^p are peak times of observed and routed outflows, respectively.

4.4 Mean Absolute Relative Criterion

MARE is a measure of error between observed and routed outflows (Toprak 2009):

$$MARE = \frac{1}{N} \sum_{t=1}^N \frac{|O_o^t - O_r^t|}{O_o^t} \quad (37)$$

4.5 Nash–Sutcliffe Criterion

NSC is recommended by the ASCE Task Committee on Definition of Criteria for Evaluation of Watershed models (McCuen et al. 2006; Perumal and Sahoo 2007):

$$NSC = \left[1 - \frac{\sum_{t=1}^N (O_o^t - O_r^t)^2}{\sum_{t=1}^N (O_o^t - O_r^t)^2} \right] \times 100 \quad (38)$$

5 Studied Case Data Set

Based on the literature review, several case studies were often considered to test the performance of all the above different versions of Muskingum models and various parameter estimation techniques. Common cases of nonlinear river flood routing model were considered to reflect the linear and nonlinear characteristics of flood routing in different regions. The Wilson dataset was a smooth unimodal process whilst the flood of Wye River in December 1960 was a non-smooth unimodal process. The Viessman and Lewis datasets and the October 1982 flood of the Wye River were multi-peak hydrological processes, and the 1961 flood of the South Canal was a hydrological process that included heavy rainfall. These were representative test sets.

5.1 Case 1: Data set by Wilson

Wilson (1974) reported the inflow and outflow hydrographs, which were smooth single-peak hydrographs, as shown in Fig. 1. The data set has been demonstrated to exhibit an obvious nonlinear relationship between the storage and weighted-flow (Mohan 1997; Yoon and Padmanabhan 1993). These data were widely used for verifying different structures of the storage equation of Muskingum models and the effectiveness of various parameters estimation techniques during the optimization stage.

5.2 Case 2: River Wyre December 1960 Flood

The second case data set were about a 1960 flood in the Wye River in the United Kingdom, which stretched 69.75 km from Erwood to Belmont and had no tributaries and very small lateral inflow (Natural Environment Research Council 1975). The inflow and outflow hydrographs were non-smooth single-peak hydrographs, as shown in Fig. 2. This case data set was first studied by O'Donnell et al. (1988) with a linear Muskingum model. This flood case was considered a good test case to test flood routing methods (Bajracharya and Barry 1997).

5.3 Case 3: Data set by Viessman and Lewis

The third case data set were introduced by Viessman and Lewis (2003). The inflow and outflow hydrographs were multiple -peak hydrographs, as shown in Fig. 3.

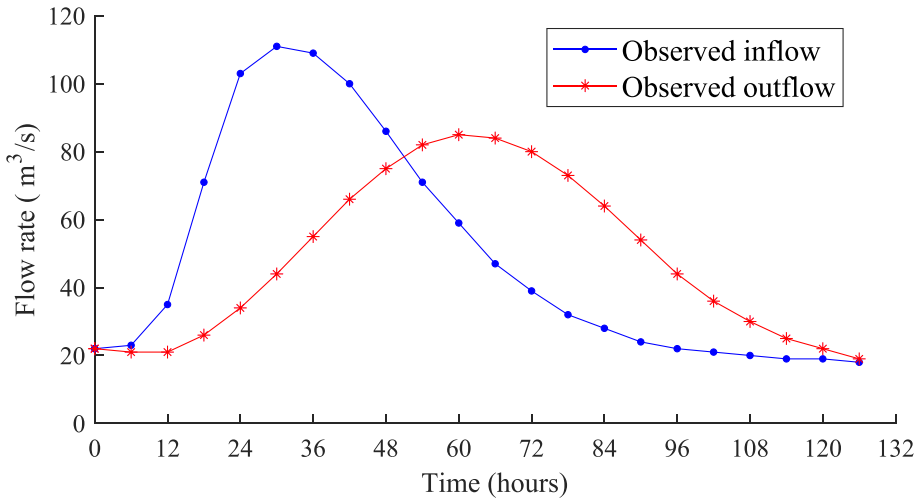


Fig. 1 Inflow and outflow hydrographs by Wilson

5.4 Case 4: River Wyre October 1982 Flood

The fourth flood event was a 1982 flood in the Wye River in the United Kingdom, which was first employed by O'Donnell (1985) for a direct three parameter Muskingum procedure incorporating lateral inflow, as shown in Fig. 4. This flood event not only exhibited a considerable increase in the flood volume between the inflow and outflow sections (lateral flow about 25 km apart), but also had a multi-peaked inflow (O'Donnell 1985; Zhang et al. 2017).

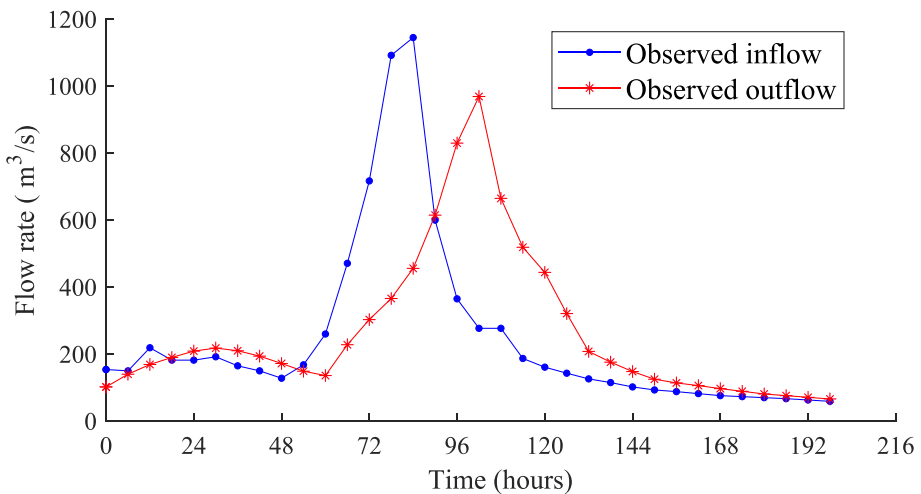


Fig. 2 Inflow and outflow hydrographs of River Wyre December 1960 flood

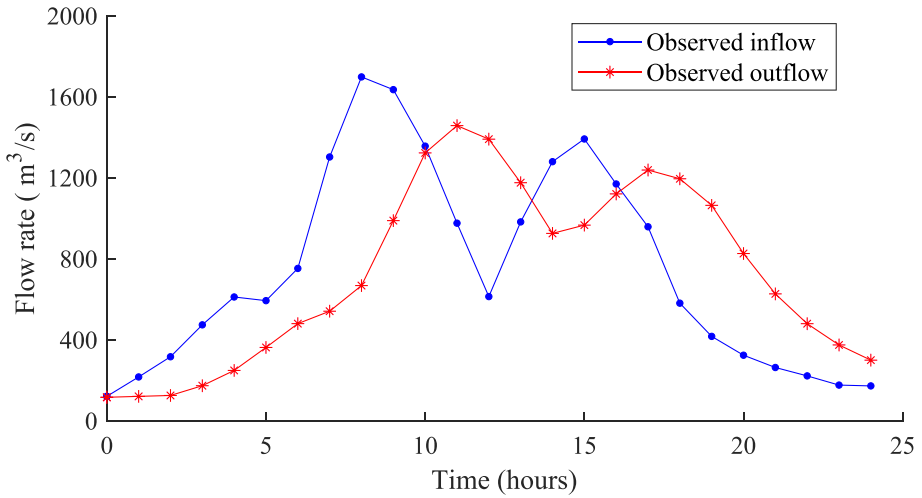


Fig. 3 Inflow and outflow hydrographs by Viessman and Lewis

5.5 Case 5: Nanyun River 1961 Flood

The fifth flood event was about a 1961 flood in the Nanyun River in China, which stretched 83.8 km from Chenggouwan to Linqing and had no tributaries in the middle. When there was heavy rainfall, there were waterlogging and water discharge, but had little impact on the flood, as shown in Fig. 5. The data set were employed with a linear Muskingum model by Wang et al. (2009).

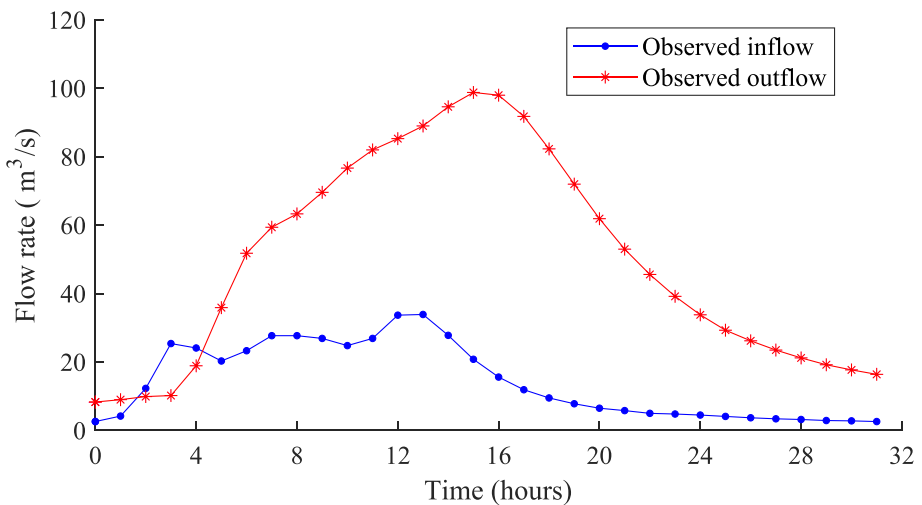


Fig. 4 Inflow and outflow hydrographs of River Wyre October 1982 flood

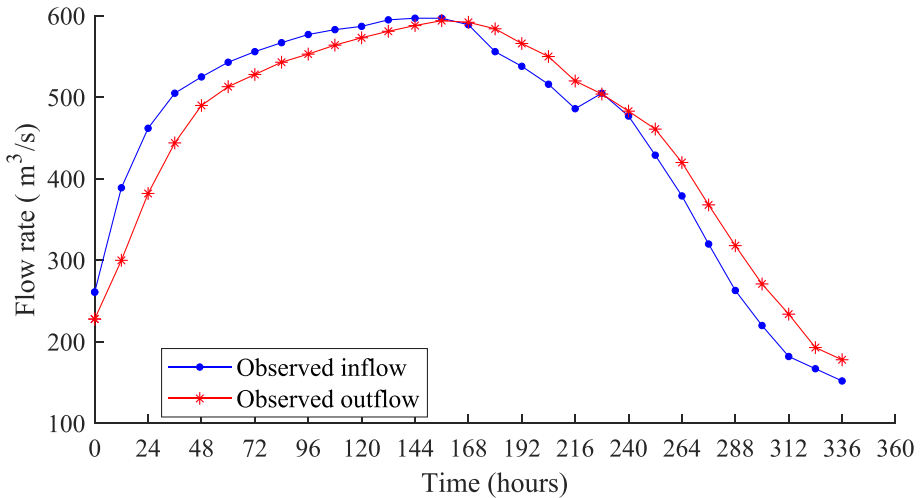


Fig. 5 Inflow and outflow hydrographs of Nanyun River 1961 flood

6 Conclusions and Future Directions

Channel flood evolution is an important aspect of water resources management. Muskingum model is one of the most widely used channel flood evolution methods. This paper reviews the research status and progress of six types of model structures, three groups of optimization techniques, six commonly used performance evaluation criteria and five reference case data sets. According to the study on different model structures, the more were the parameters, the better was the performance of flood routing model in fitting historical records. However, the optimization process becomes more complex, and the model prediction ability deteriorates. In addition, the combination of mathematical techniques and evolutionary algorithms shows high efficiency in parameter estimation. Based on this, we present our views on future directions, research questions, and hotspots related to the Muskingum model on river flood routing.

1. Some special and general Muskingum models have been developed for river flood routing. The models have different numbers of parameters, and a research question is which model can simulate routing flood more accurately for applications to different hydrograph types in practice. Field and laboratory experiments should be made in future to better understand characteristics of Muskingum models for routing flood events.
2. It was acknowledged that an increasing number of parameters in model calibration also increased the data dependency of the model and worsened the prediction ability of the model (Karahan 2014). Although some researches have done a lot of effective work for the optimal parameter estimation of Muskingum Models, calibration of their parameters was still considered to be challenging. The recent advancement in optimization technologies has provided a feasible way for practical optimization problems (Wang et al. 2020). Thus, the application of more efficient algorithms that can determine optimal solution is recommended for future studies.
3. The variations of environmental conditions can change characteristics of catchments such as catchment shape, river morphology, and vegetation, which have important effects on the channel storage. The values of parameters of Muskingum model for a given river should

be updated after any significant variations in environmental conditions. Hence, sensitivity analysis should be undertaken to investigate the influence of correlated parameters in Muskingum models on model outputs in order to adapt to impacts of environmental changes.

Appendix

Table 1

Table 1 Comparison of different models

Number	Author and year of publication	Model type	Brief description (advantages/limitations and the consequent application)
1	McCarthy (1938)	LMM	In the earliest linear Muskingum model, the nonlinear relationship between weighted traffic and storage is not significant
2	Chow (1959)	NLM1	It has more degrees of freedom than LMM, so it usually gets closer to the observed outflow
3	Gill (1978)	NLM2	It has more degrees of freedom than LMM, and further improves the fitting of the model based on NLM1. It is the most studied model based on different parameter estimation methods
4	Gavilan and Houck (1985)	NLM3	It is significantly more complex than NLM1 and NLM2. Although it increases the complexity, it greatly improves the accuracy and provides greater freedom in fitting the observation data
5	Easa (2014b)	NLM4	It is significantly more complex than NLM1 and NLM2. Although it increases the complexity, it provides greater freedom when fitting the observed data, and also greatly improves the fitting with the observed
6	Easa (2014b)	NLM5	It is the general form of the relationship between the river and the lake in the Muskingum model. In the three case studies of reference (Easa 2014b), it has achieved better results than NLM4
7	Vatankhah (2014b)	NLM6	It is a five-parameter Muskingum model, which can provide more degrees of freedom than NLM1 and NLM2 by setting some exponential parameters. Although it usually produces results closer to the observed outflow or storage rate, the model is more complex
8	Easa et al. (2014)	NLM7	It is an extension of the NLM5 model, which takes the square sum of the deviation between the observed outflow and the routing outflow (SSQ) as the objective function, so that the SSQ is further reduced, and it retains the physical meaning of the original Muskingum model parameters
9	Bozorg-Haddad et al. (2015)	NLM8	It has a more complex calibration process than other nonlinear Muskingum models, but the additional complexity may lead to substantial improvements in data fitting
10	Niazkar and Afzali (2017)	NLM9	The SSQ obtained by using the hybrid algorithm combined with the nine-parameter model is 4.52% and 99.7% lower than the previous best results, and the effect is better
11	Bozorg-Haddad et al. (2019)	NLM10	The calculation of the NLM10 model requires the values of four parameters. Compared with NLM1, the model has higher degrees of freedom in fitting the measured hydraulic data, and its ability to predict river flow is also better than other nonlinear Muskingum models

Table 1 (continued)

Number	Author and year of publication	Model type	Brief description (advantages/limitations and the consequent application)
12	Bozorg-Haddad et al. (2019)	NLM11	The calculation of the NLM11 model requires the values of four parameters. Compared with NLM2, the model has higher degrees of freedom in fitting the measured hydraulic data, and its prediction ability for river flow is also better than other nonlinear Muskingum models
13	Bozorg-Haddad et al. (2019)	NLM12	The calculation of the NLM12 model requires the values of five parameters. Compared with NLM4, the model has higher degrees of freedom in fitting the measured hydraulic data, and its ability to predict river flow is also better than other nonlinear Muskingum models
14	Bozorg-Haddad et al. (2019)	NLM13	The calculation of the NLM13 model requires the values of eight parameters. Compared with NLM8, the model has higher degrees of freedom in fitting the measured hydraulic data, and its ability to predict river flow is better than other nonlinear Muskingum models
15	Easa (2013)	NLM-VEP1	Compared with NLM2, the range of optimal exponential parameters is smaller, and the fitting of nonlinear Muskingum model is significantly improved
16	Vatankhah (2014a)	NLM-VEP2	This model is a nonlinear Muskingum model with more degrees of freedom than NLM-VEP1. Compared with the seven-parameter nonlinear Muskingum model, the accuracy is improved and the complexity is reduced
17	Easa (2015)	NLM-VEP3	Compared with the traditional three-parameter model, this model greatly improves the prediction performance of the model for flood flow
18	Easa (2014a)	NLM-VEP4	The NLM-VEP4 model is obviously more complex than the existing nonlinear models given by models NLM1 and NLM2. However, additional parameters in the new model were found to greatly improve the fitting of observed outflows
19	O'Donnell (1985)	LMM-LF	Compared with LMM, the basic two-parameter Muskingum model is extended to a three-parameter model, and a simple lateral inflow assumption is adopted, which is an advanced idea in the current environment
20	Karahan et al. (2015)	NLM-LF1	Compared with NLM2, NLM-LF1 can successfully simulate flood data with smaller lateral flow
21	Kang et al. (2017)	NLM-LF2	This nonlinear Muskingum model considering lateral flow is more advantageous than NLM4
22	Zhang et al. (2017)	NLM-VEP-LF1	The model includes a variable exponent parameter describing the nonlinearity of flood waves and a flow parameter representing the lateral flow of the studied river section. Compared with LMM and NLM2, it can generate more accurate flood routing process lines
23	Kang et al. (2017)	NLM-VEP-LF2	This is a nonlinear Muskingum model considering lateral flow, which can fit the observed outflow more accurately than NLM-LF2
24	Lu et al. (2021)	NLM-LF1-GS	Based on the improvement of NLM-LF1, the physical evolution of river flood is more reasonable
25	Lu et al. (2021)	NLM2-GS	The improved model can prevent groundwater inundation in extreme flood events and has practical significance for groundwater management and flood control

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Declarations

Ethics Approval Not applicable.

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