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A Modified Muskingum Flow Routing Model for Flood Wave Propagation during River Ice Thawing-Breakup Period

Wanlong Yang[1] • Jun Wang[1] • Jueyi Sui[2] • Fangxiu Zhang[3] • Baosen Zhang[3]

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Abstract
During the period of river ice thawing and breakup process (termed as "ice cover thawing-breakup"), vast amount of water stored in ice-covered river reach will be released comparing to that under open flow condition. The flow routing process during river ice thawing-breakup period will be different from that under open flow condition, since water stored in and channel from ice thawing-breakup process and flow routing process are very complicated. If the flow routing process during river ice thawing-breakup period can be predicted, it will very important for flood protection in the downstream river reach. In present study, water released from ice cover thawing process is considered as the lateral inflow to the channel flow during propagation process of flood wave from upstream to downstream. A model for the flood routing process during river ice thawing-breakup period has been developed based on the Muskingum hydrologic method. Using the modified Muskingum model, the routed outflow hydrograph has been determined along the Baotou Reach of the Yellow River during river ice thawing-breakup period. Results showed that the simulated hydrographs using developed model agree well with those of field measurements.

Keywords Channel-storage · Ice cover · Muskingum routing method · River ice thawing- breakup period

1 Introduction

In winter, with the presence of ice cover in northern rivers, the boundary condition of the channel flow becomes quite different from that under pure open channel condition (Note: the

✉ Jueyi Sui
jueyi.sui@unbc.ca

1 School of Civil and Hydraulic Engineering, Hefei University of Technology, Hefei, China

2 School of Engineering, University of Northern British Columbia, Prince George, Canada

3 Yellow River Institute of Hydraulic Research, Zhengzhou, China

channel flow during ice-running period is classified as a special case of open channel flow). An ice cover alters the hydraulics of an open channel by imposing an extra boundary to the flow, altering the flow velocity distribution and increasing water level compared to that under open channel flow condition. As showed in Fig. 1, under ice-covered condition, the maximum velocity is located between the channel bed and ice cover. This leads to differences in the hydraulic features. A major consequence of the appearance of ice-cover in many northern rivers is the increase in water level comparing to that under open flow condition with the same flow discharge (Beltaos 1995; Beltaos et al. 1996; Sui et al. 2005; Sui et al. 2010; Yapa and Shen 1986; Wang et al. 2019). Thus, during ice-covered period, much more water stored in covered channel comparing to that under open flow condition with the same flow discharge (Sui et al. 2005). Based on field observations of ice accumulation (ice jam) in the Hequ Reach of the Yellow River, Sui et al. (2005) studied the variations in water level under ice-covered condition. Figure 2 shows the impact of an ice cover on water level at the Toudaoguai gauge station on the Yellow River. Sui et al. (2005) pointed out that the thicker the ice cover, the higher the water level, and thus more water stored in the ice-covered channel.

The Baotou Reach of the Yellow River flows from Sanhuhe gauging station to Toudaoguai gauge station with a length of 292 km, as showed in Fig. 3.Due to its specific geomorphological conditions and hydro-meteorological features, this river reach is covered by ice from November to March. As a consequence, vast amount of ice and water stored in this river reach during ice-covered period, and water level increased a lot. During the period of ice cover thawing- breakup process, vast amount of water stored as ice in channel will be melted and released to downstream. In the mean time, water stored in channel caused by ice cover will also propagate to downstream. The discharge of water-ice mixture increases tremendously comparing to that under open flow condition. As a consequence, ice flooding occurs nearly every year along this river reach during river breakup period (Yao et al. 2007; Zhang et al. 2015). To assess flood routing process during ice cover thawing-breakup period, the Baotou Reach of the Yellow River is chosen in the present study.

The Muskingum routing method was originally developed for the flood control operation along the Muskingum River by McCarthy in 1934. On the basis of governing equations for unsteady flow, as pointed out by Cunge (1969), the Muskingum routing method is an approach for determining the relationship between the Muskingum parameters and the parameters of the convection-diffusion equation which is solved with the second order accuracy. Since the relationship between the flow discharge (Q) and channel storage (W) of a natural river is non-linear, Gill (1978) has optimized the Q~W curve and developed the non-linear

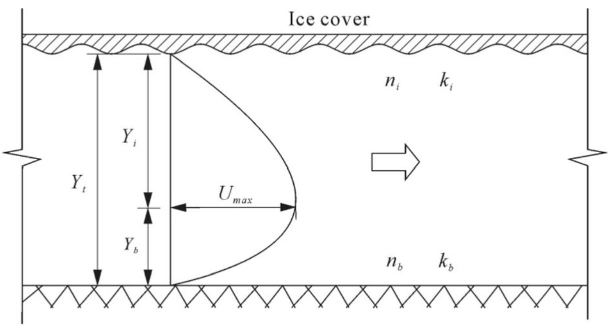

Fig. 1 Flow velocity profile under ice cover

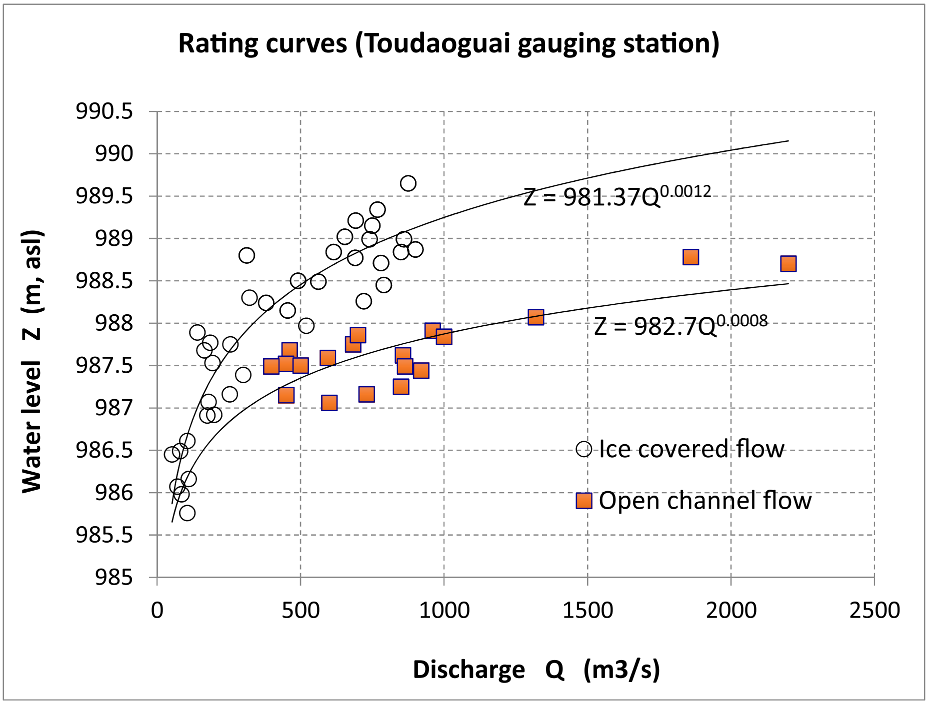

Fig. 2 Impact of ice on water level at the Toudaoguai gauging station of the Yellow River

Muskingum routing method. On the basis of the UK Flood Studies Report, O'Donnell (1985) modified the conventional Muskingum flood routing method by introducing the lateral inflows from tributaries to the main stream, and proposed a direct three-parameter Muskingum procedure. By increasing the "x" parameter in the conventional Muskingum routing method, a new Muskingum model has been extended for flood routing process along a river reach with multiple tributaries (Khan 1993).

Up to date, with respect to the flood-routing process under open channel flow condition, a lot of research results have been reported (Mohan 1997; Moghaddam et al. 2016; Niazkar and

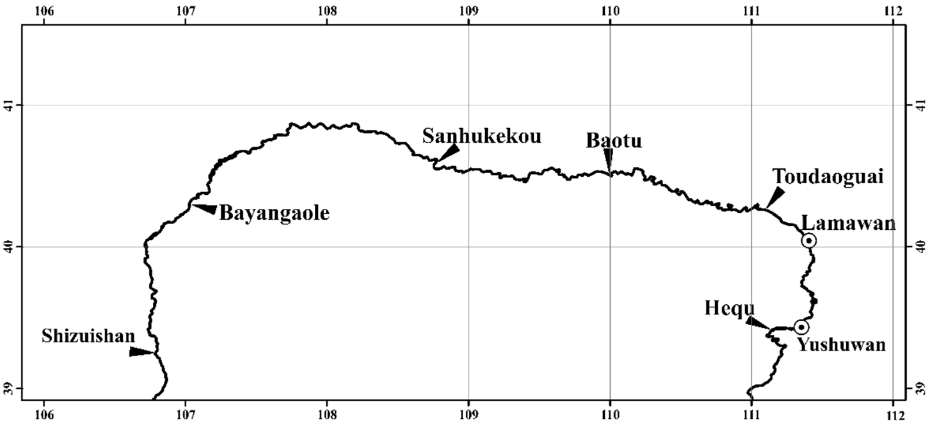

Fig. 3 The study Baotou Reach of the Yellow River

Afzali 2016). However, research work regarding flood-routing process during ice-covered period has been hardly conducted. Based on field measurements along the Ningxia-Inner Mongolia Reach of the Yellow River, Wang et al. (2018) considered the impacts of ice cover on the flood routing process. The Muskingum flood-routing method has been applied to assess the propagation of high flow during stable ice-covered period. The relationship between Muskingum parameters and roughness of ice cover has been studied. The impacts of the thickening process of ice cover on the flow routing process have been compared to those of the thawing process of ice cover. However, due to vast amount of water released from both ice cover-thawing process and channel storage, the flood routing process during ice cover thawing - breakup period has never been conducted.

Figure 4 shows the flood routing hydrograph along the Baotou-Toudaoguai Reach of the Yellow River under open flow condition in August 2016. Figure 5 shows the flood routing hydrograph along the same river reach during river ice thawing-breakup period in March 2016. The solid lines in Figs. 4 and 5 represent the upstream inflow hydrographs at the Baotou cross section (CS), and the dotted lines in Figs. 4 and 5 represent the downstream outflow hydrographs at Toudaoguai CS which is 150 km downstream of the Baotou CS. One can see from Fig. 4 that, under open flow condition, the peak flow of hydrograph at the upstream CS was flattened along the river reach and decreased significantly at the downstream Toudaoguai CS. However, during river ice thawing-breakup period, due to the release of vast amount of melted water from ice cover and channel storage caused by ice cover along this ice-covered river reach, the peak flow of the outflow hydrograph at Toudaoguai CS is much higher than that at the upstream Baotou CS. Namely, the outflow hydrograph at the downstream Toudaoguai CS has not been flattened during river ice thawing-breakup period. As a consequence, the downstream river reach may be flooded. Obviously, research work regarding flow routing process during river ice thawing-breakup period is very important for northern rivers.

In present study, water balance and channel storage along the Baotou Reach during river ice thawing-breakup period has been studied. Based on the modified Muskingum flood-routing

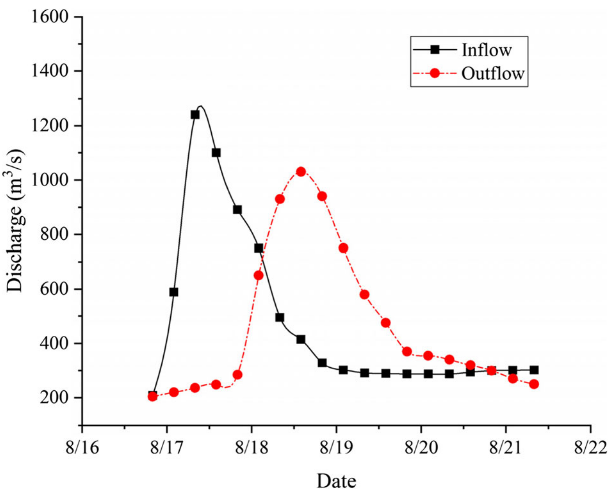

Fig. 4 Flood routing hydrograph under open-flow condition in August (inflow at Baotou CS, and outflow at Toudaoguai CS)

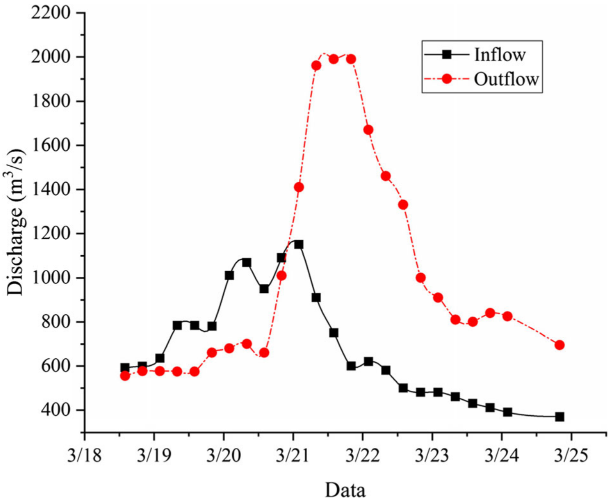

Fig. 5 Flood routing hydrograph during ice thawing-breakup period in March (inflow at Baotou CS, and outflow at Toudaoguai CS)

method considering the lateral inflow from tributaries (O'Donnell 1985), a model for flood-routing process during river ice thawing-breakup period has been developed. Using this modified Muskingum model for flood-routing process during river ice thawing-breakup period, the routed outflow hydrograph has been determined along the Baotou Reach of the Yellow River.

2 Basic Theory of the Conventional Muskingum Routing Method

Flow routing is the process of converting a hydrograph that passes through some part of a flow system. Channel routing, which caused the changes in hydrographs as the flow passes along river reaches, caused by variations in the channel geometry which result in storage effects. A widely used hydrologic method for routing flows in conveyance systems is the Muskingum method. Research work regarding this method has made a lot of progress in recent years (Bao et al. 2007; Li et al. 2012; Ostad-Ali-Askari and Shayannejad 2016).

For hydrologic routing, the relationship between the inflow rate $I(t)$, outflow rate $O(t)$, and storage W can be described by the water balance equation:

$$\frac{dW}{dt} = I(t) - O(t) \tag{1}$$

The total storage can be described as follows:

$$W = K\{xI + (1-x)O\} \tag{2}$$

Where, K is a coefficient of proportionality, also called the storage coefficient. The parameter K is the time of travel of the flood wave through the reach (has the dimension of time). The weighting factor (x) which has the range of $0 \leq x \leq 0.5$, is an indicator of river channel regulation effect that represents the degree of flattening during flood wave propagation. The value of the weighting factor (x) depends on the shape of the modelled wedge storage.

According to the Muskingum routing method, the routed outflow is described as follows:

$$O_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 O_j \tag{3}$$

where

$$\begin{cases} c_1 = \dfrac{\Delta t - 2Kx}{2K(1-x) + \Delta t} \\ c_2 = \dfrac{\Delta t + 2Kx}{2K(1-x) + \Delta t} \\ c_3 = \dfrac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t} \end{cases} \tag{4}$$

In which, I_j and I_{j+1} stand for the initial and end inflow rates during the j^{th} time interval of Δt, respectively; O_j and O_{j+1} stand for the initial and end outflow rates during the j^{th} time interval of Δt, respectively.

3 Flood Routing Model for a River Reach during River Ice Thawing-Breakup Period

3.1 Model Setup

Under open channel flow condition, both the peak flow and shape of the hydrograph will be flattened during the propagation process of a high flow from upstream to downstream. During river ice thawing-breakup period, however, vast amount of water will be released due to the thawing process of ice cover in addition to the release of huge amount of stored water in channel caused by ice cover. As a consequence, the peak flow of the outflow hydrograph at the downstream CS is much higher than that of the inflow hydrograph at the upstream CS. As showed in Fig. 5, the flow routing process during river ice thawing-breakup period is completely different from that under open channel flow condition.

During river ice thawing-breakup period, water released from the thawing process of ice cover is considered as inflow, then, the water balance equation can be written as follows,

$$(I_w + I_m)\Delta t = O\Delta t = \Delta W \tag{5}$$

In which, I_w and I_m represent the channel inflow rate from the upstream river reach and inflow rate due to ice thawing-breakup process (m^3/s), respectively; ΔW is the change of storage of water along the river reach (m^3). The inflow rate due to ice cover thawing-breakup process can be expressed as a ratio of the channel inflow rate from upstream CS, namely, $\alpha = I_m/I_w$. Note: subscript "*w*" represents flowing water from upstream CS, and subscript "*m*" represents the water released from ice cover thawing-breakup process. Thus, the total inflow rate can be described as follows,

$$I_w + I_m = (1 + \alpha) I_w \tag{6}$$

Then, Eq. 5 can be described as follows,

$$(I_w + I_m)\Delta t - O\Delta t = (1 + \alpha) I_w \Delta t - O\Delta t = \Delta W \tag{7}$$

On the basis of the modified Muskingum flood-routing model for introducing the lateral inflows from tributaries to the main stream (O'Donnell 1985), equations for describing water balance and channel storage during river ice thawing-breakup period have been derived as the follows,

$$\begin{cases} (1+\alpha)(I_n + I_{n+1})\dfrac{\Delta t}{2} - (O_n + O_{n+1})\dfrac{\Delta t}{2} = W_n - W_{n+1} \\ W = K\{x(1+\alpha)I + (1-x)O\} \end{cases} \tag{8}$$

In which, I stands for inflow rate (m^3/s); O stands for outflow rate (m^3/s); K is a coefficient of proportionality; x is the weighting factor.

Equation 8 can be described as the following finite difference scheme,

$$O_{j+1} = A_1 I_j + A_2 I_{j+1} + A_3 O_j \tag{9}$$

where,

$$\begin{cases} A_1 = \dfrac{\Delta t + 2Kx}{2K(1-x) + \Delta t}(1+\alpha) \\ A_2 = \dfrac{\Delta t - 2Kx}{2K(1-x) + \Delta t}(1+\alpha) \\ A_3 = \dfrac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t} \end{cases} \tag{10}$$

3.2 Coefficients of the Modified Muskingum Flow Routing Model during River Ice Thawing-Breakup Period

3.2.1 Inflow Ratio Rate Due to Ice Cover Thawing-Breakup Process to the Channel Inflow Rate from Upstream CS (α)

The inflow ratio due to inflow rate from ice cover thawing-breakup process to the channel inflow rate from upstream CS is defined as: $\alpha = I_m/I_w$. This coefficient describes the amount of water released during the ice thawing-breakup process. To determine this ratio (α), the water balance equation should be solved using the trial-and-error method.

3.2.2 Weighting Factor x

According to ECIWR (1978), the weighting factor x can be determined using the following formula,

$$x = x' - \frac{l}{2L} \tag{11}$$

where, L is the length of the river reach for flow routing (m); l is the characteristic length of the river (m); x' is the coefficient represents the shape of water surface profile, namely, the water surface profile of the wedge storage (above the prism storage). The weighting factor (x) ranges from 0 to 0.5. This coefficient says something about how inflow and outflow vary within a given river reach, namely the extent of flattening of the outflow hydrograph. With the increase in the weighting factor, the extent of flattening of the outflow hydrograph becomes weak. When $x = 0.5$, the inflow hydrograph at the upstream CS is the same as the outflow hydrograph at the downstream CS. For flow in natural rivers under open channel flow condition, this type of flow propagation process with a weighting factor of $x = 0.5$ will not happen. However, due to the release of vast amount of melted water from ice cover and channel storage caused by ice cover during river ice thawing-breakup period, instead of generating a flattened hydrograph at the downstream CS, a narrow sharp-crested hydrograph has been produced. Also, the peak flow of the routed hydrograph at the downstream CS is much higher than that at the upstream CS, as showed in Fig. 5. Thus, the range for the weighting factor (x) of $0 < x < 0.5$ is not applicable for the flow routing process during river ice thawing-breakup period.

3.2.3 Storage Coefficient K

For unsteady flow, the outflow rate O_i of a sub-reach can be expressed as a function of the inflow rate I_i of this sub-reach, namely, $O_i = b_i I_i$. Where, coefficient "b_i" represents the feature of propagation of flood wave along this sub-reach. Assuming the coefficient "b_i" for all sub-reaches to be the same, then,

$$O = O_w + O_m = b(I_w + I_m) \tag{12}$$

Similarly, the weighted outflow can be described as follows,

$$O_i = xI_i + (1-x)bI_i \tag{13}$$

Thus,

$$\begin{aligned} W &= K_w Q_w + K_m Q_m \\ &= K_w[xI_w + (1-\mathrm{x})bI_w] + K_m[xI_m + (1-\mathrm{x})bI_m] \\ &= [x(I_w + I_m) + (1-x)O]\left[\frac{K_w I_w + K_m I_m}{I_w + I_m}\right] \\ &= K[x(I_w + I_m) + (1-x)O] \\ &= K[x(1+\alpha)I_w + (1-x)O] \end{aligned} \tag{14}$$

Here, I_i is used as the weighting factor for determining the average value of K for a river reach with ice, namely, $K = (K_w I_w + K_m I_m)/(I_w + I_m)$.

3.3 Determination of Parameters

For the conventional Muskingum flow routing method (without the lateral inflow), the relationship between Q and W will be developed first. Then, parameters (K and x) will be determined using trial-and-error method. Afterward, coefficients C_1, C_2 and C_3 will be calculated using Eq. 4. Then, the routing hydrograph at the downstream CS will be calculated through the iterative calculation process. Comparing to the conventional Muskingum routing method, an extra parameter of α is introduced in the modified Muskingum model for flood routing along a river reach with river ice thawing-breakup process. There are three unknowns in the modified Muskingum model, namely, K, x, and α. These three unknowns cannot be determined directly as pointed out by O'Donnell (O'Donnell 1985). In present study, the relationship between Q and W will be developed first. To estimate these three unknowns (K, x, α), the trial-and-error method will be used to determine them reversely. Afterward, coefficients A_1, A_2 and A_3 in the finite difference scheme (Eq. 9) will be analyzed first, and calculated using Eq. 10. Then, the routing hydrograph at the downstream CS will be calculated through the iterative calculation process.

Regarding the determination of these parameters along a river reach using the least squares method, if the calculated results are reasonable, these calculated parameters can be further optimized for determining the routing hydrograph using the Muskingum model. However, for the flood routing process during the ice-cover thawing-breakup period, the calculation process of the parameters (K, x, α) is more complicated. Parameters (K, x, α) determined directly using the optimization method might not have the same characteristics as those of the actual parameters (K, x, α) in the conventional Muskingum model. Thus, these parameters (K, x, α) should be determined using the trial-and-error method.

For the conventional Muskingum flow routing method (without the lateral inflow), the linear relationship between Q and W can be obtained by changing the weighting factor (x). However, for the modified Muskingum model considering river ice thawing-breakup process, the parameter α has to be determined first. Then, the relationship between Q and W can be obtained by changing the weighting factor (x). Also, due to the significant change in the amount of water released during the ice thawing-breakup process, the calculated parameters after the ice thawing-breakup process are clearly different from those before the ice thawing-breakup process. Several sets of parameters are normally used for calculating the routed outflow hydrograph using the trial-and-error method. Without doubt, this kind of calculation process using the trial-and-error method is more difficult than that of the conventional Muskingum method. However, this new attempt for calculating outflow at the downstream CS is feasible. Following steps one should follow,

1). Set the initiation-time of the thawing-breakup process as the start-time for the calculation. Based on water balance equation, the ice thawing coefficient before the ice cover thawing-breakup process (α_1) and that of after the ice thawing-breakup process (α_2) are calculated;
2). Select the initial weighting factor before the initiation of the ice thawing-breakup process (x_1) and that of after the initiation of the ice thawing-breakup process (x_2). Afterward, the weighted flow (Q) can be determined using following equation: $Q = x(1+\alpha)I+(1-x)O$;
3). Then, the relationship between the weighted flow (Q) and storage (W) can be obtained, and one can plot the curves: $W \sim f_1(Q)$ and $W \sim f_2(Q)$, respectively;
4). For obtaining the solely linear relationship between the weighted flow (Q) and storage (W), both weighting factors (x_1) and (x_2) will be adjusted. Once a linear relationship between the weighted flow (Q) and storage (W) is obtained, both weighting factors (x_1) and (x_2) will be chosen for further calculation;
5). Calculate the storage coefficient K_1 (before the initiation of the ice thawing-breakup process) and K_2 (after the initiation of the ice thawing-breakup process). Note: in the $W \sim f(Q)$ linear relationship, K stands for the slope, $K = W/Q$;
6). Now, 2 sets of coefficients (K_1, x_1, α_1) and (K_2, x_2, α_2) are obtained. Then, two iterative formulas will be obtained using Eq. 9. Afterward, the outflow hydrograph at the downstream CS can be determined based on the inflow hydrograph at the upstream CS.

4 Example of Model Application

The Baotou Reach of the Yellow River between Sanhuhe gauging station and Toudaoguai gauging station is located in the northernmost section of the Yellow River in Inner Mongolia Province. The Baotou gauging station is located 150 km upstream of Toudaoguai station, as showed in Fig. 3. The distance from Sanhuhe gaging station to Baotou station is 142 km. In present study, using the modified Muskingum flow routing model, the flood routing process during the ice-cover thawing-breakup period has been investigated along the Baotou Reach. Field measurements indicate that ice-flowing along this river reach started on 26th November 2015. On 17th December 2015, this river reach was frozen and became ice-covered flow. On 21st March 2016, this ice-covered river reach became open channel flow. Ice-covered flow lasted 95 days. According to Zhang et al. (2015), the average ice cover thickness along this river reach is 0.59 m, while the maximum ice cover thickness is 1.00 m. The average width of the main channel of this river reach is 660 m, and the average with of the flood plain is 3800 m. As reported by Zhang et al. (2015), the maximum amount of water stored in this river reach during ice-covered period is 5.8×10^8 m^3. The maximum discharge during river breakup period in 2015 is 1990 m^3/s.

The modified Muskingum model for flow routing along a river reach with the ice cover thawing- breakup process was applied to calculate routed hydrographs at the downstream CS. The trial-and-error method was used to determine the coefficients and parameters which were compared to those using both the least squares method (Khan 1993) and the genetic algorithms method (Mohan 1997).

Figure 6 shows the calculated results using the least squares method and genetic algorithms method compared to results of measurement at Toudaoguai gaging station. Figure 7 shows the *W~f(Q)* linear relationship using the trial-and-error method. Figure 8 shows the calculated flood routing hydrograph at Toudaoguai gaging station using the modified Muskingum routing model in March 2016, compared to results of measurement. One can see that the calculation results using the modified Muskingum model agree well with results of measurements at Toudaoguai gaging station, comparing to those using both the least squares method and the genetic algorithms method.

Also, from the calculation results using the modified Muskingum routing model, one can see that there exists two different types of flow routing process due to effects of the different thawing-breakup process of ice cover, namely, the “shorter lag time” routing process and “longer lag time” routing process. Figure 9 shows the “shorter lag time” routing process which has a shorter lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph. For this type of flood routing process, in addition to the fast thawing-breakup process of ice cover due to a quick increase in temperature, flow with high intensity will speed up the thawing-breakup process of ice cover. During flood wave propagation process from upstream to downstream, more ice cover will get melt and broken. As a consequence, more and more water due to the thawing-breakup process of ice cover will be released. Thus, the propagation speed of flood wave is faster, and the lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph is shorter. Also, the calculated hydrograph at the downstream CS using the modified Muskingum model agrees well with the result of measurement, as showed in Fig. 9. Figure 10 shows the “longer lag time” routing process which has a longer lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph. For this type of flood routing process, the thawing-breakup process of ice cover is relatively slow, since the flow intensity and the increase in temperature are not high enough to cause a faster thawing-breakup process of ice cover. During the

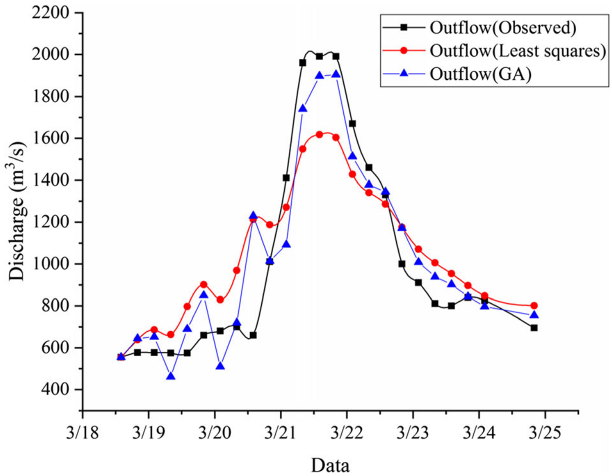

Fig. 6 Routed hydrograph using the least square and genetic algorithm method (Toudaoguai CS)

flood wave propagation process from upstream to downstream, ice cover will get melt slowly comparing to that of "shorter lag time" routing process. Also, since ice cover-thawing process is slow, the propagation process of flood wave will be slowed down due to the existence of ice cover. Thus, the lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph is longer. The dramatic increases in the peak flow of the outflow hydrograph at the downstream CS is resulted from the release of vast amount of water stored along the ice-covered river reach together with the propagated flood wave from upstream. The "longer lag time" routing process during ice cover thawing-breakup process is more complicated than the "shorter lag time" routing process. The relationship between the inflow at the upstream and the outflow at the downstream CS is not obvious, especially for the flood routing process along this long river reach.

5 Conclusions

In present study, by considering the melted water from ice cover during river ice thawing-breakup process as the lateral inflow for flood routing process, the flow routing process during river ice thawing-breakup period have been studied. The trial-and-error method is used to

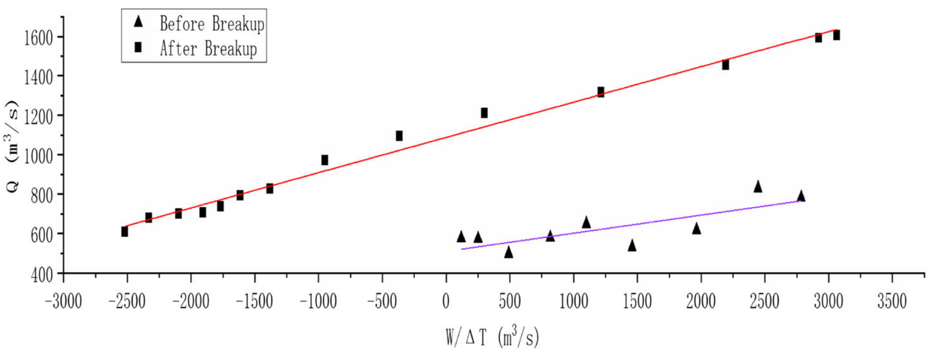

Fig. 7 Q~W linear relationship before and after the thawing-breakup process of ice cover (Toudaoguai CS)

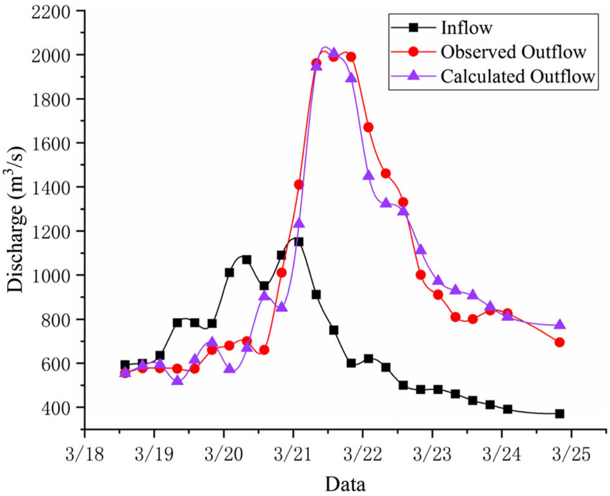

Fig. 8 Observed and routed hydrographs along the Baotou-Toudaoguan reach in March 2016 (Toudaoguai CS)

determine the parameters (K, x, and α) for the modified Muskingum routing model during river ice thawing-breakup process. Since the hydraulic condition of flow routing during river ice thawing-breakup period changes dramatically and is completely different from that under open channel flow condition, several sets of parameters (K, x, and α) will be used for determining the routed outflow hydrograph using the modified Muskingum flow routing model. Calculation results using the modified Muskingum routing model agree well with the measured hydrographs in 2014 and 2016 along the Baotou Reach of the Yellow River during river ice thawing-breakup period. Results also showed that, during river ice thawing-breakup period, if flow intensity is high together with a quick increase in temperature, ice cover will get melt faster and the propagation of flood wave from upstream to downstream is faster. As a consequence, the lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph is shorter. However, if flow intensity is not high enough and

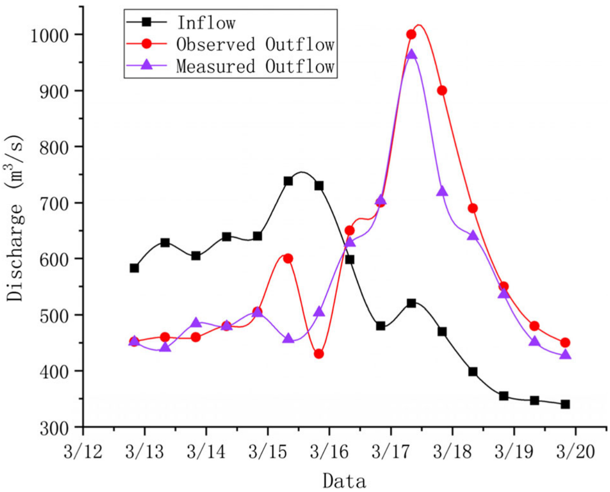

Fig. 9 Observed and routed hydrographs along the -Baotou Reach in March 2014

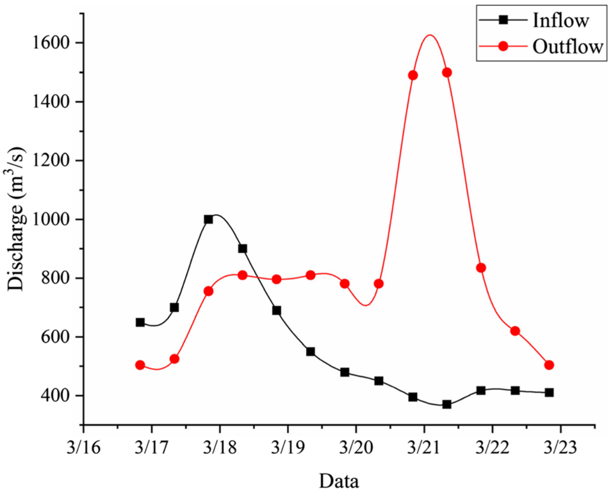

Fig. 10 Routed hydrographs using the modified Muskingum model along the Baotou-Toudaoguan reach in March 2014

the temperature doesn't increase a lot, ice cover will get melt gradually and the propagation of flood wave from upstream to downstream is slower. In this case, the lag time between the peak flow of the inflow hydrograph and that of the routed outflow hydrograph is longer. Overall, the modified Muskingum routing model for flood routing during river ice thawing-breakup process is applicable for this river reach. However, for routing process with relatively high flow intensity and "shorter lag time", the simulated results using the modified Muskingum routing model agree very well with the measured results. This research work will benefit river engineers for flood protection during river ice breakup period.

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References

Bao H, Li Z, Wang L (2007) Study of flood forecasting for upper reaches of the Huihe River above Lutaizi. J China Hydraul Eng 2007(S1):445–453

Beltaos S (ed) (1995) River ice jams. Water Resources Publications, U.S.A.

Beltaos S, Burrell B, Ismail S (1996) 1991 ice jamming along the Saint John River: a case study. Can J Civ Eng 23:381–394

Cunge JA (1969) On the subject of a flood propagation computation method. J Hydraul Res 7(2):205–230

East China Institute of Water Resources (ECIWR) (1978) Flood forecasting methods for humid areas in China. China Water and Power Press: 81–85

Gill MA (1978) Flood routing by the Muskingum method. J Hydrol 36(3):353–363

Khan MH (1993) Muskingum flood routing model for multiple tributaries. Water Resour Res 29(4):1057–1062

Li K, Fu L, Hu Y, Lu Y (2012) Determination of the value ranges of parameters C_0, C_1, and C_2 of Muskingum method. South-to-North Water Divers Water Sci Technol 10(5):43–45

Moghaddam A, Behmanesh J, Farsijani A (2016) Parameters estimation for the new four-parameter nonlinear muskingum model using the particle swarm optimization. Water Resour Manag 30(7):2143–2160

Mohan S (1997) Parameter estimation of nonlinear Muskingum models using genetic algorithm. J Hydraul Eng. https://doi.org/10.1061/(ASCE)0733-9429(1997)123:2(137)
Niazkar M, Afzali SH (2016) Application of new hybrid optimization technique for parameter estimation of new improved version of Muskingum model. Water Resour Manag 30(13):4713–4730
O'Donnell T (1985) A direct three-parameter Muskingum procedure incorporating lateral inflow. Hydrol Sci J 30(4):479–496
Ostad-Ali-Askari K, Shayannejad M (2016) Flood routing in rivers by Muskingum's method with new adjusted coefficients. Int Water Technol J 6(3):189–194
Sui J, Karney B, Fang D (2005) Variation in water level under ice-jammed condition -field investigation and experimental study. Nord Hydrol 36(1):65–84
Sui J, Wang J, He Y, Krol F (2010) Velocity profiles and incipient motion of frazil particles under ice cover. Int J Sediment Res 25(1):39–51
Wang K, Wang J, Sui J (2018) Deformation of flood wave during ice river period in the Ningxia-inner Mongolian reach of the Yellow River. J China Hydraul Eng 49(7):869–876
Wang J, Wu Y, Sui J, Bryan K (2019) Formation and movement of ice accumulation waves under ice cover–an experimental study. J Hydrol Hydromech 67(2):171–178
Yao H, Qin F, Shen G, Dong X (2007) Ice regime characteristics in the Ningxia Inner Mongolia reach of Yellow River. Adv Water Sci 18(6):893–899
Yapa PD, Shen HT (1986) Unsteady flow simulation for an ice-covered river. J Hydraul Eng 112(11):1036–1049
Zhang F, Xi G, Zhang X, Wang G, Huang R (2015) Simulation of channel-storage increment process in ice flood period. Adv Water Sci 26(2):201–211

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