An Event-Based Sediment Yield and Runoff Modeling Using Soil Moisture Balance/Budgeting (SMB) Method



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Abstract

The Soil Conservation Service Curve Number (SCS-CN) method is frequently used for the estimation of direct surface runoff depth from the small watersheds. Coupling the SCS-CN method with the Soil Moisture Balance (SMB) method, new simple 2-parameters rainfall-runoff model and 3-parametrs rainfall-sediment yield models are derived for computation of runoff and sediment yield respectively. The proposed runoff (R2) and sediment yield (S2) models have been tested on a large set of rainfall-runoff and sediment yield data (98 storm events) obtained from twelve watersheds from different land use/land cover, soil and climatic conditions. The improved runoff (R2) and sediment yield (S2) models show superior results as compared to the existing Mishra et al. (S1) and original SCS-CN (R1) models. The results and analysis justify the use of the proposed models for field applications.

Keywords Sediment yield model · Rainfall-runoff model · SMB · Watershed

1 Introduction

Estimation of runoff and sediment yield is of paramount importance in water resources, environmental engineering and hydrology. The estimates of these variables are mainly required for assessing the water resources, planning of soil and water conservation structures, and for assessing the impact of climate change on watershed output (Mishra and Singh, 1999). The

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runoff and sediment yield are affected by various watershed characteristics including soil types, land use/land cover and the hydrologic condition of the watershed.

The sediment yield modeling is a complex exercise as compared to runoff modeling. The process of sediment yield involves detachment of soil particles by rainfall and runoff and transportation of the detached particles by overland flow. Therefore, rainfall being the dominant factor affecting the soil erosion, the sediment yield is also governed by the runoff, among other factors (Ekern., 1953; Barnett and Rogers., 1966; Greer., 1971).

Despite extensive studies on the erosion process and sediment transport modeling, there exists a lack of universally accepted sediment yield formulae (Bogardi et al., 1986; Kothyari et al., 1996). Physically-based model have been developed in a coupled structure combining the component processes of detachment, transport and deposition with a rainfall-runoff model (Knisel, 1980; Leonard et al. 1987; Rode and Frede, 1997). However, these models require a large number of parameters which restricts their use for field applications in data scarce watersheds due to large input data requirement, uncertainty in specifying the parameters values, and the difference between the scales of the application, i.e., a catchment versus a field (Hadley et al. 1985; Tien et al. 1993; Kothyari and Jain, 1997).

The Soil Conservation Service Curve Number (SCS-CN) is widely used for estimation of direct runoff volume from the watersheds. The 1-parameter SCS-CN method, also referred to as Natural Resource Conservation Service Curve Number (NRCS-CN) method, was developed by USDA-ARS in 1954. This method being simple gives results with realistic accuracy and hence it is a widely used model for runoff computation. (Verma et al., 2017; Sahu et al. 2012; Tyagi et al. 2008; Mishra et al. 2006a, b; Garen and Moore, 2005; Chong and Teng., 1986; Wood and Blackburn., 1984; Williams and LaSeur., 1976; Ragan and Jackson., 1980; Hjelmfelt., 1980; and Hawkins., 1973). SCS-CN model can be applied to large watersheds with multiple land uses such as urban, forest and agricultural watershed (Singh, 1988). Mishra et al. (Mishra et al. 2006b) developed sediment yield model based on the SCS-CN method. The initial soil moisture, however, plays an important role in restructuring of the SCS-CN method as it prevents the unreasonable sudden jump in runoff and sediment yield estimation. The concept of soil moisture accounting (SMA) procedure leads to improvement in SCS-CN based models (Singh et al. 2015).

The main objectives of present paper is to (i) develop simple rainfall-runoff and sediment yield models incorporating the soil moisture balance approach with SCS-CN method (ii) apply the developed models on a large data set of rainfall-runoff-sediment yield from natural watersheds, and (iii) assess the performance and their suitability in simulating the runoff and sediment yield.

2 Existing NRCS-CN Model

The SCS-CN method couples the water balance equation (Eq. 1) with two fundamental hypothesis given by Eqs. (2) and (3) respectively, mathematically expressed as:

$$\mathbf{P} = \mathbf{I}_{\mathbf{a}} + \mathbf{F} + \mathbf{Q} \tag{1}$$

$$\frac{Q}{P-I_a} = \frac{F}{S}$$
(2)

$$I_a = \lambda S \tag{3}$$

where, P is the rainfall (mm), Q is the direct surface runoff (mm), F is the cumulative infiltration (mm), I_a is the initial abstraction (mm), S is the potential maximum retention (mm), and λ is the initial abstraction coefficient. Coupling of Eqs. (1) and (2) leads to the SCS-CN method.

$$Q = \left\{ \frac{\left(P - I_a\right)^2}{P - I_a + S}; P \ge I_a \right. \tag{4}$$

Q = 0; (Otherwise)

Coupling of Eq. (4) with Eq. (3) for $\lambda = 0.2$ enables determination of S from the rainfallrunoff data. In practice, S is derived from a mapping equation expressed in terms of curve number (CN).

$$S = \frac{25400}{\text{CN}} - 254 \tag{5}$$

The non-dimensional CN is derived from the tables given in the National Engineering Handbook, Section-4 (NEH-4) (SCS, 1956) for catchment characteristics, such as land use, types of soil, antecedent moisture condition (AMC). The CN values theoretically varies from 0 to 100 but for practical design purpose it varies from 40 to 98 (Van Mullen 1989, Mishra and Singh., 2002). The higher the CN value, the greater the runoff factor, C, or runoff potential of the watersheds, and vice versa (Sahu et al. 2012; Sahu et al. 2010; Ajmal et al. 2015; Singh et al. 2008; Singh et al. 2015; Shi et al. 2009).

2.1 Mishra Et al. (Mishra et al. 2006b) Model

The Mishra et al. (Mishra et al. 2006b) model is based on SCS-CN method for computation of sediment yield from natural watersheds. The model is developed by coupling three hypotheses, (i) the runoff coefficient, C (= Q/P) equals the degree of saturation, Sr (= $F/S = V_w/V_v$), (where, V_w is the volume of water, V_v is the volume of void), (ii) the relationship between potential maximum retention, S and Universal Soil Loss equation (USLE), expressed mathematically as, $S = n(1 - S_{ro})/(1 - n)\rho_s RKSLCP$, (where n = soil porosity, S_{ro} = initial degree of saturation, ρ_s = density of solids, and the terms R,K, L,S, C and P are same as in USLE), and (iii) the sediment delivery ratio (DR) is equal to the runoff coefficient (C) i.e. DR = C. The sediment yield model developed by Mishra et al. (Mishra et al. 2006b) is mathematically expressed as,

$$Y = A \frac{(P-0.2S)}{P+0.8S}$$
(6)

where Y, is the sediment yield (KN) A is the potential maximum erosion (KN/ha),

3 Proposed Rainfall-Runoff and Sediment Yield Models

The proposed model is based on first hypothesis of traditional CN model which is,

$$\frac{Q}{P-I_a} = \frac{F}{S}$$
(7)

Equation (7) shows that the ratio of actual runoff to potential runoff is equal to the ratio of actual retention to potential retention. Incorporating static and dynamic infiltration components in Eq. 7 (Mishra and Nema, 1998; Mishra and Singh, 2003; Tyagi et al. 2008), it can be rewritten as,

$$\frac{Q}{P-I_a-F_c} = \frac{F_d}{S}$$
(8)

where, the sum of the dynamic portion of infiltration (F_d , occurring mainly due to capillary) and static potion of infiltration (F_c , occurring largely due to gravity) yields F. Simplification of Eq. 8 yields,

$$Q = \frac{(P - I_a - F_c)^2}{P - I_a - F_C + S}$$
(9)

where Q is the direct surface runoff (mm) and I_a is the initial abstraction. Assuming initial abstraction (I_a) equal to zero, Eq. 9 yields,

$$Q = \frac{\left(P - F_c\right)^2}{P - F_c + S} \tag{10}$$

Differentiating Eq. 10 with respect to time, t, yields

$$q = \frac{dP}{dt} \frac{(P-F_c)^2}{P-F_c+S}$$
(11)

After simplification of Eq. (11), we get,

$$q = p \frac{(P-F_c)(P-F_c+2S)}{(P-F_c+S)^2}$$
 if $P > F_c$ (12)

q = 0 for $P \le F_c$, where q = dQ/dt, p = dp/dt. Soil moisture accounting (SMA) procedure is based on the notion that higher the moisture store level, higher the fraction of rainfall that is converted into runoff. If the moisture store level is full, all the rainfall become runoff (Michel et al. 2005). The moisture store level depends on the numerous properties of watershed such as soil, hydrological condition, land use/land cover, hydraulic conductivity, porosity and void ratio.

The SMA model can be an analytically expressed as,

$$V = V_0 + P - Q \tag{13}$$

where, V is the soil moisture storage at any time t during a storm event, P is the accumulated rainfall up to the time t, Q is the corresponding runoff, and V_0 is the initial soil moisture. Differentiating Eq. 13 with respect to time t,

$$\frac{\mathrm{dV}}{\mathrm{dt}} = \mathbf{p} - \mathbf{q} \tag{14}$$

Substituting Eq. (10) into Eq. (13) yields

$$V = V_0 + P - \frac{(P - F_c)^2}{P - F_c + S}$$
(15)

Simplification of Eq. (15) yield

$$V = V_0 + \frac{P(S + F_c) - Fc^2}{P - F_c + S}$$
(16)

Mathematical interpretation from Eq. (16) for $(P-F_c)$, $(P-F_c+2S)$ and $(P-F_c+S)^2$ Substituting into Eq. (12) yields

$$p\frac{(P-F_c)(P-F_c+2S)}{(P-F_c+S)^2} = \frac{\{V-(V_0+F_c)\}}{S} \left[2 - \frac{V-(V_0+F_c)}{S}\right]$$
(17)

Coupling of Eq. (10), Eq. (12) and Eq. (13), where $V' = V_0 + F_c$, it is mathematically expressed as

$$q = p \frac{V - V'}{S} \left[2 - \frac{V - V'}{S} \right] \quad \text{if } V > V' \tag{18}$$

$$q = 0$$
 otherwise

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}t} \left[1 - \left[\frac{V - V'}{S} \right] \right]^2 \tag{19}$$

Simplification of Eq. (19) we get

$$\frac{\mathrm{dV}}{\left(\frac{\mathrm{V}-\mathrm{S}-\mathrm{V}'}{\mathrm{S}}\right)^2} = \frac{\mathrm{dp}}{\mathrm{dt}}\,\mathrm{dt} \tag{20}$$

Mathematical interpretation of Eq. (20) yields

$$\frac{\mathrm{dV}}{\left[\mathrm{V}-\mathrm{S}-\mathrm{V}'\right]^2} = \frac{\mathrm{pdt}}{\mathrm{S}^2} \tag{21}$$

Integration of Eq. (21) with respect to time t and using upper (V, V₀) and lower limit (t, 0), yields

$$\int_{V_0}^{V} \frac{dV}{\left[V - S - V'\right]^2} = \int_{0}^{t} \frac{Pdt}{S^2}$$
(22)

After integration of Eq. (22) yields

$$\frac{1}{(S-V-V')} - \frac{1}{S-V-V_0} = \frac{P}{S^2}$$
(23)

Replacing V by $(V_0 + P - Q)$ from Eq. (23) yield

$$\frac{1}{\left(S - (V_0 + P - Q) - V'\right)} - \frac{1}{S - (V_0 + P - Q) - V_0} = \frac{P}{S^2}$$
(24)

Simplification of Eq. (24) yield

$$Q = P \left[1 - \frac{(S + V' - V_0)^2}{S^2 + P(S + V' - V_0)} \right]$$
(25)

The mathematical formulation of model can be summarized by the following set of model and their relevant hypothesis:

(i) if
$$(V_0 + P) \leq V'$$
 then $Q = 0$ (26)

(ii) if $(V_0 + P) > V'$ then

$$Q = P \left[1 - \frac{(S + V' - V_0)^2}{S^2 + P(S + V' - V_0)} \right]$$
(27)

Substituting $V' = V_0 + F_c$ into Eq. (27) yields

$$Q = P \left[1 - \frac{(S + V_0 + F_c - V_0)^2}{S^2 + P(S + V_0 + F_c - V_0)} \right]$$
(28)

Simplification of Eq. (28) yield

$$Q = P \left[1 - \frac{(S + F_c)^2}{S^2 + P(S + F_c)} \right]$$
(29)

where, F_C is the static infiltration is directly proportional to minimum infiltration and storm duration. Eq. (29) is the proposed rainfall-runoff model. The static infiltration can be calculated using the equation proposed by Tyagi et al. (2008) and Sahu et al. (2012)

$$F_{\rm C} = f_{\rm c} T \tag{30}$$

where f_c is the minimum infiltration (mm/h), and T is the storm duration (h) are constant for all the watersheds (Shi et al. 2017). Simplification of Eq. (30) yield

$$\frac{Q}{P} = \left[1 - \frac{(S + F_c)^2}{S^2 + P(S + F_c)}\right]$$
(31)

where Q/P is the runoff coefficient, Eq. (31) substituting into Eq. (32) yields

$$Y = AC_r \tag{32}$$

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where, Y, A and C_r are respectively, sediment yield, potential maximum erosion and runoff coefficient,

$$Y = A \left[1 - \frac{(S + F_c)^2}{S^2 + P(S + F_c)} \right]$$
(33)

Equation (33) is the proposed simple 3-parameters of sediment yield model based on SMA procedure. Mathematical formulation of sediment yield and runoff models included parameters in proposed (S2 and R2) and existing models (S1 and R1) are summarized in Table.1

4 Model Applications

4.1 Hydro-Meteorological Data

The proposed sediment yield and runoff models are applied on large set of rainfall-runoff and sediment yield data of 98 events on Indian and the USDA-ARS watersheds. The proposed model was employed on four Indo-German Bilateral Project (IGBP) Indian watersheds, and remaining the USDA-ARS watersheds. The Indian watersheds categorize into four watershed such as Karso watershed (27.93 Km²), Banha watershed (17.51 Km²), Nagwa watershed (92.46 Km²) in Hazaribagh district, Bihar, India, and Mansara watershed (8.70 Km²) in Barabanki district, Uttar Pradesh, India, and the USDA-ARS watersheds categorized as W2 Treynor (0.33 Km²), three subwatersheds of the Goodwin creek (GC) experimental watershed, namely, W6 (1.25 Km²), W7 (1.66 Km²), W14 (1.66 Km²), Cincinnati (3.0×10^{-4} Km²), In account, three North Appalachian Experimental Watersheds (NAEW) of USDA-ARS, namely, 123 (5.50×10^{-3} Km²), 129 ($1.10 \times$ 10^{-2} Km²) and 182 (0.28 Km²) watersheds (Kalin and Hantush, 2003). The hydrological data of rainfall, runoff and sediment yield data of Karso, Banha, Nagwa and Mansara these watersheds are available in SWCD (1991; 1993; 1994; 1995; 1996). Similarly the runoff, sediment, and rainfall data of Goodwin Creek sub-watersheds are available on WWW at URL: http://msa.ars.usda. gov/ms/oxford/ nsl/cwp unit/Goodwin.html. The other details of rainfall-runoff, sediment yield and hydro-meteorology are given in Table 2. In the present study of numerous climatic conditions, average annual rainfall, soils, land use/land cover and number events are presented in Table 2 and Fig. 1

4.2 Performance Evaluation Criteria

From the academic, scientific and practical point of view, the goal of any watershed models is to provide results near to precision with acceptable accuracy (Seibert 2001). The precision is

Model	Parameters	Model formulation for computing runoff (Q)
R1	S	Equations (3) , (4) and (5)
R2	S, Fc	Equations (27), (30) and (31) Model formulation for computing sediment yield (Y)
S1	A, S	Equation (6)
S2	A, S, Fc	Equations (33) and (34)

Table 1 Model formulations

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Tab	He 2 Hydro-climatic characte	cristics of the wa	tersheds sel	ected for the stud	y (source: 1	Mishra et al. 2	0006b)	
No S	Watershed/Size/ Location	Climate	Av. Annual Rainfall (mm)	Soils	Av. Slope (Percent)	Land Use (Percent)	Source of Watershed Details/Rainfall-Runoff-Sediment Yield data	No. of Events Used in the Study
1. (1)	(2) Nagwa (92.46 km ²) Hazaribagh, Bihar, India (85° 16' 41" and 85° 23' 50" E) (23° 59' 33" and 24° 05' 33" N''	(3) Sub-humid, tropical	(4) 1076	 (5) Sandy loam, silty clay, clay loam, loam 	(6) 2.3	(7) AG = 64 FO = 6 OS = 9 WL = 21	(8) SWCD (1991; 1993; 1994)	4)
6	Karso (27.93 km ²) Hazaribagh, Bihar, India (85 ⁰ 24' 20" and 85 ⁰ 28' 06"E) (24 ⁰ 16' 47" and 24 ⁰ 12' 18" N)	Sub-humid, tropical	1243	Light sandy loam	7.3	AG=49 FO=41 OS=10	SWCD (1991; 1993; 1994; 1995; 1996)	6
3.	Banha (17.51 km ²) Hazaribagh, Bihar, India (85° 12' 02" and 85° 16' 05" E) (24° 13' 50" and 24° 17' 00" N)	Sub-humid, tropical	1277	Sandy loam, Loam, clay loam	3.5	AG=32 FO=35 WL=18 GR=15	SWCD (1993; 1994; 1995; 1996)	16
4.	Mansara (8.70 km ²) Barabanki, Uttar Pradesh, India (81° 23' 42" and 81° 26' 15" E) (26° 41' 04" and 26° 43' 15" N)	Semi-arid, sub-tropical	1021	Loam, sandy loam, sandy	-	AG = 84 OS = 16	SWCD (1994; 1996) Agriculture Dept. (1990)	=
5.	W2 Treynor (0.33 km ²) Treynor, Iowa, USA (Not known)	Not known	814	Silt loam	×	AG = 95 GR = 5	Bradford (1988); Vanliew and Saxton (1984); Kalin et al. 2003; 2004)	9
6.	× .	Humid	1440	Silty, silt loam	5	AG = 35		7

Tab	sle 2 (continued)							
No No	Watershed/Size/ Location	Climate	Av. Amnual Rainfall (mm)	Soils	Av. Slope (Percent)	Land Use (Percent)	Source of Watershed Details/Rainfall-Runoff-Sediment Yield data	No. of Events Used in the Study
	W6 Goodwin Creek (1.25 km ²) Batesville, Mississippi, USA (89° 51' 44.665" E) (34° 16' 16 082" M)					GR = 23 Idle = 10 FO = 32	Blackmarr (1995); http://msa.ars.usda.gov/ms/oxford/nsl/- cwp_unit/Goodwin.html	
7.	W7 Goodwin Creek W7 Goodwin Creek (1.66 km ²) Batesville, Mississippi, USA (34° 51' 34.479" E) (34° 15' (0 347" N)	Humid	1440	Silty, silt loam	4	AG = 28 GR = 49 Idle = 3 FO = 20	Blackmarr (1995); http://msa.ars.usda.gov/ms/oxford/nsl/- cwp_unit/Goodwin.html	7
×	W14 Go.dovin Creek(1.66 km ²) Batesville, Mississippi, USA (89° 52' 53.252" E) (34° 15' (77.040" N)	Humid	1440	Silty, silt loam	Ś	AG = 34 $GR = 40$ $Idle = 9$ $FO = 17$	Blackmarr (1995); http://msa.ars.usda.gov/ms/oxford/nsl/- cwp_unit/Goodwin.html	7
6	Cincinnati (3.0 × 10 ⁻⁴ km ²) Asphalt pavement at milestone 2.6 of 1-75, U.S.A. (Not known)	Not known	1020	Asphalt pavement	0.4	urban = 100	Sansalone and Buchberger (1997); Soil (1982); Sansalone et al. (1998); Li et al. (1999)	11
10.	(5.50×10 ⁻³ km ²) (5.50×10 ⁻³ km ²) Coshocton, Ohio, USA (81° 47' 20" E), (40° 22' 23" N)	Not known	Not known	Silt loams	0.1	AG=100	Tien et al. (1993); Kelly et al. (1975)	S

Tak	le 2 (continued)							
No	Watershed/Size/ Location	Climate	Av. Annual Rainfall (mm)	Soils	Av. Slope (Percent)	Land Use (Percent)	Source of Watershed Details/Rainfall-Runoff-Sediment Yield data	No. of Events Used in the Study
11.	129 NAEW (1.10 × 10 ⁻² km ²) Coshocton, Ohio, USA (81° 47' 52" E), (40° 22' 10° N)	Not known	Not known	Silt loams	17	GR = 100	Tien et al. (1993); Kelly et al. (1975)	Ś
12.	182 NAEW (0.28 km ²) Coshocton, Ohio, USA (81° 46' 55" E), (40° 21' 36" N)	Not known	Not known	Silt loams	L	GR = 90 FO = 10	Tien et al. (1993); Kelly et al. (1975)	٢
No	e: $AG = Agriculture; FO = F$	orest; OS = Ope	n scrub; GR =	= Grass/Pasture;	WL = Waste	land; NAEW	∕ = North Appalachian Experimental Watersheds	



Fig. 1 Study watersheds (a) Nagwa; (b) Karso; (c) Banha; (d) Mansara; (e) W2 Treynor; (f) W6Goodwin Creek;
(g) W7 Goodwin Creek; (h) W14 Goodwin Creek (i) Cincinnati (j) 123 NAEW; (k) 129 NAEW; and (l) 182 NAEW

defined as how close the observed values are to each other, i.e. acceptable accuracy as for how close an observed value is to a computed value. The factor influence of precision and acceptable accuracy are catchment characteristics and observed data (Seibert 2001). Several statistics tools are utilized to assess these models quantification performance (Moriasi et al. 2007). The performance evaluation of proposed sediment yield model, runoff model and other existing SCS-CN model is evaluated based on Nash-Sutcliffe Efficiency (NSE) (Nash and Sutcliffe, 1970), percent bias (PBIAS), root mean square error (RMSE), and normalized root mean square error (nRMSE).

The performance evaluation criteria of models used, is mathematically expressed as.

$$NSE = \left[1 - \frac{\sum_{l=1}^{N} \left(Q_{obs} - Q_{comp} \right)^{2}{}_{i}}{\sum_{i=1}^{N} \left(Q_{obs} - \overline{Q_{obs}} \right)^{2}{}_{i}} \right] \times 100$$
(34)

where Q_{obs} is the observed sediment yield, $\overline{Q_{obs}}$ is the mean of the observed sediment yield, Q_{comp} is the computed sediment yield, N is the numbers of observations. NSE may be vary from minus infinity to 100%.

Higher NSE indicate a good model performance and vice versa (Mishra et al. 2006b). NSE is categorized into three groups i.e., very good when NSE is greater than 75%, satisfactory when NSE is between 36 to 75% and unsatisfactory when NSE is lower than 36% (Tyagi et al. 2014). Accordingly, Ritter and Munoz-Carpena (2013) established watershed model performance rating in which a NSE < 65% (Unsatisfactory) was deemed a lower threshold. Other model performance rating were acceptable (65 % \leq NSE < 80%), good (80 % \leq NSE < 90%), and very good (NSE \geq 90%). Similarly, the PBIAS quantifies a model's tendency to underestimate or overestimate, where a value of zero (optimum) shows perfect fit. PBIAS it is mathematically expressed as

$$PBIAS = \left[\frac{\sum_{i=1}^{N} (Q_{obs} - Q_{comp})_i}{\sum_{i=1}^{N} (Q_{obs})_i}\right] \times 100$$
(35)

PBIAS is the basic performance evaluation criteria of the model and is defined as the ratio of difference between computed and observed sediment yield to the summation of observed sediment yield and it is expressed in percentage. Similarly, RMSE is basic criteria for model assessment; the lower value of RMSE indicates better performance and vice versa. It means RMSE = 0 indicate a good agreement between observed runoff and computed runoff or observed sediment yield and computed sediment yield. Analytically, this can be expressed as

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Q_{obs} - Q_{comp})^{2}}_{i}$$
(36)

nRMSE is another most widely used statistical indices for model performance evaluation (Santhi et al. 2001; Van Liew et al. 2003), it is an analytically expressed as

$$nRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N} \left(Q_{obs} - Q_{comp}\right)^{2}}_{i}}{\overline{Q_{obs}}}$$
(37)

S. No	Parameters	Physical description	Ranges	
			Minimum	Maximum
1	А	Potential maximum erosion	0.0	59000.0
2	S	Potential maximum retention	0.0	310.0
3	Fc	Static infiltration	0.0	101.5

Table 3 The models parameter and their range are summaries as given below

Model	Parameters	Mean	Median	Minimum	Maximum	90% confide	ence level
						Lower	Upper
R1	S	114.21	118.04	7.87	233.68	84.53	143.88
R2	S	68.90	56.13	22.15	172.85	46.43	91.38
	Fc	12.37	12.10	0.40	24.33	9.05	15.69
S1	А	23718.01	5714.74	0.009	197950.01	-2703.09	50139.12
	S	114.21	118.99	7.87	233.68	84.54	143.88
S2	А	20347.72	5010.07	0.016	170093.23	-2400.18	43095.63
	S	68.90	56.13	22.15	172.85	46.43	91.38
	Fc	12.37	12.10	0.40	24.33	9.05	15.69

Table 4 Statistical range of parameters obtained from model application in twelve watersheds

5 Results and Discussion

5.1 Parameter Estimation

In the present research work, an analytical model is developed for assessment of rainfall-runoff and sediment yield models, parameter of which were optimized using the non-linear Marquardat (1963) algorithm. This algorithm is an elegant and improved version of the nonlinear optimization and provides a smooth variation between the two extremes of the inverse-Hessian method and the steepest descent method. The models parameter and their range are summaries as given below in Table 3.

The computed parameters of potential maximum retention (S) for proposed rainfall-runoff (R2) and sediment yield model (S2) was varied from 22.15 to 172.85 mm, and, the static infiltration (Fc) was varied from 0.40 to 24.33 mm/h. The existing SCS-CN model (R1) of potential maximum retention (S) was varies from 7.87 to 233.68 mm from all the watersheds. The range of variation of potential maximum erosion (A) for proposed sediment yield model (S2) was varied from0.016 to 170093.23 KN and existing Mishra et al. (2006b) model (S1)

S. No	Name of WS	Proposed	l rainfall-s	ediment yi	eld model	Mishra e	et al. (2006b) sec	liment yield	l model
		PBIAS (%)	RMSE (mm)	nRMSE (mm)	Eff. (%)	PBIAS (%)	RMSE (mm)	nRMSE (mm)	Eff. (%)
1	Karso	0.062	0.71	0.0018	84.31	1.19	13.48	0.03	84.51
2	Banha	0.035	0.56	0.001	74.55	0.208	3.32	0.00	75.21
3	Nagwa	0.013	0.475	0.00	91.13	0.62	22.35	0.016	91.78
4	Mansara	-1.48	2.78	0.049	80.03	1.58	2.97	0.052	81.45
5	Cincinnati	0.50	4.58	0.017	89.40	1.35	113.88	0.42	76.15
6	W2 Treynor	0.062	0.29	0.001	83.46	0.71	3.37	0.017	85.43
7	W6 GWC	-0.90	0.57	0.024	80.54	2.68	1.71	0.07	81.03
8	W7 GWC	-0.71	1.54	0.019	80.32	2.33	5.04	0.06	80.20
9	W14 GWC	0.32	0.32	0.008	87.97	1.34	1.32	0.03	90.04
10	182	-0.49	0.09	0.013	79.05	0.35	0.07	0.009	81.20
11	129	-11.11	0.011	0.25	88.42	0.0	0.0	0.0	90.95
12	123	-0.66	0.002	0.015	84.73	-0.13	0.0	0.00	84.04

 Table 5
 Comparative analysis between proposed rainfall-sediment yield and Existing Mishra et al. (2006b) models

of parameters of potential maximum retention (S) and potential maximum erosion (A) was varied from 7.87 to 233.68 mm, 0.009 to 197950.01 KN are respectively. The statistical range for models R1, R2, S1, and S2 are summarized in Table 4. Parameter A depends on the numerous factor i.e. rainfall intensity, land use/ land cover, land slope, types of soil, rainfall amount and duration of rainfall occurred on the water-shed. The highest runoff producing watershed, similarly produce highest sediment yield, some of the watershed which is consistent with the paved nature. On the other hand, the watershed characteristics in terms of its subtropical, semi-arid climate, and alluvial soils. The other watersheds falling in between have fairly good to good runoff potential.

The NRCS-CN method is widely used for computation of sediment yield and runoff from natural watershed using soil moisture accounting procedure (Michel et al. 2005; Sahu et al. 2007). The unavailability of any reliable initial SMA procedure in the model leads to inefficient sediment yield and runoff computation which result in overall under performance of model (Brocca et al. 2008). From the proposed sediment yield the computed S and F_c values was used for computation of runoff the natural watersheds and similarly the computed S value of sediment yield is higher than runoff model (Mishra et al. 2006b).

In the proposed model, existing Mishra et al. (2005), and NRSC-CN models, the performance is based on four statistical indices and subsequently NSE, RMSE, nRMSE and PBIAS. The NSE of proposed sediment yield model (S2) of individual watershed are varied from 84.31% for Karso, 74.55% for Banha, 91.13% for Nagwa, 80.03% for Mansara, 89.40% for Cincinnati, 83.46% for W2 Treynor, 80.54% for W6, 80.32% for W7, 87.97 for W14, 79.05% for 182, 88.42% for 129 and 84.73 for 123 watersheds respectively as shown in Table. 4. PBIAS of proposed sediment yield model was found to vary from 0.062% for Karso, 0.035% for Banha, 0.013% for Nagwa, -1.48% for Mansara, 0.50% for Cincinnati, 0.062 for W2, -0.92% for W6, -0.71% for W7, 0.32% for W14, -0.49% for 182, -11.11 for 129 and -0.66% for 123 watershed, respectively as shown in Table 5.

The RMSE were used for performance evaluation of S2 model. The obtained values are 0.71 mm for Karso, 0.56 mm for Banha, 0.47 mm for Nagwa, 2.78 mm for Mansara, 4.58 mm for Cincinnati, 0.29 mm for W2, 0.57 mm for W6, 1.54 mm for W7, 0.32 mm for W14,

S. No	Name of WS	Proposed	d rainfall-	runoff mod	el	Existing	SCS-CN model	nRMSE Ef (mm) (% 1.80 45 2.16 1. 2.02 13 2.64 58 0.98 87 1.68 -2 1.57 36 1.82 15 1.87 -8 2.19 -8 1.85 -1	
		PBIAS (%)	RMSE (mm)	nRMSE (mm)	Eff. (%)	PBIAS (%)	RMSE (mm)	nRMSE (mm)	Eff. (%)
1	Karso	32.69	6.98	0.98	70.77	-59.92	12.79	1.80	45.62
2	Banha	26.41	18.89	1.05	70.86	-54.07	38.67	2.16	1.76
3	Nagwa	43.40	10.68	1.15	71.36	-76.25	18.76	2.02	13.55
4	Mansara	7.07	1.37	0.23	92.48	-63.58	15.46	2.64	58.79
5	Cincinnati	49.85	7.54	1.65	66.43	-29.55	4.47	0.98	87.34
6	W2 Treynor	22.21	3.58	0.54	78.44	-68.43	11.05	1.68	-26.10
7	W6 GWC	28.98	6.01	0.77	82.22	-59.16	12.27	1.57	36.26
8	W7 GWC	46.24	17.76	1.22	58.28	-68.71	26.40	1.82	15.97
9	W14 GWC	33.79	7.50	0.89	64.46	-70.80	15.71	1.87	-8.67
10	182	32.55	10.90	0.86	64.82	-82.92	27.78	2.19	-80.28
11	129	21.82	5.89	0.49	55.23	-82.81	22.39	1.85	-102.94
12	123	34.77	5.84	0.77	80.39	-89.18	14.99	1.99	-69.26

Table 6 Comparative analysis between proposed rainfall-runoff and Existing SCS-CN models



Fig. 2 Comparison between sediment yield (S2) and runoff (R2) models from Karso (a), Banha (b), Nagwa (c), Mansara (d), Cincinnati (e), W2 Treynor, (f), W6Goodwin Creek (g), W7 Goodwin Creek (h), W14 watershed (i), 182 NAEW (j), 129 NAEW (k), 123 NAEW (l)

0.09 mm for 182, 0.011 mm for 129 and 0.002 mm for 123 watersheds, the proposed sediment yield model (S2) of lowest RMSE as compared to S1 model as shown in Table 4. Similarly nRMSE of S2 model are 0.0018 mm for Karso, 0.001 mm for Banha, 0.00 mm for Nagwa, 0.049 mm for Mansara, 0.017 mm for Cincinnati, 0.001 mm for W2 Treynor, 0.024 mm for W6 GWC, 0.019 mm for W7 GWC, 0.008 mm for W14, 0.013 mm for 182, 0.25 mm for 129 and 0.015 mm for 123 watershed respectively as shown in Table 5.



Fig. 2 (continued)

The computed values of potential maximum retention (S) and static infiltration that were used for the S2 model were also used to compute the runoff of R2 model as shown in Table 5. Similarly, performance of R2 model were evaluated using statistical indices used for NSE, PBIAS, RMSE, and nRMSE same technique were used for R1 model. Among all the watersheds, the NSE of R2 model is observed to be superior as compared to R1 model from all the watersheds as shown in Table 6. However, in some of the watershed the performance were inferior due to deposition of sediment yield (Mishra et al. 2006a, b). The PBIAS of R2



Fig. 2 (continued)

model were computed on the basis of observed runoff and computed runoff the PBIAS of R2 model are lowest as compared to R1 model form all the watersheds as shown in Table 6. Accordingly RMSE of R2 model, the R2 model of performance is superior as compared to R1 model from the respective watersheds.

The nRMSE of R2 model varies from 0.23 to 1.15 mm from all the watersheds and R1 model of nRMSE varies from 0.98 to 2.64 mm respectively as shown in Table 6. Finally,



Fig. 2 (continued)

sediment yield and runoff model are compared for the twelve watersheds via scatter plots in sequence to visualize the model reliability for application of the present study. For model reliability the observed sediment yield and computed sediment yield are plotted on both sides of line of perfect fit, similarly for runoff model the observed runoff and computed runoff are plotted on both sides of line of perfect fit (Fig. 2).

6 Conclusions

Improved models of sediment yield and runoff have been developed by incorporating soil moisture budgeting. The developed models, Mishra et al. (2006b) model and original SCS-CN model were applied to a large set of rainfall-runoff and sediment yield data from 4 Indian watersheds and 8 USDA-ARS watersheds. The developed sediment yield (S2) and runoff (R2) models predict the sediment yield and runoff, respectively with the NSE of 84.31 and 70.77% for Karso, 74.55 and 70.86% for Banha, 91.13 and 71.36% for Nagwa, 80.03 and 92.48% for Mansara, 89.40 and 66.43% for Cincinnati, 83.46 and 78.44% for W2, 80.54 and 82.44% for W6. 80.32 and 58.28% for W7. 87.97 and 64.46% for W14, 79.05 and 64.82% for 182, 88.42 and 55.23% for 129 and 84.73 and 80.39% for 123 watersheds. The computed values of potential maximum retention (S) and static infiltration (F_c) used for the S2 model were also used to compute the runoff (R2 model). The proposed SMA based sediment yield (S2) and runoff (R2) models are simple and have only 3-paramters and 2-parameters respectively. The proposed runoff (R2) model computes higher NSE from all the watersheds as compared to original SCS-CN (R1) model. The proposed sediment yield and runoff models are simple and can easily be adapted for field applications in computing sediment yield and runoff from hydrologically similar watersheds in the field.

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Compliance with Ethical Standards

Conflict of Interest We have no conflicts of interest to disclose.

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