Application of Parameter-Less Rao Algorithm in Optimization of Water Distribution Networks Through Pressure-Driven Analysis

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Abstract

Water distribution networks (WDNs) connect consumers to the source of water. The primary goal of optimizing WDNs is to minimize the network costs as WDNs entail high construction costs. This paper presents a newly developed parameter-less Rao algorithm for optimization of WDNs. The methodology is based on a pressure and discharge dependent penalty. It is written in python code by linking to a hydraulic model of WDN implemented in EPANET. The algorithm is applied and tested on three benchmark networks, namely Two-loop (TL), New York Tunnel (NYT) and Goyang (GY) and a real WDN of School of Planning and Architecture (SPA), Bhopal, India. Rao algorithm employs two approaches to hydraulic modelling, demand-driven analysis (DDA) and pressure-driven analysis (PDA), as the DDA-Rao algorithm and the PDA-Rao algorithm. PDA-Rao algorithm outperforms DDA-Rao algorithm in terms of convergence. PDA-Rao algorithm saved 1.7% network cost for the NYT network, while the best-known least-cost values were obtained for TL and GY networks. It is seen that the Rao algorithms are efficient, easy to apply and do not require any parameter tuning, which reduces a large number of computational efforts.

Keywords Water distribution network · Pressure-driven analysis · Optimization · Rao algorithm · Evolutionary algorithm

1 Introduction

Every water distribution network (WDN) consists of various elements such as pipes, junctions, reservoirs, tanks and valves. The main aim of a WDN is to supply water at the required pressure and flow rate. As WDNs involve high construction costs, there is a high requirement for a cost-effective network design. The cost of the network increases with increasing the size of its elements. Thus, the selection of optimum pipe diameters is required for the optimal design of WDN. Even for a small pipe network, there are thousands of pipe combinations available. For example, there are

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10^8 pipe size combinations for a small eight-pipe network with ten available pipe sizes. Therefore, it is difficult to get the optimum results using classical optimization techniques.

The use of stochastic techniques is vastly suggested in the literature for problems that have large search spaces. Savic and Walters (1997), Wu and Simpsons (2002), and Kadu et al. (2008) are only few of many researchers who used genetic algorithms (GA) in this area. An extensive literature is available on application of several other evolutionary algorithms (EAs) in WDN optimization. For instance, simulated annealing (SA) (Cunha and Sousa 1999), ant colony optimization (ACO) (Maier et al. 2003), shuffled frog leaping algorithm (SFLA) (Chung and Lansey 2009), harmony search (HS) (Geem 2006), particle swarm optimization (PSO) (Suribabu and Neelakantan 2006), differential evolution (DE) (Suribabu 2010; Dong et al. 2012), developed swarm optimizer (DSO) (Sheikholeslami and Talatahari 2016). In recent optimization studies, Fallah et al. (2019) applied improved crow search (ICS) algorithm and Pankaj et al. (2020) used self-adaptive cuckoo search (SACS) algorithm.

Most of these algorithms are evolutionary algorithms or metaphor-based algorithms inspired by the natural phenomenon. In addition to standard parameters (population size and number of generations), they require some algorithm-specific parameters. For example, mutation, crossover and selection in GA; inertia weight, cognitive and social parameter in PSO; cooling factor and elasticity of acceptance in SA; number of nests, shifting and step size control parameter in cuckoo search (CS). Tuning of these algorithm-specific parameters is crucial. Improper tuning either results in local optima or increased computational efforts. In addition, they involve a complex working process that is challenging to understand for new designers and researchers.

Addressing these problems, Rao (2020) recently introduced three algorithms, called Rao-1, Rao-2 and Rao-3. These algorithms are algorithm-specific parameter-less, metaphor-less and straightforward. In this study, Rao algorithm uses two approaches to hydraulic modelling, demand-driven analysis (DDA) and pressure-driven analysis (PDA), as DDA-Rao algorithm and PDA-Rao algorithm.

In the conventional hydraulic analysis called demand-driven analysis (DDA), it is assumed that the demands are known functions of time and are independent of pressure. DDA has produced effective solutions in many models under normal conditions. Although, it performs inadequately when abnormal conditions are considered. On the contrary, pressure-driven analysis (PDA) incorporates a relationship between demand and pressure. In PDA, if some pressure is available above the minimum required level, it is assumed that a portion of demand will be supplied at the node. Under abnormal conditions, PDA predicts better network performance as it considers both nodal requirements and pressure requirements (Babu 2021).

In the present work, a new technique has been applied in the optimization of water distribution networks. A newly developed parameter-less Rao algorithm is integrated with pressure-driven analysis as well as demand-driven analysis. The results obtained using DDA-Rao and PDA-Rao algorithms are compared to those of previously applied algorithms for validation purposes. Also, the convergence speed is shown using convergence plots for all case studies. The comparative study based on the modified performance index shows that the PDA-Rao algorithm performs exceptionally well in all performance aspects and outperforms many evolutionary algorithms.

2 Optimization Procedure

This work aims to analyze the applicability of a newly developed Rao algorithm to optimize discrete, non-linear and constrained WDN design problems. The objective function is to minimize the network cost by meeting all the constraints. The function variables are the pipe sizes and the head at each node.

2.1 Simulation Model

The hydraulic analysis is carried out using EPANET 2.2 (Rossman et al. 2020) as the simulation model. EPANET 2.2 is linked to a Python code using Water Network Tool for Resilience (WNTR). WNTR is a Python package, which contains several sub-packages. Each sub-package contains modules that include classes, methods and functions. These classes are used to generate water network models and run simulations (Klise et al. 2017). The hydraulic simulation models are used to analyze the pressure at nodes and flow velocity in pipes to satisfy the demand at each node.

2.1.1 Pressure-demand relationship

The PDA-Rao algorithm uses the pressure-demand relationship proposed by Wagner et al. (1988) as shown below:

$$Q_{j,avl} = Q_{j,req} \qquad\qquad\qquad \text{if } H_{j,avl} \geq H_{j,req} \tag{1}$$

$$Q_{j,avl} = Q_{j,req} * \left(\left(H_{j,avl} - H_{j,min} \right) / \left(H_{j,req} - H_{j,min} \right) \right)^{(1/2)} \quad \text{if } H_{j,min} < H_{j,avl} < H_{j,req} \tag{2}$$

$$Q_{j,avl} = 0 \qquad\qquad\qquad \text{if } H_{j,avl} < H_{j,min} \tag{3}$$

$Q_{j,avl}$ and $Q_{j,req}$ are available and required discharge at node j, respectively. $H_{j,avl}$, $H_{j,req}$ and $H_{j,min}$ are the available pressure head, required pressure head (nominal head) and minimum pressure head at node j, respectively.

2.1.2 Objective Function

The objective of this work is to minimize the network cost, which is a function of pipe diameter and pipe length, as shown below:

$$C = \sum_{i=0}^{np} C(D_i) * L_i \tag{4}$$

$C(D_i)$ is the cost per unit length and L_i is the length of pipe i; C is the cost of the network; np is the number of pipes in the network. The minimization of the objective function is carried out satisfying the following constraints:

2.1.3 Conservation of Mass

For each node, continuity of flow must be satisfied:

$$\sum Q_1 - \sum Q_2 = Q_j, \forall \, j \, \epsilon \, nn \tag{5}$$

Q_1 is discharge going towards a specific node; Q_2 is discharge coming out from the specific node; Q_j is the water demand at node j; nn is the number of nodes.

2.1.4 Conservation of Energy

The loss of head around a closed loop of pipe should be equal to zero or the conservation of the energy equation for each loop should satisfy the network design as shown in Eq. (6).

$$\sum H_f = 0 \tag{6}$$

where H_f is head loss due to friction in pipe (m), which is calculated using the Hazen-Williams formula:

$$H_f = \omega L Q^{\alpha} / C_{hw}{}^{\alpha} D^{\beta} \tag{7}$$

where L is the length (m) and D is the diameter (m) of the pipe; Q is the discharge (m³/s); α and β are the exponents; ω is the numerical conversion constant; C_{hw} is the roughness coefficient.

2.1.5 Available Pipe Sizes

The diameter of the pipes should be selected from a set of commercially available sizes and are thus discrete:

$$D_i \, \epsilon \, \{D_1, D_2 \dots, D_{ns}\}, \forall \, i \, \epsilon \, ns \tag{8}$$

$D_1, D_2\dots, D_{ns}$ is the set of commercially available pipe sizes; ns is the number of candidate pipe sizes.

2.1.6 Required Pressure Head

The available pressure head at each node should be greater than the required pressure head.

$$H_{j,avl} \geq H_{j,req}, \forall \, j \, \epsilon \, nn \tag{9}$$

2.2 Optimization Model

The optimization model is constructed using Rao algorithm in Python. Rao algorithms are recently introduced by Rao (2020). 'The key advantage of Rao algorithms is that the algorithm-specific parameters do not need to be tuned to search for the optimal solution of design problems. Its working procedure is easy to understand and execute' (Rao and Pawar 2020).

2.2.1 Rao Algorithm

The Rao algorithm works by moving closer to the best solution and away from the worst. With random interactions, the solution moves throughout the population. The basic procedure of Rao algorithm in optimization of WDN is shown in Fig. 1.
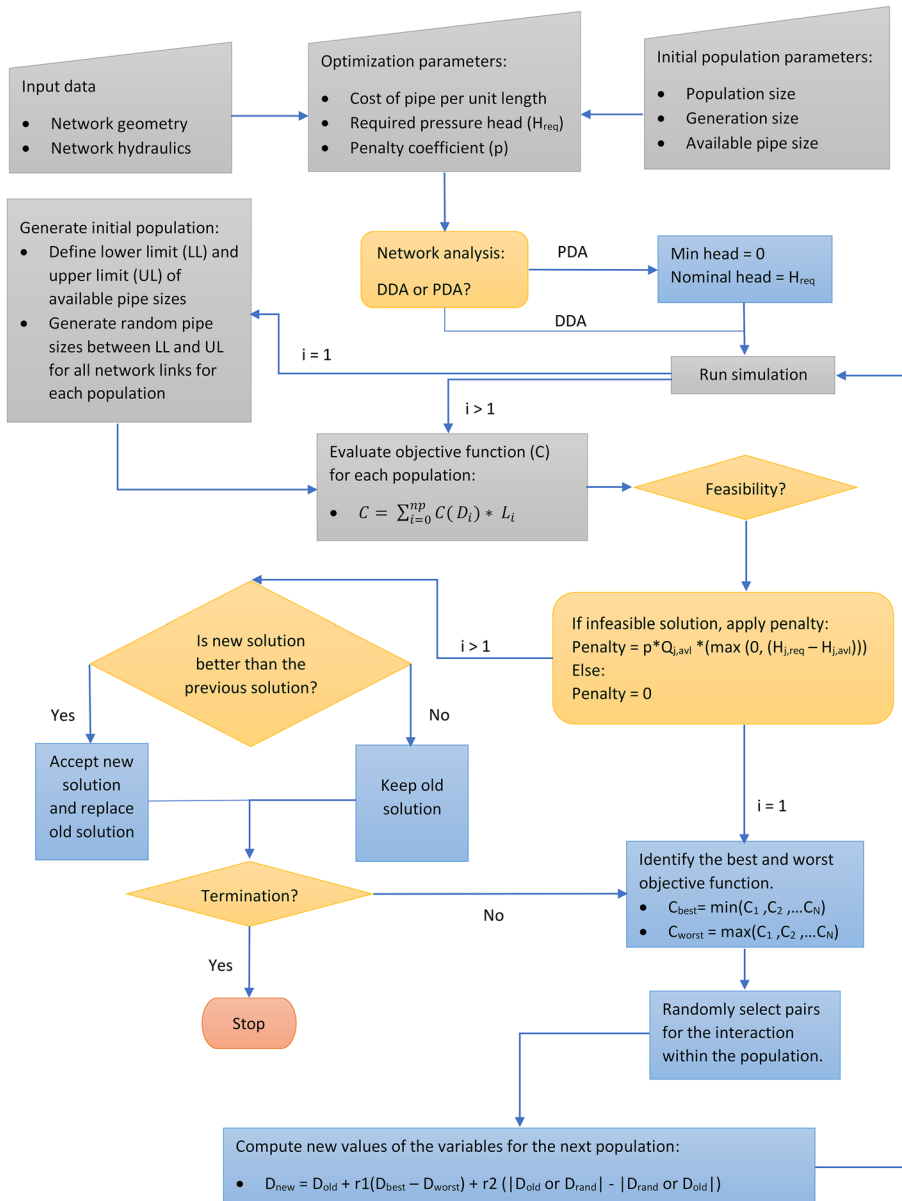

Fig. 1 Flowchart of Rao algorithm in optimization of water distribution networks

At first, initial population of the diameters is generated using random numbers considering upper and lower bound values. These diameters are used to find the value of the objective function, C. The minimum (best) and maximum (worst) cost candidates are selected from the range of solutions. The diameters corresponding to the best and worst solutions are used to generate a new population using one of the three equations of the Rao algorithms. 'In the Rao algorithms, the Rao-1 algorithm improves the result by considering the difference between the best and worst solutions; the Rao-2 and Rao-3 algorithms improve the result by considering not only the difference between the best and worst solutions but also the random interactions of the candidate solutions. The difference between Rao-2 and Rao–3 algorithms lies in considering the absolute values of the variables in the respective equations' (Rao and Pawar 2020). Rao-2 algorithm promotes more diversity because of the random interactions. Also, absolute values of variables should be considered to find the resilient solution as the value of pipe diameter is bound to be a real number. Therefore, the Rao-2 algorithm is applied in the present work, as shown in Eq. (10).

$$D_{new} = D_{old} + r1\left(D_{best} - D_{worst}\right) + r2(\left|D_{old} \text{ or } D_{rand}\right| - \left|D_{rand} \text{ or } D_{old}\right|) \quad (10)$$

D_{new} is the diameter for new population; D_{old} is the diameter for old population; D_{best}, D_{worst} and D_{rand} are the diameters corresponding to the best, worst and random solutions, respectively; r1 and r2 are random numbers.

2.2.2 Pressure and Discharge Dependent Penalty

After calculating the cost, the feasibility of the solution is checked. The feasibility criterion is the fulfilment of the required pressure head at each node. If the solution found is infeasible, then a penalty is applied to the cost function. Using DDA, the deficiencies in pressure heads are measured assuming full supply at nodes. In contrast to actual values, this approach results in higher deficiencies. Whereas in PDA, both nodal and pressure requirements are considered to provide better performance (Abdy Sayyed et al. 2019). Thus, a pressure and discharge dependent penalty based on equivalent energy costs (Kadu et al. 2008) is used in this study, as shown below:

$$\text{Penalty} = p * Q_{j,avl} * (\max(0, (H_{j,req} - H_{j,avl}))) \quad (11)$$

where, p is the penalty coefficient, measured to pump a unit quantity of water into a unit head, as suggested by Kadu et al. (2008).

2.3 Modified Performance Index (MPI)

A performance index (PI) suggested by Deep and Thakur (2007) is used to measure the effectiveness of the successful run for the given objective function. A modification is made to obtain a better comparison following the parameters provided by previous researchers. The modified performance index (MPI) illustrates the algorithm's degree of reliability, efficiency and accuracy. The reliability of PDA-Rao algorithm is defined as the frequency with which it converges to the near-optimal solution, which is measured by calculating the success rate. If the optimum cost obtained in a run is within 1% accuracy of the best-known value for that problem, then a run is considered a successful run (Deep and Bansal 2009).

The relative performance of an algorithm using MPI can be calculated as:

$$MPI = \frac{1}{Np} \sum_{i=1}^{Np} (k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3 + k_4\alpha_4 + k_5\alpha_5)_i \qquad (12)$$

where, $\alpha_1 = (S_{ri}/T_{ri})$; $\alpha_2 = (LAf_i/Af_i)$; $\alpha_3 = (LAt_i/At_i)$; $\alpha_4 = (LMf_i/Mf_i)$; $\alpha_5 = (LMt_i/Mt_i)$.

Sr_i = Number of successful runs of ith problem.

Tr_i = Total number of runs of ith problem.

The following terms in the calculation of the MPI are defined for the successful runs of the ith problem:

LAf_i = Least of average function evaluation (AFE) numbers documented for the entire range of applied algorithms.

Af_i = AFE number of the current algorithm.

LAt_i = Least of average computational time of an entire range of applied algorithms.

At_i = Average computational time of the current algorithm.

LMf_i = Least of minimum function evaluation (MFE) numbers documented for the entire range of applied algorithms.

Mf_i = MFE numbers of the current algorithm.

LMt_i = Least of minimum computational time of an entire range of applied algorithms.

Mt_i = Minimum computational time of the current algorithm.

Np = Total number of problems analyzed.

k_1, k_2, k_3, k_4 and k_5 are weights such that, $\sum_{i=1}^{5} k_i = 1$

3 Case Studies

The TL, NYT and GY benchmark networks are used to evaluate the performance of PDA-Rao algorithm. Further, this algorithm is used to solve a real WDN of School of Planning and Architecture (SPA) located in Bhopal region of India. Figure 2 depicts the schematic diagrams of all of the network cases.

The Two-loop network is a gravity-fed network firstly used by Alperovits and Shamir (1977). Later, it has been widely used by researchers (e.g., Savic and Walters 1997; Cunha and Sousa 1999; Babu and Vijayalakshmi 2013) in the field of optimization. The New York Tunnel network was initially presented by Schaake and Lai (1969). As Suribabu (2010) reiterated, the problem aims to determine the installation of new pipes parallel to the existing network to fulfil the pressure head requirement at critical nodes (node 16—20). The Goyang network is a pumped network presented by Kim et al. (1994) in South Korea. The hydraulic data for this network are taken from Geem (2006). The School of Planning and Architecture (SPA) network is a gravity-fed network with a total pipeline length of 1890 m. Demand at each location is measured using 2 h of supply per day, taking into account acceptable requirements. The available pipe diameters and the corresponding rates for SPA network are taken following CPWD GOI (2019).

The available pipe diameters for all case study networks are given in Table 1. The Hazen–William's constants ($\omega = 10.667$, $\alpha = 1.852$ and $\beta = 4.871$) are identical for all networks and equal to the values used by EPANET 2.2. The developed module was run on an Intel (R) Core (TM) i5-8250U CPU @ 1.60 GHz with 8.00 GB RAM, using the Python programming language.
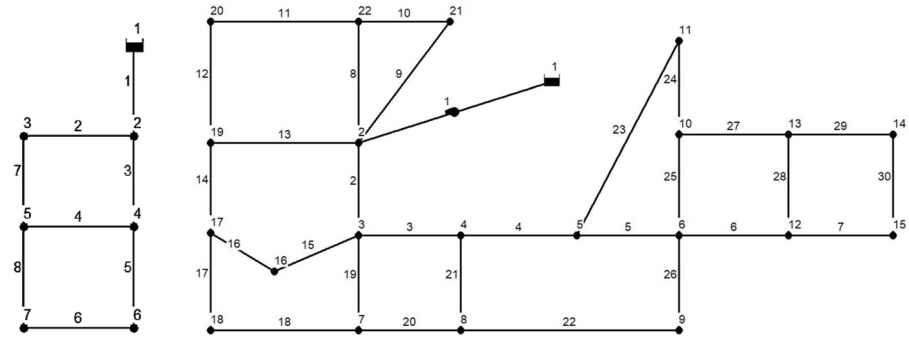
a. *Two-loop network*

c. *Goyang network*
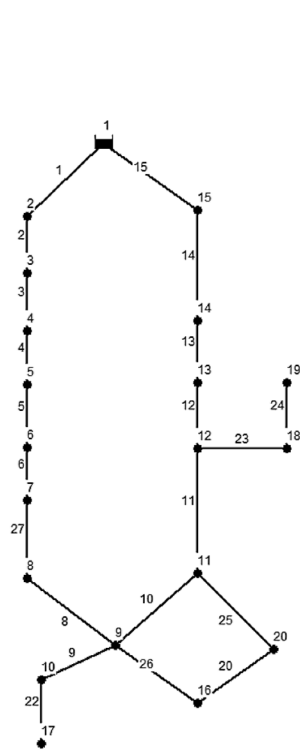
b. *New York Tunnel network*
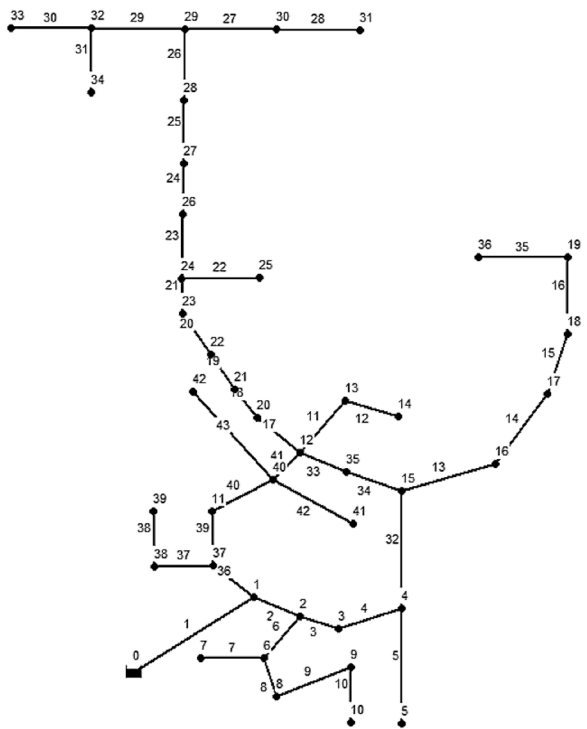
d. *School of Planning and Architecture network*

Fig. 2 Schematic diagram of case study networks: **a** Two-loop network, **b** New York Tunnel network, **c** Goyang network, and **d** School of Planning and Architecture (SPA) network

4 Results and Discussions

A simple procedure to find the minimum cost of a given water distribution network is suggested in this paper using a newly developed Rao algorithm while satisfying constraints. The solutions obtained using DDA-Rao and PDA-Rao algorithms are compared with those

Table 1 Available pipe sizes for Two-loop, New York Tunnel, Goyang and SPA networks

Two-loop Network (Alperovits and Shamir 1977)		New York Tunnel Network (Geem 2006)		Goyang Network (Geem 2006)		SPA Network (CPWD GOI 2019)	
Diameter (mm)	Cost ($/m)	Diameter (mm)	Cost ($/m)	Diameter (mm)	Cost (Won/m)	Diameter (mm)	Cost (Rs/m)
25.4	2	0.0	0	80	37,890	15	48.55
50.8	5	914.4	306.77	100	38,933	20	77.80
76.2	8	1219.2	439.65	125	40,563	25	113.60
101.6	11	1524.0	577.46	150	42,554	32	163.65
152.4	16	1828.8	725.10	200	47,624	40	228.70
203.2	23	2133.6	876.03	250	54,125	50	375.25
254.0	32	2438.4	1036.80	300	62,109	100	765.00
304.8	50	2743.2	1197.57	350	71,524	150	1150.00
355.6	60	3048.0	1368.18			200	1575.00
406.4	90	3352.8	1538.79			250	2200.00
457.2	130	3657.6	1712.68			300	2750.00
508.0	170	3962.4	1893.14			350	3350.00
558.8	300	4267.2	2073.59			400	4300.00
609.6	550	4572.0	2260.61			450	5000.00
		4876.8	2447.63			500	6270.00
		5181.6	2637.92			600	7560.00
						700	10,500.00
						750	11,350.00
						800	11,450.00
						900	13,900.00
						1000	15,550.00

of other optimization algorithms based on minimum cost achieved and AFE numbers. For both techniques, the population size is fixed to 20, and the number of iterations differs based on the network size and search space.

The minimum feasible cost of the Two-loop network is stated as $419,000 by various researchers (Savic and Walters 1997; Geem 2006; Babu and Vijayalakshmi 2013). PDA-Rao algorithm converged to the minimum cost within 33 iterations in only 33 s. Thus, the MFEs for the PDA-Rao algorithm is 660 compared to 6,500 for GA (Savic and Walters 1997) and 5,000 for SA (Cunha and Sousa 1999). Comparative results of PDA-Rao algorithm are presented in Table 2, which are obtained over 30 runs. Both DDA-Rao and PDA-Rao algorithms got the best-known value of network cost in 4,000 and 840 AFE numbers, respectively. However, AFE numbers for the same network were reported as 65,000 using

Table 2 Comparison of optimal results

Reference	Algorithm	Cost	Average function evaluations	Optimal diameters (mm)
Two-loop network				
Alperovits and Shamir (1977)	LP	$497,525	25,000	[508—457.2, 203.2—152.4, 457.2—203.2, 152.4—406.4, 304.8—254, 152.4, 152.4—101.6]
Savic and Walters (1997)	GA	$419,000	65,000	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
Cunha and Sousa (1999)	SA	$419,000	25,000	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
Babu and Vijayalakshmi (2013)	PSO-GA	$419,000	1,300	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
Pankaj et al. (2020)	SACS	$419,000	3,600	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
Present work	DDA-Rao	$419,000	4,000	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
	PDA-Rao	$419,000	840	[457.2, 254, 406.4, 101.6, 406.4, 254, 254, 25.4]
New York Tunnel network				
Dandy et al. (1996)	GA	$38.80 million	200,000	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3048, 2133.6, 2438.4, 2133.6, 1828.8, 0, 1828.8]
Savic and Walters (1997)	GA	$37.13 million	100,000	[0, 0, 0, 0, 0, 0, 2743.2, 0, 0, 0, 0, 0, 0, 0, 2438.4, 2438.4, 2133.6, 1828.8, 0, 1828.8]
Lippai et al. (1999)	GA	$38.13 million	100,000	[0, 0, 0, 0, 0, 3352.8, 0, 0, 0, 0, 0, 0, 0, 0, 2438.4, 2438.4, 2133.6, 1828.8, 0, 1828.8]
Wu and Simpson (2002)	GA	$38.80 million	30,000	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3048, 2133.6, 2438.4, 2133.6, 1828.8, 0, 1828.8]
Geem (2006)	HS	$36.66 million	6,000	[0, 0, 0, 0, 0, 2438.4, 0, 0, 0, 0, 0, 0, 0, 0, 2438.4, 2438.4, 2133.6, 1828.8, 0, 1828.8]
Present work	DDA-Rao	$36.33 million	3,000	[0, 0, 0, 0, 0, 2438.4, 0, 0, 0, 0, 0, 0, 0, 0, 2133.6, 2743.2, 1828.8, 1524, 0, 2133.6]
	PDA-Rao	$36.03 million	1,920	[0, 0, 0, 0, 0, 3352.8, 0, 0, 0, 0, 0, 0, 0, 0, 1828.8, 2743.2, 1828.8, 1828.8, 0, 1828.8]

Table 2 (continued)

Reference	Algorithm	Cost	Average function evaluations	Optimal diameters (mm)
Goyang network				
Kim et al. (1994)	NLP	179,142,700 Won	*	[200, 200, 125, 125, 100, 100, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 100, 125, 80, 80, 80, 80, 80, 80, 80, 80, 100, 80]
Geem (2006)	HS	177,135,800 Won	10,000	[150, 150, 125, 150, 100, 100, 80, 100, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80]
Present work	DDA-Rao	177,010,359 Won	1,400	[200, 125, 125, 100, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80]
	PDA-Rao	177,010,359 Won	890	[200, 125, 125, 100, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80]

*Data not available

GA (Savic and Walters 1997), 25,000 using SA (Cunha and Sousa 1999) and 1,300 using PSO-GA (Babu and Vijayalakshmi 2013). Therefore, a 35.38% reduction in computational cost is achieved in solving the TL network problem.

In the existing design of New York Tunnel network, the pressure head value does not satisfy the requirement of pressure for multiple nodes. Thus, various algorithms are applied to extend the existing network considering new pipes parallel to the existing pipes. As reported by Geem (2006), the optimum cost is $36.66 million for this network. DDA-Rao algorithm converged to an upgraded solution ($36.33 million) in only 40 iterations. However, PDA-Rao algorithm converged to $36.03 million, which is the least-cost among the results shown in Table 2 for NYT network. It is obtained in only 64 iterations and 1,280 MFEs. As a result, the PDA-Rao algorithm outperformed all other algorithms in terms of convergence as well as cost, with a 68% reduction in computational cost and a 1.7% reduction in network cost. The pressure head values at critical nodes for the optimum solutions of DDA-Rao and PDA-Rao algorithms are shown in Table 3.

Table 3 Pressure head at critical nodes of New York Tunnel network

Node	Demand (m³/s)	Existing head (m)	Required head (m)	Available head (m)	
				DDA-Rao	PDA-Rao
16	4.81	59.265	79.244	79.866	79.506
17	1.63	74.901	82.902	83.020	82.917
18	3.32	44.901	77.720	81.143	81.219
19	3.32	25.194	77.720	78.226	78.299
20	4.81	61.862	77.720	78.866	79.698

a. Two-loop network

b. New York Tunnel network

c. Goyang network

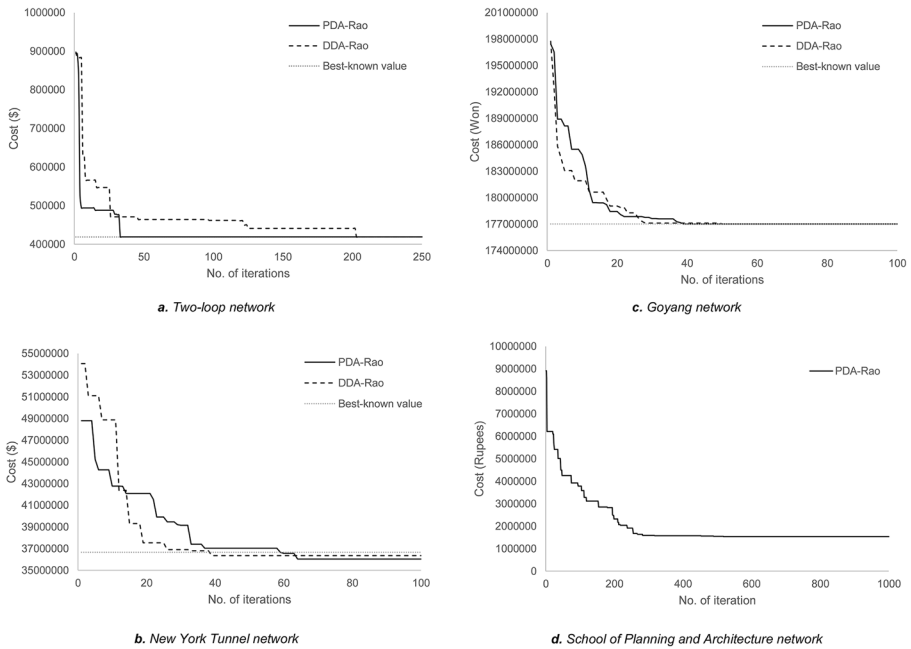*d. School of Planning and Architecture network*

Fig. 3 Convergence of Rao algorithm: **a** Two-loop network, **b** New York Tunnel network, **c** Goyang network, and **d** School of Planning and Architecture (SPA) network

According to Kim et al. (1994), the original and optimum network cost of the Goyang network is 179,428,600 Won and 179,142,700 Won, respectively. PDA-Rao algorithm converged to 177,010,359 Won as the minimum cost of this network in only 39 iterations. PDA-Rao algorithm shows 0.71% reduction in network cost with 8.9% computational cost compared to HS algorithm (Geem 2006), as shown in Table 2.

Convergence curves for DDA-Rao and PDA-Rao algorithms are plotted for the best value over 30 runs for all benchmark networks, as shown in Fig. 3. They demonstrate that the PDA-Rao algorithm converges faster than the DDA-Rao algorithm.

The statistical results for the TL, NYT and GY networks are shown in Table 4, which include minimum, maximum and average values of the cost obtained in 30 runs. As per the results, median of the cost is very close to the optimum cost, which shows the accuracy of PDA-Rao algorithm. The AFE number, MFE number and computation time required by PDA-Rao algorithm demonstrate the requirement of less computational efforts and good efficiency.

The MPI is used to observe the consolidated effect of success rate, function evaluation numbers and computation time on the PDA-Rao algorithm. Table 5 provides a review of studies related to the application of evolutionary algorithms in the minimum cost calculation of one or more of the three benchmark WDNs. The PI and MPI are plotted by reviewing and comparing the results of these studies to the proposed PDA-Rao algorithm. Firstly, these studies are compared to get the least of the MFE numbers, AFE

Table 4 Statistical results of PDA-Rao algorithm

Benchmark Network	Two-loop	New York Tunnel	Goyang
Reference	Alperovits and Shamir (1977)	(Schaake and Lai 1969)	(Geem 2006)
No. of decision variables	8	34	30
No. of available diameters	14	15	8
Size of total search space	$1.475 * 10^9$	$1.934 * 10^{25}$	$1.238 * 10^{27}$
No. of runs	30	30	30
No. of successful runs	23	18	29
Success rate	77%	60%	94%
Minimum cost	$419,000	$36,036,869	177,010,359 Won
Maximum cost	$441,000	$38,301,655	180,284,765 Won
Average cost	$421,500	$36,855,324	177,388,497 Won
Median cost	$420,000	$36,627,206	177,204,977 Won
Standard deviation	4507.4	544,126.1	627,083.6
Population size	20	20	20
Maximum allowed iterations	1000	1000	1000
Minimum function evaluations	660	1280	340
Average function evaluations	840	1920	890
Minimum computational time (s)	33	69	31
Average computational time (s)	83	121	98

numbers and computation times. Then the ratio of these least values to that of PDA-Rao algorithm are measured to calculate MPI using Eq. (12).

For a clear visualization of performance, a method is adopted as suggested by Deep and Thakur (2007). In this method, equal weights are assigned to all the terms except for one term. Hence, PI or MPI becomes a function of one variable. A geometric graph is obtained by drawing the MPI and PI on ten scaled bars, as shown in Fig. 4. The proximity of this graph to the best-performance value is an indicator of the performance of the algorithm. The graph suggests that, compared to HS (Geem 2006), the PDA-Rao algorithm consistently outperforms other optimization algorithms in solving WDN problems.

The value of the performance index for PDA-Rao algorithm varies between 0.69 to 0.94. It demonstrates the competence of the PDA-Rao algorithm compared to conventional high-performance algorithms (genetic algorithms, particle swarm optimization algorithm, harmony search algorithm and simulated annealing algorithm) used by various researchers to optimize WDN.

After validating the approach using three benchmark networks, PDA-Rao algorithm was applied to a real WDN of School of Planning and Architecture (SPA). The hydraulic results

Table 5 Review of various studies and evolutionary algorithms used for the optimization of water distribution networks

Reference	Algorithm	Benchmark WDN
Kim et al. (1994)	NLP	GY
Dandy et al. (1996)	GA	NYT
Savic and Walters (1997)	GA	TL, NYT
Cunha and Sousa (1999)	SA	TL
Wu and Simpson (2002)	fmGA	NYT
Maier et al. (2003)	ACO	NYT
Liong and Atiquzzaman (2004)	SCE	TL
Suribabu and Neelakantan (2006)	PSO	TL
Suribabu and Neelakantan (2006)	GA	TL
Geem (2006)	HS	TL, NYT, GY
Tolson et al. (2009)	HD-DDS	NYT
Suribabu (2010)	DE	NYT
Dong et al. (2012)	DE	TL, NYT, GY
Dong et al. (2012)	GA	TL, NYT, GY
Sedki and Ouazar (2012)	PSO	TL, NYT
Sedki and Ouazar (2012)	PSO-DE	TL, NYT
Babu and Vijayalakshmi (2013)	PSO-GA	TL
Moosavian and Roodsari (2014)	SLC	NYT
Sadollah et al. (2015)	IMBA	NYT
Sheikholeslami and Talatahari (2016)	DSO	GY
Reca et al. (2017)	GA	TL
Fallah et al. (2019)	GSA	NYT
Abdy Sayyed et al. (2019)	GA	TL
Sirsant and Reddy (2021)	SADE-DP	TL, NYT

NLP non-linear programming, *GA* genetic algorithm, *SA* simulated annealing, *fmGA* fast messy GA, *SFLA* shuffled frog leaping algorithm, *ACO* ant colony optimization, *SCE* shuffled complex evolution, *PSO* particle swarm optimization, *HS* harmony search, *HD-DDS* hybrid discrete dynamically dimension search, *DE* differential evolution, *SLC* soccer league competition, *IMBA* improved mine blast algorithm, *DSO* developed swarm optimizer, *GSA* gravitational search algorithm, *SADE* self-adaptive differential evolution, *DP* dynamic programming

for this network for the best of 5 test runs are given in Table 6. The population size and the maximum number of iterations are 15 and 1000, respectively. In PDA, the nominal head shall be provided at the demand nodes for the total supply. The required pressure head is taken as the nominal head, which is 13 m, and the minimum pressure head for the flow through pipes is 0 m. PDA-Rao algorithm converged to a minimum cost solution of Rs.15,40,519 for SPA network. The number of function evaluations and computation time to achieve the minimum cost solution were 7785 MFEs and 651 s, respectively.
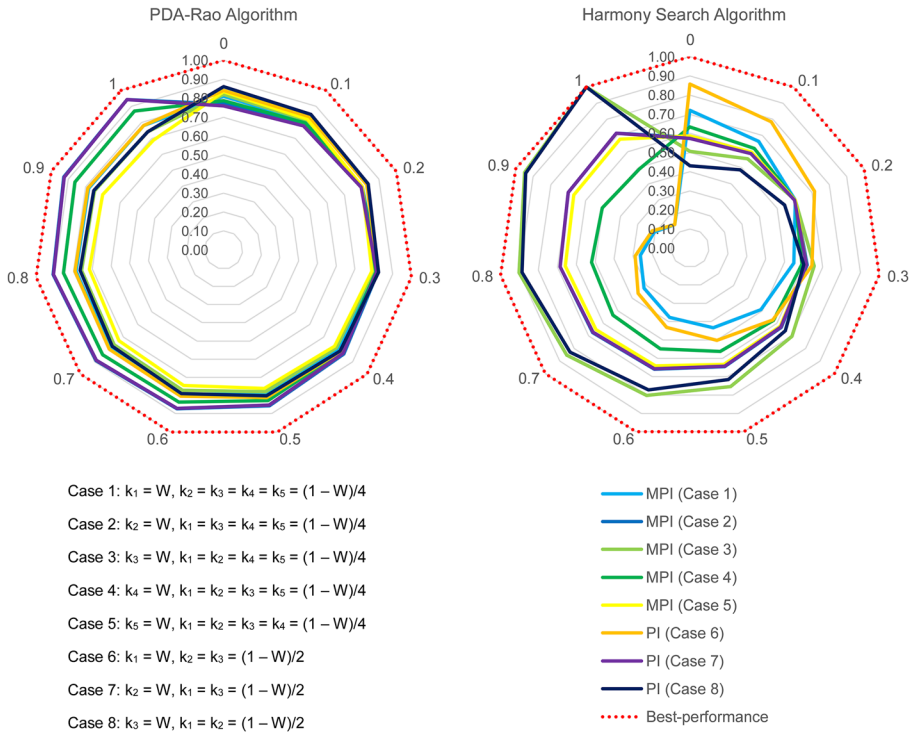
PDA-Rao Algorithm

Harmony Search Algorithm

Case 1: $k_1 = W$, $k_2 = k_3 = k_4 = k_5 = (1 - W)/4$

Case 2: $k_2 = W$, $k_1 = k_3 = k_4 = k_5 = (1 - W)/4$

Case 3: $k_3 = W$, $k_1 = k_2 = k_4 = k_5 = (1 - W)/4$

Case 4: $k_4 = W$, $k_1 = k_2 = k_3 = k_5 = (1 - W)/4$

Case 5: $k_5 = W$, $k_1 = k_2 = k_3 = k_4 = (1 - W)/4$

Case 6: $k_1 = W$, $k_2 = k_3 = (1 - W)/2$

Case 7: $k_2 = W$, $k_1 = k_3 = (1 - W)/2$

Case 8: $k_3 = W$, $k_1 = k_2 = (1 - W)/2$

— MPI (Case 1)
— MPI (Case 2)
— MPI (Case 3)
— MPI (Case 4)
— MPI (Case 5)
— PI (Case 6)
— PI (Case 7)
— PI (Case 8)
····· Best-performance

Fig. 4 Modified performance index (MPI) and performance index (PI) plotted using various weight combinations for PDA-Rao algorithm and Harmony search algorithm

Table 6 Hydraulic results of the School of Planning and Architecture (SPA) network using the PDA-Rao algorithm

Pipe	Length (m)	Diameter (mm)	Node	Elevation (m)	Demand (L/s)	Available head (m)
1	64.5	150	1	522.55	0	17.2
2	38	100	2	521.80	1.11	16.9
3	29	150	3	520.80	1.11	17.9
4	48	100	4	520.86	1.11	17.4
5	52	50	5	521.95	2.22	14.6
6	20	50	6	522.08	0	15.2
7	15	25	7	522.08	0.14	15.1
8	15	50	8	522.32	0	14.0
9	12	50	9	521.95	0	13.5
10	22.5	300	10	521.95	3.33	13.5
11	18	25	11	522.30	0	17.3
12	22.5	25	12	523.55	0	15.4
13	61.5	20	13	522.80	0	14.8
14	60	32	14	522.30	0.56	13.7
15	93	40	15	522.98	2.77	15.1
16	93	40	16	523.50	0	10.9

Table 6 (continued)

Pipe	Length (m)	Diameter (mm)	Node	Elevation (m)	Demand (L/s)	Available head (m)
17	36	200	17	521.68	0	12.4
18	36	200	18	521.58	0	12.3
19	39	150	19	521.58	0	12.1
20	38	200	20	524.05	0	14.9
21	39	100	21	523.80	0	15.1
22	42	100	22	524.30	0	14.5
23	33	100	23	524.60	0	14.2
24	44	200	24	524.55	0	13.4
25	47	250	25	523.05	6.4	14.6
26	36	150	26	524.30	0	13.7
27	68	100	27	524.10	0	13.9
28	47	40	28	523.55	0	14.4
29	60	100	29	524.85	0	13.1
30	56	50	30	524.00	0.5	13.9
31	23	350	31	523.55	0.75	13.8
32	99	100	32	524.85	0	13.1
33	39	15	33	524.85	0.07	13.1
34	39	50	34	524.80	0.28	13.2
35	69	200	35	523.28	0	14.8
36	9	150	36	520.00	0.28	13.7
37	18	250	37	523.25	0	16.4
38	21	50	38	522.05	0	17.6
39	54	200	39	523.25	1.11	16.2
40	27	250	40	523.12	0	16.5
41	28.5	100	41	522.50	0.83	15.6
42	88.5	40	42	523.00	0.83	16.1
43	90	50				

5 Conclusions

The present work deals with the application of a metaphor-less and algorithm-specific-parameter-less Rao algorithm in optimization of WDNs. The results obtained using GA, SA and Rao algorithms were identical and optimum for the TL network. For the GY network, best-known values were obtained and a 0.71% reduction in cost was observed using PDA-Rao compared to HS (Geem 2006). At the same time, results were significantly improved for the NYT network with a 1.7% reduction in best-known network cost. Moreover, the minimum cost solution for the real-world SPA network was found in 7785 function evaluations, which is negligible compared to the size of search space $(7.17 *10^{56})$.

In conclusion, the PDA-Rao algorithm was computationally inexpensive in solving problems with large search spaces. Significantly, the methodology is straightforward to understand and execute, shows better convergence and gives more accurate results with less computational effort. It can therefore be easily applied to other extensive WDN

optimization problems. From the statistical analysis, the probability of getting acceptable results using PDA-Rao algorithm is 0.77, 0.60, and 0.97 for TL, NYT and GY networks, respectively. It suggests that the probability of achieving the global best solution using PDA-Rao algorithm is extremely high. Further, this algorithmic rule can also be applied in multi-objective optimization of WDNs with slight modifications.

Data Availability Data, codes, and models used are available upon request.

Declarations

Competing Interests The authors have no conflicts of interest to declare which are relevant to the content of this article.

References

Abdy Sayyed MAH, Gupta R, Tanyimboh T (2019) Combined flow and pressure deficit-based penalty in GA for optimal design of water distribution network. ISH J Hydraul Eng 1–11. https://doi.org/10.1080/09715010.2019.1604180

Alperovits E, Shamir U (1977) Design of optimal water distribution systems. Water Resour Res 13(6):885–900. https://doi.org/10.1029/WR013i006p00885

Babu KJ (2021) Fictitious component free-pressure deficient network algorithm for water distribution network with variable minimum and required pressure-heads. Water Resour Manage 35:2585–2600. https://doi.org/10.1007/s11269-021-02852-0

Babu K, Vijayalakshmi D (2013) Self-adaptive PSO-GA hybrid model for combinatorial water distribution network design. J Pipeline Syst Eng Pract 4(1):57–67. https://doi.org/10.1061/(ASCE)PS.1949-1204.0000113

Chung G, Lansey K (2009) Application of the shuffled frog leaping algorithm for the optimization of a general large-scale water supply system. Water Resour Manage 23:797–823. https://doi.org/10.1007/s11269-008-9300-6

CPWD GOI (2019) CPWD Delhi Analysis of Rates (Civil Works) (Vol. 2). Jain Book Agency, New Delhi, India

Cunha M, Sousa J (1999) Water distribution network design optimization: simulated annealing approach. J Water Resour Plan Manag 125(4):215–221. https://doi.org/10.1061/(ASCE)0733-9496(1999)125:4(215)

Dandy G, Simpson A, Murphy L (1996) An improved genetic algorithm for pipe network optimization. Water Resour Res 32(2):449–458. https://doi.org/10.1029/95WR02917

Deep K, Bansal J (2009) Mean particle swarm optimization for function optimization. Int J Comput Intell Stud 1(1):72–92. https://doi.org/10.1504/IJCIStudies.2009.025339

Deep K, Thakur M (2007) A new mutation operator for real coded genetic algorithms. Appl Math Comput 193(1):211–230. https://doi.org/10.1016/j.amc.2007.03.046

Dong XL, Liu SQ, Tao T, Li SP, Xin KL (2012) A comparative study of differential evolution and genetic algorithms for optimizing the design of water distribution systems. J Zhejiang Univ Sci A 13(9):674–686. https://doi.org/10.1631/jzus.A1200072

Fallah H, Kisi O, Kim S, Rezaie-Balf M (2019) A new optimization approach for the least-cost design of water distribution networks: improved crow search algorithm. Water Resour Manage 33(10):3595–3613. https://doi.org/10.1007/s11269-019-02322-8

Geem Z (2006) Optimal cost design of water distribution networks using harmony search. Eng Optim 38(03):259–277. https://doi.org/10.1080/03052150500467430

Kadu M, Gupta R, Bhave P (2008) Optimal design of water networks using a modified genetic algorithm with reduction in search space. J Water Resour Plan Manag 134(2):147–160. https://doi.org/10.1061/(ASCE)0733-9496(2008)134:2(147)

Kim JH, Kim TG, Kim JH, Yoon YN (1994) A study on pipe network system design using non-linear programming. J Korean Water Resour Assoc 27(4):59–67

Klise K, Bynum M, Moriarty D, Murray R (2017) A software framework for assessing the resilience of drinking water systems to disasters with an example earthquake case study. Environ Model Softw 95:420–431. https://doi.org/10.1016/j.envsoft.2017.06.022

Liong SY, Atiquzzaman M (2004) Optimal design of water distribution network using shuffled complex evolution. J Inst Eng (singapore) 44(1):93–107

Lippai I, Heaney J, Laguna M (1999) Robust water system design with commercial intelligent search optimizer. J Comput Civ Eng 13(3):135–143. https://doi.org/10.1061/(ASCE)0887-3801(1999)13:3(135)

Maier H, Simpson A, Zecchin A, Foong W, Phang K, Seah H, Tan C (2003) Ant colony optimization for design of water distribution systems. J Water Resour Plan Manag 129(3):200–209. https://doi.org/10.1061/(ASCE)0733-9496(2003)129:3(200)

Moosavian N, Roodsari BK (2014) Soccer league competition algorithm: a novel meta-heuristic algorithm for optimal design of water distribution networks. Swarm Evol Comput 17:14–24. https://doi.org/10.1016/j.swevo.2014.02.002

Pankaj B, Naidu M, Vasan A, Varma MR (2020) Self-adaptive cuckoo search algorithm for optimal design of water distribution systems. Water Resour Manage 34:3129–3146. https://doi.org/10.1007/s11269-020-02597-2

Rao R (2020) Rao algorithms: three metaphor-less simple algorithms for solving optimization problems. Int J Ind Eng Comput 11:107–130. https://doi.org/10.5267/j.ijiec.2019.6.002

Rao RV, Pawar R (2020) Constrained design optimization of selected mechanical system components using Rao algorithms. Appl Soft Comput 89:106–141. https://doi.org/10.1016/j.asoc.2020.106141

Reca J, Martínez J, López R (2017) A hybrid water distribution networks design optimization method based on a search space reduction approach and a genetic algorithm. Water 9(11):845. https://doi.org/10.3390/w9110845

Rossman L, Woo H, Tryby M, Shang F, Janke R, Haxton T (2020) EPANET 2.2 User Manual. Washington, DC: U.S. Environmental Protection Agency

Sadollah A, Yoo DG, Kim JH (2015) Improved mine blast algorithm for optimal cost design of water distribution systems. Eng Optim 47(12):1602–1618. https://doi.org/10.1080/0305215X.2014.979815

Savic D, Walters G (1997) Genetic algorithms for least-cost design of water distribution networks. J Water Resour Plan Manag 123(2):67–77. https://doi.org/10.1061/(ASCE)0733-9496(1997)123:2(67)

Schaake J, Lai F (1969) Linear programming and dynamic programming applications to water distribution network design. Rep 116, Hydrodynamics Laboratory, Department of Civil Engineering, MIT: Cambridge, Mass

Sedki A, Ouazar D (2012) Hybrid particle swarm optimization and differential evolution for optimal design of water distribution systems. Adv Eng Inform 26(3):582–591. https://doi.org/10.1016/j.aei.2012.03.007

Sheikholeslami R, Talatahari S (2016) Developed swarm optimizer: a new method for sizing optimization of water distribution systems. J Comput Civ Eng 30(5):04016005. https://doi.org/10.1061/(ASCE)CP.1943-5487.0000552

Sirsant S, Reddy MJ (2021) Optimal design of pipe networks accounting for future demands and phased expansion using integrated dynamic programming and differential evolution approach. Water Resour Manage 35(4):1231–1250. https://doi.org/10.1007/s11269-021-02777-8

Suribabu CR (2010) Differential evolution algorithm for optimal design of water distribution networks. J Hydroinf 12(1):66–82. https://doi.org/10.2166/hydro.2010.014

Suribabu CR, Neelakantan TR (2006) Design of water distribution networks using particle swarm optimization. Urban Water J 3(2):111–120. https://doi.org/10.1080/15730620600855928

Tolson BA, Asadzadeh M, Maier HR, Zecchin A (2009) Hybrid discrete dynamically dimensioned search (HD-DDS) algorithm for water distribution system design optimization. Water Resour Res 45(12). https://doi.org/10.1029/2008WR007673

Wagner J, Shamir U, Marks D (1988) Water distribution reliability: simulation methods. J Water Resour Plan Manag 114(3):276–294. https://doi.org/10.1061/(ASCE)0733-9496(1988)114:3(276)

Wu Z, Simpson A (2002) A self-adaptive boundary search genetic algorithm and its application to water distribution systems. J Hydraul Res 40(2):191–203. https://doi.org/10.1080/00221680209499862