Sustainable Management and Environmental Protection for Basin Water Allocation: Differential Game-based Multiobjective Programming

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Abstract

It is important to manage water resources via emergy theory and its implementation mechanism for water dispatching while considering both economic development and environmental needs. The contribution of this paper is to propose a multi-objective dynamic differential game that can determine the optimal tax rate (OTR), optimal trading quantity of water (OTQW) in each province, and optimal bargain price (OBP) to balance resource consumption, economic development and environmental protection. Considering the sustainability of the ecological environment, the quantification of negative sewage value and net carbon emission constraints are introduced into the water allocation. To maximize the target revenue functions, minimize the net carbon emission constraints and obtain sustainable equilibrium solutions in this triple-level game, the costate function, Hamiltonian, and Lagrangian multiplier method are introduced. Then, a water dispatching structure is constructed for error correction between the predicted and actual runoff of the YRB. Taking the Yellow River Basin as an example, the validity of the proposed framework and solution method were verified under different hydrological years, with 6.5%~9.1% higher economic benefits than other schemes and 4.3%~5.9% lower net carbon emissions than other schemes. Compared to previous studies, this scheme can better meet the requirements of sustainable development and environmental protection.

Keywords Sustainability · Coordinated development · Water allocation · Multi-objective programming

1 Introduction

1.1 Problem Statement

Due to climate warming, environmental pollution and population growth, global water shortages have seriously affected the sustainable development of human society (Liu

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et al. 2020; He et al. 2018). The contradiction between rising water demands and limited water allocations is becoming increasingly prominent (Huang et al. 2021; Janjua and Hassan 2020). Such phenomena not only are attributable to the limitations and uncertainty of water resources in some countries or regions but also involve intricate interactions among the state taxation departments, water management authorities, and regulators of water markets (Ebrahimi et al. 2021; Hillman et al. 2012). In the economic, social, and ecological fields, scientifically formulating sustainable water allocations (SWAs) for insufficient water resources in a manner that meets water demands is one of the most effective approaches for addressing water shortages (Bajany et al. 2021; Zheng et al. 2022). Considering the sustainability of the ecological environment, the basin SWA is based on a desire to minimize net carbon emissions. The water resource system of the river basin has a unique operating mechanism. On the one hand, the state taxation department not only tends to maximize the tax revenue resulting from the SWA in the river basin but also hopes to scientifically and reasonably control the scale of sewage discharge in the basin. On the other hand, the administrative management authorities of the basin and provinces tend to maximize their respective profits for SWAs. Moreover, reasonable water dispatch is key to realizing the SWA in the river basin. Therefore, formulating an efficient SWA and water dispatch for the river basin can balance resource consumption, economic development and environmental protection and coordinate the relationship between the state taxation department, the administrative management authority, and provinces/regions (Qin et al. 2012; Kazemi et al. 2020).

1.2 Literature Review

1.2.1 Research Status of Water Allocation Schemes

To promote equitable, efficient and sustainable water use, it is necessary to formulate water resource allocation schemes (Xu et al. 2019). The development of numerical methods (Zhao et al. 2019) has stimulated many successive studies based on multivariate constraints and linear optimization methods. For example, Shen et al. (2021) constructed a three-layer model to address the synergistic configuration of complex systems of regional water resources. Liu et al. (2014) proposed multi-models with uncertain parameters to improve water allocation. Suo et al. (2013) introduced a mathematical theory with fuzzy sets and random events to program and calculate the optimal solutions for water allocation in an urban water ecosystem. Boelens et al. (2014) developed a matrix shortest-path algorithm with an elite strategy for optimal water rights transactions. Since the coordinating role of administrative authorities in water resource management is recognized, analytical models related to maximizing the comprehensive value of water resources (CVWR) have been widely adopted. For example, Degefu et al. (2017) introduced a two-level game model based on administrative management and formulated a multistakeholder cooperation structure. Nicklow et al. (2010) proposed a bilevel interactive method based satisfaction evaluation in water rights transactions. The above studies have improved the efficiency of water utilization and achieved relatively optimized water allocation schemes (WASs). To date, few studies related to SWA have focused on target revenue optimization and coordination among the objectives of the state taxation department, administrative management authority, and provinces/regions (Shen 2018).

1.2.2 Research Status of Water Dispatching

Moving from sustainable water allocation to real-time water dispatching reflects the complexity of coupling between water resource systems and socioeconomic systems (Cosgrove and Loucks 2015). Since the middle and late twentieth century, domestic and foreign scholars have explored real-time water dispatching based on initial water allocations. For example, Yeh et al. (1992) applied Bayesian theory and a mixed reasoning structure to develop runoff forecasting tools with hidden user experience and formulated a real-time computer-based decision support system for water dispatching. Green and Hamilton (2000) developed a real-time optimal dispatching model combining linear programming and dynamic programming and applied it to two reservoir systems. However, these studies were based on the initial water allocation system and did not consider the error correction of water dispatching according to the predicted runoff, actual runoff, and real-time results of the sustainable water allocation system (Araral and Wu 2016).

1.3 Research Gap

Many studies related to SWA and water dispatching have been conducted using different numerical analyses and application scenarios (Tayfur 2017). Even though the methods presented above are efficient and effective for handling single-objective water allocation systems, they remain invalid for handling the coupling relationship between the tax rate and target revenue for the entire basin and provinces/regions in the river basin, which requires equilibrium solutions for multiobjective optimal allocation (Bagatin et al. 2014). In addition, previous water allocation and dispatching schemes are not based on solving the sustainability among compromised resource consumption, economic development and environmental protection of the basin, and a real-time adjustment mechanism of error correction between the predicted runoff and actual runoff is not taken into account (Medeiros et al. 2017).

1.4 Motivations and Objectives

The Yellow River Basin (YRB), which is the second largest economic development district in China, is faced with severe water shortages. The SWA in the YRB has the ability to balance resource consumption, economic development and environmental protection and calculate the optimal tax rate (OTR), optimal trading quantity of water (OTQW) in each province, and the optimal bargain price (OBP). According to the SWA rules (Wang et al. 2019), the administrative management authorities and each province should follow the state taxation department's management strategies in accordance with the tax rate settings for water rights transactions (WRT).

In conclusion, the motivations and objectives of SWA and water dispatching are presented below:

1. The basin's primary motivation is to minimize net carbon emissions.
2. The objective of the state taxation department is to maximize the sum of the tax revenue and the negative value of the eco-environment (NVE).
3. The objective of the administrative management authority in the YRB is to maximize the TRF, which is composed of the CVWR in the basin and the tax revenue.
4. The objective of each province in the YRB is to maximize the sum of the CVWR in each province and the tax revenue.

5. The objective of water dispatching in the YRB is to perform an error correction between the predicted runoff and actual runoff.

1.5 Contributions

In comparison with previous achievements, the contributions in this research are listed below:

1. A multiobjective SWA game model (MSWA) in the YRB is formulated based on optimal differential game theory. Combined with the initial conditions, the Hamiltonian function and Lagrangian multiplier, the OTR, OTQW in each province, and OBP are calculated with minimization constraints for the net carbon emissions. The equilibrium solutions are analysed and formulated according to the objectives of the state taxation department, administrative management authority, and each province/region.
2. According to the proposed MSWA in this study, an implementation mechanism of water dispatching in the YRB is presented based on the error correction between the predicted runoff and actual runoff.

2 Case Study

2.1 Study Area

With China's increasing emphasis on protecting water ecosystems and water cycle safety, the central government has implemented strict policies regulating the amount of groundwater that can be extracted from each province in the YRB (Fig. 1). The extreme imbalance
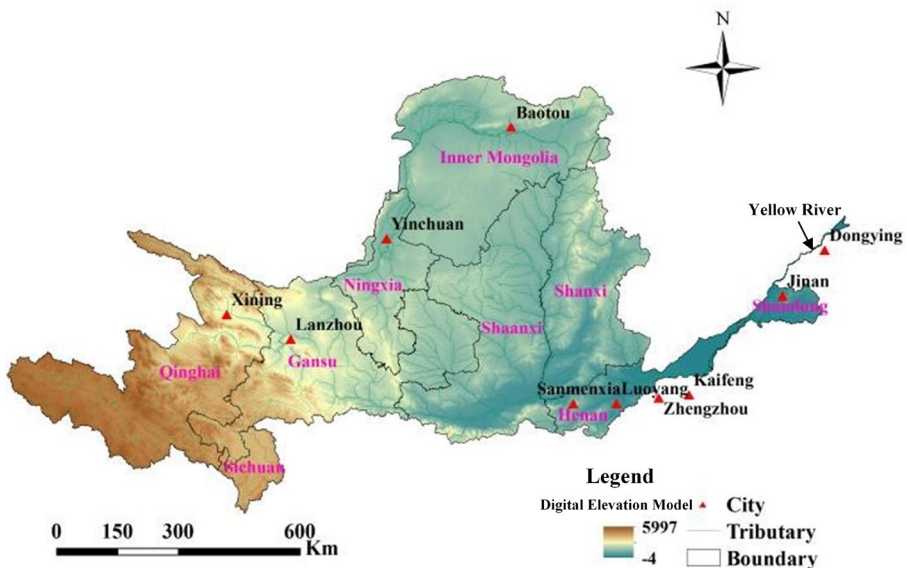

Fig. 1 Distribution of provinces and major cities in the YRB

in regional economic development has caused severe water shortages in some provinces of the YRB. Purchasing water rights from economically underdeveloped provinces in the YRB and transferring them to economically developed provinces has become an important approach in alleviating water shortage pressures.

2.2 Data Sources

Water consumption datasets and YRB provincial economic and social development indicators from 2012 to 2021 were adopted as inputs for the SWA model and the water dispatching implementation mechanism.

3 Methodology

3.1 Game Modelling for Sustainable Water Allocation

3.1.1 Model Structure

The model structure of the SWA in the YRB based on the constraints of net carbon emission thresholds, tax rates, administrative management authority of the basin (AMAB), and markets is presented below (Fig. 2).

As shown in Fig. 2, to balance resource consumption, economic development and environmental protection, the constraint of net carbon emission minimization is adopted as the overall constraint and input of the sustainable model. The state taxation department takes the tax revenue from water rights transactions and the negative value of sewage (that is, NVE) as the main elements for constructing the TRF in the first-level game. To balance the tax revenue and control pollution emissions, the OTR is solved to maximize the value of the TRF. In the second-level game process, the AMAB takes into account the CVWR and the payment of the transaction tax and formulates the TRF to calculate the OTQW in each province during the game period. The TRF of each province in the third-level game consists of the CVWR and the amount of tax to be deducted. Then, the value of the TRF is maximized to calculate the OBP of the basin. Based on dynamic optimal control theory and the relational equations constructed by the multiobjective game model, the OTR, OTQW in each province, and OBP can be solved.

The symbols and definitions in the game model are shown in Table 1.

According to the framework of the multiobjective "tax rate-trading quantity of water-bargain price" game model, the formalized definition of the game can be presented as follows.

Definition 1 The "tax rate-trading quantity of water-bargain price" triple-level game G_D has four elements $J(N, \Theta, B_s, U)$, where:

1. $N = \{$province 1 (Qinghai), province 2 (Sichuan), ……, province n (Shandong)$\}$.
2. $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$ and $n = 9$. $\Theta_i = \{\Theta_i = 0, \Theta_i = 1\}$, $\Theta_i = 0/1$ denotes that province i is a seller/purchaser.
3. $B_{SET} = s(t) = \{\left(b_1(t), R_{tax}(t)\right), \left(b_2(t), R_{tax}(t)\right), \cdots, \left(b_n(t), R_{tax}(t)\right)\}$. If $q_i(t) - e_i(t) > 0$, $b_i(t) = q_i(t) - e_i(t)$ represents the quantity of water purchased by province i from the water rights exchange at time t. If $q_i(t) - e_i(t) \leq 0$, $b_i(t) = e_i(t) - q_i(t)$ denotes the quantity of water sold by province i to the water rights exchange at time t.
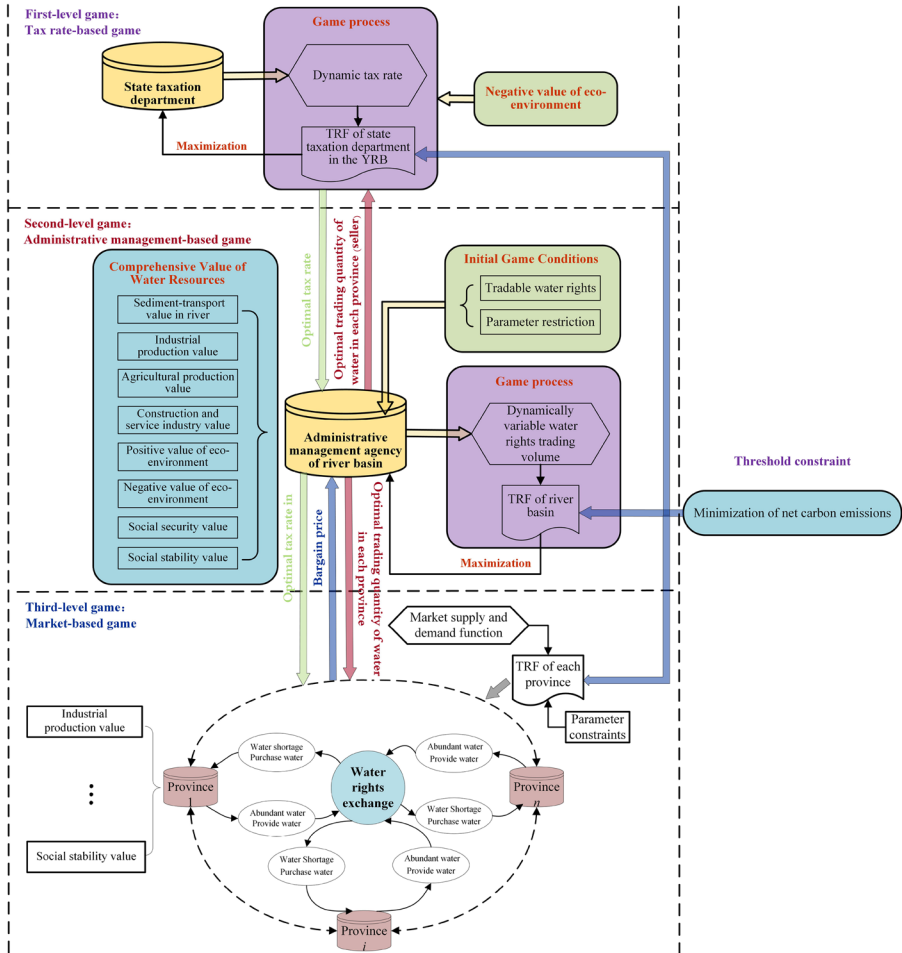
Fig. 2 Model structure of sustainable water allocation in the YRB

4. $U = (J_t(s(t)), J(s(t)), J_i(s(t)))$.

3.1.2 Tax Rate-based Game

The CVWR of each province is directly related to the quantity of water that it holds. Let $R_i(q_i(t))$ be the instantaneous value of water $q_i(t)$ held by province i. The unit water resource value of province i in terms of water for industrial production, agricultural production, construction and the service industry, social consumption, and the ecological environment outside the river course are IW_i, AW_i, CW_i, SW_i, and EW_i, respectively. The water distribution ratios of the above water departments are η_{IW_i}, η_{AW_i}, η_{CW_i}, η_{SW_i}, and η_{EW_i}. WI_i represents the unit sewage treatment consumption value. It is assumed that k_{IW_i}, k_{AW_i}, k_{CW_i}, and k_{SW_i} denote the proportions of sewage generated by industrial production, agricultural production, construction and the service industry, and social consumption. Then, we have

Table 1 Definitions of the symbols

Symbol	Definition	Symbol	Definition
N	The set of players in the game	Δt	Infinitesimal time interval
$B_{SET} = s(t)$	The set of strategies at time t	Θ	Type space of province i
U	The set of the TRF in the multi-objective game	$J_t(s(t))$	TRF of the state taxation department
n	The number of provinces participating in water rights transactions in the YRB	$J(s(t))$	TRF of the YRB
$e_i(t)$	The initial water rights of province i at time t	$R_{tax}(t)$	Tax rate of the YRB at time t
$q_i(t)$	The instantaneous water rights held by province i at time t	$\rho(t)$	Bargain price at time t
T	The game period	$J_i(s(t))$	TRF of the province i
$S(t)$	The accumulated amount of sewage discharged into the environment at time t	$M_i(t)$	Water requirement threshold of all water departments at time t

$\eta_{WI_i} = k_{IW_i} \cdot \eta_{IW_i} + k_{AW_i} \cdot \eta_{AW_i} + k_{CW_i} \cdot \eta_{CW_i} + k_{SW_i} \cdot \eta_{SW_i}$. TE denotes the ecological environmental value of the whole river course.

Each province generates a fixed proportion of untreated sewage $S(t)$ during production. According to the analysis method of dynamic differentiation, an instantaneous increase in $S(t)$ can be calculated as follows.

$$\begin{cases} \frac{dS(t)}{dt} = \sum_{j=1}^{n} k_j q_j(t) - \delta S(t) \\ S(0) = 0 \end{cases} \tag{1}$$

where k_j represents the proportion of untreated sewage at time t to $q_j(t)$ of province j, and δ is the natural decomposition rate of untreated sewage.

Untreated sewage causes continuous damage to the environment. This portion of the loss can be linearly quantified with the quantified correlation coefficient of ξ_i. The state taxation department balances the increases in taxation with the control of sewage discharge by adjusting the tax rate paid by the sellers (provinces) in water rights transactions. On this basis, the TRF $J_t(s(t))$ for constructing a game based on tax rate is as follows.

$$J_t(s(t)) = \int_0^T e^{rt} \left[\left(R_{tax}(t) \cdot \rho(t) \sum_{i=1}^{N_s} (e_i(t) - q_i(t)) \right) - \sum_{i=1}^{n} (\eta_{WI_i} \cdot WI_i \cdot q_i(t) + \xi_i \cdot S(t)) \right] dt - \sum_{i=1}^{n} e^{-rT} d_i S(T)$$

$$s.t. \begin{cases} \frac{dS(t)}{dt} = \sum_{j=1}^{n} k_j q_j(t) - \delta S(t), \\ S(0) = 0, \\ \sum_{j=1}^{n} q_j(t) = \sum_{j=1}^{n} e_j(t), \\ \int_0^T q_i(t) dt \geq \int_0^T M_i(t) dt, \ 1 \leq i \leq n. \end{cases} \tag{2}$$

where r is the discount rate, d_i denotes the value of loss caused by the units of water pollution at the end of game period T, and $e^{-rT} d_i S(T)$ represents the terminal cost of untreated sewage.

In the tax rate-based game, the equilibrium strategy $R_{tax}^*(t)$ that maximizes the TRF $J_t(s(t))$ of the first-level game satisfies the following constraints:

$$\max_{R_{tax}^*(t)} \left\{ \int_0^T e^{rt} \left[\left(R_{tax}(t) \cdot \rho(t) \sum_{i=1}^{N_s} (e_i(t) - q_i(t)) \right) - \sum_{i=1}^{n} (\eta_{WI_i} \cdot WI_i \cdot q_i(t) + \xi_i \cdot S(t)) \right] dt - \sum_{i=1}^{n} e^{-rT} d_i S(T) \right\}$$

$$\tag{3}$$

3.1.3 Administrative Management-based Game

$R_i(q_i(t))$ is a convex function; that is, $R_i''(q_i(t)) < 0$. Thus, $R_i(q_i(t))$ can be expressed as $R_i(q_i(t)) = \varsigma_i \cdot q_i(t) - \psi_i q_i^2(t)$, where ς_i and ψ_i are the coefficients of convex functions, and $\varsigma_i = c_i (\eta_{IW_i} \cdot IW_i + \eta_{AW_i} \cdot AW_i + \eta_{CW_i} \cdot CW_i + \eta_{SW_i} \cdot SW_i + \eta_{EW_i} \cdot EW_i - \eta_{WI_i} \cdot WI_i)$. Under the condition that water rights trading is allowed among provinces in the YRB, the instantaneous TRF of province i can be calculated by $\pi_i(t) = R_i(q_i(t)) - \rho(t)[q_i(t) - e_i(t)]$.

It can be assumed that the imposition of a water rights transaction tax does not affect the original water rights sellers and purchasers. Therefore, an administrative management-based game can be constructed to determine the N_s sellers and N_p purchasers without considering the impact of the water rights transaction tax. Then, by combining the N_s sellers,

N_p purchasers, water rights transaction tax rate $R_{tax}(t)$, and other related parameters, the TRF of the basin is constructed as follows.

$$J(s(t)) = \sum_{i=1}^{n} \left(\int_0^T e^{-rt} \left[\pi_i(t) - \xi_i S(t) \right] dt - e^{-rT} d_i S(T) \right)$$

$$- \int_0^T e^{-rt} \left(R_{tax}(t) \cdot \rho(t) \sum_{i=1}^{N_s} \left(e_i(t) - q_i(t) \right) \right) dt + TE$$

$$s.t. \begin{cases} \frac{dS(t)}{dt} = \sum_{j=1}^{n} k_j q_j(t) - \delta S(t), \\ S(0) = 0, \\ \sum_{j=1}^{n} q_j(t) = \sum_{j=1}^{n} e_j(t). \end{cases} \tag{4}$$

In the administrative management-based game, the equilibrium strategy $s^*(t) = \{b_1^*(t), b_2^*(t), \cdots, b_n^*(t)\}$ that maximizes the basin's TRF $J(s(t))$ should satisfy the constraints below:

$$\max_{s^*(t)} \left\{ \begin{array}{l} \sum_{i=1}^{n} \left(\int_0^T e^{-rt} \left[\pi_i(t) - \xi_i S(t) \right] dt - e^{-rT} d_i S(T) \right) \\ - \int_0^T e^{-rt} \left(R_{tax}(t) \cdot \rho(t) \sum_{i=1}^{N_s} \left(e_i(t) - q_i(t) \right) \right) dt + TE \end{array} \right\} \tag{5}$$

As shown in Eq. (5), the AMAB is responsible for adjusting the game strategy set $s(t)$ to maximize the value of the TRF for the entire basin. The second-level game (administrative management-based game) is a dynamic nonzero-sum game with complete and perfect information—that is, the players of the game can estimate the revenue of each province through known strategies, $s(t)$.

3.1.4 Market-based Game

WRTs can be considered commodity transactions under restricted conditions and are similar to other commodity transactions in their price fluctuations; that is, prices are affected by market supply and demand. Provinces with high water-saving costs can alleviate the shortages of water resources by purchasing a certain quantity of water from provinces with low water-saving costs.

Let $x_i(t)$ be the quantity of water available to province i at time t. $u_i(t)$ represents the quantity of presale water provided by seller i to the water rights exchange at time t. The quantity of prepurchased water provided by purchaser i to the water rights exchange at time t is denoted by $v_i(t)$. $r_i(t)$ represents the quantity of water obtained by province i through water-saving measures at time t. According to the historical water-saving data, the total water-saving cost function $f_{ws}^i(t)$ of province i at time t can be expressed as

$$f_{ws}^i(t) = \alpha_i r_i^2(t) + \beta_i r_i(t) + \chi_i \tag{6}$$

where α_i, β_i, and χ_i represent the water-saving coefficients of province i.

The instantaneous value of the variation trend of $x_i(t)$ can be expressed as

$$\frac{dx_i(t)}{dt} = -u_i(t) + r_i(t) \tag{7}$$

The purchaser dynamically adjusts their purchasing arrangement in accordance with the OTQW in the first-level game. To maximize their own interests, each seller chooses to dynamically change the quantity of water sold to the water rights exchange. The initiation of WRTs in the YRB is controlled by sellers. Considering the pricing law of the relationship between water supply and demand, the bargain price is a convex function of the sellers and a concave function of the purchasers. Therefore, the function of the bargain price can be expressed as

$$\rho(t) = p_0 - \omega_s(u_1(t) + u_2(t) + \ldots + u_{N_s}(t))^3 + \tau_d(v_1(t) + v_2(t) + \ldots + v_{N_p}(t))^3 \tag{8}$$

where ω_s and τ_d are the correlation coefficients of the relationship between the water supply and demand, and $\tau_d \ll \omega_s$; that is, the bargain price is mainly affected by the seller's market, and p_0 is the benchmark price of the transaction.

In the market-based game, the TRF of each seller in the YRB is as follows:

$$J_i(s(t)) = \int_0^T e^{-rt}[(1 - R_{tax}(t))\rho(t)u_i(t) - f_{ws}^i(t)]dt + e^{-rT}\kappa_i x_i(T) + TE_i$$
$$s.t. \begin{cases} \frac{dx_i(t)}{dt} = -u_i(t) + r_i(t), \\ \int_0^T \left(e_i(t) - q_i^*(t)\right)dt = \int_0^T u_i(t)dt. \end{cases} \tag{9}$$

where κ_i represents the comprehensive value of the unit of water at the end of the game period [0, T]. TE_i denotes the eco-environmental value in the corresponding channel of province i. $\int_0^T \left(e_i(t) - q_i^*(t)\right)dt = \int_0^T u_i(t)dt$ indicates that the total trading quantity of water from seller i in the second-level game is consistent with the results of the first-level game in game period T.

In the market-based OBP game (third-level game), the equilibrium strategy $\rho^*(t)$ that maximizes the basin's TRF $J_i(s(t))$ should meet the following constraints:

$$\max_{u_i, r_i} \left\{ \int_0^T e^{-rt}[(1 - R_{tax}(t))\rho(t)u_i(t) - f_{ws}^i(t)]dt + e^{-rT}\kappa_i x_i(T) + TE_i \right\} \tag{10}$$

3.1.5 Net Carbon Emission Constraints

Due to the impact of sustainability considerations on both economic benefits and environmental protections, net carton emission constraints during basin water allocation should be introduced. The objective of these constraints is to minimize the total net carbon emissions of the basin. The objective constraints can be expressed as:

$$\min f = \sum_{i=1}^n \left(C_i A_i q_i^* - S_i q_i^*\right) \tag{11}$$

where q_i^* is the water allocation of province i (10^8 m^3); C_i refers to the carbon emission coefficient of province i (kgCO$_2$/tce); A_i is the energy consumption per cubic metre of water (kgce/m^3); and S_i is the carbon sequestration coefficient of sector i (kgCO$_2$/tce).

3.2 Implementation Mechanism of Water Dispatching

3.2.1 Initial Water Rights Allocation

The main function of the initial water allocation is to allocate the initial water rights proportionally to the provinces in the basin in strict accordance with the "Comprehensive Planning of the Yellow River Basin (2012~2030)" and the principles of "abundance increase and withered decrease". Then, the initial water allocation for each month/dekad is given as Eqs. (12)–(16).

$$LYR = \sum_{i=1}^{n} \sum_{j=1}^{12} LIWR_i^j + WRIR \tag{12}$$

$$LIWR_i^j = \sum_{e=1}^{3} LIWR_i^j(e) \tag{13}$$

$$LIWR_i = \sum_{j=1}^{12} LIWR_i^j \tag{14}$$

$$PIWR_i^{k,j}(e) = LIWR_i^j(e) \cdot \frac{PYR^k}{LYR} \tag{15}$$

$$PIWR_i^{k,j} = \sum_{t=1}^{3} PIWR_i^{k,j}(e) \tag{16}$$

where i denotes the provincial number in the basin, j represents the monthly serial number, the current transaction year is k, e denotes the dekad's serial number, n is the number of provinces in the basin, LYR denotes the average annual total water resources of the basin, $LIWR_i^j$ represents the initial water allocation in horizontal years for province i in the basin in the jth month, $WRIR$ denotes the total quantity of water resources reserved in the river channel in horizontal years, $LIWR_i^j(e)$ represents the initial water allocation of province i in the jth month and e^{th} dekad, $LIWR_i$ represents the initial water allocation of province i in horizontal years, PYR^k denotes the annual runoff calculated by runoff prediction, the runoff of the YRB in horizontal years is LYR, $PIWR_i^{k,j}(e)$ denotes the predicted initial water allocation of province i in the kth year, j^{th} month, and e^{th} dekad, and $PIWR_i^{k,j}$ represents the predicted initial water allocation of province i in the kth year and j^{th} month.

3.2.2 Real-time Water Dispatching of the Province

The predicted OTQW in each province can be formulated by combining Eq. (12)–(16) in the initial water rights allocation, the quantification of the CVWR, and the multiobjective game model. The ratio between the predicted monthly runoff values based on the BP neural network and time series is adopted to refine the predicted value of the OTQW in each province. The above results and the initial water allocation serve as the foundations for real-time water dispatching during the transaction month in the basin.

At the end of real-time water dispatching in the transaction month, the actual initial water allocation and OTQW are calculated according to the actual runoff of the river channel. The error between the actual value and the predicted value is added to the water allocation indicator for the next month as the water shortage in the current transaction month. Therefore, the water allocation indicator is composed of the predicted quantity of water in the current month based on the monthly runoff prediction and the water shortage in the previous month.

Based on the above analysis, the relationship between the parameters in real-time water dispatching according to the multiobjective game is as follows:

$$PYR^{k,j} = PYR^k \cdot \frac{LMR^j}{LYR} \tag{17}$$

$$PIWR_i^{k,j,m} = PIWR_i^{k,j} \cdot \frac{PMR_i^{k,j}}{PYR^{k,j}} \tag{18}$$

$$PIWR_i^{k,j,m}(e) = PIWR_i^{k,j}(e) \cdot \frac{PMR_i^{k,j,m}}{PYR^{k,j}} \tag{19}$$

$$PIWR_i^{k,j,m} = \sum_{t=1}^{3} PIWR_i^{k,j,m}(e) \tag{20}$$

$$AIWR_i^{k,j}(e) = LIWR_i^j(e) \cdot \frac{AMR^{k,j}}{LMR^j} \tag{21}$$

$$AIWR_i^{k,j} = \sum_{t=1}^{3} AIWR_i^{k,j}(e) \tag{22}$$

$$SIWR_i^{k,j} = AIWR_i^{k,j} - PIWR_i^{k,j} \tag{23}$$

$$STWR_i^{k,j} = ATWR_i^{k,j} - PTWR_i^{k,j} \tag{24}$$

$$SYWR_i^k = SIWR_i^{k,12} + STWR_i^{k,12} \tag{25}$$

$$WRAI_i^{k,j} = PIWR_i^{k,j,m} + SIWR_i^{k,j-1} + STWR_i^{k,j-1} + SYWR_i^{k-1} \cdot \frac{LMR^j}{LYR} \tag{26}$$

$$\begin{aligned} WRAI_i^{k,j}(e) = {}& PIWR_i^{k,j,m}(e) + \left(SIWR_i^{k,j-1} + STWR_i^{k,j-1} \right) \\ & \cdot \frac{LIWR_i^j(e)}{LIWR_i^j} + SYWR_i^{k-1} \cdot \frac{LIWR_i^j(e)}{LIWR_i} \end{aligned} \tag{27}$$

$$WRAI_i^{k,j} = \sum_{t=1}^{3} WRAI_i^{k,j}(e) \tag{28}$$

where m is the parameters obtained from monthly runoff prediction, $PYR^{k,j}$ represents the predicted runoff in the kth year and j^{th} month converted from the annual runoff prediction, LMR^j denotes the horizontal annual runoff in the jth month, $PIWR_i^{k,j,m}$ is the predicted initial water allocation of province i in the kth year and j^{th} month converted from the monthly runoff prediction, $PMR_i^{k,j,m}$ represents the predicted runoff of province i in the kth year and j^{th} month converted from the monthly runoff prediction, $PIWR_i^{k,j,m}(e)$ denotes the predicted initial water allocation of province i in the kth year, j^{th} month, and e^{th} dekad, $AMR^{k,j}$ is the actual runoff of the river channel in the kth year and j^{th} month, $AIWR_i^{k,j}(e)$ represents the actual initial water allocation of province i in the kth year, j^{th} month and e^{th} dekad, $AIWR_i^{k,j}$ is the actual initial water allocation of the province i in the kth year and j^{th} month, $SIWR_i^{k,j}$ denotes the initial water allocation shortage of province i in the kth year and j^{th} month, $ATWR_i^{k,j}$ represents the actual optimal trading quantity of the water of province i based on the parameter $AMR^{k,j}$, $PTWR_i^{k,j}$ denotes the predicted optimal trading quantity of the water of province i based on the parameter $PIWR_i^{k,j,m}$, $STWR_i^{k,j}$ represents the shortage of the water rights transaction of province i in the kth year and j^{th} month, $SYWR_i^k$ denotes the water shortage of the annual settlement, $WRAI_i^{k,j}$ is the water allocation indicator of province i in the kth year and j^{th} month, and $WRAI_i^{k,j}(e)$ represents the water allocation indicator of province i in the kth year, j^{th} month, and e^{th} dekad. The OTQW in this section refers to the purchase volume, which is a negative number when province i is a seller.

4 Results and Discussion

4.1 Comparison and Analysis of Water Allocation Schemes

4.1.1 Performance Comparison Between the Initial WAS and MSWA

The performance of different water allocation schemes is compared and analysed by taking two different trading periods of one month and half a year as examples where the monthly trading period is August 2021 and the half-year trading period is the first half of the year. The value curve of the TRF in this study and the initial WAS of the Yellow River are shown in Fig. 3 under different trading periods. The results of the OTR, OTQW in each province, and OBP are calculated in the MSWA model based on the Hamilton function and Lagrange function. If the value of the seller's TRF changes, the MSWA model dynamically adjusts the trading quantity of water in each province. These adjustment processes are performed according to the strategy in Sect. 3. From the perspective of the TRF value, compared with the initial WAS of the YRB, the MSWA in this study is obviously superior to that of the initial WAS.

4.1.2 Performance Analysis of Different WASs

1. **Calculation and comparison of different WASs**
   To analyse the performance of the MSWA model and other WASs that do not adopt game methods, comparisons were made between the MSWA model, the administrative

and market-based regional water resource allocation problem (AM-RWRAP), and the sustainable water allocation and water rights trading (SWA-WRT) model (Bekchanov et al. 2015; Wang et al. 2022). Here, each WAS applied to the water allocation in the YRB, and the constraints of this study and the quantification method for the CVWR were adopted simultaneously. The experiments were repeated 20 times for each scheme to account for the averages of all results, and then the performance of different parameters was analysed. Finally, the advantages of the OTR, OTQW, and OBP in the MSWA were verified through numerical calculations and comparison.

The optimal water consumption, trading quantity of water, shadow price, and value of the TRF (Eq. (10)) in each province of the YRB based on the SWA-WRT are listed in Fig. 4. When combining Fig. 4 with Fig. 3, the MSWA and SWA-WRT are shown to be consistent in the calculation results of the sellers in the YRB. In fact, according to the calculation results, the AM-RWRAP and MSMA are also consistent in the seller's determination. To compare and analyse the performance of each WAS, the TRF of the MSWA in Sect. 3 is accurately computed in the simulated experimental environment as a unified measurement. The trading period is considered months and half a year. The calculation results of the MSWA, SWA-WRT, and AM-RWRAP from different emphases are presented in Fig. 5.

Combined with the TRF values of the YRB (Eq. (4)), in the SWA-WRT and AM-RWRAP, the results of the monthly trading period of ¥74.04 billion and the semi-annual trading period of ¥415.16 billion in the MSWA model are both optimal values.

The economic value of water in the SWA-WRT and AM-RWRAP is 15.3% ~ 42.1% higher than that in the MSWA (see Fig. 5a). Compared with other schemes, the value of the standard deviation of the MSMA among the economic value, social value, negative value of the ecological environment, and positive value of the ecological environment is the smallest. This indicates that this scheme can coordinate and optimize ecological, environmental, economic, and social values to improve strategic trade-offs in efforts
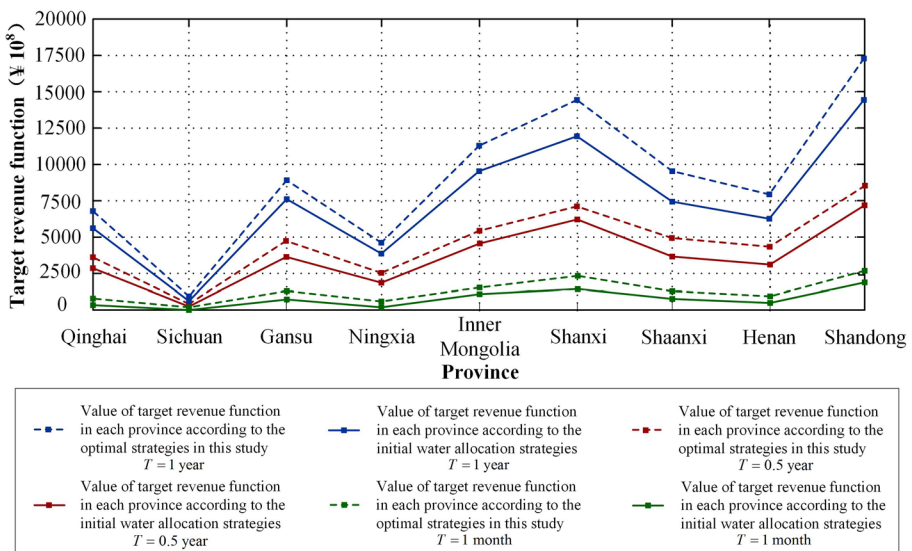

Fig. 3 Curve of the TRF in each province in 2021

towards basin water sustainability. Due to the introduction of the tax rate mechanism and constraints of net carbon emissions, the value of net carbon emissions in the MSWA is 4.3%~5.9% lower than that in the other schemes. This is in line with the concept of the coordinated development of water utilization, the economy, society, and the ecological environment; moreover, the comprehensive value of water in the MSWA is higher than that of SWA-WRT and AM-RWRAP (see Fig. 5b, c). The tax rate varies dynamically between 0.06 and 0.15 (see Fig. 5d). When the tax rate of the MSMA model increases to 0.09, the decreasing trend of the carbon emission value basically tends to be stable, and the CVWR begins to decline significantly; that is, the optimal tax rate is 0.09. A comparison between Figs. 4 and 5 shows that the MSWA in this study is more feasible for WAS.

2. **Discussion: Advantages of the MSWA model**

   Compared with the previous WAS, the MSWA model accounts for the tax rate, trading quantity of water, and bargain price as the game's target parameters in a triple-level game that maximizes water utilization efficiency while ensuring that wastewater discharge is controlled and tax revenue is increased. This is in line with the actual requirements of the MSWA in the YRB. At the same time, previous sustainable water allocation game models have mostly adopted complete information static games, and the corresponding trading strategies lack flexibility, while the MSWA model in this study can be dynamically adjusted according to variable trading periods and the revenues of different trading strategies. Therefore, the MSWA model has obvious advantages as a quantification method for the CVWR in this study.

4.2 Results: Statistics and Analysis of Water Dispatching

Taking into account the initial water allocation in the horizontal years and the annual runoff predictions for the river basin, the principle of "abundance increase and withered decrease" and the quantification method of the CVWR in the YRB are adopted to complete the MSWA game. Then, the transaction results are converted proportionally into the
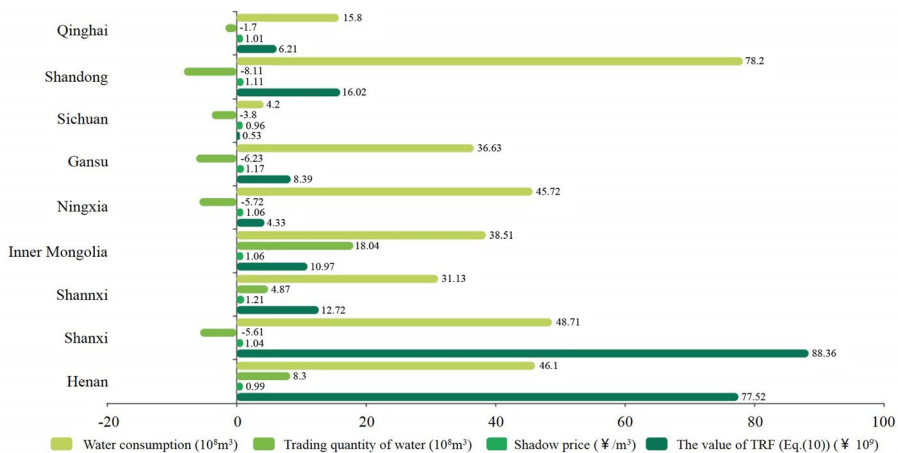

Fig. 4 The results of SWA-WRT-based sustainable water allocation in 2021
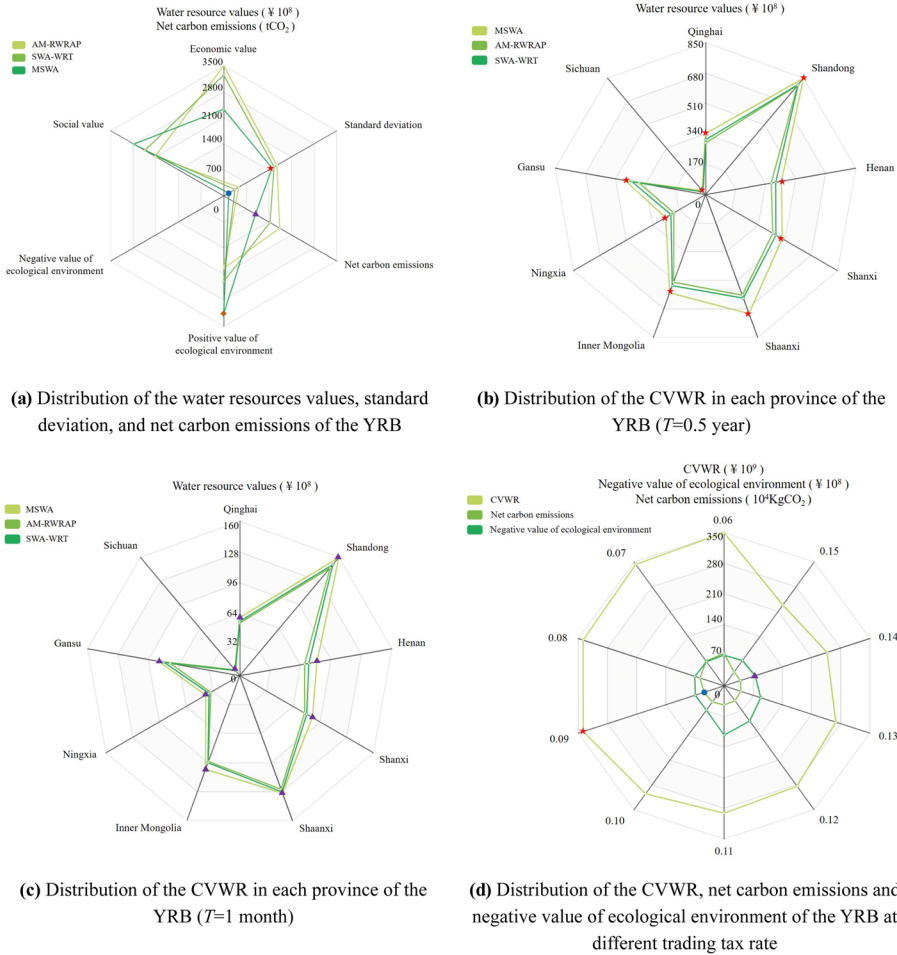
(a) Distribution of the water resources values, standard deviation, and net carbon emissions of the YRB

(b) Distribution of the CVWR in each province of the YRB (*T*=0.5 year)

(c) Distribution of the CVWR in each province of the YRB (*T*=1 month)

(d) Distribution of the CVWR, net carbon emissions and negative value of ecological environment of the YRB at different trading tax rate

Fig. 5 Comparison of the distribution of water resource values and net carbon emissions in 2021

predicted monthly runoff values. Real-time water dispatching is realized through "ten-day dispatching—monthly adjustment—annual settlement", and the current month's water shortage is included in the next month's water dispatching.

Taking Henan Province as an example, the statistical results of water dispatching for each month based on the MSWA in the YRB are listed in Fig. 6. Figure 6 shows the initial water allocation for each month in Henan Province in horizontal years, the initial water allocation for each month in Henan Province in the predicted year, the predicted quantity of water for each month, and the water allocation indicators. Figure 6 shows that the YRB in 2021 was a dry year. The initial water allocation of each month of the predicted year is converted on a proportional basis according to the horizontal year. Thus, the initial water allocation of the predicted year is significantly less than the initial water allocation in the horizontal years. The predicted quantity of water and water allocation indicators for each month in the water rights transaction are converted based on the annual runoff predictions and the monthly runoff predictions in equal proportions. At the same time, the water
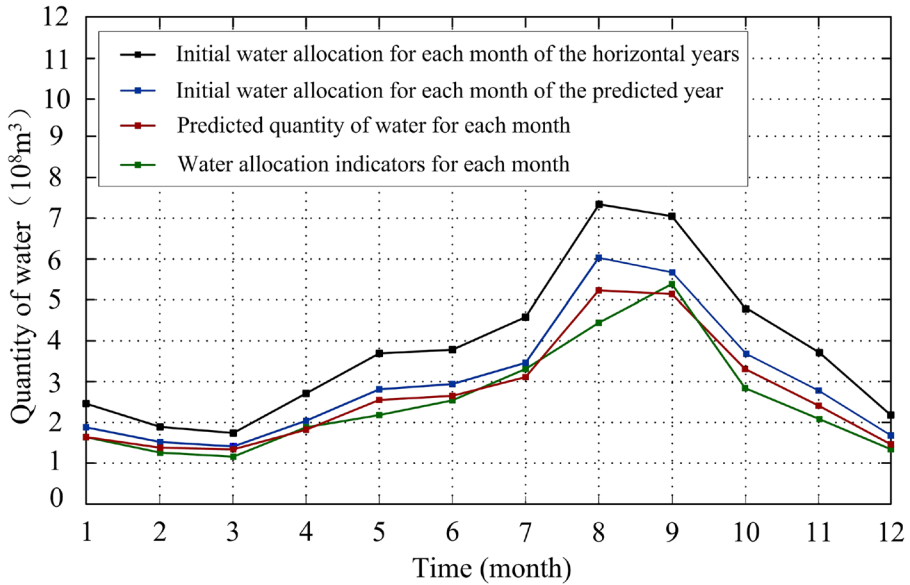
Fig. 6 Monthly water dispatching for Henan Province in 2021

shortage of the previous month is introduced into the water allocation indicator of the current month. Therefore, the predicted quantity of water and water allocation indicators for each month are significantly different.

The quantified results of water dispatching in different schemes are presented in Fig. 7. As seen in Fig. 7, the value of the TRF in the MSWA during each month is greater than that in the other two schemes. The comparison results show that AM-RWRAP and SWA-WRT focus more on the economic benefits of water resources, while the MSWA-based implementation mechanism of water dispatching takes into account the sustainability and coordinated development of the economic society and ecological environment. Simultaneously, the CVWR for the implementation mechanism of water dispatching in this study is superior to that of AM-RWRAP and SWA-WRT. Therefore, it can be concluded that this study has obvious advantages in the MSWA and the implementation mechanism of water dispatching in the YRB.
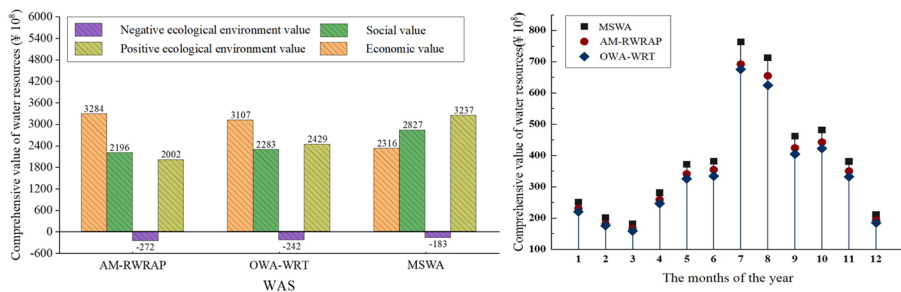

Fig. 7 Comparison of the CVWR in different WASs in 2021

5 Conclusions

In this work, a framework of water resource management for threshold-constrained differential-game error-correction multiobjective sustainable water allocation and implementation mechanism of water dispatching (MSWA-IMWD) was constructed by considering the interactions among compromise resource consumption, economic development and environmental protection. The MSWA formulates a multiobjective triple-level model with the objective of maximizing the TRF of multiple decision-makers to calculate the OTR and OTQW in each province, as well as the OBP in the YRB. In the process, the MSWA simultaneously maximizes the tax revenue from water rights transactions, minimizes net carbon emissions, and controls the scale of sewage discharge. The IMWD is constructed for error correction between the predicted runoff and actual runoff of the YRB. The MSWA-IMWD is effective for determining the optimal strategies of water allocation and water dispatching in the YRB.

The experimental comparison and data analysis according to the MSWA-IMWD model showed that a) in terms of the value of the TRF, the economic benefits of MSWA were $6.5\% \sim 9.1\%$ higher than the initial WAS and previous studies; b) the net carbon emissions in this study were $4.3\% \sim 5.9\%$ lower than other schemes; c) due to the introduction of the tax-rate mechanism and the consideration of sewage and carbon emission control, the standard deviation of economic value, social value, and negative ecological environment value in this paper were the lowest, which indicates that the economic, social, and ecological environmental value of water resources in each province of the YRB were more balanced and sustainable than those in other WASs. In conclusion, the modelling takes the net carbon emissions and negative sewage value into consideration and thus promotes coordinated and balanced development that benefits economic, social, and ecological environment values in the YRB.

Availability of Data and Materials The data that support the findings of this study can be found online at http://www.yellowriver.gov.cn/doi.org/10.1016/0165-1889(89)90011-0.

Declarations

Ethics Approval N/A.

Consent to Participate N/A.

Consent to Publish N/A.

Competing Interests The authors declare that there are no financial or non-financial competing interests related to this article. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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