




# Uncertainty Analysis of Flood Control Design Under Multiple Floods

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Received: 3 December 2020 / Accepted: 10 January 2022 / Published online: 22 February 2022  
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## Abstract

Hydraulic engineering built on tributaries at the confluence of main and branch streams are significant to river management and runoff regulation. The Flood Control Design Level (FCDL) calculations for these works are directly influenced by tributary floods and supporting effects from the mainstream. However, the determination of design level under main and tributary floods has not been well investigated. To address this issue, the authors proposed a Copula-based approach to analyze the design level under multiple runoff discharge with a case study of the Guiping Shipping Hub(GPSH). The proposed method is compared with the conventional multivariate hydrological elements analysis approach, and the sampling uncertainty is also studied. The results showed that the joint distribution of main and tributary floods is well modeled by Clayton Copula, with PE3s as the best-fit marginal distributions. Furthermore, the different roles of main and branch fluxes in design level calculation can be identified by the offered Flood Control return period(FCRP). And the design levels conducted by the FCRP can avoid the situation over-or-under performed by the OR or AND RP. Moreover, flood combinations uncertainty analysis indicates that the uncertainty of the joint design combinations decreases with the increase of sample size  $n$  but increases with the rise of the design  $T$ . Finally, the 95% confidence interval and standard deviation of the design level calculated by FCRP are smaller than that of OR RP, which means the FCRP can reduce uncertainty under multiple floods. These results suggest that the proposed FCRP provides an appropriate approach for determining the design level under combined fluxes and serves as a reference for engineering practice.

**Keywords** Multiple floods · Copula · Flood control design level · Return period · Uncertainty analysis

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## 1 Introduction

In the past decades, with the explosive growth of the urban population, more and more cities and towns along the river have built sluices, weirs, and other water conservancy projects to meet the needs of urban water and economic construction. For these sluice and weir projects, the determination of the design water level has an important engineering significance for the safe operation (Alila and Mtiraoui 2002; Ren et al. 2016) and thus must be calculated scientifically and reasonably. In general, the design level determination under a single flood can be treated by flood frequency analysis by fitting the most optimal univariate probability distribution of flood peak sequence (Modarres 2008). However, in the case of multiple floods, such as the main and tributary confluence, the design level calculation analysis cannot simply combine the different probability distributions of main and branch floods, especially when the river characteristics are dramatically altered by climate change and human activity. Hence, further investigation of the design level determination under multiple floods has a great engineering significance.

To our knowledge, the design water level driven by the combined effect of multiple floods has received little attention in the literature; besides, there is no effective calculation method for the engineering practice. Some engineering examples, such as the case study of GPSH in this paper, the design level is performed by flood regulating calculation with the Empirical combination method (ECM). The ECM usually uses the flood discharge of mainstream that satisfies the design RP combines the average flood discharge of tributary or vice versa. Though the ECM can be treated as a practical approach, particularly when it is insufficient to measure flood discharge samples, the applicability is still worth considering due to the following debates. On the one hand, the ECM comes from the summary of engineering practice and lacks a scientific theoretical basis. On the other hand, the ECM cannot accurately distinguish the different characteristics of main and tributary floods due to connecting the multiple floods simply; this would lead to uncertainty in determining the design water level and cause potential risks.

Copula can characterize the different effects of multivariates by connecting them in an appropriate form with a strong scientific basis. Because of its simple construction and flexible structure, the copulas have become the most popular mathematical modeling tool for multivariate hydrological frequency analysis (Favre et al. 2004; Salvadori and Michele 2004; Shiau and Modarres 2009; Zhang and Singh 2007a). Copulas have been widely applied in the multivariate hydrological risk frequency analysis such as flood peak discharge and volume (Duan et al. 2016; Li et al. 2013a, b; Sraj et al. 2015; Tsakiris et al. 2015; Zhang and Singh 2007b), duration and intensity of drought (Abdi et al. 2016; Mirakbari et al. 2010; Rad et al. 2017; Xu et al. 2015), the combination of rain and tide in the coastal area (Lian et al. 2013; Xu et al. 2014; Tu et al. 2018), and suspended sediment analysis (Shojaeezadeh et al. 2018; Peng et al. 2020). In another study, Dodangeh et al. (2020) investigated the flood frequency of interconnected rivers by applying Copulas. The results show that the binary distribution is more appropriate than the univariate distribution for flood frequency analysis at the confluence of two rivers. Therefore, for the FCDL investigation at the intersection of main and tributary, the Copula would be a proper approach to characterize the different influences of multiple floods.

Based on copula theory, the design level investigation driven by the combined action of main and tributary in the study can be concluded as follow, (1) modeling the main and tributary floods with an appropriate copula; (2) determining the design flood combination according to AND and OR RPs; (3) flood regulating calculation. However, both OR and

AND RPs correspond to numerous flood combinations for a given  $T$ . Despite the Same-Frequency (Li et al. 2013a, b) and Most Likely (Salvadori et al. 2011) combinations being widely applied, the most appropriate flood combination to determine the design level is still controversial. Thus, further investigation should be conducted to avoid the controversy of selecting flood combinations for design level determination under multiple floods.

The main purpose of this study is to investigate the design level driven by the combined action of main and tributary floods and thus evaluate the uncertainty of the FCDL calculation caused by samplings. To reach this purpose, the FCRP performed by the bivariate Copula was proposed to identify the different roles of main and tributary fluxes on design level determination. The design levels conducted by the proposed FCRP were compared with the conventional AND and OR RPs. Furthermore, the increasing concern of uncertainty analysis due to the sample sizes  $n$  was also discussed. This paper might be the first to focus on the design level analysis under main and tributary floods in literature, and the results would be of an innovative reference significance for calculating design levels driven by multiple floods.

## 2 Theory and Method

### 2.1 Copula Theory

For bivariate cases,  $F(x)$  and  $F(y)$  are cumulative marginal distribution functions of the two continuous random variable sequences  $X$  and  $Y$ , respectively. According to Sklar (1959), the joint distribution function  $F_{XY}(x, y)$  of the random variables  $x$  and  $y$  can be expressed as,

$$F_{XY}(x, y) = C[F(x), F(y)] = C[u, v; \theta] \tag{1}$$

where  $C[u, v; \theta]$  is a bivariate Copula function with a parameter  $\theta$ .

There are Ellipse copulas (Fang et al. 2002), Plackett copulas (Plackett 1965), and Archimedean copulas three main types of bivariate Copulas applied in hydrological analysis, in which the Archimedean Copulas are the most widely performed due to their simple symmetrical structure and ease of calculation (Brahimi et al. 2011; Mou et al. 2018; Nelsen 2000). Thus, four Archimedean copulas, Clayton, Frank, Gumbel-Hougaard (GH), and Ali-Mikhail-Haq (AMH) copulas presented in Table 1, have been applied to model the main and tributary sequences in the study. The parameter  $\theta$  is performed by solving the relationship between  $\theta$  and Kendall correlation coefficient  $\tau$ .

**Table 1** Four commonly used Archimedean Copula functions

Copula type	Bivariate Copula function
Clayton	$C(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}; \theta \in (0, \infty)$
Frank	$C(u, v) = -\frac{1}{\theta} \ln \left\{ 1 + \frac{[\exp(-\theta u) - 1][\exp(-\theta v) - 1]}{\exp(-\theta) - 1} \right\}; \theta \in R$
GH	$C(u, v) = \exp \left\{ -[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta} \right\}; \theta \in [1, \infty]$
AMH	$C(u, v) = \frac{uv}{1 - \theta(1-u)(1-v)}; \theta \in [-1, 1]$

For bivariate hydrological probability analysis, the *OR* and *AND* RPs are defined as follow,

$$T_O = \frac{1}{P(X > x \cup Y > y)} = \frac{1}{1 - C(u, v)} \tag{2}$$

$$T_A = \frac{1}{P(X > x \cap Y > y)} = \frac{1}{1 - u - v + C(u, v)} \tag{3}$$

According to the *OR* and *AND* RPs, there would be numerous flood combinations with a given *T*; this indicates no one-to-one correspondence between design *T* and design flood combination. The risks associated with varying combinations of the flood are extremely diverse, and the design water levels carried out by different flood combinations are various, which will result in inconsistent flood protection for water projects. An increasing body of research suggests that the threat caused by the flood is related to the characteristics of the flood itself and the features of the flood discharge boundary.

The coupling effect of floods and different flooding boundary conditions can produce a variety of flood control characteristic parameters, such as flood control levels of floodgates and dams, and flood control storage capacity, the difference between the water depth after the jump of the stilling pool and the water depth of the channel, and the flood surface of the track. In the paper, the average interval time when the flood control parameter *f* determined by different flood combinations exceeds its specific flood control parameter *F* is defined as the FCRP. The design water level *z* under the combined action of main and tributary floods is considered as the flood control parameter; thus, the dangerous events can be described as,

$$E_{x,y}^F = \{H(x, y) > Z\} \tag{4}$$

where *x* and *y* are main and tributary flood peak discharges, respectively, *H* is a method for flood regulation calculation, *Z* is a design water level under a specific return period, *m*.

The recurrence period corresponding to  $E_{x,y}^F$  is the FCRP, which is,

$$T_F = \frac{1}{P\{H(x, y) > Z\}} = \frac{1}{1 - F_Z(z)} \tag{5}$$

where  $F_Z(z)$  is the cumulative distribution function of the design water level.

In multivariate hydrological analysis, the most likely combination is the most concerned, that is,

$$(u_m, v_m) = \arg \max f(u, v) \tag{6}$$

$$f(u, v) = c(u, v)f(x)f(y) \tag{7}$$

where  $c(u, v)$  is the joint distribution probability density function,  $f(x)$  and  $f(y)$  are the marginal distribution probability density functions of main and tributary floods, respectively.

### 2.2 Method for Flood Regulation Calculation

Figure 1 shows the relationship between the runoff discharges of Guigang(GG) and Dahuangjiangkou(DHJK) stations corresponding to the bottom water level of GPSH. The design water level of GPSH could be performed by flood regulation calculation as follow,

$$Q = \sigma \varepsilon MB \sqrt{2gH_0}^{\frac{3}{2}} \tag{8}$$

where  $\sigma$  is submerged coefficient,  $\varepsilon$  is lateral shrinkage coefficient,  $M = \sqrt{2gm}$ , in which  $m$  is discharge coefficient,  $B$  is the net width of a single gate,  $m$ ,  $H_0$  is the total hydraulic head of the wire crest,  $m$ .

### 2.3 Uncertainty Measurement

The horizontal average offset ( $D_X$ ,  $m^3/s$ ), vertical average offset ( $D_Y$ ,  $m^3/s$ ), area of confidence interval ( $S$ ,  $m^3/s \cdot m^3/s$ ), and average Euclidean distance ( $d$ ,  $m^3/s$ ) (Pham-Gia and Huang 2001; Liu et al. 2010; Yin et al. 2018) are applied to investigate the sampling uncertainty quantitatively. In this paper,  $D_X$  and  $D_Y$  are the estimated deviations between main and tributary flood discharges and design values derived from the measured sample sequences, respectively.  $S$  and  $d$  are used to measure the spatial distance between the design value point data and the measured sample series design value. ContourSizes Functions of the R program perform the confidence interval area  $S$ , and the other three metrics are expressed as follow,

$$D(X) = \frac{1}{N} \sum_{i=1}^N |x_{Ti} - \hat{x}_T|, D(Y) = \frac{1}{N} \sum_{i=1}^N |y_{Ti} - \hat{y}_T| \tag{9}$$

$$d = \frac{1}{N} \sqrt{\sum_{i=1}^N (x_{Ti} - \hat{x}_T)^2 + (y_{Ti} - \hat{y}_T)^2} \tag{10}$$

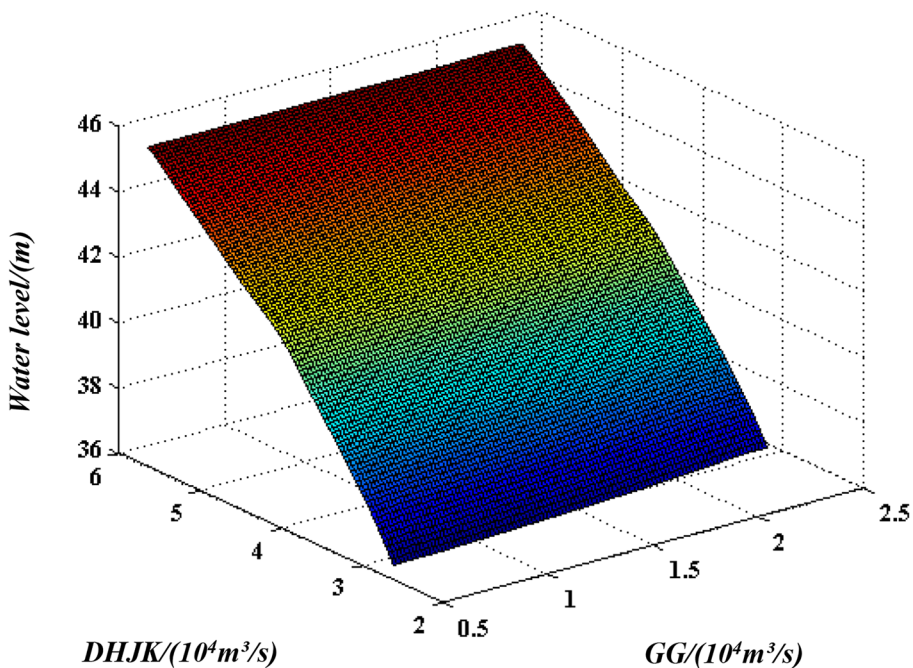


Fig. 1 Runoff from GG and DHJK stations corresponding to the natural water level at the bottom of the GPSH

where  $N$  is the number of repeated samplings, which is taken as 1000 in the paper,  $(\hat{x}_T, \hat{y}_T)$  is the most likely design combination of the measured sequence. The smaller the index value, the smaller the uncertainty.

## 2.4 Steps for FCDL Calculation and Sampling Uncertainty Analysis

Combined with the Monte Carlo random simulation approach, the specific steps to calculate the design water level are,

1. Determine the optimal marginal distribution  $F(x)$ ,  $F(y)$  of the measured flood sequences, and the optimal joint distribution model  $C(u, v)$ .
2. Generate two random numbers  $n_1$  and  $n_2$  at  $(0, 1)$ , letting  $n_1 = u$ , according to  $n_2 = C(v|u) = \frac{\partial}{\partial v} C(u, v)$  obtain a set of related probabilities  $u$  and  $v$ .
3. Convert  $u$  and  $v$  into flood combinations  $x$  and  $y$ , a water level  $Z$  is obtained after flood regulation calculation.
4. Repeat steps 1)~3)  $N$  times to get  $N$  water levels  $Z$ , then sort the water levels  $Z$ , calculate the design water level follow  $P(Z > z) = 1/T$  with a given  $T$ .

The specific steps to investigate the sampling uncertainty under the combined action of main and tributary floods are as follows:

- (a) The same as step 1) – 2) aforementioned to obtain flood sequences  $X$  and  $Y$  with a sample size  $n$ .
- (b) Repeat step *a* for  $N$  times to get  $N$  flood combinations  $X$  and  $Y$  with sample size  $n$ , and  $N$  Copula function parameter  $\theta$ .
- (c) Calculate  $N$  most likely combinations  $(x_m, y_m)$  according to the Eqs. (6) and (7) with a given  $T$ .
- (d) Obtain the  $(1-\alpha)$  confidence interval area for the  $N(x_m, y_m)$  with a given certain significance level  $\alpha$  by applying the Kernel density estimation.
- (e) Obtain  $N$  design water levels  $z_{OR}$  performed by flood regulation calculations on the  $N$  flood combinations  $(x_m, y_m)$ .

## 3 Case Study

The GPSH project is located at the Yujiang River section at the Yujiang and Qianjiang rivers intersection in Guiping City, Guangxi, China (Fig. 2). The engineering is mainly composed of shipping lock, overflow dam, and sluice, in which the length of the overflow dam is 296 m, the width of the total overflow surface is 238 m with 17 holes set to discharge. Thus the net width of each hole is 14 m, and the height of the weir crest is 21 m. The design recurrence period of the sluice of GPSH is once every 100 years with a design flood level is 43.48 m, obtained after flood regulation calculation by performing the combinations of a 100-year flood peak at DHJK station and 0.2 times the flood peak value.

Fifty-eight years (1953–2010) measured flood peak discharge data of the GG and DHJK hydrographic stations were used in the paper. The data collected from the GG station is defined as the  $Y$  series. The mainstream flood sequence  $X$  is conducted by subtracting the measured sample data of the GG station from the DHJK station.

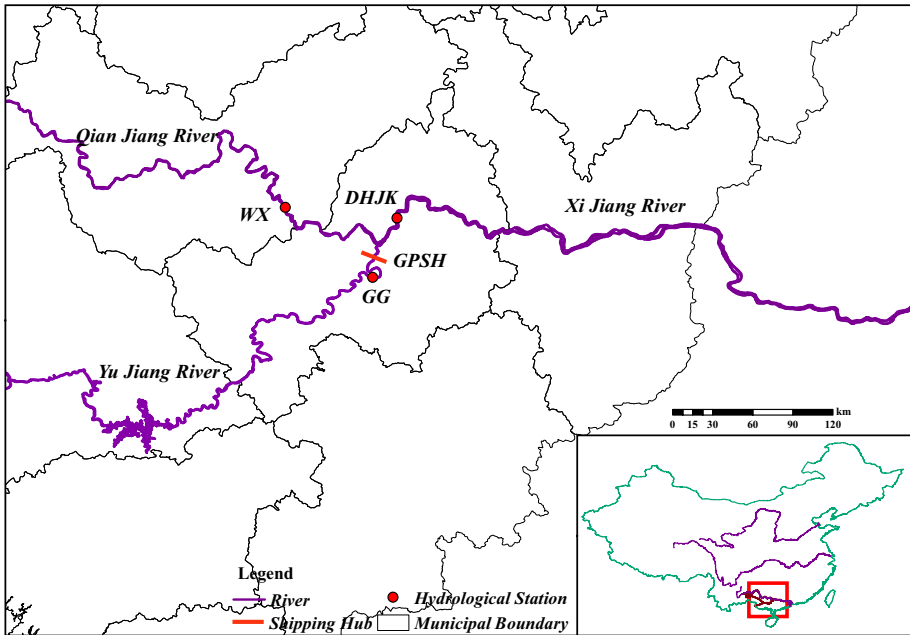


Fig. 2 Overview of the study area and location map of GPSH

## 4 Result and Discussion

### 4.1 Selection of Marginal Distribution and Copula Distribution

PE3, Generalized Extreme Value (GEV), and Weibull are adopted in the paper to model the marginal distributions. Parameters of marginal distributions are conducted by Moment estimation; the K-S Test is applied to examine the fitting degree of the sample theoretical and empirical distributions. The critical value of the K-S test in the paper is 0.179, all the marginal distributions shown in Table 2 pass the K-S test. Therefore, according to the AIC and RMSE criteria, both main and tributary flood samples are fitted by PE3 distribution.

As shown in Table 3, the joint distribution of main and tributary floods is fitted by Clayton Copula for its minimal RMSE and AIC values of 0.05 and -346.258, respectively.

Thus, the joint Copula model for main and tributary floods can be described as follow,

$$C_{Clayton}(u, v) = (u^{-0.377} + v^{-0.377} - 1)^{-1/0.377} \tag{11}$$

Table 2 selection of marginal distribution

Distribution	DHJK Station			GG Station		
	K-S	AIC	RMSE	K-S	AIC	RMSE
PE3	0.084	-392.110	0.030	0.096	-407.373	0.028
GEV	0.101	-376.671	0.032	0.103	-388.301	0.033
Weibull	0.108	-389.122	0.033	0.110	-375.367	0.037

**Table 3** Copula function fitness evaluation result

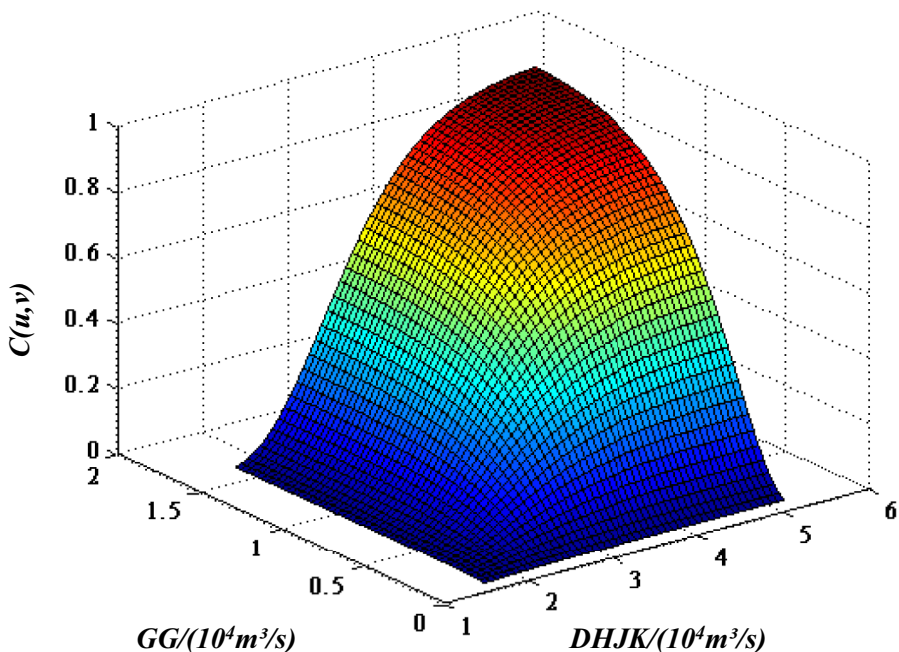
Copula function	Parameter $\theta$	RMSE	AIC
G-H Copula	1.189	0.055	-334.261
Clayton Copula	0.377	0.050	-346.258
Frank Copula	0.592	0.051	-342.727
A-M-H Copula	1.450	0.053	-339.622

According to the analysis above, the joint distribution probability of main and tributary floods can be plotted as Fig. 3,

#### 4.2 Analysis of Joint Characteristics of Main and Tributary Floods

The contours of OR and AND RPs with design  $T=100, 50, 20, 10, 5,$  and  $2$  are plotted in Fig. 4. It can be noted that the RP contours are symmetrically distributed with the  $45^\circ$  line; this may indicate that the main and tributary floods have the same effect on the design level calculation of the sluice. However, in actual projects, the impact of main and tributary floods on the FCDL determination of hydraulic engineering is usually diverted with the river characteristics. Therefore, though Fig. 4 shows the correlation between the main and tributary floods to a certain extent, it fails to deliver the various impact of the main and tributary floods on calculating the design water level of the project.

Figure 5a shows the contours carried out by FCRP; as the design  $T$  increases, the shapes tend to be more perpendicular to the x-axis; this indicates that the main flood has a much greater impact on the sluice's FCDL calculation than the tributary flood. And the effect of



**Fig. 3** Joint distribution probability of main and tributary floods



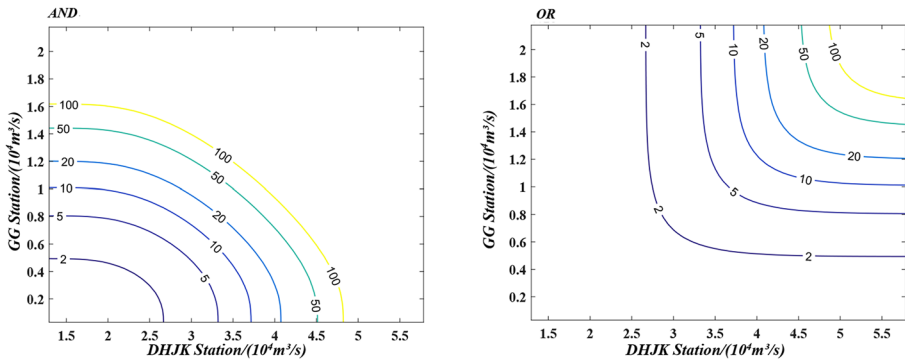


Fig. 4 T-level curves of AND and OR recurrence period

the mainstream increases with increasing design  $T$ . The results show that FCRP can better characterize different floods' impacts in design level calculations than OR and AND, which consider the same effects of main tributary floods.

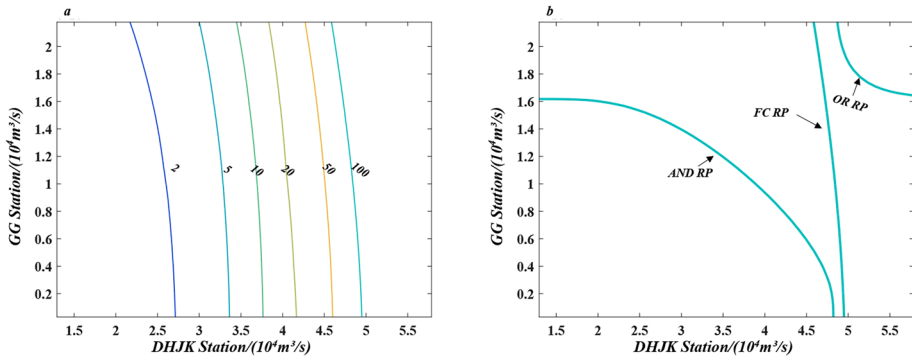
For the FCRP, flood combinations of the curve correspond to the same water level  $Z$ , while the OR and AND recurrence periods are conducted to the same  $T$ . Figure 5b describes the distribution of the main and tributary flood combinations contour conducted by OR, AND, and FCRP three different standards with the design  $T=100$ . It is worthy to note that the FCRP contour is between OR and AND ones; this indicates that the design water level performed by the FCRP can avoid the over-or-under designed water levels by OR or AND RPs, respectively. Therefore, the FCRP proposed in the study provides an appropriate approach to conducting multiple floods' design water levels.

### 4.3 Uncertainty Analysis of Main and Tributary Flood Combinations

According to Serinaldif (2013), the OR RP is recommended to investigate the multi-variate hydrological uncertainty analysis. Thus, OR RPs with design  $T$  of 20 and 50 years are chosen to evaluate the sampling uncertainty in this paper. The measured sample length of main and tributary flood combinations is 58; compared with the measured samples and simplicity, the simulated sample lengths were 58, 100, and 200, respectively.

The parameter uncertainty estimation is an essential part of the sampling uncertainty assessment. Uncertainty analysis of parameters is a statistically based mathematical analysis that requires random sampling for a large number of parameters. Table 4 shows the interval distribution of the joint distribution parameters based on Monte Carlo simulation with different sample sizes at 95% confidence conditions. It can be seen that the amplitude of the parameters decreases as the sample size increases.

As shown in Fig. 6, for the same  $T$ , the binary confidence interval of the joint design value decreases with the increasing sample size. In contrast, the binary confidence interval increases as the  $T$  increases with the same sample size  $n$ . Moreover, it can also be seen that at  $T=50$  and  $n=58$ , the most likely combinations scatter from the contour of  $T=10$  to 100. In contrast, at  $T=20$  and  $n=200$ , the most likely varieties are mainly concentrated between the  $T=10$  and the  $T=50$  curves. These findings indicate that when the sample size  $n$  is



**Fig. 5** a. T-level curves of FCRP; b. Comparison of three different recurrence periods

smaller and the design recurrence period is larger, the uncertainty of the design value is also greater.

The four uncertainty evaluation indexes were performed to characterize the sampling uncertainty under multiple floods, and the results are listed in Table 5. It can be concluded that  $D_x$ ,  $D_y$ ,  $d$ , and the area of 95% confidence interval are reduced by 42.5%, 42.9%, 44.7, and 73.9% respectively at  $T=20$  with increasing sample size  $n$ , and at  $T=50$  it is reduced by 39.4%, 46.3%, 41.1%, and 68.3% respectively.

### 4.4 Uncertainty Analysis of Design Level

For the safety of hydraulic engineering, the most important control factor is the design water level rather than the flood discharge; thus, the uncertainty of the flood combinations due to sampling can be considered to uncertainty analysis of the design water level. Here, the design level uncertainty conducted by the OR RP and FCRP was further investigated. Following the flood regulation calculation of the GPSH, the  $N$  design levels corresponding to the most likely combinations of OR RP and the flood protection levels under the FCRP criteria can be obtained separately for each set of parameters. Table 6 shows the design level estimation results of the GPSH under different criteria of recurrence period. It is noted that the 95% confidence interval and standard deviation of the design value calculated by the FCRP are less than the OR RP with the same sample size. Take the sample size  $n=58$  and  $T=20$  as an example; the interval length and standard deviation of the FCRP are 14.1% and 14.4%, respectively, smaller than the OR return period; this indicates the design water levels conducted by the FCRP are more reliable. Therefore, the FCRP can effectively avoid the problem of uncertainty in the design of flood protection under the action of multiple floods due to the sample length.

**Table 4** 95% confidence interval for joint distribution parameters of different sample sizes

Sample size $n$	Joint distribution parameter $\theta$	Parameter amplitude
58	(0.390, 0.423)	8.243%
100	(0.375, 0.399)	6.488%
200	(0.383, 0.401)	4.513%

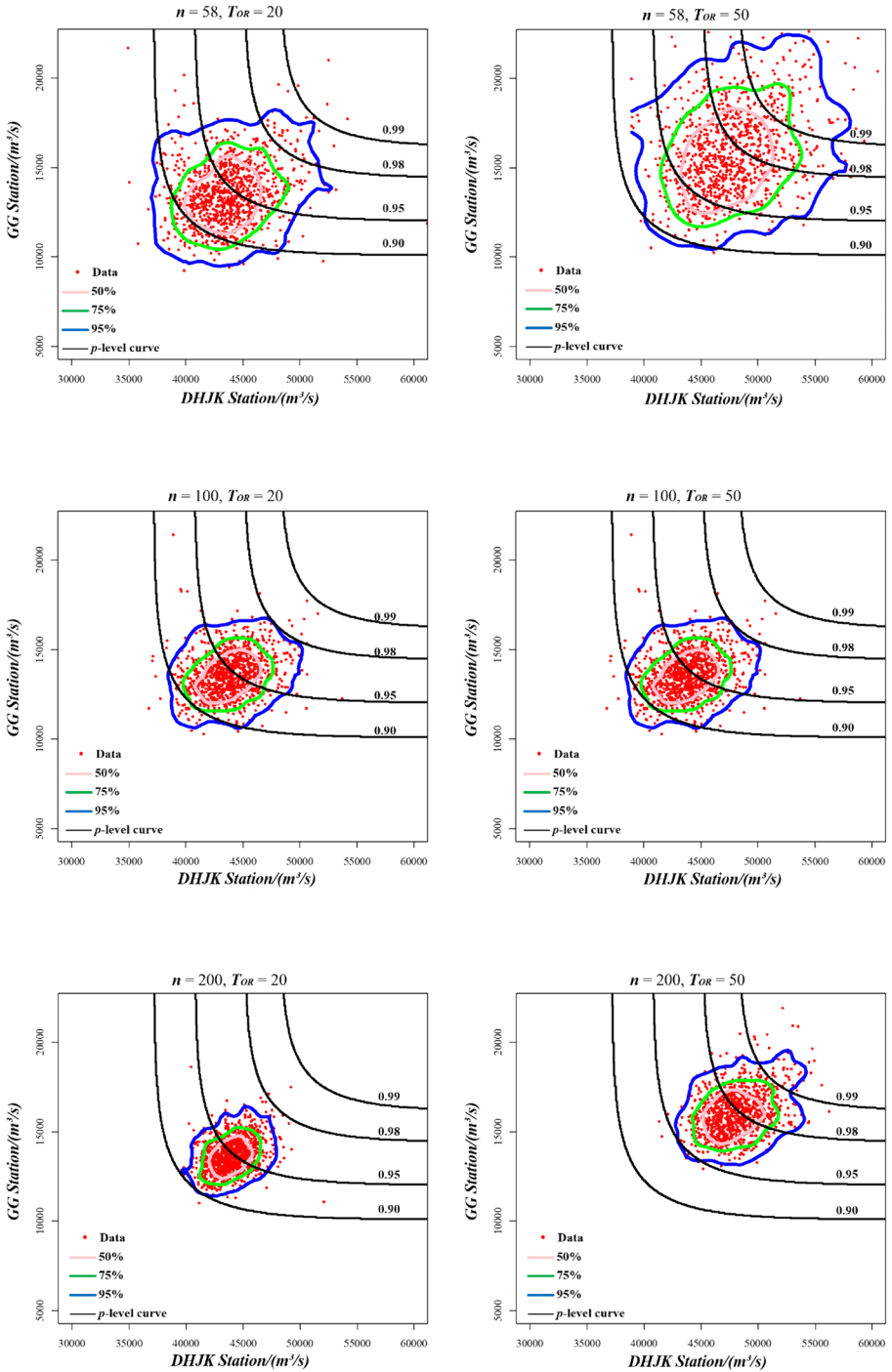


Fig. 6 Binary confidence interval graph of joint design values

**Table 5** Calculation Results of Uncertainty Index of Main and Tributary Flood Combination

<i>T</i> / year	<i>n</i>	<i>D<sub>v</sub>'</i> / (m <sup>3</sup> /s)	<i>D<sub>v</sub>'</i> / (m <sup>3</sup> /s)	<i>d</i> / (m <sup>3</sup> /s)	Confidence interval area/(10 <sup>7</sup> × m <sup>3</sup> /s · m <sup>3</sup> /s)		
					50%	75%	95%
20	58	2292.088	1514.948	114.025	1.880	3.722	9.447
	100	1853.864	1094.652	86.538	1.058	2.111	4.928
	200	1317.647	864.390	63.038	0.550	1.117	2.470
50	58	2914.282	2004.538	140.806	3.115	6.658	15.034
	100	2318.090	1605.967	118.578	1.890	3.755	9.146
	200	1764.757	1076.690	82.917	1.022	2.128	4.779

Furthermore, when *n* is less than 100, the interval length of both design standards exceeds 60 mm. As a result, the standard deviations are greater than 0.5; a short size of the flood sample series creates a large uncertainty in the FCDL determination of hydraulic engineering. Therefore, if necessary, the flood sample lengths need to be extended to enhance the reliability of the results.

The box diagrams of the design water level for the FCRP and OR RP are further described in Fig. 7. It can be seen that the length of the design level box becomes shorter as the sample size *n* increases when *T* is the same, while the length of the design level box becomes longer as *T* increases when *n* is the same. Those findings indicate that the larger the sampling *n*, or the smaller the design *T*, the more stable the calculation results, consistent with the analysis of the conclusion in Table 6. Furthermore, the median levels of the box diagram for the FCRP are smaller than that of the OR RP under the same condition, suggesting the fact that design water levels conducted by the FCRP are smaller than those for the OR RP, which is consistent with the analysis in Fig. 5b.

**Table 6** Estimation of flood control level of sluice under different criteria of recurrence period

<i>T</i> /year	Standard	<i>n</i>	<i>Z</i> /(calculated by measured data, m)	<i>Z</i> /(expected design value, m)	95% Confidence interval	Interval length/ (mm)	Standard deviation
20	OR RP	58	42.33	42.41	(42.36, 42.46)	97	0.780
		100		42.44	(42.40, 42.48)	76	0.609
		200		42.50	(42.44, 42.50)	53	0.429
	FCRP	58	41.62	42.47	(41.43, 41.51)	85	0.682
		100		42.49	(42.45, 42.52)	64	0.515
		200		42.53	(42.51, 42.56)	44	0.355
50	OR RP	58	43.54	43.51	(43.45, 43.57)	116	0.933
		100		43.53	(43.48, 43.58)	95	0.766
		200		43.61	(43.57, 43.64)	70	0.562
	FCRP	58	42.71	42.64	(42.59, 42.69)	105	0.847
		100		42.66	(42.61, 42.70)	83	0.665
		200		42.69	(42.66, 42.72)	61	0.493

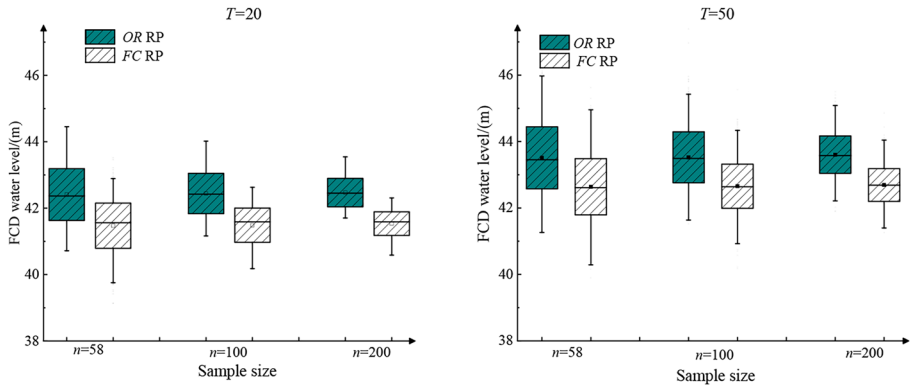


Fig. 7 Comparison of design levels box diagrams for FCRP and OR RP

### 5 Conclusion

Analysis of FCDL determination under the combined action of main and tributary floods is extremely important for the safe operation of hydraulic engineering. Still, it has attracted little attention in the literature. The paper proposes the FCRP based on the Copula to investigate the design level driven by multiple floods with a case study of GPSH. Four symmetric Archimedean with three marginal distributions were applied to model the main and tributary floods. The sampling uncertainty to the flood combinations and design water level calculation under multiple floods is further analyzed. Some main conclusions can be drawn from the study,

1. The Clayton Copula with PE3 and PE3 marginal distributions is the best-fit joint distribution for main and tributary floods.
2. The proposed FCRP can identify the different roles of main and tributary floods on design level calculation compared with the ECM and conventional AND and OR RPs. The design level conducted by the FCRP can avoid the situation over-or-under performed by the OR or AND RP.
3. The uncertainty of the joint design combinations under the effect of multiple floods decreases with the increase of sample size  $n$  but increases with the rise of the design  $T$ .
4. The 95% confidence interval and standard deviation of the FCDL calculated by FCRP are smaller than that of OR RP, which means the FCRP can reduce the uncertainty of design level calculation under the condition of multiple floods.

This paper provides innovative ideas for the design of flood protection under the combined effect of multiple floods, which can be used as a reference for practical projects.

**Acknowledgements** We are grateful for the support received as part of the projects funded by the National Natural Science Foundation of China (Grant Numbers, 41807197), the Natural Science Foundation of Guangxi (Grant Numbers, 2018GXNSFAA138042), and the Innovation Team Project of Estuarine and Coastal Protection and Management (Grant Numbers, Y220013).

**Authors' Contributions** All authors contributed to the study's conception and design. Material preparation, data collection, and analysis were performed by Y.M. Huang, Y.J. Li, M. L. L. Xiao, F.W. Gan, J. Jiao. This

revised version of the manuscript was written by Y.M. Huang, and reviewed by L. Xiao, F.W. Gan, J. Jiao. All authors read and approved the final manuscript.

**Funding** This research was partly supported by the National Natural Science Foundation of China (Grant Numbers, 41807197), the Natural Science Foundation of Guangxi (Grant Numbers, 2018GXNS-FAA138042), and the Innovation Team Project of Estuarine and Coastal Protection and Management (Grant Numbers, Y220013).

**Data Availability** The hydrological time-series data used to support the findings of this study are available from the corresponding author upon reasonable request.

## Declarations

**Consent to Participate** The authors declare that they consent to participate.

**Consent to Publish** The authors declare that they consent to publish.

**Conflict of Interest** The authors declare that they have no conflicts of interest in this work.

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