Water Resources Management (2019) 33: 1167–1183
https://doi.org/10.1007/s11269-018-2173-4

Leakage Estimation in Water Distribution Network: Effect of the Shape and Size Cracks

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Received: 28 November 2017 Accepted: 17 December 2018 /
Published online: 2 January 2019

Abstract

The definition of the relationship between the leak outflow, the total head at the leak and other relevant parameters such as pipe stiffness, leak dimension and shape has been object of extensive studies. The attention to a correct estimation of leakages, *leak law*, is crucial for the management of water distribution systems. Water utilities, in fact, can reduce leakage levels through the leak detection or more usually by pressure management. In the last cases, the known of the relationship between leak discharge and pressure is fundamental. In recent decades, the use of the Torricelli equation has been questioned, because some experimental results showed that it can yield unsatisfactory results, and other formulations have been suggested to model water leakages in water distribution networks. To investigate the effectiveness of the formulations suggested by different authors, an experimental campaign was carried out at the Environmental Hydraulic Laboratory of the University of Enna (Italy) for leaks of different shape and size in polyethylene pipes. The results of the laboratory experiments contribute to clarify the applicability of the leak law for circular and rectangular leaks and suggest that Torricelli formulation is valid in absence of leak area deformation. Furthermore, the analysis contributes to the knowledge of the coefficients of the leak laws used to estimate the leakage outflow.

Keywords Water distribution network · Leakages · Water losses · Emitter law

1 Introduction

In recent decades, the changing scenario in the availability and the use of water has made the efficiency of water distribution systems (WDSs) management a topic of great importance, particularly in terms of leakage detection and control. In fact, pipes in water distribution systems are susceptible to water leaks, that cannot be directly observed due to the pipes being buried.

The existence of leaks is highly costly, not only in terms of water wastage but also due to increasing costs of pumping to balance inefficient energy distribution through the

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network and due to the cost for the detection (see Meniconi et al. 2011). Moreover, it is an environmental and potentially a health and safety issue in low pressure conditions, due to contamination by intrusion of unwanted physical, biological or chemical agents.

Water leakages deeply influence the management of water networks, water utilities, in fact, spend lots of resources on leak control and leak detection. Recently, Ferrante et al. (2014) analysed different leak detection techniques, in steady state as well as in transient conditions. The authors observed that even though a leakge should be easy to detect by any technique, the dimensions of the leak cannot be considered as a reliable way to determine the leak detectability. In recent decades the definition of the relationships which relates the leak outflow and the relevant hydraulic parameters has received more and more increasing attention. For each leak, the pressure modification in the distribution network affects the rate of water loss through the leakage, as indicated by several studies (Farley and Trow 2003; Van Zyl and Clayton 2007; Walski et al. 2006). Leakages depend on the area and shape, the pressure inside and outside the pipe as well as pipe material. Basically, the higher the pressure, the larger the leak flow and vice versa. Pressure management is, in fact, a common practice to reduce the leakage rate from the network and the rate of new failures which occur under high pressure conditions (Thornton 2002). In leakage control techniques based on pressure reduction, the hydraulic model for leaks affects the reliability of the estimated cost-effectiveness of a system improvement; so the need for the definition of the relationship between leakage and functioning conditions in a damaged pipe, able to correctly capture the leak outflow for rigid and deformable pipes in different crack geometry conditions, is really felt (Brunone and Ferrante 2004; Ferrante 2012). As pointed out by (Ferrante et al. 2014), a correct definition of the leak law is crucial for prediction of leakages throughout mathematical models and for a correct management of WDNs even though is particularly difficult to use a single law valid in all the points of a network distribution. The authors, in fact, showed that leak law derived from laboratory experiments cannot be easily applied for the estimation of the leak law in WDNs.

Various studies focused the attention on leaks estimation and it is nearly impossible to cite the entire literature findings on this issue, therefore here the attention is mainly focused on the analysis of the behavior of different types of leak openings (e.g. round holes, longitudinal and transverse cracks) in pressurised pipes, taking into account the effect of rigid and deformable materials (uPVC, steel, cast iron, polyethylene, etc.). Despite the great efforts to find a unifying theory for leakages, the research field is still open and requires further numerical as well as experimental analysis to improve the state-of-the-art. In order to do this, the main objective of the proposed research is to investigate on the most conventional head-leakage law, through laboratory experiments, analysing different leak shapes. Specifically, thanks to experimental campaigns the coefficients of the leak outflow can be derived and validated and used in numerical models. In order to improve the knowledge of global leak laws (see Ferrante et al. 2014), the experiments were carried out in a water distribution network at laboratory scale.

2 State-of-the-Art of the Head-Leakage Relationship

In the past, the most conventional law to calculate the leak flow rate was the orifice equation, derived analytically for an orifice on the thin horizontal (or vertical) wall of a constant head reservoir, also known as Torricelli's formula:

$$Q_L = C_L A_L \sqrt{2gH} \tag{1}$$

where Q_L [m^3/s] is the leak outflow; C_L is the non dimensional discharge coefficient; A_L [m^2] is the orifice (leak) area; g [m/s^2] is the gravity acceleration; H [m] is the total head in the tank. The product $C_L A_L$ was later defined as effective area A_E. Equation 1 has been widely used in the literature to interpret the leakage in the pressurised pipe systems, replacing the total head H with the pressure head h (the use of h instead of H is not a relevant issue in pressurised pipe systems where the contribution of the velocity head is almost negligible, therefore H will be used hereafter). In Eq. 1 and in the following, H is referred to the leak elevation and a leak free efflux in atmosphere is considered. Van Zyl and Cassa (2014) observed that the square root relationship between flow rate and pressure head in Eq. 1 is only valid for turbulent flow. For laminar flow , in fact, the discharge coefficient C_L also becomes a function of the pressure head, effectively resulting in linear relationship between flow rate and pressure head.

The proportionality of the leak discharge to the square root of the pressure head inside the pipe has been questioned by many authors (Walski et al. 2006; Greyvenstein and van Zyl 2007; Van Zyl and Clayton 2007; Cassa et al. 2010). To interpret data coming from laboratory tests and field measurements, Thornton and Lambert (2005) suggested the use of the power law equation:

$$Q_L = a_I H^{b_I} \tag{2}$$

used by the International Water Association Water Loss Task Force (in the following IWA), that includes the Torricelli's equation if $a_I = C_L A_L \sqrt{2g}$ and $b_I = 0.5$ (Thornton 2003). Field studies have shown that the sensitivity of the volume of water leaked for unit time Q_L to pressure, expressed by b_I in Eq. 2, is often considerably higher than the theoretical orifice value of 0.5, typically spanning from 0.5 to 2.79, with a mean value of 1.15 (Farley and Trow 2003), suggesting the dependence on H on the effective area $C_L A_L$ (Van Zyl and Clayton 2007; Cassa et al. 2010). Thornton and Lambert (2005) analysed the leak exponent b_I for unplasticised polyvinyl chloride (uPVC), polyethylene and metal pipes. The authors observed that for flexible pipes the exponent is about 1.5, on the other hand $b_I = 0.5$ for rigid pipes. In the past, several researches focused the attention on the definition of proper values for the leakage exponent and field tests have found system leakage exponents substantially higher than 0.5. These analysis can be roughly divided, as suggested by Ferrante et al. (2014) and Schwaller and van Zyl (2015), in two main categories: studies carried out at laboratory scale where single leaks are analysed (see among others Greyvenstein and van Zyl 2007; Walski et al. 2009; Ferrante 2012) and researches dealing leakages at district scale (see among others Greyvenstein and Zyl 2007; Walski et al. 2009; Al-Ghamd 2011; Ferrante 2012). Unfortunately, as pointed out in the analysis of Ferrante et al. (2014) and Schwaller and van Zyl (2015), the results achieved at district scale and those obtained in laboratory for a single leak cannot be easily compared. In general the values of the exponent b_I at district scale is higher than 0.5, achieving also values of about 2.79 (Farley and Trow 2003).

Experimental results showed a strong correlation between the pipe material stress-strain and the pressure-leak discharge relationship (Ferrante et al. 2011; Massari et al. 2012; Ferrante 2012; Cassa et al. 2010; Greyvenstein and van Zyl 2007; Thornton and Lambert 2005; Van Zyl and Clayton 2007; Walski et al. 2006) and the relationship is much more complicated for plastic pipes due to viscoelastic deformation (Ferrante 2012; Massari et al. 2012).

Leak area deformation depends not only on pipe material but also on leak shape and pipe thickness. Assuming that the possible deviation from the orifice's formula might be justified in terms of dependence of the effective area by the pressure, Ferrante (2012) proposed a modified version of the IWA law, indicated as the author states as IWAM:

$$Q_L = a_M H^{0.5} + c_M H^{b_M} \tag{3}$$

Greyvenstein and van Zyl (2007) and Van Zyl and Clayton (2007) analyzed the effects of pipe material, leak geometry and the surrounding soil. They stated that the material along with the leak geometries have an important role on leakage. In particular, they studied the influence of the pressure on leaks in uPVC pipes, asbestos cement and mild steel for different opening geometries, using the power law function proposed by Thornton and Lambert (2005). The results pointed out highest exponents in corroded steel pipes and longitudinal cracks. More recently, other Authors contributed to the explanation of this phenomenon (Cassa et al. 2010; Greyvenstein and van Zyl 2007; Thornton and Lambert 2005; Van Zyl and Clayton 2007; Walski et al. 2006).

May (1994) was probably the first who considered the leak area variation due to pressure. The author represented the leakage as sum of two terms, one related to leaks with constant area (fixed area term) and the other related to joints and leaks that expand with pressure (expanding area term), leading to the Fixed and Variable Area Discharges concept (FAVAD). Following the FAVAD theory, Cassa et al. (2010) considered three types of leak openings, namely longitudinal, circumferential and spiral cracks finding that, regardless of the geometry of the leak, the effective area played a larger role than expected on the basis of the orifice law and the effect of pressure on the leakage exponent is significant in pipes with cracks: the effective area increases linearly with pressure and the slope of the linear relationship depends on loading state, pipe dimensions and pipe material properties. In their finite element analysis, Cassa et al. (2010) assumed a linear elastic behaviour of the pipe material, leading to the area-pressure relationship:

$$A_L(h) = A_0 + mH \tag{4}$$

where A_L is the leak area , A_0 the initial leak opening for $H = 0$ in the pipe, and m the slope of the pressure-area line, which depends on the shape and the geometry of leak opening. Substituting Eqs. 4 in 1 results in the following equation for the leakage rate as a function of pressure:

$$Q_L = C_L\sqrt{2g}(A_0 H^{0.5} + mH^{1.5}) \tag{5}$$

The Cassa formulation is also know in literature as:

$$Q_L = a_C H^{0.5} + b_c H^{1.5} \tag{6}$$

with $a_C = C_L A_E \cdot \sqrt{2g}$ and $b_C = C_L m \cdot \sqrt{2g}$.

To investigate on the effect of pressure on leaks in steel and polyethylene pipes, Ferrante et al. (2011) conducted several experimental tests on a longitudinal leak, analysing the dependence of the effective area $A_E = C_L A_L$ on the total head H. Based on their results an analogy seems to apply between the dependence of A_E on H and the constitutive laws given by the rheology of the pipe material. A logical chain seems to give a physical meaning to this analogy, relating pressure inside the pipe - pipe stresses - pipe strains - leak deformation - leak effective area variation - leak discharge (Massari et al. 2012).

Massari et al. (2012) introduced a time dependence on the observed phenomenon and explored the viscoelastic behavior of a leak in a polyethylene pipe by using pressure, discharge and strains data collected at the Water Engineering Laboratory (WEL) of Perugia (Italy) with the aim of investigating and analysing the leak head-discharge relationship. Cassa and Zyl (2014) explored analytically the power law and the FAVAD equation finding that b_I tends to 0.5 when the system pressure tends to zero and 1.5 when the system pressure tends to infinity. Moreover, they defined a more consistent way to characterise the pressure response of leaks by means of a new dimensionless leakage number, defined as the ratio between the variable and fixed portions of a leak. In the end, they found a relationship between the IWA equation and the FAVAD (Fixed and Variable Area Discharges) formulation in terms of a simple equation which links the leakage number and the leakage exponent, that is between the IWA formulation and the FAVAD theory.

More recently, De Marchis et al. (2016) carried out laboratory experiments aimed to analyse the relationship between pressure and leak outflow in secondary pipes, focusing the attention on longitudinal leaks in two polyethylene pipes which have different rigidity (i.e., different nominal pressure, PN), providing new insights on the estimation of the coefficients of the head-discharge laws.

In the following, experimental results are used to assess the effectiveness of different leak head-discharge relationships showing that deviations from the orifice formula can be ascribed to the leak area deformation as a function of the pressure.

3 Experimental Setup

3.1 Facility

The experiments were conducted at the Environmental Hydraulic Laboratory of the University of Enna Kore (Italy), where a small water distribution network is located. The WDN is composed of three main loops (M=3) of high-density polyethylene (HDPE 100 PN16) pipes (nine internal nodes, N=9, one external node, S=1, and thirteen pipes, L=13) having nominal diameter (DN) of 63 mm. Each pipe is about 45 m long. For a detailed description of the

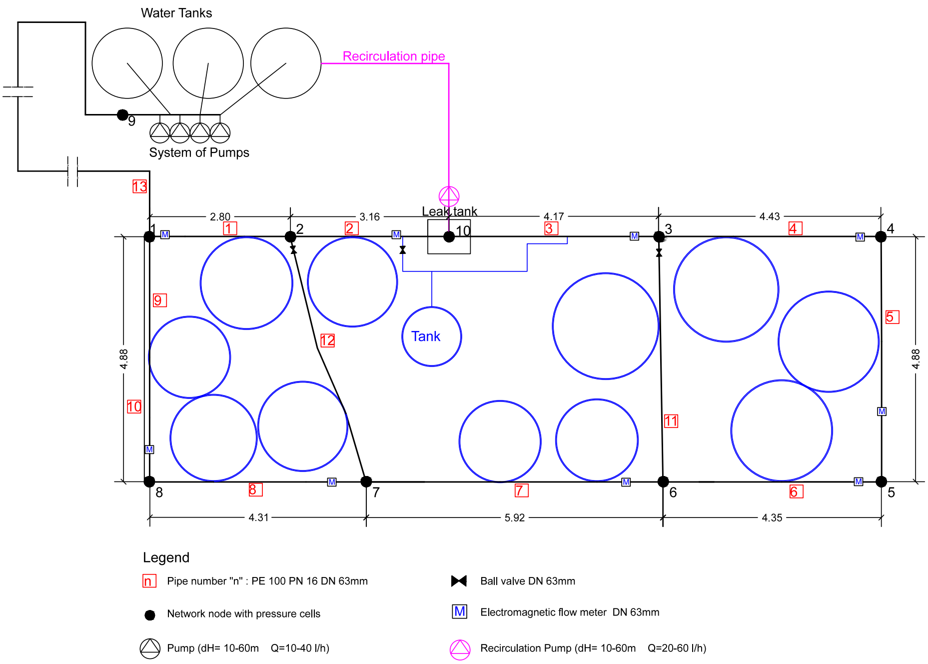

Fig. 1 Layout of the experimental water distribution network

facility see De Marchis et al. (2016). In Fig. 1 is reported a schematic representation of the WDN. As reported in Fig. 1, the network is equipped with pressure transducers, multi-jet water metres and electromagnetic flow meter.

To investigate at a local scale the relationship between the leak outflow Q_L and the pressure, H inside the pressurised pipe several tests were conducted considering rectangular transversal and circular cracks of different sizes. The presence of cracks has been taken into account replacing a small portion of the pipe 4-6 with a leak trunk, approximately 70 cm long (located in the in Fig. 1). The leak discharges into a free surface tank and thanks to a recirculation system, discharges are pumped to supply water tanks.

The experimental facility described above allows to easily simulate leaks with different shapes and sizes as well as the behaviour of different pipe materials by substitution of the leak trunk. The leak discharge is computed by means of two electromagnetic flow metres, with measurement accuracy of 0.4%, placed at the distance of 1 m upstream (UD) and downstream (DD) of the leak trunk. Discharge measurements at UD and DD allowed the evaluation of the leak outflow Q_L. The pipe between UD and DD was horizontal. The pressure is measured by means of a piezoresistive pressure transducer (p), with a 6-bar full scale (f.s.) and an accuracy of 0.1% f.s. placed upstream from the leak at approximately 1 m.

3.2 Leakages

In order to investigate the relationship between leak outflow and water head, as a function of the crack shape, several experiments were conducted considering cracks of different shape and size. Specifically, two sets of cracks were investigated: circular and rectangular transversal cracks, artificially generated, see Fig. 2. In Table 1 details about the leakages are reported. The two sets of experiments were designed with the aim to have different geometries maintaining almost the same leak area: for example the test cases T_1 with a rectangular leak of 1.5 x 10 mm, has an area $A_0 = 0.007$ equal to that considered in the test case C_1 (circular track), having a diameter equal to D = 4.35 mm, see Table 1. In the present analysis longitudinal cracks were neglected since this kind of geometries have already been analysed in a similar study carried out by De Marchis et al. (2016). Anyway, the experimental results will be analysed in the light of those achieved for longitudinal leakages.

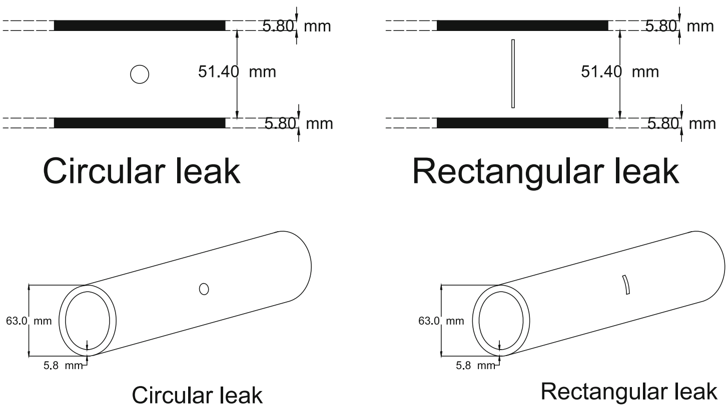

Fig. 2 Schematic representation of the pipes where artificial cracks are generated, with circular and rectangular shapes

Table 1 Details about the leak characteristics

Test case	Shape	Length (D/L) [mm]	A_0 [mm^2]	A_0/A_P
T_1	Rectangular	10	15	0.007
T_2	Rectangular	20	30	0.014
T_3	Rectangular	30	45	0.022
T_4	Rectangular	40	60	0.029
T_5	Rectangular	50	75	0.036
T_6	Rectangular	60	90	0.043
C_1	Circular	4.35	15	0.007
C_2	Circular	6.18	30	0.014
C_3	Circular	7.57	45	0.022
C_4	Circular	8.74	60	0.029
C_5	Circular	9.77	75	0.036
C_6	Circular	10.70	90	0.043

Length represents the diameter for circular leaks, whereas represent the transverse dimension for the rectangular leak. For the rectangular shape the width of the leak is equal to 1.5 mm. A_0 is the leak area at the atmospheric pressure. A_P is the internal pipe diameter

For each experimental test the head at the inlet node of the network was modified by means of the pump station from 1.0 to 5.0 bar, with a step of 0.2 bar, in order to have a detailed variation of the leak outflow as a function of the water head upstream. For all the experiments the leak discharge and water pressure were measured simultaneously with a sampling frequency of 2 seconds (0.5 Hz). Measurements were collected after the steady-state conditions for water head and leak discharge were reached, approximately after 5 minutes.

4 Results and Discussion

The achieved results will be discussed as follow. Initially, the instant measured quantities are presented. Specifically, the measured flow rate and pressure are plotted, thus to show the collected data used to estimate the men flow rate discharging through the leaks. Later, the attention is focused on the estimation of the leak flow by means of the IWA, Cassa and Torricelli's formulation, tuning the coefficients of the known laws to fit experimental data. Finally, some considerations are made about the variation of the leak area with pressure.

4.1 Pressure and Flow Rate Measurements

Figure 3a–b shows the variation in time of the measured water head H, obtained for the test cases C_1 (left panel) and T_2 (right panel). 21 lines are obtained chancing the pressure at the pump station between 1.0 to 5.0 bar with a step of 0.2 bar. As expected, increasing the pressure at the inlet an increase of the water head at the leakage node is observed. Figure 3a–b displays that in the water distribution network the steady state is achieved for each experiment. In fact, data fluctuate around the mean value but all the lines are almost horizontal, confirming that the chosen period is sufficient to obtain a mean constant value. The other cases have the same behaviour and thus, for the sake of brevity, are not shown

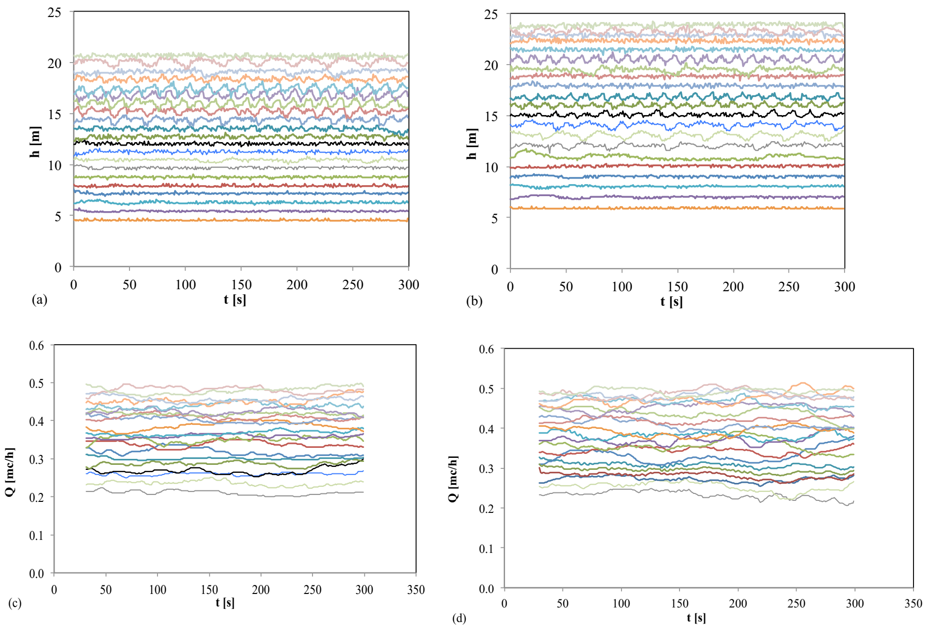

Fig. 3 Variation of the water head and leakage discharge, in the leak node, in time. Panel **a** is referred to the water head for the test case C_1. Panel **b** is referred to the water head for the test case T_2. Panel **c** is referred to the leakage discharge for the test case C_1. Panel **d** is referred to the leakage discharge for the test case T_2.The lowest line is achieved with a pressure at the pump station of 1.0 bar, whereas the upper line for 5.0 bar. The other lines differ for 0.2 bar one each other

here. The same consideration applies to the leakage flow rate. Figure 3c–d shows, in fact, that the measured leak flow discharge oscillates around a mean values, even though an almost horizontal pattern can be observed. Moreover, the steady state condition was verified through statistical test on trend. Data were collected with a frequency of 0.5 Hz.

4.2 Leakages Flow Rate Estimate

For each experiment, the instantaneous values of the pressure and discharge, like those shown in Fig. 3, were averaged in time, thus to achieve a couple of values to be plotted in the plane Q_L vs H. Figure 4 shows water leakages, plotted in l/s, against water head, measured in m. Each panel is referred to a specific transverse crack length. Figure 4a plots data achieved for the test case T_1, Fig. 4f plots data achieved for the test case T_6. Similarly, in Fig. 5 the leakage discharges versus water head for the circular cracks are plotted. In Figs. 4 and 5 the mean values is plotted within the standard deviation. Specifically the error bars plot the values $m.v. \pm 2\ s.d.$ of the leakages.

Overall, Figs. 4 and 5 show that increasing the pressure the leakage flow rate increases, as expected. Moreover, data achieved for lower crack dimension (i.e. TC_1, TC_2) have a somewhat linear trend, whereas as the pressure increases (i.e TC_5, TC_6) a power law trend is observed. In the figures the mathematical laws, described above, are superposed to the circular and transversal data. The comparison between data and analytical results shows a perfect overlap with the classical Torricelli's formula (1) as well as with the formulations of IWA (2), IWA modified (3) and by Cassa et al. (2010) (6). The overlap was obtained

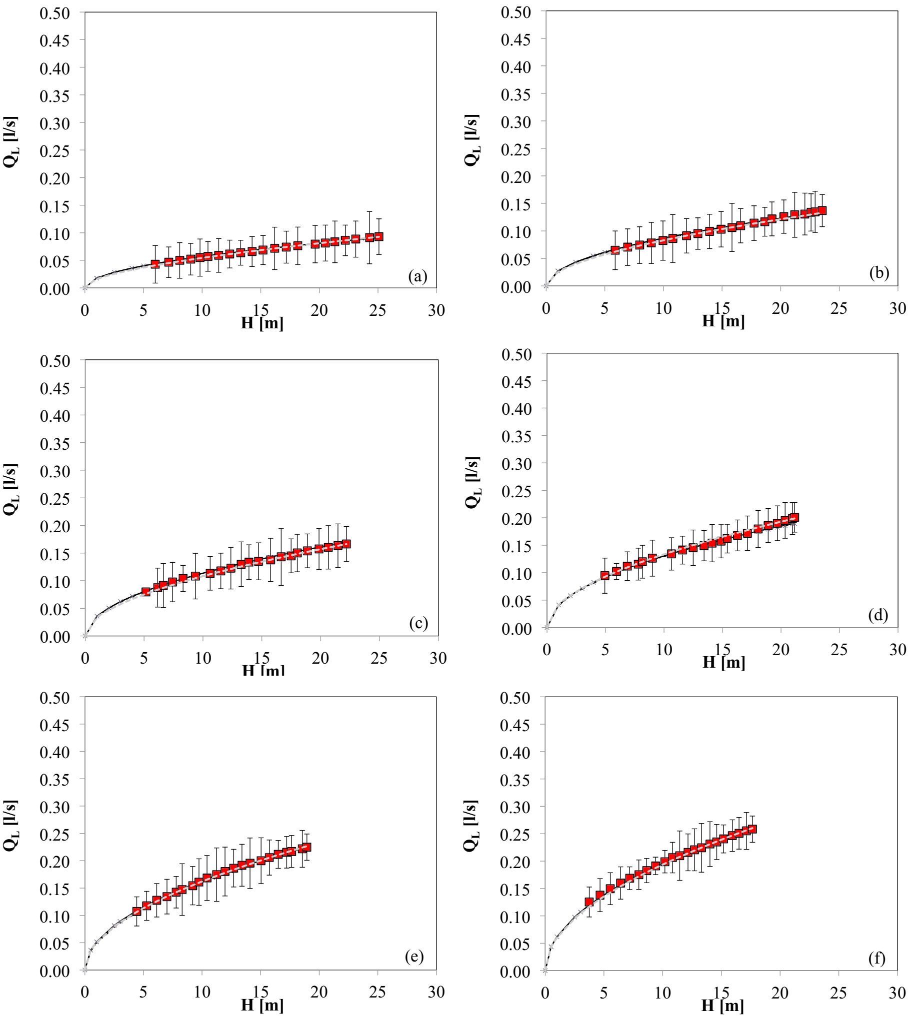

Fig. 4 Head-leak discharge variation for pipe trunks belonging to the transverse linear cracks. **a** case T_1; **b** case T_2; **c** case T_3; **d** case T_4; **e** case T_5; **f** case T_6. ■: experimental data achieved with a pressure step of 0.2 bar. × Eq. 1; Black dot-dashed line Eq. 2; Grey dashed line: Eq. 3; black line: Eq. 6. The error bars show the value of two times the standard deviation ($s.d.$) from the mean value ($m.v.$), $m.v.\pm$ 2 $s.d.$

through a calibration procedure of the coefficients for each mathematical formulation. The best fit of the data was obtained changing the coefficients of the leak laws thus to minimises the sum of squared residuals. A discussion about the coefficients will be reported below.

Several authors highlighted the effect of the deformation of the leak area on the discharge and tried to find the relationship between leak area deformation and pressure. Here, in order to better understand the effect of the head H on the effective area A_E variation, according to Ferrante (2012) and Ferrante et al. (2013), a power law has been considered:

$$A_E = a_{II} H^{b_{II}} \tag{7}$$

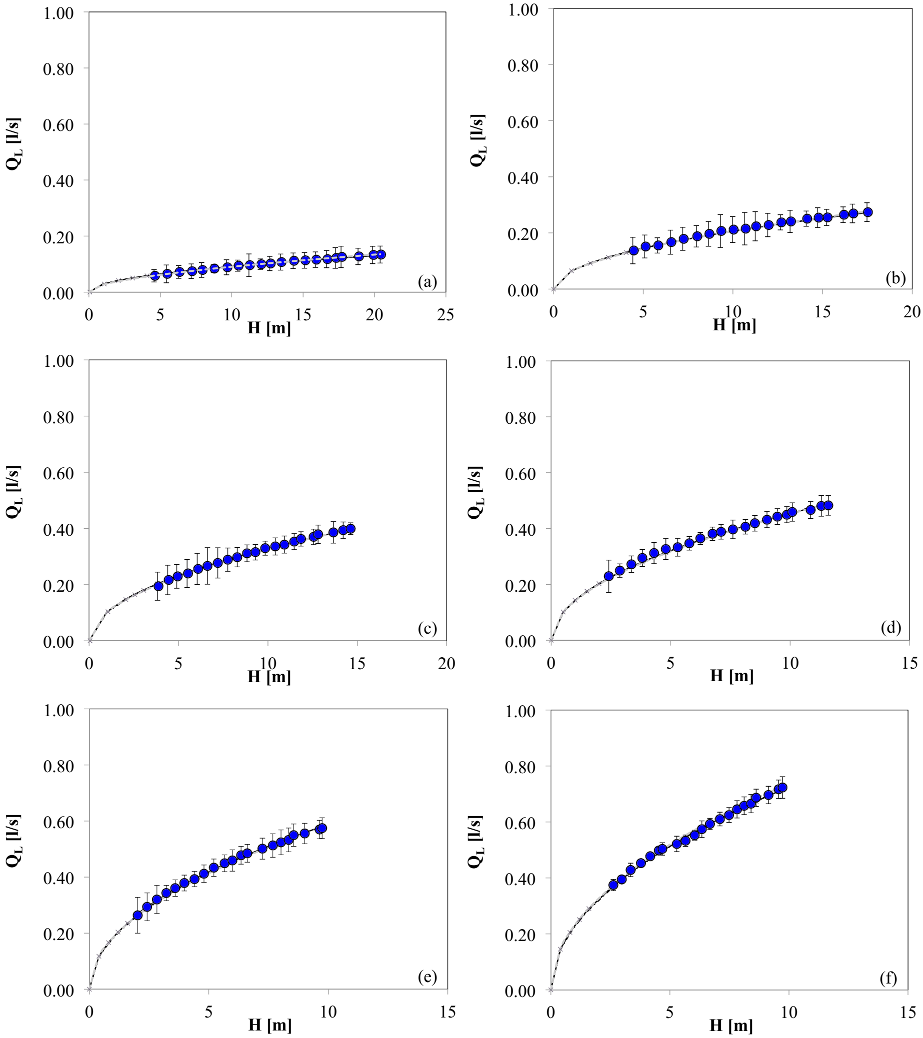

Fig. 5 Head-leak discharge variation for pipe trunks belonging to the circular cracks. **a** case C_1; **b** case C_2; **c** case C_3; **d** case C_4; **e** case C_5; **f** case C_6. •: experimental data achieved with a pressure step of 0.2 bar. × Eq. 1; Black dot-dashed line Eq. 2; Grey dashed line: Eq. 3; black line: Eq. 6. The error bars show the value of two times the standard deviation ($s.d.$) from the mean value ($m.v.$), $m.v. \pm 2s.d.$

with $a_{II} = a_I/\sqrt{(2g)}$ and $b_{II} = (b_I$-0.5). The formulation introduced by Ferrante (2012) and Ferrante et al. (2013) assumes that the possible deviation from the orifice's formula can be justified in terms of the variation of the effective area with the pressure. The attention was focused on the effective area instead of the leak area because it is not easy to separate the variation of C_L from the variation of A_L (Ferrante 2012).

Another formulation was proposed by Cassa et al. (2010) where the effective area varies linearly with the water head H and already reported in Eq. 4. Figures 6 and 7 show the measured variation of the effective area A_E with the head H evaluated by Eq. 1, that is

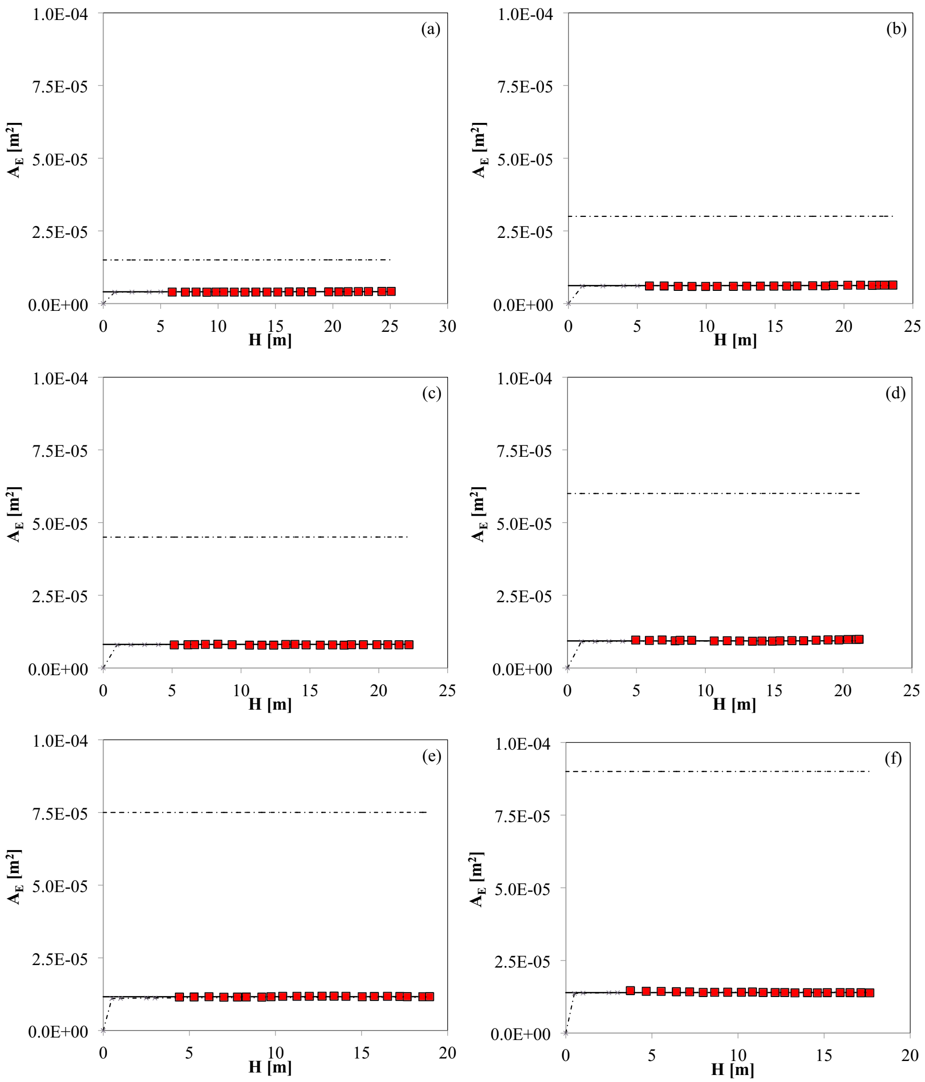

Fig. 6 Head-leak area variation for pipe trunks belonging to the transverse linear cracks. **a** case T_1; **b** case T_2; **c** case T_3; **d** case T_4; **e** case T_5; **f** case T_6. ■: experimental data achieved with a pressure step of 0.2 bar. × Eq. 7; dot-dashed: Eq. 4; black line: leak area

$A_E = Q_L/(2gH)^{0.5}$, together with fitted variations of A_E with H obtained from IWA and CAS leak head-discharge relationships.

Interestingly, the leak effective area is not affected by the pressure. All the data are, in fact, aligned with a sub-horizontal line. This can be explained considering the low values of the leak area (see Table 1). The results is in agreement with those achieved by De Marchis et al. (2016) for leak length lower than 50 mm, where the ratio A_0/A_P is ≈ 0.04. The main difference between the proposed experiments and those conducted by De Marchis et al. (2016) is the leak shape. Here, in fact, leaks have circular shape or rectangular shape with the

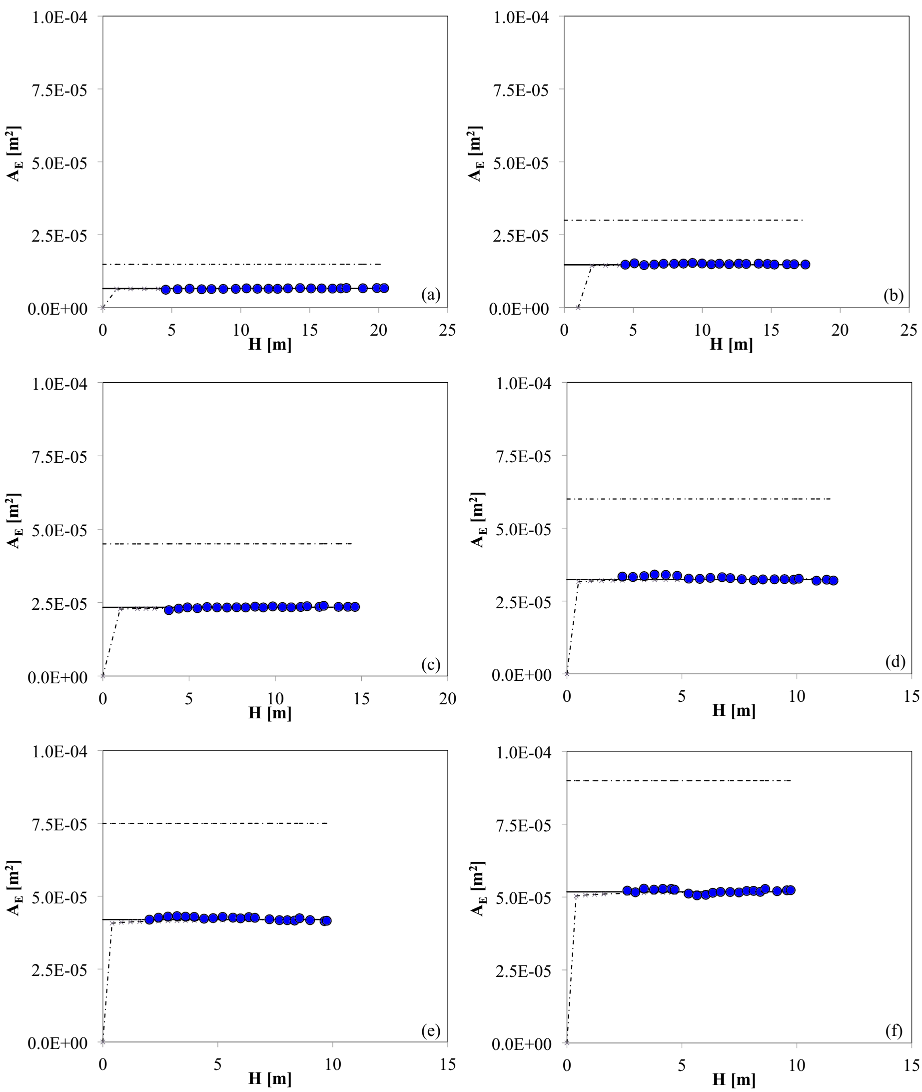

Fig. 7 Head-leak area variation for pipe trunks belonging to the circular linear cracks. **a** case C_1; **b** case C_2; **c** case C_3; **d** case C_4; **e** case C_5; **f** case C_6. •: experimental data achieved with a pressure step of 0.2 bar. × Eq. 7; dot-dashed: Eq. 4; black line: leak area

main dimension aligned in the transverse direction. The different kind of leak deformation between longitudinal cracks and transverse/circular orifice can be explained considering the constitutive material laws of the pipe in HDPE materials.

In their analysis, De Marchis et al. (2016) observed that when the leak effective area increases with the head, Torricelli's formula fails, whereas the proposed results confirm that data are well interpreted by Torricelli when the effective area is constant. In all the cases shown in Figs. 4 and 5, in fact, the Torricelli's formula overlaps the experimental data.

This specific behaviour can be explained considering that the formulation of Torricelli, here identified with Eq. 1, does not take into account the effective area rather, it considers only the leakage area. In the proposed experiments, leak area and effective are identical whereas in the analysis of De Marchis et al. (2016), increasing the pressure a deformation of the leak area is observed.

5 Parametric Analysis of the Head-Leakage Relationship

In the previous section the experimental results have been compared to the most known head-leakage equations. It was demonstrated that all the proposed mathematical formulations overlap the laboratory data. In this section the variation with the leak area of the coefficients in the Eqs. 1, 2, 3 and 6 are analysed. The modelling parameters have been fitted to the experimental data. The fitting procedure was obtained tuning the coefficients thus to maximise the value of the Nash-Sutcliffe (NS) model. All the coefficients are summarised in Table 2 for the rectangular and circular leaks, respectively . Below it is reported a discussion about the coefficient of each mathematical formulation.

5.1 Torricelli's and IWA Law

Figure 8a plots the coefficients of the Torricelli's formulation (1), below reported:

$$Q_L = C_L A_L \sqrt{2gH}$$

Results clearly show that C_L follows different laws for the circular and the rectangular transverse leaks. Basically, the circular cracks follows the equation $C_L = 0.30{\cdot}(A_L)^{0.15}$, whereas the transverse rectangular shapes follow the equation $C_L = 0.65{\cdot}(A_L)^{-0.35}$ (see the dashed lines in the figure). Moreover, in Fig. 8b–c the coefficients of the IWA formulation (2) are plotted. Here the IWA equation is reported for clarity:

$$Q_L = a_I H^{b_I}$$

The values of b_I are constant for all the experiments and equal to 0.51. This value confirms the validity of the Torricelli's formula and the result is in agreement with other literature findings obtained over circumferential (see among others Greyvenstein and van Zyl 2007). Higher values of b_I where observed in rectangular longitudinal leaks (see De Marchis et al. 2016 and literature therein reported). In the researches where b_I is higher than 0.5, a deformation of the leak area was observed, increasing the pressure. Here, as shown in Figs. 6 and 7, the effective area A_E isn't influenced by the increasing water head. It can be argue, thus, that not only the discharge coefficients are affected by leak deformation but also the exponent of the power law has a clear dependence. This result confirms previous finding about the effectiveness of the Torricelli's law, which can be used to correctly estimate the leakages when the leak area does not vary with the pressure.

On the other hands, the a_I coefficient grows linearly with the leak area. Considering the absence of leak deformation, this result is coherent with the growth of characteristic length of the leak shape (the diameter D for the circular cracks and/or the size of the elongated direction L of the rectangular cracks). The slope of the linear trend is higher for circular leak than for transverse cracks. The results is coherent with the highest discharge values observed for the circular holes, as observed comparing data from Figs. 4 and 5.

Table 2 Parameters of the head-discharge relationships obtained for the rectangular transverse and circular leaks

Law	Coeff.	$A_0 = 15\ [mm^2]$	$A_0 = 30\ [mm^2]$	$A_0 = 45\ [mm^2]$	$A_0 = 60\ [mm^2]$	$A_0 = 75\ [mm^2]$	$A_0 = 90\ [mm^2]$
Transverse cracks							
IWA	a_I	1.75E-05	2.65E-05	3.50E-05	4.15E-05	4.90E-05	5.80E-05
IWA	b_I	0.51	0.51	0.51	0.51	0.51	0.51
Cassa	a_c	1.79E-05	2.74E-05	3.59E-05	4.22E-05	5.02E-05	5.98E-05
Cassa	b_c	8.86E-09	8.86E-09	8.86E-09	8.86E-09	8.86E-09	8.86E-09
Cassa	m	2.00E-09	2.00E-09	2.00E-09	2.00E-09	2.00E-09	2.00E-09
Cassa	C_L	0.27	0.206	0.18	0.159	0.151	0.15
Torricelli	C_L	0.27	0.206	0.18	0.159	0.151	0.15
IWAM	a_M	1.50E-05	2.40E-05	3.20E-05	3.90E-05	4.90E-05	5.90E-05
IWAM	b_M	1.00	1.00	1.00	1.00	1.00	1.00
IWAM	c_M	7.0E-07	7.0E-07	7.0E-07	7.0E-07	7.0E-07	7.0E-07
Circular cracks							
IWA	a_I	2.83E-05	6.36E-05	1.01E-04	1.41E-04	1.82E-04	2.25E-04
IWA	b_I	0.51	0.51	0.51	0.51	0.51	0.51
Cassa	a_c	2.90E-05	6.51E-05	1.04E-04	1.43E-04	1.86E-04	2.29E-04
Cassa	b_c	8.86E-09	8.86E-09	8.86E-09	8.86E-09	8.86E-09	8.86E-09
Cassa	m	2.00E-09	2.00E-09	2.00E-09	2.00E-09	2.00E-09	2.00E-09
Cassa	C_L	0.44	0.49	0.52	0.54	0.56	0.576
Torricelli	C_L	0.44	0.49	0.52	0.54	0.56	0.576
IWAM	a_M	1.50E-05	2.40E-05	3.20E-05	3.90E-05	4.90E-05	5.90E-05
IWAM	b_M	1.00	1.00	1.00	1.00	1.00	1.00
IWAM	c_M	7.0E-07	7.0E-07	7.0E-07	7.0E-07	7.0E-07	7.0E-07

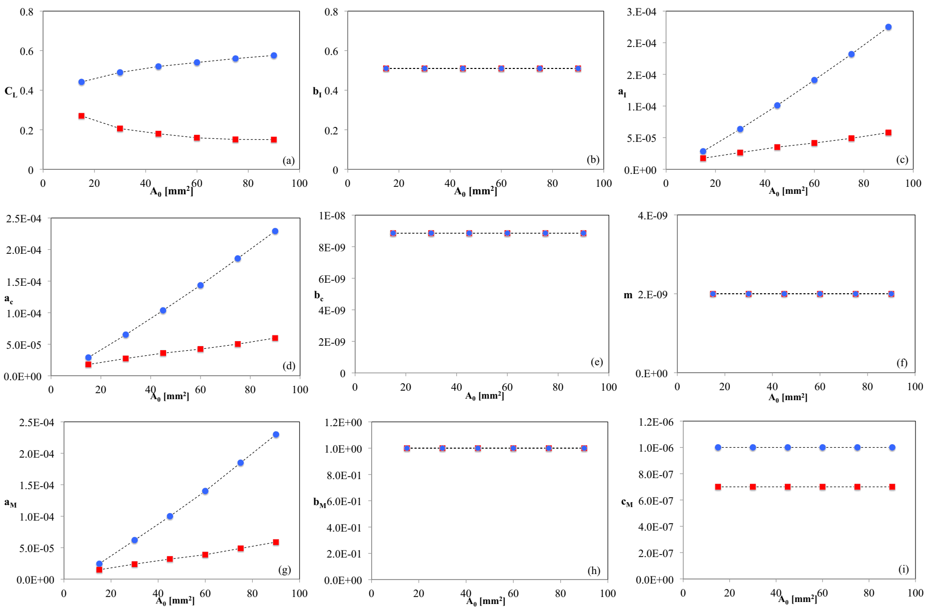

Fig. 8 **a** Analysis of the Torricelli's formula parameter C_L (1) with the crack dimension. **b–c** Analysis of the parameters of the IWA formulation (2) with the crack dimension. **d–f** Analysis of the parameters of the Cassa formulation (6) with the crack dimension. **g–i** Analysis of the parameters of the modified IWA formulation (3) with the crack dimension. ●: Circular leaks. ■: Transverse rectangular leaks

5.2 Cassa Law

In Fig. 8d–f the parameters of the Cassa and co-authors formulation are analysed. To improve the clarity in the reading, the formulation is here rewritten as:

$$Q_L = a_C H^{0.5} + b_c H^{1.5} \qquad \text{with} \qquad a_C = C_L A_E \cdot \sqrt{2g} \text{ and } b_C = C_L m \cdot \sqrt{2g}.$$

The results confirm the findings of the other equations. The coefficient a_C has, in fact, the same shape and similar values achieved for a_I and a_M. Coherently with the coefficients b_I and b_M, b_C has the same value for all the experiments. Finally, the coefficient m, representing the head-area slope (see Eq. 4), assume the same value for all cases, confirming that circular and transverse leaks, at least in the present configuration, are not affected by deformation with the pressure.

5.3 IWA Modified Law

In Fig. 8g–i the coefficients of the modified version of the IWA law are plotted. In the following Eq. 3 is reported:

$$Q_L = a_M H^{0.5} + c_M H^{b_M}$$

Coherently with the result observed for the a_I coefficient of the IWA formulation, the coefficient a_M of Eq. 3 has a linear variation with the leak area and has values close to a_I. The same consideration reported above for the IWA coefficient applies here.

The coefficient b_M assumes a constant value equal to 1.0 for all the cracks analysed, whereas the coefficient c_M has different values for circular and transverse cracks, even though the values doesn't vary with the leak area. It is worthwhile to specify that the values of c_M was obtained for a value of Nash Sutcliffe coefficient equal to 0.999, considering that the highest level of NS is equal to 1. Similar consideration apply for the coefficients of Cassa formulation.

6 Conclusions

In this study, the effect of geometry which characterises the leak area was investigated through experimental analysis. Results were achieved at the Environmental Hydraulic Laboratory of the University of Enna (Italy). Basically, the head-discharge law as well as the head-effective area relationship was analysed in a secondary pipe of a water distribution network. In order to do this, the laboratory experiments investigated high-density polyethylene pipe with small pipe diameter (DN 63 mm) and Pressure Nominal (PN) equal to 16 bar. Two different leak shapes were artificially generated in leak trunks, with circular and rectangular geometry. Rectangular cracks were elongated in the transverse direction with respect to mean flow direction. All the acquired data were used to investigate on the validity of different head-discharge laws. Specifically, the Torricelli's formulation, the IWA equation, a modified version of the IWA law and the formulation proposed by Cassa and co-authors were deeply analysed. All these formulation are ruled by one or more parametric coefficients. The experiments were used to calibrate these coefficients in order to achieve the best fitting with the experimental results. The experimental campaign was conducted investigating six different leaks for each shape, with different diameter of the circular cracks and different length for the transverse leak. Each leak trunk was analysed for 21 different pressures, in the range between 2 to 5 bar. The circular and rectangular holes were designed thus to have the same leak area. In this way, the comparison is not affected by the leak area and the shape effect is isolated. The results pointed out that two cracks having same leak area and same pressure conditions have different discharges. Basically, circular leaks cause higher level of outflow than rectangular transverse leaks. Based on the achieved results, it is evident that the effective area variation with the total head can be neglected for circular and rectangular transverse leak geometry. This result explains why the experimental data are well modelled by the Torricelli's formula. Nevertheless, further experiments are required, considering different materials and pipe diameters, to confirm the observatins. In the absence of leak area deformation, the exponent of the power law (IWA formulation) $b_I = 0.51$ and the discharge coefficient a_I linearly increases as the leak area grows and the slope of the linear trend is higher for circular leak than for transverse cracks, coherently with the highest discharge values observed for circular holes. The same consideration reported above for the IWA formulation applies to IWAM and Cassa formulations. Therefore, in the presence of circular and transversal cracks the comparable effectiveness of the different head-discharge laws is evident, at least for the studied configurations. The present study contributes to the practical optimization of the coefficients in the IWA and CAS relationships with regard to small diameter pipes. To confirm this result, further studies are needed to extend the range of experimental pressures and diameters tested.

Compliance with Ethical Standards

Conflict of interests None

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