

# An Optimal Operation Model for Hydropower Stations Considering Inflow Forecasts with Different Lead-Times

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# Abstract

To make full use of inflow forecasts with different lead times, a new reservoir operation model that considers the long-, medium- and short-term inflow forecasts (LMS-BSDP) for the real-time operation of hydropower stations is presented in this paper. First, a hybrid model, including a multiple linear regression model and the Xinanjiang model, is developed to obtain the 10-day inflow forecasts, and ANN models with the circulation indexes as inputs are developed to obtain the seasonal inflow forecasts. Then, the 10-day inflow forecast is divided into two segments, the first 5 days and the second 5 days, and the seasonal inflow forecast is deemed as the long-term forecast. Next, the three inflow forecasts are coupled using the Bayesian theory to develop LMS-BSDP model and the operation policies are obtained. Finally, the decision processes for the first 5 days and the entire 10 days are made according to their operation policies and the three inflow forecasts, respectively. The newly developed model is tested with the Huanren hydropower station located in China and compared with three other stochastic dynamic programming models. The simulation results demonstrate that LMS-BSDP performs best with higher power generation due to its employment of the long-term runoff forecast. The novelties of the present study lies in that it develops a new reservoir operation model that can use the long-, medium- and short-term inflow forecasts, which is a further study about the combined use of the inflow forecasts with different lead times based on the existed achievements.

Keywords Hydropower station · Different lead-times · Inflow forecasts · LMS-BSDP model

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# 1 Introduction

Inflow forecasts have been proved to be a promising tool for improving the efficiency of reservoir operation (McCollor and Stull 2008). In recent years, many studies have identified the value of inflow forecasts for reservoir decision-making. For example, Li et al. (2010) investigated the use of short-term inflow forecasts in flood control. Xu et al. (2013) incorporated medium-term inflow forecasts from the Quantitative Precipitation Forecasts (QPF) into the operation models for cascaded hydropower reservoirs, and the results showed that the efficiency and reliability of reservoir operation improved. Hamlet et al. (2002) illustrated that increased hydropower revenue is directly attributed to use of long-lead forecast information. Kim and Palmer (1997) investigated the value of seasonal flow forecasts in hydropower generation using a Bayesian Stochastic Dynamic Programming (BSDP) model. Mujumdar and Nirmala (2007) investigated the value of inflow forecasts for a multi-reservoir system using BSDP model. All of these studies found that forecast-based decision-making yields more economic benefits than conventional operating rules (Maurer and Lettenmaier 2004). However, these studies are mainly focused on the application of inflow forecast with a single forecast horizon.

Inflow forecasts with different lead-times have their own merits and faults. For example, short-term forecasts usually have a limited forecast horizon, but a high accuracy, which means small uncertainties. While long- and medium-term usually have a long forecast horizon, but a low accuracy, indicating large uncertainties. To take advantage of the long horizon of long- and medium-term forecasts and the high accuracy of short-term forecasts, long-, medium- and short-term inflow forecasts are usually used in combination in reservoir operations. The main challenge of this method is how to develop operation models that consider the combined use of forecasts with various forecast horizons (Eum et al. 2011). Yao and Georgakakos (2001) developed an integrated forecast-decision system for Folsom Lake (California) to assess the sensitivity of reservoir performance to inflow forecasts with different lead times. Their system includes three models pertinent to turbine load dispatching, short-range energy generation scheduling and long/mid-range reservoir management. The inflow forecasts are used in the corresponding models. The results show that reliable inflow forecasts is beneficial to reservoir performance. Tang et al. (2010a) presented two hybrid SDP models to investigate the potential value of inflow forecasts with various lead times in hydropower generation. In their models, the inflow forecasts of dry seasons and wet seasons are treated differently. The results shows that including inflow forecasts with various lead times is beneficial to the hydropower generation. Xu et al. (2014) presented a Two Stage Bayesian Stochastic Dynamic Programming (TS-BSDP) model for real-time operation of cascaded hydropower systems to handle the combined use of short-term forecasts (i.e., inflow forecast of 1-5 days) and medium-term forecasts (i.e., inflow forecasts of 6-10 days). The results showed that the combined use of forecasts with different forecast horizons can improve system performance in terms of power generation and system reliability. However, long-term forecasts (i.e., seasonal inflow forecasts) are not considered in the operation model. Therefore, when an operation decision to increase release is made with this model, if the seasonal forecast for the remainder of the flood season runoff is large, then there will be abandoned water and the release cannot be further increased. In such cases, the system has low water utilization efficiency because water resources are wasted. Meanwhile, if the seasonal forecast for the remainder of the flood season runoff is small, then there will be a lower power generation because the water level would be maintained at a low level. Therefore, long-term forecasts should be considered in the operation model to potentially improve power generation.

The main purpose of the present study is to develop an operation model that effectively couples long-, medium- and short-term forecasts. First, a hybrid inflow models, including a multiple linear regression model and the Xinanjiang model (Zhao 1992), is used to forecast the 10-day inflows, and the ANN models with the circulation indexes as inputs are used to forecast the seasonal inflows. Then the 10-day inflow forecast is partitioned into two periods, i.e., the first 5 days (i.e., the short-term inflow forecast) and the second 5 days (i.e., the medium-term inflow forecast), according to the TS-BSDP model proposed by Xu et al. (2014), and the seasonal inflow forecast is deemed as the long-term forecast. Finally, using Bayesian theory, two different strategies are developed to incorporate the three inflows, and a reservoir operation model that considers the long-, medium-and short-term inflow forecasts (LMS-BSDP) is developed. The Huanren hydropower station, located in China, is used to demonstrate the newly developed model. The novelties and advances of the present study are that it conducts a further study about the combined use of the inflow forecasts with different lead times based on the achievements of Xu et al. (2014) and develops a new reservoir

# 2 Reservoir Operation Model that Considers the Long-, Mediumand Short-Term Inflow Forecasts

operation model that can use the long-, medium- and short-term inflow forecasts.

### 2.1 Objective Function

The aim of the operation policy in this study is to maximize the expected total power production, under the condition that the stability of the power supply is guaranteed. Thus, the objective function can be shown as below.

$$f_{opt}^{n}(S_t, Q_t) = Max \left[ \sum_{t=1}^{T} E[B(S_t, Q_t, S_{t+1})] \right]$$

$$\tag{1}$$

$$B(S_t, Q_t, S_{t+1}) = \left[ b(S_t, Q_t, S_{t+1}) - \alpha \{ Max(e - b(S_t, Q_t, S_{t+1}), 0) \}^{\beta} \right] \cdot \Delta t$$
(2)

where, *t* is period index.  $t = 1, 2, \dots, T$ . *n* is the number of the time periods between the current period *t* and the last period *T*.  $S_t$  and  $S_{t+1}$  are the storage at the beginning and end of period *t*, respectively.  $Q_t$  is the observed inflow during period *t*.  $B(\cdot)$  is the power generation at period *t* corresponding to an initial reservoir storage ( $S_t$ ), an inflow ( $Q_t$ ) and a final storage volume ( $S_{t+1}$ ).  $b(\cdot)$  is the output at period *t*, and *e* is the firm output.  $\alpha$  and  $\beta$  are penalty factors, which are used to punish the reservoir performance, i.e. let the value of  $B(\cdot)$  decrease correspondingly, when the value of  $b(\cdot)$  is less than the firm capacity.  $\Delta t$  (*h*) is the time for decision interval.

#### 2.2 Recursive Equations

In classical SDP models, the optimal releases for a specified time horizon can be determined by solution of (Tejada-Guibert et al. 1995):

$$f_{opt}^{n}(S_{t},H_{t}) = Max \left\{ \sum_{Q_{t}} P_{Q_{t}|H_{t}} \cdot \left[ B_{t}(S_{t},Q_{t},S_{t+1}) + \sum_{H_{t+1}} P_{H_{t+1}|H_{t},Q_{t}} \cdot \left[ f_{opt}^{n-1}(S_{t+1},H_{t+1}) \right] \right] \right\}$$
(3)

where  $f_{opt}^{n}(\cdot)$  is the expected power generation from the current period *t* to *T*.  $f_{opt}^{n-1}(\cdot)$  is the expected power generation from the next period *t* + 1 to *T*.  $P_{O_t|H_t}$  is the conditional probability

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for a flow of  $Q_t$  during period t, given a specific hydrologic state variable ( $H_t$ ), i.e., the previous inflows, the short-, medium- and long-term inflow forecasts.

All the models used in the present study, including the proposed LMS-BSDP model and other alternative models, can be formulated from Eq. (3) by employing a different set of hydrologic state variables. For example, in dynamic programming (DP) model, it is assumed that the inflow can be forecasted perfectly. It can be deemed that DP model employs no hydrologic state variables. SDP-Q<sub>t-1</sub> employs the previous period's inflow  $Q_{t-1}$  as a hydrologic state variable, the current period's inflow  $Q_t$  is used as a hydrologic state variable in the SDP-Q<sub>t</sub> model, the TS-BSDP model employs the current period's inflow  $Q_t$  and the forecast of the next period's inflow  $F_{t+1}$  as the hydrologic state variables, while the LMS-BSDP model employs the current period's inflow  $Q_t$ , the forecast of the next period's inflow  $F_{t+1}$  and the seasonal inflow forecasts  $FL_{t+1}$  as the hydrologic state variables. The recursive equations of the four models, i.e., DP model, SDP-Q<sub>t-1</sub> model, SDP-Q<sub>t</sub> model and TS-BSDP model, are shown in SUPPLE-MENTARY MATERIAL. The details of LMS-BSDP model are shown below.

Same as in the TS-BSDP model, the Short-term Forecast (SF, 1–5 days) is used directly as the current period's inflow with the assumption that it is accurate, and the Medium-term Forecast (MF, 6–10 days) is deemed as the next period's inflow. Due to the fact that in Northern China, the inflow in the flood season (from May to October) is larger than that in the non-flood season (from November to next April) and that the remainder of the flood season inflow has an influence on the reservoir operation decisions, the seasonal flow forecast, which is defined as the inflow from the next period (which should be in the flood season) through October 31, is taken as a hydrologic state variable by the LMS-BSDP model. It should be noted that the seasonal flow forecast is deemed as the long-term forecast, and it is only available for the flood season and is updated for every period in the flood and non-flood seasons. Take *T* represents the final time step and the recursive equations for every time step are shown below. As shown in Fig. 1a, the non-flood season include period 1 to period *n*, and period *m* + 1 to period *T*, while the flood season include period n + 1 to period *m*. Periods *n* and *m* are the last period in April and October, respectively.

(1) When t = T

There is no MF and seasonal flow forecast at time step T. To calculate the power generation, the storage at the end of the period is set to  $S_{\min}$ , which is the minimum of the reservoir storage.



Fig. 1 Description of inflow processes (a) and decision-making strategies (b-c) for LMS-BSDP

Due to that the iteration stops only when it reaches a steady state, so the value of the  $S_{min}$  doesn't affect the optimization results. So the recursive equation of MF is equal to that of SF.

$$\begin{cases} SF f_{opt}^n(S_t, Q_t) = Max\{B_t(S_t, Q_t, S_{\min})\} \\ MF f_{opt}^n(S_t, Q_t) = Max\{B_t(S_t, Q_t, S_{\min})\} \end{cases}$$
(4)

Where  $_{SF}f_{opt}^{n}(\cdot)$  and  $_{MF}f_{opt}^{n}(\cdot)$  represent the maximum expected generation of hydropower for the SF, the MF, respectively.

(2) When t = T - 1

There are two periods *T*-1 and *T* to calculate, and there is MF for period *T*-1 but no seasonal flow forecast. The expected power generation for the SF at period *T*-1 is determined by the power generation at period *T*-1, the expected power generation for SF at period T and the inflow posterior flow transition probability  $P_{jmk}^{t+1}$  i.e.,  $P_{Q_{t+1}|Q_t,F_{t+1}}$ , which is the probability that the flow  $Q_{t+1}$  in time period t+1 belongs to the class interval k, given that the flow  $Q_t$  in time period t belongs to the class interval j and the flow forecast  $F_{t+1}$  in time period t+1 belongs to the class interval m. The expected power generation for MF at *T*-1 is determined by the expected power generation for SF at period T and the inflow posterior flow transition probability  $P_{jmk}^{t+1}$ . The expected power generation for the forecast horizon (FH) is determined by the power generation and inflow forecast of SF and MF. The relative recursive equation of SF, MF and FH are shown below.

$$\begin{cases} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmk}^{t+1} \cdot_{SF} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}) \right\} \\ MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = Max \left\{ P_{jmk}^{t+1} \cdot_{SF} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}) \right\} \\ FH f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = \frac{Q_{t}}{Q_{t} + F_{t+1}} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) + \frac{F_{t+1}}{Q_{t} + F_{t+1}} MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) \end{cases}$$
(5)

Where,  $_{FH}f^n_{opt}(\cdot)$  represents the maximum expected generation of hydropower for the entire FH.

(3) When  $m \le t \le T - 2$  and  $1 \le t < n - 1$ 

The period is in non-flood season. There are at least two periods that should be considered for calculation, and there is MF for period t but no seasonal flow forecast. The proposed LMS-BSDP model employs the current period's inflow  $Q_t$  and the forecast of the next period's inflow  $F_{t+1}$  as the hydrologic state variables in the non-flood season, which is the same as TS-BSDP model. Replacing  $H_t$  in Eq. (3) with  $Q_t$  and  $F_{t+1}$ , and replacing  $H_{t+1}$  with  $Q_{t+1}$  and  $F_{t+2}$ , respectively, the recursive equations for the first 1–5 days (SF) can be written as:

$$SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1})$$

$$= Max \left\{ \sum_{Q_{t}} P_{Q_{t}|Q_{t}, F_{t+1}} \cdot \left[ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}, F_{t+2}} P_{Q_{t+1}, F_{t+2}|Q_{t}, F_{t+1}, Q_{t}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right] \right\}$$

$$= Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}, F_{t+2}} P_{Q_{t+1}, F_{t+2}|Q_{t}, F_{t+1}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right\}$$

$$(6)$$

Same as the study of Karamouz and Vasiliadis (1992), two assumptions are adopted here: the forecasts for different periods are stochastically independent of each other and the forecast is stochastically dependent on the inflow of the current period and the inflow of the previous

period. Therefore, according to the condition probability formula, the above equation can be reduced to:

$$SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}} \sum_{F_{t+2}} P_{Q_{t+1}, F_{t+2} | Q_{t}, F_{t+1}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right\}$$
(7)  
$$= Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}} P_{Q_{t+1} | Q_{t}, F_{t+1}} \cdot \sum_{F_{t+2}} P_{F_{t+2} | Q_{t+1}, F_{t+1}, Q_{t}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right\}$$
$$= Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}} P_{Q_{t+1} | Q_{t}, F_{t+1}} \cdot \sum_{F_{t+2}} P_{F_{t+2} | Q_{t+1}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right\}$$
$$= Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmk}^{t+1} \cdot \sum_{n} P_{kn}^{t+2} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right] \right\}$$

Equation (7) shows that the expected performance for SF at period t is determined by the performance of SF at time step t  $B_t(S_t, Q_t, S_{t+1})$ , the expected performance for SF at time step t+1, the posterior flow transition probability  $P_{jmk}^{t+1}$  and the predictive probability of forecasts  $P_{kn}^{t+2}$ , i.e.,  $P_{F_{t+2}|Q_{t+1}}$ , is the probability that the flow forecast  $F_{t+2}$  in time period t+2 belongs to the class interval n, given that the flow  $Q_{t+1}$  in time period t+1 belongs to k. The expected performance for MF at period t is determined by the last three factors. The performance of the entire FH at period t is determined by the system performance and inflow forecast of SF and MF. Thus, the recursive equation can be shown as below.

$$\begin{cases} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmk}^{t+1} \cdot \sum_{n} P_{kn}^{t+2} \cdot SF f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right\} \\ MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = Max \left\{ P_{jmk}^{t+1} \cdot \sum_{n} P_{kn}^{t+2} \cdot SF f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right\} \\ FH f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) = \frac{Q_{t}}{Q_{t} + F_{t+1}} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) + \frac{F_{t+1}}{Q_{t} + F_{t+1}} MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}) \end{cases}$$
(8)

(4) When  $n \le t < m - 1$ 

The period is in flood season. There are also at least two periods that should be considered for calculation, and there are both MF and seasonal flow forecast for period t. So the current period's inflow  $Q_t$ , the forecast of the next period's inflow  $F_{t+1}$  and the seasonal inflow forecasts  $FL_{t+1}$  are employed as the hydrologic state variables. Replacing  $H_t$  in Eq. (3) with  $Q_t$ ,  $F_{t+1}$  and  $FL_{t+1}$ , and replacing  $H_{t+1}$  with  $Q_{t+1}$ ,  $F_{t+2}$  and  $FL_{t+2}$ , respectively, the recursive equations for the first 1–5 days (SF) can be written as:

$$SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1})$$

$$= Max \left\{ \sum_{Q_{t}} P_{Q_{t}|Q_{t}, F_{t+1}, FL_{t+1}} \cdot \left[ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}, F_{t+2}, FL_{t+2}} P_{Q_{t+1}, F_{t+2}, FL_{t+2}|Q_{t}, F_{t+1}, FL_{t+1}, Q_{t}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right] \right] \right\}$$

$$= Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}, F_{t+2}, FL_{t+2}} P_{Q_{t+1}, F_{t+2}, FL_{t+2}|Q_{t}, F_{t+1}, FL_{t+1}} \cdot \left[ f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right] \right\}$$

$$(9)$$

The two assumptions used in the non-flood season are also adopted here. And it is assumed in the present study that the medium-term forecast for next period is

$$SF \int_{opt}^{n} (S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) \\ = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}, F_{t+2}, FL_{t+2}} P_{Q_{t+1}, F_{t+2}, FL_{t+2}|Q_{t}, F_{t+1}, FL_{t+1}} \cdot \left[ f_{opt}^{n-1} \left( S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2} \right) \right] \right\} \\ = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{Q_{t+1}} P_{Q_{t+1}|Q_{t}, F_{t+1}, FL_{t+1}} \cdot \sum_{FL_{t+2}} P_{FL_{t+2}|Q_{t+1}, FL_{t+1}} \cdot \sum_{F_{t+2}} P_{F_{t+2}|, FL_{t+2}, Q_{t}, Q_{t+1}, F_{t+1}, FL_{t+1}} \cdot \left[ f_{opt}^{n-1} \left( S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2} \right) \right] \right\} \\ = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{P_{t+2}} P_{L_{t+2}|Q_{t+1}, FL_{t+1}} \cdot \sum_{FL_{t+2}} P_{FL_{t+2}|Q_{t+1}, FL_{t+1}} \cdot \sum_{F_{t+2}} P_{FL_{t+2}|Q_{t+1}, FL_{t+2}, Q_{t+1}, FL_{t+2}, PL_{t+2}, Q_{t+1}, FL_{t+2}, Q_{t+1}, FL_{t+2}, PL_{t+2}, Q_{t+1}, FL_{t+2}, PL_{t+2}, PL_{$$

Equation (10) shows that the expected performance for SF at period t is determined by the performance of SF at period t  $B_t(S_t, Q_t, S_{t+1})$ , the expected performance for SF at period t+1, the posterior flow transition probability  $P_{imak}^{t+1}$ ,  $P_{krn}^{t+2}$  and the predictive probability of forecasts  $P_{kr}^{t+2}$ . The expected performance for MF at period t is determined by the last four factors. The performance of the entire FH at period t is determined by the performance and inflow forecast of SF and MF. Thus, in the flood season, the recursive equations can be shown as below.

$$SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmqk}^{t+1} \cdot \sum_{r} P_{kr}^{t+2} \cdot \sum_{n} P_{krn}^{t+2} \cdot SF f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right\}$$

$$MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ P_{jmqk}^{t+1} \cdot \sum_{r} P_{kr}^{t+2} \cdot \sum_{n} P_{krn}^{t+2} \cdot SF f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right\}$$

$$(11)$$

$$F_{H}f^{n}_{opl}(S_{t}, \mathcal{Q}_{t}, F_{t+1}, FL_{t+1}) = \frac{\mathcal{Q}_{t}}{\mathcal{Q}_{t} + F_{t+1}} SFf^{n}_{opl}(S_{t}, \mathcal{Q}_{t}, F_{t+1}, FL_{t+1}) + \frac{F_{t+1}}{\mathcal{Q}_{t} + F_{t+1}} MFf^{n}_{opt}(S_{t}, \mathcal{Q}_{t}, F_{t+1}, FL_{t+1})$$

where,  $P_{imak}^{t+1}$ , i.e.,  $P_{Q_{t+1}|Q_t,F_{t+1},FL_{t+1}}$ , is the probability that the flow  $Q_{t+1}$  in time period t+1 belongs to class interval k, given that the flow  $Q_t$  in time period t belongs to class interval j, the flow forecast  $F_{t+1}$  in time period t+1 belongs to class interval m and the seasonal flow forecast  $FL_{t+1}$  in time period t+1 belongs to class interval q.  $P_{kr}^{t+2}$ , i.e.,  $P_{FL_{t+2}|Q_{t+1}}$ , is the probability that the seasonal flow forecast  $FL_{t+2}$  in time period t+2 belongs to class interval r, given that the flow  $Q_{t+1}$  in time period t+1belongs to class interval k.  $P_{krn}^{t+2}$ , i.e.,  $P_{F_{t+2}|Q_{t+1},FL_{t+2}}$ , is the probability that the flow forecast F in time period t+2 belongs to class interval n, given that the flow  $Q_{t+1}$  in time period t+1 belongs to class interval k and the seasonal flow forecast  $FL_{t+2}$  in time period t+2 belongs to class interval r.

(4) When t = m - 1

The period is in flood season. The periods from m-1 to T should be considered for calculation. However, there are no seasonal flow forecast for periods m+1 to T. Therefore, compared to Eq. (11), the expected performance for SF at period t is determined by the performance of SF at period t  $B_t(S_t, Q_t, S_{t+1})$ , the expected performance for SF at period t+1, the posterior flow transition probability  $P_{jmqk}^{t+1}$  and the predictive probability of forecasts  $P_{kn}^{t+2}$ . The expected performance for MF at period t is determined by the last three factors. The performance of the entire FH at period t is determined by the performance and inflow forecast of SF and MF. Thus, the recursive equations can be shown as below.

$$\begin{cases} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ B_{t}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmqk}^{t+1} \sum_{n} P_{kn}^{t+2} \cdot_{SF} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right\} \\ MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ P_{jmqk}^{t+1} \cdot \sum_{n} P_{kn}^{t+2} \cdot_{SF} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}) \right\} \\ FH f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = \frac{Q_{t}}{Q_{t} + F_{t+1}} SF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) + \frac{F_{t+1}}{Q_{t} + F_{t+1}} MF f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) \end{cases}$$

$$(12)$$

(4) When t = n - 1

The period is in non-flood season. The periods from *n*-1 to *T* should be considered for calculation. However, there are seasonal flow forecast for periods n + 1 to *T*. Therefore, compared to Eq. (8), the expected performance for SF at period *t* is determined by the performance of SF at period  $t B_t(S_t, Q_t, S_{t+1})$ , the expected performance for SF at period t + 1, the posterior flow transition probability  $P_{jmk}^{t+1}$ ,  $P_{km}^{t+2}$  and the predictive probability of forecasts  $P_{kr}^{t+2}$ . The expected performance for MF at period *t* is determined by the last four factors. The performance of the entire FH at period *t* is determined by the performance and inflow forecast of SF and MF. Thus, in the flood season, the recursive equations can be shown as below.

$$\int_{SF} f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ B_{l}(S_{t}, Q_{t}, S_{t+1}) + \sum_{k} P_{jmk}^{t+1} \cdot \sum_{r} P_{kr}^{t+2} \cdot \sum_{n} P_{km}^{t+2} \cdot S_{F} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right\}$$

$$MFf_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = Max \left\{ P_{jmk}^{t+1} \cdot \sum_{r} P_{kr}^{t+2} \cdot \sum_{n} P_{km}^{t+2} \cdot S_{F} f_{opt}^{n-1}(S_{t+1}, Q_{t+1}, F_{t+2}, FL_{t+2}) \right\}$$

$$(13)$$

$$FHf_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) = \frac{Q_{t}}{Q_{t} + F_{t+1}} S_{F} f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1}) + \frac{F_{t+1}}{Q_{t} + F_{t+1}} M_{F} f_{opt}^{n}(S_{t}, Q_{t}, F_{t+1}, FL_{t+1})$$

#### 2.3 Decision Strategies

The decision-making strategies is determined according to the relationship between the decision horizon (DH) and the forecast horizon (FH). The inflow forecast used in the LMS-BSDP model are the Short-term Forecast (SF, 1–5 days) and the Medium-term Forecast (MF, 6–10 days) and the long-term forecast (LF) for the runoff from time step t+1 through October 31. The SF and MF are the two periods partitioned from the entire FH inflow (10 days) (Xu et al. 2014). So the time step t of LMS-BSDP is 5 days and two decision-making strategies are developed. The first one, shown in Fig. 1b, is defined as Strategy D, in which the SF at time step t and the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time step t, the MF and LF at time step t+1 are used to determine the operation policies for time steps t and t+1. The decision-making strategies adopted by DP, SDP-Q<sub>t-1</sub>, SDP-Q<sub>t</sub>, TS-BSDP are shown in SUPPLEMENTARY MATERIAL.

#### 2.4 Performance Metrics

To compare the performance of DP,  $SDP-Q_{t-1}$ ,  $SDP-Q_t$ , TS-BSDP and SML-BSDP, two commonly used metrics are adopted here: Annual hydropower generation (AHG) and

reliability. Annual hydropower generation is the most important performance indicator of the hydropower system. Reliability is the ratio of the periods that the output is not lower than the firm capacity and the all operation periods. They are listed below.

$$AHG = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{M} B_{ij}$$

$$\tag{14}$$

$$Reliability = \frac{n}{N \times M} \times 100\%$$
(15)

where, *N* is the total number of years, *M* is the total number of the periods in a year,  $B_{ij}$  is the power generation at the period *j* of year *i*, *n* is the total number of the periods whose output is lower than the firm capacity.

# 3 Case Study

#### 3.1 Study Area

The Hun River cascaded hydropower reservoirs system, located on the Hun River, is one of the most important water resource systems in northeast China. This hydropower reservoirs system consists of three reservoirs: the Huanren reservoir, the Huilong reservoir and the Taipingshao reservoir, the locations and characteristics of which are referred to the Fig. 3 and Table 1 in the paper of Xu et al. (2014), respectively. Among these three reservoirs, the Huanren reservoir is the upstream reservoir, providing the main water for hydroelectric production, and thus is responsible for the total hydropower generation of the system. Huilong and Taipingshao are the downstream reservoirs, and daily regulation reservoirs. Therefore, the Huanren reservoir is selected as a case study to verify the newly developed model in this study. The Huanren reservoir covers a total drainage area of approximately 10,400 km<sup>2</sup>. The basin has two distinct seasons: non-flood and flood. Precipitation varies significantly between these two seasons, with about 70 to 80% of the precipitation occurring in the flood seasons (from May to October).

Models	Variables	Inflow for decision	(a)		(b)	
			AHG (MWH)	Reliability (%)	AHG (MWH)	Reliability (%)
DP			513.61	94.44	517.38	97.22
SDP-Qt	$Q_t$	Observed	480.03	88.66	482.34	95.83
TS-BSDP	$\widetilde{Q}_t, F_{t+1}$	Observed	483.07	93.29	483.96	93.06
LMS-BSDP	$\tilde{Q}_{t}, F_{t+1}, FL_{t+1}$	Observed	485.32	92.13	486.69	91.2
SDP-Q <sub>t-1</sub>	$\overline{Q}_{t-1}$	Observed	472.8	82.18	467.4	83.33
SDP-Qt	$\tilde{Q}_t$	Simulated	473.42	88.43	468.47	91.2
TS-BSDP	$Q_t, F_{t+1}$	Simulated	477.49	90.97	472.89	88.43
LMS-BSDP	$Q_t, F_{t+1}, FL_{t+1}$	Simulated	478.46	90.05	476.35	86.11

 Table 1
 Simulation results of the optimization models (a) when the decision horizon is 5 days and (b) when the decision horizon is 10 days

### 3.2 Inflow Forecasts

In the present study, to implement all the models in cases when decision horizons are 5 days and 10 days, the short-term inflow forecast (SF, 1–5 days), the medium-term

5 days and 10 days, the short-term inflow forecast (SF, 1-5 days), the medium-term inflow forecast (MF, 6-10 days), the entire forecast horizon inflow (FH, 1-10 days) and the long-term inflow forecast (LF, seasonal inflow forecast) are needed. To obtain these inflow forecasts, different models are developed. The details are shown below.

Two models, a multiple linear regression model for the non-flood seasons and the Xinangjiang model for the flood seasons, which were proposed by Xu et al. (2014), are used to predict the average inflow for SF (1–5 days), MF (6–10 days) and FH (1–10 days). It should be noted that the parameters of the two models are recalibrated in the present study. The data series from 1968 to 2000 is treated as the calibration dataset to train the models, and the series from 2001 to 2015 is treated as the validation dataset to evaluate model performance. According to the calibrated models, the medium-term QPFs from 2001 to 2015 in the global forecast system (GFS), which were developed by the U.S. National Center for Environmental Prediction, are applied to forecast the inflows.

Similar to the study of Xu et al. (2014), the simulation results of SF, MF and FH are shown in Fig. 2. It can be seen that the forecast models perform well during the



Fig. 2 A comparison of simulated and observed inflows with different lead-times

calibration and verification periods, with the Nash-Sutcliffe efficiency coefficients (NSEs) being equal to or greater than 0.9. During the forecast periods, all the NSEs show a decrease because of the uncertainty of GFS. The NSEs for SF, MF and FH are 0.83, 0.55, and 0.78, respectively. This shows that MF inflow forecasts are most uncertain. The forecasted inflows of calibration and validation periods will be used to develop operation models and to derive the operation policies and those of forecasting periods are used to test the performances of the operation models.

To obtain the seasonal inflow forecast, the flood season (from May to October) is divided into 5-day periods, and ANNs are developed for every period, with the 74 circulation indexes and the previous inflow as inputs. The 74 circulation indexes are downloaded freely from the website of China National Climate Center (http://ncc.cma.gov.cn). To develop the ANNs models, first candidate inputs are selected from the previous inflow and the 74 circulation indexes in the period from January in previous year to 2 month before the forecast time, using the linear correlation analysis (LCA) (Sudheer et al. 2002), and then the candidate inputs are added one by one according to their orders until the performance of the model shows no significant improvements. The data series is partitioned into the training dataset (1968–1991), the cross-validation dataset (1991-2000) and the validation dataset (2001-2015) to train and evaluate the models, respectively. According to the national criteria for inflow forecasts in China, if the absolute value of the relative error is no more than 20% of the mean annual deviation, or the grade difference is no more than 1, then the seasonal inflow forecast is deemed as qualified. The qualified rate of deviation and grade for every period (i.e., every ANN model) are shown in Fig. 3. It can be seen that for periods in training dataset, the qualified rate of deviation varies from 82.14 to 100%, the qualified rate of grade varies from 82.14 to 96.43%, and for periods in cross-validation dataset, the qualified rate of deviation is 100%, the qualified rate of grade varies from 81.82 to 100%. All these results indicate that these ANN models can be used in the validation dataset. In validation dataset, the number of periods whose qualified rate of deviation is more than 80% is 28, which is 82.35% of the total periods, and the number of periods whose qualified rate of grade is more than 80% is 30, which is 88.23% of the total periods. The results of validation dataset also reveal that the ANN models show a satisfactory performance.



Fig. 3 Performance of every periods. a The qualified rate of deviation, b The qualified rate of grade

184

# 4.1 Operation Policies

To obtain the operation polices, all the developed models iterate, using the backward recursive equations, until the ending storage reaches a steady state. During the iterations, the penalty factors  $\alpha$  and  $\beta$  are set to 1 and 2 respectively, same as the study of Tang et al. (2010b).

To calculate the probability used in the SDP models, the inflow forecasts should be discretized. The SF, MF and the seasonal inflow forecast of the same period in different years are first collected, then a heuristic procedure, i.e., different intervals are tested, is used to determine the intervals of the three inflow. Finally, due to the better performance, the intervals for SF, MF, and LF are determined as 6, 6, and 3, respectively. That is, for each period, the inflows of SF and MF are discretized into 6 intervals, 15%, 30%, 45%, 60%, 75%, and 90%, denoted as m = 1, 2, ..., 6 and n = 1, 2, ..., 6, respectively, the seasonal inflow forecast is discretized into 3 intervals, denoted as k = 1, 2, 3. Besides, storage is discretized into 29 intervals with an increment of 30 Mm<sup>3</sup>.

Figure 4 shows the operation polices of LMS-BSDP for different decision styles and periods, in the cases when the interval of the seasonal inflow forecast k is 3 and the interval of MF n is 5. Figure 4a and c are the operation policies derived using Strategy D for 5 days, August 11–15 and September 1–5, respectively. Figure 4b and d are derived using Strategy E for 10 days, August 11–20, and September 1–10, respectively.



Fig. 4 The operation policy of LMS-BSDP for different decision styles and periods. a for August 11-15 using Strategy D, b for August 11-20 using Strategy E, c for September 1-5 using Strategy D, d for September 1-10 using Strategy E

### 4.2 Simulation Results

To test the performance of the models used in this study, their operation policies are used to make operation decision of the period from 2001 to 2015 for Huarnren reservoir in cases when decision horizons are 5 days and 10 days. In the test, both the observed and simulated inflows to the Huanren reservoir from 2001 to 2015 are used. It should be known that the observed inflow case is used as a reference to the simulated inflow case and it assumes that inflows can be perfectly forecasted. The results of the observed inflow case is expected to reveal the maximum efficiency and reliability that could be achieved based on accurate information. The results are shown in Table 1. It can be seen that, for the same model, the results obtained using the observed inflow are better than those obtained using the simulated inflow. This is because the inflow forecast obtained using GFS is uncertain and cannot match the observed inflow accurately, thus leads to the worse results. For the same model, when the observed inflow is used to make decision, the power generation obtained when the decision horizon is 5 days is larger than that obtained when the decision horizon is 10 days. On the other hand, when simulated inflow is used to make decisions, power generation in cases when the decision horizon is 5 days is smaller than in the cases when it is 10 days. The reason is that when the observed inflows are used, the perfect inflow forecast is used, so the 10 days lead-time can provide more time to increase the output and reduce the release, while when the simulated inflows are used, the 10 days inflow forecast is more uncertain, which leads to smaller power generation.

Table 1 also show that  $SDP-Q_{t-1}$  performs worst in all cases and has the lowest AHG and reliability.  $SDP-Q_t$  performs better than  $SDP-Q_{t-1}$ , which implies that applying the inflow forecast to power station operation can improve the AHG and reliability of the power station. Compared to  $SDP-Q_t$ , TS-BSDP achieves a better AHG and reliability, indicating that the combined use of medium- and short-term inflow forecasts can further improve the performance of the power station because forecast uncertainties are dealt with differently. Due to the employment of the long-term inflow forecast, LMS-BSDP performs better than TS-BSDP with a better AHG, which shows that the combined use of inflow forecasts with different lead-times can enhance power station performance.

As shown above, the overall performance of LMS-BSDP is better than that of TS-BSDP. LMS-BSDP uses the SF, MF inflows and seasonal inflow to determine the operation policy of the SF, while TS-BSDP only uses the SF and MF inflows. To illustrate the influence of the seasonal inflow forecast, the operation processes from period 23 to 48 of 2010 are shown as an example in Fig. 5. Figure 5a shows the inflows and releases for hydroelectric generation, and Fig. 5b shows water levels and spillage. It can be seen that there is a significant difference in the release at period 25 between LMS-BSDP and TS-BSDP. The LMS-BSDP model increases release to reduce storage due to the high seasonal inflow forecasted at period 26. It is the same case for period 41.

When the simulated inflow is used, the power generation of LMS-BSDP is smaller than that of DP by 35.15 MWH and 41.03 MWH when decision horizons are 5 days and 10 days, respectively. The reason is that in the DP model, the observed inflows are used to make decisions, and thus the conditional probability matrix in Eq. (3) is the unit matrix composed of 0 and 1. While in LMS-BSDP model, the simulated



Fig. 5 The decision process of Strategy D for the periods from early April to late October in 2010

inflows are used to make decisions, i.e., the simulated 10-day inflow and the simulated seasonal inflow for the flood seasons. All the simulated inflows have uncertainty, and thus the conditional probability matrix in Eq. (3) is not the unit matrix. The results indicate that accurate inflow forecasts can help to improve the conditional probability matrix and achieve more power generation.

# **5** Conclusions

This paper investigates the combined use of inflow forecasts with different lead-times to improve the overall system performance of hydropower station in terms of hydropower generation and reliability. Based on the TS-BSDP model, which can quantify the uncertainties of medium- and short-term inflow forecasts, a new reservoir operation model that considers the long-, medium- and short-term inflow forecasts (LMS-BSDP) is developed for real-time hydropower station operation. The newly developed LMS-BSDP is tested on the Huanren hydropower station in China and is compared with four other models. The main findings are summarized below.

- (1) In both cases when the decision horizons are 5 and 10 days, SDP-Q<sub>t</sub>, TS-BSDP and LMS-BSDP perform better than SDP-Q<sub>t-1</sub> with higher hydropower generation and reliability, the reason is that the uncertainty of inflow forecasts are better considered in the former three models than in SDP- Q<sub>t-1</sub>.
- (2) When the decision horizon is 5 days, all the reliabilities of LMS-BSDP are more than 90%, and the hydropower generation is 2.25 MWH and 0.97 MWH more

than that obtained using TS-BSDP, when the observed inflow and simulated inflow are used, respectively. And when the decision horizon is 10 days, all the reliabilities of LMS-BSDP are more than 86% and the hydropower generation is increased by 2.73 MWH and 3.46 MWH, respectively. All these demonstrate that LMS-BSDP can achieve a better performance with less abandoned water and more hydropower generation due to the employment of the long-term inflow forecast.

(3) Among all the models used in the present study, DP performs best with the highest hydropower generation and reliabilities. The reason is that in this model, it is assumed that the inflows are known previously, although this is impossible in real reservoir operations. This reveals that further studies are needed to get more accurate inflow forecast for achieving more power generation.

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