

# The Impact of Uncertainty Factors on Optimal Sizing and Costs of Low-Impact Development: a Case Study from Beijing, China

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**Abstract** The facility allocation optimization of Low-impact development (LID) optimization has been used widely to prevent and tackle urban storm water pollution. However, uncertainties existing in nature and human society would influence the size and total cost of LID. To study the influence of the uncertainties on LID optimization allocation, the research develops the model of LID optimization allocation under uncertainty. The principle of the model is establishing primarily the LID optimization model based on certain numbers and identifying the uncertainties. Hence, the model integrates the uncertainty programming, including interval programming, fuzzy programming, stochastic programming, chance constraint programming (CCP) and scenario programming. The model of LID optimization allocation under uncertainty is established with the conditions. The developed uncertainty model tackles multiple types of uncertainties, and the results of the model are in the interval form in multiple scenarios. The model analyses the effects of uncertainties on the size and total cost of LID in this way. The study shows that the uncertainties in rainfall, infiltration rate, release coefficient, funds and unit price all have a significant influence on the size and total cost of LID when these uncertainty factors overlay. A higher violation probability of CCP corresponding to LID sizing results to a wider interval number of the corresponding uncertainty. The developed method of the study is universal, and the method could be extended to other cases of LID optimization allocation to speculate the influence of uncertainties.

**Keywords** Low-impact development · Optimization · Uncertainty · Sizing · Cost

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## 1 Introduction

Global urbanisation has resulted in the continuous expansion of city areas. Following this phenomenon, huge areas of impervious underlying surfaces in city districts exist, which seriously ward off rain water infiltration, leading to a series of urban problems caused by rainfall, such as rainfall flood, waterlogging and storm water pollution (Damodaram and Zechman 2013; Jia et al. 2014).

Many concepts and methods for urban rain water management have been developed to solve these problems, such as Best Management Practices (BMPs), Sustainable Urban Drainage Systems (SUDS) (Zhou 2014), Water Sensitive Urban Design (WSUD) (Coutts et al. 2013), Sustainable Infrastructure (SI) (Tan and Dur 2010), Low Influence Development (LID) and Green Infrastructure (GI) (Debo and Reese 2003). LID is a useful strategy to solve urban storm water and storm water pollution problems, its principle is to use source control techniques to reduce runoff and pollution loads (Martin-Mikle et al. 2015), and LID includes bioretention, grass swale, rain barrels, vegetable filter step (Herendeen et al. 2009).

Determining the size of the LID in each location and confirming the total cost are two important factors for LID planning and allocation. A reasonable facility capacity can reduce construction fund whilst achieving the rainfall control target (Chen et al. 2016). At present, the methods for setting LID capacity include SUSTAIN software (Mao et al. 2016) and linear programming (Faucette et al. 2009; Loáiciga et al. 2015).

The key factors for determining the LID capacity and planning budget include urban rainfall, urban surface rain water runoff, permeability coefficient of each LID, release coefficient of each LID, implementation cost of each facility and budget (Loáiciga et al. 2015). However, there are many uncertainties in nature and human society; Generally, uncertainty factors influencing LID sizing are categorised as either hydrological-hydraulic uncertainty factors or economic uncertainty factors (Gu et al. 2016a; b). Hydrological-hydraulic uncertainty factors include surface runoff uncertainty, infiltration rate and permeability rate of the LID. The surface runoff uncertainty should be considered firstly, because the surface rain water runoff is determined by the volume of rainfall and rain water permeability. Among them, the volume of rainfall are uncertainties that follow stochastic probability distribution (Gabellani et al. 2007), and determines the design capacity of LID facility, though certain studies have confirmed that the infiltration rate of the LID follows lognormal distribution (Mishra et al. 1989). The release coefficient of water in LID facility is similar to the release coefficient of water in pipe, and this coefficient has been confirmed to be in the form of probability distribution (Kapelán et al. 2003), but in related research obeys the normal distribution (Kapelán et al. 2003). The cost and budget of the LID remain uncertain regarding economic uncertainty. These uncertain factors would influence LID sizing, implementation effect and final cost.

Solving the uncertainties in linear optimization is necessary when using optimization to calculate the scale of LID facility. At present, many studies have focused on optimization hybrid with uncertainty (Li et al. 2016b; Ren et al. 2017). Uncertainties are categorized as interval, stochastic and fuzzy uncertainties. When tackling the uncertainties of optimization programming, the corresponding programming to these uncertainties are interval, stochastic and fuzzy programming (Guo and Huang 2011). In some cases, many types of uncertainties in optimization programming exist, requiring the use of hybrid inexact linear programming integrated with many types of uncertainties in solving the problem (Gu et al. 2013). However, the programming may pose a complex and serious problem when connections exist in these multiple uncertainties. The chance constraint programming (CCP) can solve the problem effectively despite

the existence of connections, because it does not require all constraints in the programming to be totally satisfied; instead, the constraints can be satisfied in proportion to cases with given probabilities (He et al. 2008; Li et al. 2016a). When meeting more complicated uncertainty cases, such as when both sides of the CCP are uncertainties, the CCP programming would be too complicated to solve. Some uncertainty programming, such as Interval-Parameter Chance-Constraint Mixed-Integer programming, have been developed to solve these cases (Huang et al. 2011). In conclusion, the complicated uncertainty program of linear programming is handled by integrating uncertainty linear optimization programming with CCP programming, the integrated model evaluates the rational scales of decision variables effectively, and the impact of uncertainties on LID facility sizing is estimated by transplanting the uncertainty optimization method into the sizing optimization of for LID facility.

Given the increasingly severe urban storm water and urban water pollution, the LID optimization has attracted much attention, and this research on sizing and total cost of LID planning plays an important role in it. The uncertainty in hydrology and economy influences the effect of LID optimization, nevertheless most of the researches on urban rainfall control are under certain condition. Though some studies on uncertainty exist, these uncertainty studies are mostly focused on the characteristics of hydrology or hydraulics of urban rainfall, and few studies focus on LID facility. In addition, the study of LID planning under uncertainty is still at its initial stage.

At present, the studies of LID planning do not address the following problems. 1) how does the uncertainty factor affect the LID facility capacity and LID planning costs? 2) how can a reasonable LID planning under uncertainty be designed? The solution to these problems indicates that importance of reasonably planning LID in complex circumstances.

Corresponding to the problems above, this study has three main aims. 1) To establish the optimization formation for LID optimization in certain circumstance; 2) To distinguish the uncertain factors that affect LID facility and 3) To develop the optimization model for LID sizing and total LID planning costs based on uncertainty.

## 2 Methodology

### 2.1 Defining the Sizing and Consumption of LID in Certain Environment

#### 2.1.1 Style of Typical LID and its Working Principle

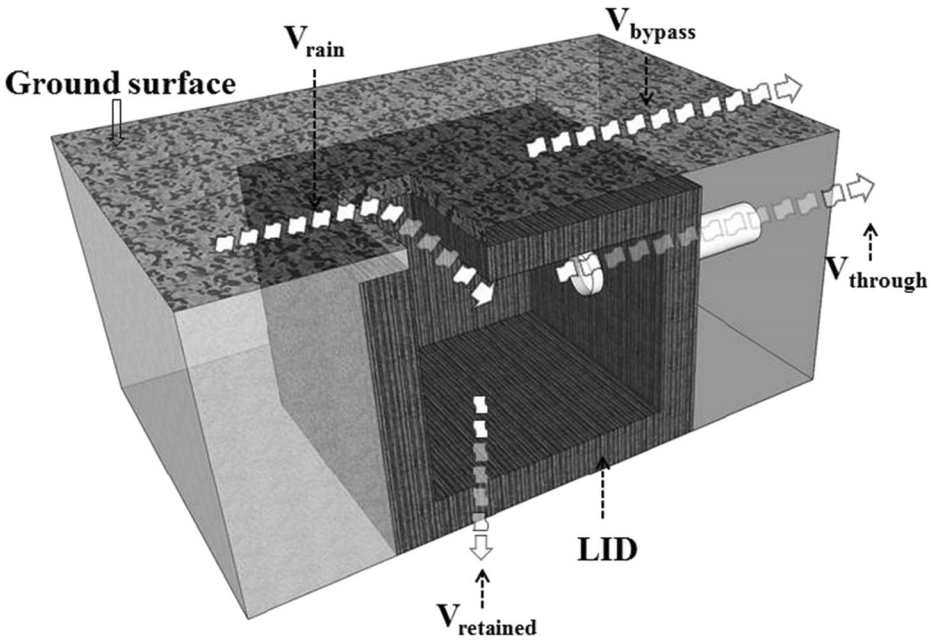
LID facility alleviates urban storm water and reduces pollution in rain water runoff by suspending rainfall runoff. The principle of typical LID is that all or part of the runoff could be suspended in LID facility when the runoff bypasses the LID site. In retention space ( $V_{retained}$ ) of LID facility, rain water seeps into sand or soil, where the pollution is also absorbed. If the volume of influent rain water exceeds the retention capacity, then the excess part of the storm water ( $V_{through}$ ) passes through the LID facility. Whereas if the volume of influent rain water exceeds the sum volume of  $V_{retained}$  and  $V_{through}$ , the excess parts ( $V_{bypass}$ ) spill over the LID facility.

In Fig. 1.

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$V_{rain}$	the volume of water arrives at the LID site, and it has the concentration C of pollution.
$V_{retained}$	the volume of water retained on site by LID facility during the design storm.
$V_{through}$	the volume of water flows through the LID facility during the design storm.
$V_{bypass}$	the volume of water spills over the LID facility, and it have the concentration C of pollution.

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**Fig. 1** The schematic of a typical LID

*2.1.2 Relationship Between Elements of LID*

The water balance equation of LID facility is  $V_{bypass} = V_{rain} - V_{retained} - V_{through}$  whilst the retention space is full of water.

The  $V_{retained}$  and  $V_{through}$  are discussed respectively as follows:

$V_{retained}$  is represented as

$$V_{retained,ij} = a_{ij} \cdot k_{ij} + c_{ij} \tag{1}$$

$i$  represents the LID facility of type  $i$ .

$j$  represents the location site of LID facility.

$a_{ij}$ , represent the known characteristics of the LID  $i$  on site  $j$ .

$c_{ij}$

$k_{ij}$  represents the unknown decision variables of LID  $i$  of site  $j$ , and the decision variable refers to the different means in different scenarios.

When the LID has no retention space, then  $V_{retained} = 0$ , and  $a_{ij} = 0$ ,  $c_{ij} = 0$ .

When the LID has retention capacity and its bottom is impermeable or low permeable, then  $a_{ij} = 1$ ,  $c_{ij} = 0$ , and the  $k$  represents the retention volume.

When the LID facility are composed of permeable materials, such as soil, sand, the  $a_{ij}$  and  $c_{ij}$  are dependent on the geometry of LID facility, hydrology characteristics of permeable material and the duration of urban rainfall. Suppose the bottom of the LID facility is rectangle, and the permeability process continues during the duration of rainfall, in addition, the permeability of the bottom material is  $f$ , then the  $V_{retained, ij} = L_{ij} \cdot W_{ij} \cdot f \cdot \Delta t$ . In this case,  $a_{ij} = L_{ij} \cdot f \cdot \Delta t$ ,  $c_{ij} = 0$ ,  $k_{ij} = W_{ij}$ .

As to percolation well, suspended rain water permeates through the wall and bottom of the facility, in this case, the permeability area is calculated as the rectangular area of the wall plus the circle area of the bottom (assuming the bottom is impermeable) (Loaiciga and Church 2010)

$V_{through.ij}$ , the retention space of the LID facility is assumed to be equal to linear reservoir with the effective volume  $V_{ij}$  (Amarocho 1973). Thus, the linear release model, which is widely utilised in hydrological-hydraulic, is used to simulate the volume of water flow-through the LID channel (U.S. Corps of Engineers 2000).

$$V_{through.ij} = r_{ij} * V_{ij} * \Delta t \tag{2}$$

where

- $r_{ij}$  release coefficient (unit:1/time), and it is determined by experiment
- $V_{ij}$  effective storage volume
- $\Delta t$  duration of the design storm

The volume of  $V_{through}$  could be written as

$$V_{through.ij} = b_{ij} * k_{ij} + d_{ij} \tag{3}$$

The  $b_{ij}$  and  $d_{ij}$  are coefficients of j - th LID facility in i - th site.

Assume that the j - th LID facility is in site i, its effective retention volume is  $V_{ij}$ , the linear release model shows that its real-time change rate of rain water volume is  $dV_{ij}/dt$  and is equal to  $r_{ij} * V_{ij}$ ; hence by using finite-different form, the  $b_{ij} = r_{ij} * \Delta t$ ,  $k_{ij} = V_{ij}$ ,  $d_{ij} = 0$ .

If the depth ( $D_{ij}$ ) of the LID facility is unknown, wherein the  $D_{ij}$  is the decision variable, and the width ( $W_{ij}$ ) and length ( $L_{ij}$ ) are known, the  $b_{ij} = r_{ij} * W_{ij} * L_{ij} * \Delta t$  and  $k_{ij} = D_{ij}$ ,  $d_{ij} = 0$ .

In case of percolation, the volume of flow-through has no connection with  $D_{ij}$ , that is,  $D_{ij}$  is not the decision variable, which means  $b_{ij} = 0$  and  $d_{ij} = r_{ij} * \Delta t * V_{ij}$  (Table 1).

In general, the volume of  $V_{through}$  is a small portion compared with the volume of  $V_{retained}$  and the volume of  $V_{bypass}$ .

The LID facility, which has both water retention volume and permeable wall, could assimilate the pollution in rainfall, which flows into the retention space of LID facility.

$$\frac{V_{retained.ij}}{V_{through.ij} + V_{bypass.ij} + V_{retained.ij}} \geq \epsilon_{ijr} \tag{4}$$

Where

- $V_{retained.ij}$  the storage volume of j - th facility in site i
- $V_{through.ij}$  the volume of dredge water of j-th facility in site i
- $V_{bypass.ij}$  the volume of water spills over the j-th facility in site i

**Table 1** the eigenvalues and the variables of  $V_{through.ij}$

	$b_{ij}$	$d_{ij}$	$k_{ij}$
LID facility	$r_{ij} * \Delta t$	0	$V_{ij}$
$D_{ij}$ is the decision variable	$r_{ij} * W_{ij} * L_{ij} * \Delta t$	0	$D_{ij}$
$D_{ij}$ is not the decision variable	0	$r_{ij} * \Delta t * V_{ij}$	-

$\epsilon_{ijr}$  the efficiency of  $r$  pollution reduction for rainwater flows through  $i$  site

When the LID facility has no  $V_{retained}$  and its flow-through facility has certain ability of pollution reduction such as grass swale, then its efficiency of pollution reduction is

Where 
$$\frac{V_{through.ij} \cdot C_{through.ijr}}{V_{bypass.ij} \cdot C_{rain.r} + V_{through.ij} \cdot C_{through.ijr}} \geq \epsilon_{ijr} \tag{5}$$

$V_{bypass. ij}$  the volume of rainwater flows through  $j$  facility in  $i$  site  
 $C_{through. ijr}$  the pollution load in rainwater passes through  $j$  facility in  $i$

### 2.1.3 Optimization Model of LID Sizing and Finance

Assuming that the type of LID facility in each site is predetermined, the linear programming is used to forecast the LID facility sizing. The objective of the programming are 1) To effectively reduce the volume of surface storm water and reduce the rain water pollution to standard and 2) To minimise the cost of installation, operation and maintenance. Optimal LID sizing and total cost are achieved by satisfying the objectives above.

**Objective Function** The unknown capacity of LID facility at site  $i$  is set as  $k_i$ , unit variable cost of the LID facility is equal to  $UC_i$ , and fixed charge, which includes installation charge and maintenance charge and so on is  $FC_i$ . The objective function of the study is to minimise the total cost of the LID facility in the study area.

$$\text{Minimise } Z = \sum_{i=1}^n (UC_i \cdot k_i + FC_i) \tag{6}$$

$UC_i$  the unit variable cost of LID facility  
 $FC_i$  the fixed cost of the LID facility that is dependent of its size  
 $k_i$  the sizing of LID facility at site  $i$

**Capacity Constraint** The capacity of the LID facility at each site must be greater than the minimum permissible capacity and less than the maximum permissible capacity.

$$k_{mini} \leq k_i \leq k_{maxi} \tag{7}$$

**Budgetary Constraint** The total cost for installation, operation and maintenance may not exceed the maximum available budget.

$$\sum_{i=1}^n (UC_i \times k_i + FC_i) \leq B \tag{8}$$

**Volume Constraint** The total volume of retained rain water in LID facility and the volume of flow-through rain water may not exceed the volume of rain water that arrives at  $i$  site.

According to the equation  $V_{retained \cdot i} = a_i \times k_i + c_i$ ,  $V_{through \cdot i} = b_i \times k_i + d_i$ .

The constraint is solved as

$$(a_i + b_i) \times k_i \leq I_i - (c_i + d_i) \quad i = 1, 2 \dots n \tag{9}$$

**Volume Constraint of Special Capacity of LID Facility** The surface rain water volume reduction of LID facility at site  $i$  may not be less than the certain percentage of volume of the rain water that arrives at site  $i$ , and the surface rain water volume reduction of all LID facilities in the study area may not be less than the certain percentage of rain water volume in the study district. The above mentioned volume reduction pertains to the sum volume of both retained rain water and flow-through rain water.

$$a_i \times k_i + c_i + b_i \times k_i + d_i \geq \text{percentage } I_i \tag{10}$$

$$\sum_{i=1}^n (a_i \times k_i + c_i + b_i \times k_i + d_i) \geq \text{percentage} \sum_{i=1}^n I_i \tag{11}$$

Water quality constraint

1. When the LID facility is underlain by impervious materials and has pollution removal ability, the rain water contaminant level in LID facility is  $EMC_i$
2. When the LID facility has retention capacity with infiltration effect, all the pollution in storage space ( $V_{retained}$ ) can be absorbed, thus the contaminant level in LID facility is equal to 0.

Therefore, in the case of 1, the pollution reduction rate is

$$\frac{V_{retained} \cdot EMC_2}{(V_{through} + V_{bypass}) \cdot EMC_1 + V_{retain} \cdot EMC_2} \geq \text{percentage } i \tag{12}$$

where

$EMC_1$  the rain water contaminant level

$EMC_2$  contaminant level of rain water in retention space

In case of 2 the pollution reduction rate is

$$\frac{V_{retained}}{V_{through} + V_{bypass} + V_{retain}} \geq \text{percentage } i \tag{13}$$

The optimal sizing of the LID facility is solved through the objective function and constraint programming.

### 2.2 Construction of the uncertainty optimization model

The general uncertainty optimization model could be expressed as:

$$\text{Max}f^{\pm} = \sum_{i=1}^n C_i^{\pm} \cdot X_i^{\pm} \tag{14}$$

Subject to

$$\begin{aligned} \sum_{i=1}^m A_{ij}^{\pm} \cdot X_i^{\pm} &\leq B_j^{\pm}, \quad j = 1, 2, \dots, m_1; i = 1, 2, \dots, n \\ x_i^{\pm} &\geq 0, \quad i = 1, 2, \dots, n \end{aligned}$$

For the general form of uncertainty model,  $X_i^{\pm}$ ,  $A_{ij}^{\pm}$ ,  $B_j^{\pm}$  and  $C_i^{\pm}$  are uncertainty numbers represented as interval, these interval uncertainties numbers have deterministic and closed boundary, the ‘-’ represents the lower bound of the uncertainty, and the ‘+’ represents the upper bound. The uncertainty could be interval uncertainty number or stochastic uncertainty number. In the case of stochastic number, the interval is the confidence interval of probability numbers (Huang et al. 2001). However, when some uncertainty numbers in objective function or constraint programming appear in the fuzzy set form, the optimization programming could be represent as follow.

$$\text{Max}f^{\pm} = \sum_{i=1}^n \left( C_i^{\pm} + \tilde{C}_i' \right) \cdot X_i^{\pm} \tag{15}$$

Subject to

$$\begin{aligned} \sum_{i=1}^m A_{ij}^{\pm} \cdot X_i^{\pm} &\leq B_j^{\pm}, \quad j = 1, 2, \dots, m_1; i = 1, 2, \dots, n \\ \sum_{i=1}^m \tilde{A}_{ig}' \cdot X_i^{\pm} &\leq \tilde{B}_g', \quad g = 1, 2, \dots, m_2; i = 1, 2, \dots, n \\ x_i^{\pm} &\geq 0, \quad i = 1, 2, \dots, n \end{aligned}$$

$\tilde{A}_{ig}'$  and  $\tilde{C}_i'$  are fuzzy numbers of objective function and constraint programming, the fuzzy number can be represented in the form of membership function, and the  $\alpha$ -cut could be used to represent the distribution interval for fuzzy uncertainty number, the presentation function of the  $\alpha$ -cut is (Gu et al. 2013)

$$\tilde{A}_{\alpha} = \left\{ x \mid \mu_{\tilde{A}}(x) \geq \alpha, x \in X \right\} \tag{16}$$

$\mu_{\tilde{A}}$  is membership function,  $X$  is universe set of elements;  $\alpha$  is the membership,  $\tilde{A}_{\alpha}$  is the sets that all components of  $X$  is greater than or equal to  $\alpha$ .

Multiple types of uncertainty could be tackled effectively by optimization function integrated with interval number, stochastic number and fuzzy number, however in some practical problems, dynamic and varying interactions exists between these uncertainties, and this feature reflected in the equations is the coefficient randomness in the right side of the equation, in this circumstance, the constraint would be violated, thus the chance



constraint programming (CCP) was developed to deal with this situation. The CCP can reflect the violation in uncertainty constraint, and it also could be represented in certain form. The two conditions of the CCP set are (1) defining a series of probabilities  $p_i$  ( $p_i \in [0, 1]$ ), (2) the constraint should be satisfied with at least a probability of  $(1 - p_i)$ . On the premise satisfying the conditions above, the CCP could be represented as follow (Charnes and Cooper 1983; Huang and Loucks 2000).

$$\Pr\{t|A_i(t) \cdot X \leq B_i(t)\} \geq 1 - p_i, \quad A_i(t) \in A(t), \quad i = 1, 2, \dots, m \tag{17}$$

The constraint (17) is always nonlinear, however it could be converted to linear programming, and in linear programming form,  $A_i(t)$  is a certain number,  $B_i(t)$  is a stochastic number, and the converted linear version is (Wu et al. 2015).

$$\Pr A_i(t) \cdot X \leq \tilde{B}(t)^P, \quad A_i(t) \in A(t), \quad i = 1, 2, \dots, m \tag{18}$$

where  $\tilde{B}(t)^P = \{B_i(t)^{p_i} | i = 1, 2, \dots, m\}$

By integrating the uncertainty optimization model with CCP, the integrated optimization formation could be represented as follow:

$$\text{Max } f^\pm = \sum_{i=1}^n \left( C_i^\pm + \tilde{C}_i' \right) \cdot X_i^\pm \tag{19}$$

Subject to

$$\begin{aligned} & \sum_{i=1}^m A_{ij}^\pm \cdot X_i^\pm \leq B_j^\pm, \quad j = 1, 2, \dots, m_1; i = 1, 2, \dots, n \\ & \Pr \left[ \sum_{i=1}^n \tilde{A}_{ig}' \cdot X_i^\pm \leq \tilde{B}_g^\pm \right] \geq Q_g, \quad g = 1, 2, \dots, m_2; i = 1, 2, \dots, n \\ & x_i^\pm \geq 0, \quad i = 1, 2, \dots, n \end{aligned}$$

Where  $Q_g$  is fuzzy tolerance measures, and  $0 \leq Q_g \leq 1$ .

The fuzzy values  $\tilde{A}_{ig}'$  and  $\tilde{C}_i'$  in the formulation could be represented in triangular fuzzy value or trapezoid fuzzy value, with the  $\alpha$ -cut method, the fuzzy values could be represented as  $\tilde{A}_{ig}' = (A_{ig}^-, A_{ig1}, A_{ig2}, A_{ig}^+)$  and  $\tilde{C}_i' = (C_i^-, C_{i1}', C_{i2}', C_i^+)$ , and its distribution interval could be defined with the  $\alpha$ -cut method, and the interval value are  $[(1-\alpha) \cdot A_{ig}^- + \alpha \cdot A_{ig1}, (1-\alpha) \cdot A_{ig}^+ + \alpha \cdot A_{ig2}]$  and  $[(1-\alpha) \cdot C_i^- + \alpha \cdot C_{i1}', (1-\alpha) \cdot C_i^+ + \alpha \cdot C_{i2}']$  respectively (Iskander 2005).

The uncertainty optimization could be solved by interactive algorithm, the interactive algorithm could analyze and tackle the interaction between uncertainty values, its principle is to use two stage method to solve the uncertainty model, the two stage method is (1) establishing the sub-model  $f^+$  which maximize the objective function. (2) establishing the

sub-model  $f^-$  which minimize the objective function according to the result of the sub-model  $f^+$  (Wu et al. 2015). Therefore, the uncertainty model could be converted to two sub-models:

$$\text{Max}f^+ = \sum_{i=1}^{k_1} (C_i^+ + (1-\alpha) \cdot C_i^{\prime+} + \alpha \cdot C_{i2}^{\prime}) \cdot X_i^+ + \sum_{i=k_1+1}^n (C_i^+ + (1-\alpha) \cdot C_i^{\prime+} + \alpha \cdot C_{i2}^{\prime}) \cdot X_i^- \quad (20)$$

Subject to

$$\sum_{i=1}^{k_1} A_{ij}^- \cdot X_i^+ + \sum_{i=k_1+1}^n A_{ij}^- \cdot X_i^- \leq B_j^+, \quad j = 1, 2, \dots, m_1$$

$$\text{Pr} \left[ \sum_{i=1}^{k_1} [(1-\alpha)A_{ig}^- + \alpha \cdot A_{ig1}] \cdot X_i^+ + \sum_{i=k_1+1}^n [(1-\alpha)A_{ig}^- + \alpha \cdot A_{ig1}] \cdot X_i^- \leq B_g^+ \right] \geq Q_g,$$

$$g = 1, 2, \dots, m_2, x_i^+ \geq 0, i = 1, 2, \dots, k_1, x_i^- \geq 0, i = k_1 + 1, k_1 + 2, \dots, n$$

Where  $x_i^+$  ( $i = 1, 2, \dots, k_1$ ) are interval variables with positive coefficients;  $x_i^-$  ( $i = 1, 2, \dots, k_1$ ) are interval variables with negative coefficients; Solutions of  $x_{ihopt}^+$  ( $i = 1, 2, \dots, k_1$ ) and  $x_{ihopt}^-$  ( $i = 1, 2, \dots, k_1$ ) can be obtained.

The sub-model  $f^-$  could be formulated as follow:

$$\text{Max}f^- = \sum_{i=1}^{k_1} (C_i^- + (1-\alpha) \cdot C_i^{\prime-} + \alpha \cdot C_{i1}^{\prime}) \cdot X_i^- + \sum_{i=k_1+1}^n (C_i^- + (1-\alpha) \cdot C_i^{\prime-} + \alpha \cdot C_{i1}^{\prime}) \cdot X_i^+ \quad (21)$$

Subject to

$$\sum_{i=1}^{k_1} A_{ij}^+ \cdot X_i^+ + \sum_{i=k_1+1}^n A_{ij}^+ \cdot X_i^- \leq B_j^-, \quad j = 1, 2, \dots, m_1$$

$$\text{Pr} \left[ \sum_{i=1}^{k_1} [(1-\alpha)A_{ig}^+ + \alpha \cdot A_{ig2}] \cdot X_i^+ + \sum_{i=k_1+1}^n [(1-\alpha)A_{ig}^+ + \alpha \cdot A_{ig2}] \cdot X_i^- \leq B_g^- \right] \geq Q_g,$$

$$g = 1, 2, \dots, m_2, 0 \leq x_i^+ \leq x_{ihopt}^+, \quad i = 1, 2, \dots, k_1, x_i^- \geq x_{ihopt}^-, \quad i = k_1 + 1, k_1 + 2, \dots, n$$

Where the solutions of  $x_{ihopt}^-$  ( $i = 1, 2, \dots, k_1$ ) and  $x_{ihopt}^+$  ( $i = 1, 2, \dots, k_1$ ) can be obtained through solving sub model (20). Thus the results could be get through combing the two sub-model.

$$x_{iopt}^\pm = [x_{iopt}^-, x_{iopt}^+], \forall i; \quad (22)$$

$$f_{opt}^\pm = [f_{opt}^-, f_{opt}^+] \quad (23)$$

### 3 Case Study

#### 3.1 Overview of the Case Study

Due to the accelerated development of urbanisation, the urban impermeable land surface has increased rapidly leading to urban storm inundation in many Chinese cities during a rain storm, in response, ‘sponge city construction standard’ was introduced by the Chinese government to guide urban planning. ‘Sponge city’ means to decrease urban rainfall flood and rainfall pollution through the LID facility. When planning the LID facility, all the benefits and costs of rainfall management should be considered, therefore, the LID facilities should be planned rationally for a good tradeoff between cost and benefit. However, environmental and human uncertainties are unavoidable in the planning and these uncertainties would influence the planning of LID, thus investigating the influence of uncertainty on the LID sizing and budget and developing the model of LID facility planning under uncertainty to avoid or reduce risks is meaningful. This study develops a straightforward hydrologic model to research the influence of uncertainty on LID sizing, and the study also develops the model of LID facility planning under uncertainty. In the study, where and what type of BMPs are used are predetermined, and the climate and soil of Beijing district are taken as the research background.

##### 3.1.1 Analysis of LID Facility

The study takes the Beijing urban district as research background. Beijing is located at 30° N, 119° E, and has a north temperate continental monsoon climate, having hot and rainy summers. The one-year rainfall is 40 mm, and the two-year rainfall is 70.9 mm.

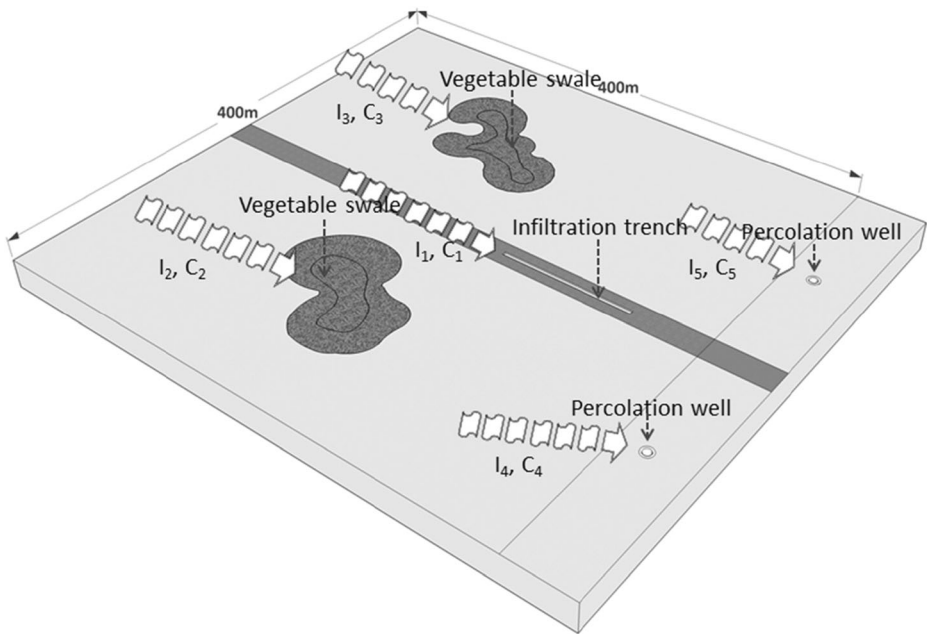
The study chooses a square district with sides of 400 m, the runoff coefficient of the total area is 40% (including the pervious area, impervious area and drainage), five places where LID facility will be installed to handle the rainstorm and rain water pollution were chosen, and in each place, only one LID facility will be installed. The LID facilities could be classified into three types, that is, infiltration trench, vegetated swales and percolation well. Site 1 is a street across the district where the corresponding LID facility is an infiltration trench overlain by porous pavement. Sites 2 and 3 are public open space where the corresponding LID facilities are vegetated swales. Sites 4 and 5 are spaces downslope from the street transect and public space, and the corresponding facilities are percolation wells (Fig. 2).

In infiltration well and vegetable swale, the retained water percolates through the bottom surface to the underlying soil, and in percolation well, the retained water percolates through both the inside wall and the bottom surface of the vessel to the surrounding soil outside, and thus according to the formation  $V_{retained, ij} = a_{ij} \cdot k_{ij} + c_{ij}$  and the description of different type of  $V_{retained}$ , the eigenvalues and variables of the three types of the LID facilities are given in Table 2.

The depths of the infiltration trench and the vegetable swale are neglected, and the length of the infiltration trench and the radius of the percolation well are set to be the fixed values, that is, the length of the infiltration trench is 100 m, radius of the percolation well is 0.3 m, and the percolation well has no retention volume.

Therefore, according to the function expression of  $V_{through}$  which  $V_{through, ij} = b_{ij} \cdot k_{ij} + d_{ij}$ , and the description for function relationship of three types of  $V_{through}$ , its corresponding eigenvalues and the variables are given in Table 3.

The ability of LID facility planning to control the surface runoff of the one-year rainfall or two-year rainfall is the standard, whereas the key factors for the LID facility sizing include the



**Fig. 2** The layout of LID allocation

volume of surface rain water runoff, infiltration capacity, release coefficient of the LID sizing and the budget, and the volume of surface rain water runoff is determined by the rainfall and the runoff coefficient of the study area. The influence factors are given in Table 4.

### 3.1.2 Uncertainty Factor

Uncertainties inevitably exist because of varying natural and human factors. The uncertainties affecting the sizing of the LID facility is classified into two categories: hydrologic/hydraulic uncertainty factors and economic uncertainty factors.

The hydrologic/hydraulic uncertainty includes infiltration rate of the LID facility  $f$ , release rate of the LID facility  $r$ , the rainfall which relates to the construction standard of the LID facility and the probability violation for the capacity volume. The infiltration rate of infiltration trench obeys the normal distribution, and its distribution interval is [0.0184, 0.0342] (unit: mm), the infiltration rate of the vegetative swale, composed of sandy soil and sandy loam soil, is 0.0142 mm/s and 0.0069 mm/s, respectively. Thus, the distribution interval of the rate is [0.006, 0.0142] (unit: mm), for the percolation well, its infiltration rate obeys the logarithmic normal distribution with a distribution interval of [0.0147, 0.02] (unit: mm).

**Table 2** the eigenvalues and the variables of the  $V_{retained, ij}$  for three types of the LID facilities

	$V_{retained, ij}$	$a_{ij}$	$c_{ij}$	$k_{ij}$
Inf	$L_{ij} \cdot W_{ij} \cdot f_{inf} \cdot \Delta t$	$L_{ij} \cdot f_{inf} \cdot \Delta t$	0	$W_{ij}$
veg	$S_{ij} \cdot f_{veg} \cdot \Delta t$	$f_{veg} \cdot \Delta t$	0	$S_{ij}$
per	$D_{ij} \cdot 2\pi \cdot r_{ij} \cdot f_{per} \cdot \Delta t + \pi \cdot r_{ij}^2 \cdot f \cdot \Delta t$	$2\pi \cdot r_{ij} \cdot f_{per} \cdot \Delta t$	$\pi \cdot r_{ij}^2 \cdot f \cdot \Delta t$	$D_{ij}$

**Table 3** the eigenvalues and the variables for the  $V_{through, ij}$  of three types of the LID facilities

	$V_{through, ij}$	$b_{ij}$	$d_{ij}$	$k_{ij}$
<i>inf</i>	$r_{inf} \cdot L_{ij} \cdot W_{ij} \cdot \Delta t$	$r_{inf} \cdot L_{ij} \cdot \Delta t$	0	$W_{ij}$
<i>veg</i>	$r_{veg} \cdot \Delta t \cdot S_{ij}$	$r_{veg} \cdot \Delta t$	0	$S_{ij}$
<i>per</i>	$r_{per} \cdot \Delta t \cdot \pi \cdot r_{ij}^2$	0	$r_{per} \cdot \Delta t \cdot \pi \cdot r_{per}^2$	$D_{ij}$

Looking at the release coefficient of the LID facility, the character of the rain water flowing through the LID facility obeys the character of the water body flowing in a pipeline, thus, the hydrological uncertainty of the rain water flowing in LID facility is similar to the hydrological uncertainty of water flowing in a pipeline, and the flow velocity of the rain water in LID facility obeys the logarithmic normal distribution. In the study, the velocity characters of the three types of LID facilities are: for infiltration trench, the mean value is  $1.9167 \times 10^4$  mm/s, and the standard deviation is  $1.9167 \times 10^5$  mm/s; for vegetable swale, the mean value is  $6.3611 \times 10^4$  mm/s, and the standard deviation  $6.3611 \times 10^5$  mm/s; for percolation well, the mean value is  $7.7778 \times 10^4$  mm/s and the standard deviation  $7.7778 \times 10^5$  mm/s.

The study takes one-year rainfall and two-year rainfall volume in 24 h as standards of the designed rainfall corresponding to the sizing of the LID setting.

Economic uncertainty could be classified as either budget uncertainty or cost uncertainty.

For budget uncertainty, the economic budget is always in fuzzy set distribution, the fuzzy set distribution of the study is [6.5, 7.8] (unit:  $10^4$  euro).

For cost uncertainty, the cost distribution is based on the cost of a certain LID facility project which has been established in Beijing, and its uncertainty factors and cost distribution is given in Table 5.

The probability violation for the constraint on the LID facility capacity is uncertain because despite the rainfall standard being at a constant value because of the climate variability, the precipitation is the uncertainty value treated as probability distribution, and the summer precipitation in Beijing district are in the form of normal distribution. Thus, considering that the observed rainfall always exceeds the retention capacity of LID facility, which is established according to fix rainfall value, rain water runoff could not be totally disposed. However the CCP can handle this situation, the violation probability of the CCP set in the study are  $P1 = 0.9$ ,  $P2 = 0.8$ ,  $P3 = 0.7$ , and these uncertainties could be categorised as project uncertainty.

### 3.1.3 Model Formulation

To address the influence of uncertainty on the total cost and the capacity of the LID facility, this study establishes the corresponding optimization model and focuses on two aspects: 1. establishing the optimization model based on the certain feature, 2. integrating the optimization model with uncertainty.

**Table 4** the influence factor

Influence factor	
Hydrological/hydraulic factor	Economic factor
Infiltration/release rate	Budget
Rainfall	
Surface rainwater runoff	

**Table 5** the uncertainty and its distribution interval

Hydrological/hydraulic uncertainty	Infiltration rate (mm/s)	Release coefficient (mm/s)
<i>inf</i>	[0.06629, 0.12311]	[1.1342, 2.4917]
<i>veg</i>	[0.025, 0.51]	[4.4453, 8.2684]
<i>per</i>	[0.0530, 0.0985]	[5.4444, 10.1111]
<i>P1</i>	0.9	
<i>P2</i>	0.8	
<i>P3</i>	0.7	
1Y	45 mm	
2Y	70.9 mm	
Economic uncertainty	Fix cost	Float cost
<i>inf</i>	[7500, 10,000]	[17,875, 24,625]
<i>veg</i>	[1500, 2000]	[71.5, 98.5]
<i>per</i>	[1500, 2000]	[193.3, 140.3]
budget	[500,000, 600,000]	

Five aspects should be comprehensively considered when establishing the LID model based on the certain value: 1. establishment of objective function, 2. Capacity constraint of LID facility, 3. constraint of budget, 4. runoff retention constraint, 5. pollution control constraint.

Regarding uncertainty integration, the uncertainties of the study include 1. The model parameters including interval parameter (the infiltration rate of the vegetable swale, cost), fuzzy parameter (cost and budget) and the stochastic parameter which is in the form of different probability distribution (infiltration rate of the infiltration trench and percolation well, release coefficient). 2. The model referred to in the scenario analysis (one-year rainfall, two-year rainfall) and probability violation.

The study integrates the model with interval planning, fuzzy planning, stochastic planning, scenario analysis and CCP, by doing this, the model can effectively tackle the uncertainty in the optimization and eventually obtain the optimization scheme for the LID facility planning under uncertainty (Fig. 3).

The model based on the certainty factor is.

Objective function

$$\text{Minimize } Z = \sum_{i=1}^n (UC_i \cdot k_i + FC_i) \tag{24}$$

Capacity constraint for LID facility

$$k_{\text{mini}} \leq k_i \leq k_{\text{maxi}} \tag{25}$$

Budget constraint

$$\sum_{i=1}^n (UC_i \times k_i + FC_i) \leq B \tag{26}$$

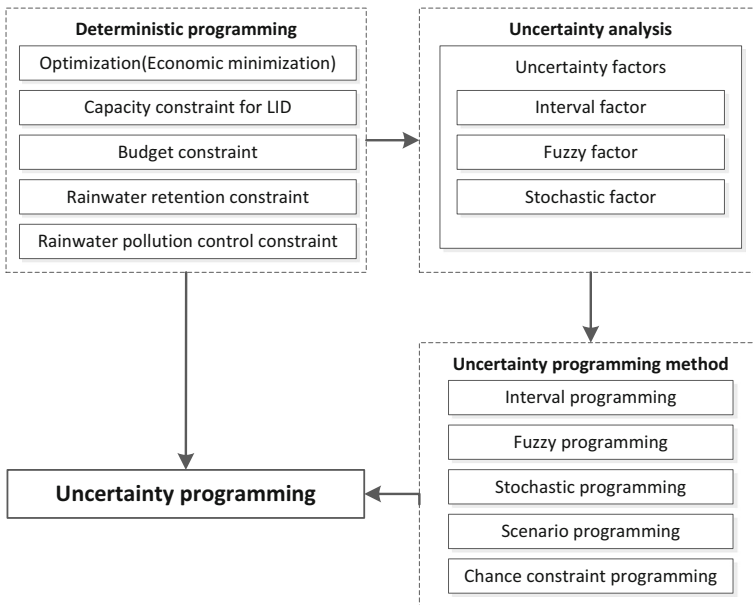


Fig. 3 The flow diagram

The constraint for the volume of rainwater retention for single LID facility

$$peri \cdot I_i \leq (V_{retained,i} + V_{through,i}) \leq I_i \tag{27}$$

The constraint for the volume of rainwater retention for all LID facilities

$$\sum_{i=1}^n (V_{retained,i} + V_{through,i}) \geq peri \sum_{i=1}^n I_i \tag{28}$$

The constraint for pollution in surface runoff

$$\frac{V_{retained,i}}{I_i} \geq peri \tag{29}$$

Where

- $V_{retained}$  The volume of rainwater retention in LID facility during the rainfall
- $V_{through}$  The volume of rainwater flow through the LID facility during the rainfall

$UC_i$	The unit construction cost for LID facility at $i$ site
$k_i$	The fixed cost related to the capacity for the LID facility at $i$ site
$k_i$	The sizing of the LID facility at $i$ site
$I_i$	The runoff arrived at $i$ site
$B$	The maximum capacity

### 3.1.4 Establishing the Optimization Model Based on the Uncertainty

The integrated model refers to the interval value, stochastic value and fuzzy value, and the stochastic value and the fuzzy value is in the form of interval value, where the stochastic value takes the 95% confident interval value and the fuzzy value takes the  $\alpha$  – cut interval value, and the '+' denotes the upper value, and the '-' denotes the lower value.

The infiltration rates of the LID facility  $f(f_{veg}, f_{per}, f_{inf})$  obey the normal distribution, thus its corresponding  $a, c$  obeys the normal distribution. The release coefficient  $r(r_{veg}, r_{per}, r_{inf})$  obeys the lognormal distribution, and its corresponding  $b, d$  also obeys the lognormal distribution.

The integrated model is as follows:

Objective function

$$\text{Minimize } Z^\pm = \sum_{i=1}^n (\widetilde{UC}_i \cdot k_i + FC_i^\pm) \tag{30}$$

Constraints

$$\begin{aligned}
 & k_{mini} \leq k_i \leq k_{maxi} \\
 & \sum_{i=1}^n (\widetilde{UC}_i \cdot k_i + FC_i^\pm) \leq B^\pm \\
 & (\text{percentage } I_i)^P \leq a_i^\pm \times k_i + c_i^\pm + b_i^\pm \times k_i + d_i^\pm \leq (I_i)^P \\
 & \sum_{i=1}^n (a_i^\pm \times k_i + c_i^\pm + b_i^\pm \times k_i + d_i^\pm) \geq \left( \text{percentage } \sum_{i=1}^n I_i \right)^P \\
 & \frac{a_i^\pm \times k_i + c_i^\pm}{I_i} \geq \text{percentage } i
 \end{aligned}$$

## 4 Results and Discussion

1. In the infiltration rate, release coefficient, budget and cost significantly influenced the cost of the LID facility planning and the capacity of the LID facility, and their effect is greater when these factors are combined.

According to the optimization of the LID facility planning under uncertainty, the total cost of the LID facility planning and the sizing of each LID facility corresponding to the one-year and two-year rainfall are listed in Table 6.



**Table 6** LID for one-year and two-year rainfall

		Sizing				
		1	2	3	4	5
1Y	P1	[0.4425, 5.4773]	[0.7512, 1.3956]	[100, 1966.3465]	[10.2672, 19.0729]	[10.2672, 19.0729]
	P2	[0.4466, 5.7419]	[0.7892, 1.4660]	[100, 2065.5674]	[10.7856, 20.0357]	[10.7856, 20.0357]
	P3	[0.4496, 5.9327]	[0.8168, 1.5168]	[100, 2137.1127]	[11.1594, 20.7299]	[11.1594, 20.7299]
2Y	P1	[0.5662, 8.4952]	[1.1840, 2.1988]	[141.4041, 3098.0872]	[16.1801, 30.0541]	[16.1801, 30.0541]
	P2	[0.5859, 8.9121]	[1.2436, 2.3096]	[148.5393, 3254.4157]	[16.9969, 31.5710]	[16.9969, 31.5710]
	P3	[0.6001, 9.2126]	[1.2868, 2.3896]	[153.6843, 3367.1391]	[17.5858, 32.6647]	[17.5858, 32.6647]

In Table 6, each datum are in the interval form. Results show that for the interval date of the cost according to one-year rainfall, the number of upper level is 12.37 times the lower level, and for the date according to two-year rainfall, the number of upper level is more than 15 times the lower level. For the interval data of sizing of vegetable swale facility, according to the one-year rainfall, the number of upper level is 19.66 times the lower level according, and according to the two-year rainfall, the number of upper level is 21.90 times the lower level. These interval numbers arise from the uncertainty of infiltration rate, release coefficient, budget and unit cost of the facility. When these uncertainties are occur simultaneously, a significant impact exists on the budget and sizing of the LID facility.

2. The greater the P, the larger the values of the upper and lower levels of the corresponding interval.

Because the different P corresponds to different volume of surface runoff arrived at the LID facility, the greater the P, the higher the volume of the surface runoff, thus the higher the corresponding total cost and capacity of the LID facility and the larger values of the upper level and the lower level of the corresponding uncertainty interval.

The Table 7 shows that the P1, P2, P3 arise from the hydrological uncertainty, the different probabilities lead to different interval distribution, and the greater the P, the greater the interval distribution (Fig. 4).

3. By contrasting the volume of one-year rainfall to the volume of two-year rainfall, the different volume of arrived surface runoff leads to a different distribution interval, and the higher the volume of arrived surface runoff, the larger the value of the interval number.

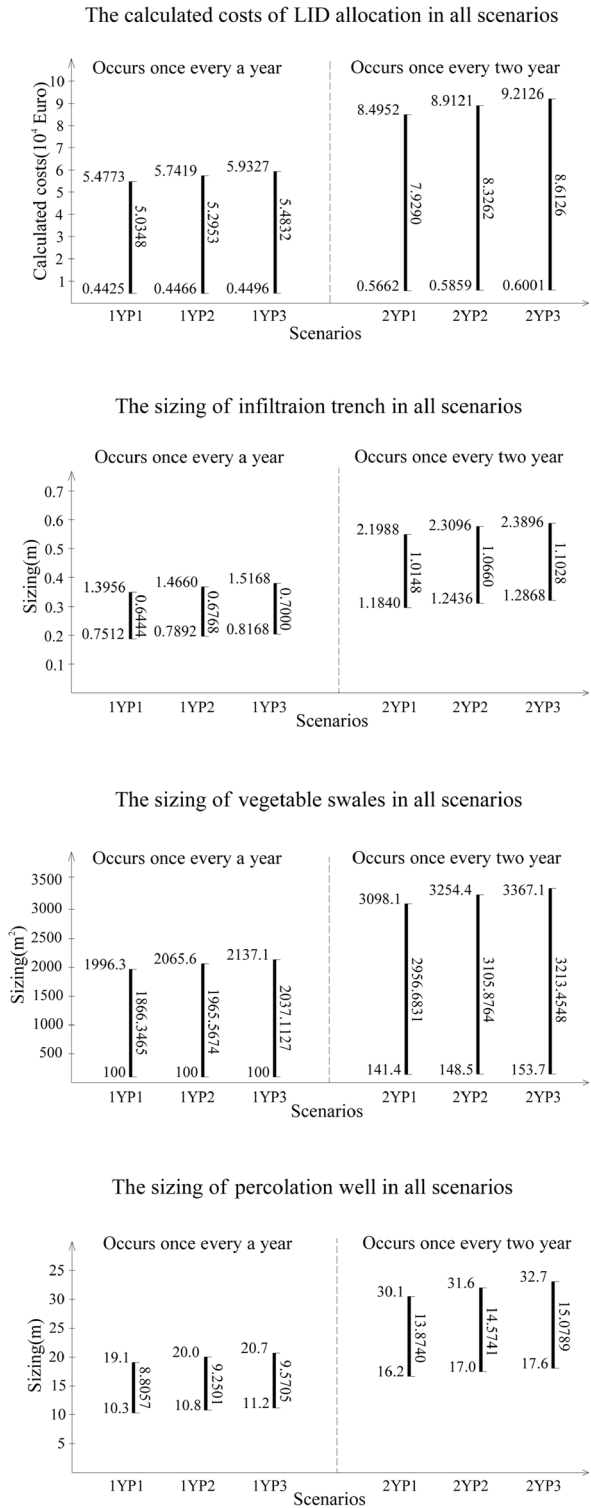
The features corresponding to two-year rainfall are all greater than the features corresponding to one-year rainfall, and these features include budget, which is the value of the upper level and the lower level of interval of the LID facility capacity, because the volume of surface runoff corresponding to two-year rainfall is larger than the volume corresponding to one-year rainfall and the values of total cost and the capacity corresponding to two-year rainfall are greater than the values corresponding to one-year rainfall.

4. It is inevitable that hydrological/hydraulic uncertainty exists in nature and human society leading to the uncertainty in LID facility planning. The study identifies the uncertainty factor on the premise of establishing a model based on the certain value. On the basis of this, the study

**Table 7** the difference of economy and sizing between the upper and lower level

			The difference of economy between the upper and lower level				
			The difference of sizing between the upper and lower level				
			1	2	3	4	5
1Y	P1	5.0348	0.6444	1866.3465	1866.3465	8.8057	8.8057
	P2	5.2953	0.6768	1965.5674	1965.5674	9.2501	9.2501
	P3	5.4832	0.7000	2037.1127	2037.1127	9.5705	9.5705
2Y	P1	7.9290	1.0148	2956.6831	2956.6831	13.8740	13.8740
	P2	8.3262	1.0660	3105.8764	3105.8764	14.5741	14.5741
	P3	8.6126	1.1028	3213.4548	3213.4548	15.0789	15.0789

**Fig. 4** The difference of economy and sizing between the upper and lower level in different scenario



develops the optimal model for LID facility planning, and obtains the interval uncertainty results. Except for some extreme events, the outcome of the optimal planning must be in the interval results. Addressing and reducing these uncertainties in optimization will effectively minimise the effects of uncertainties and makes the results reasonable.

5. The study integrates uncertainty planning with the optimization model, thus handling the uncertainty in the optimization effectively, the uncertainty planning of the study includes interval planning, stochastic planning, fuzzy planning, scenario analysis and CCP, among them, the interval planning addresses the uncertainty values of vegetable swale and soil infiltration rate which are in the form of interval number, the stochastic planning handles the uncertainty values of infiltration trench, infiltration rate of the percolation well and the three types of release coefficient of LID facility which are in the form of fuzzy number, the fuzzy planning handles the uncertainty of the budget and the cost, and the scenario analysis reflects the rainfall standard for one-year and two-year, whereas the CCP effectively reflects the probability violation caused by the stochastic distribution of the actual rainfall volume.
6. In general, the uncertainty of the LID facility planning arises from the hydrological/hydraulic factor, different climates, different geographic and geological conditions, and different types of LID facility planning lead to the different planning results. Hence, the different situations should be considered in the specific study.

## 5 Conclusion

1. The study develops the optimal allocation model for LID facility under uncertainty. The principle of the model is identifying the uncertainty, integrating the optimization model with uncertainty planning, and obtaining the interval results for the planning budgets and sizing for LID facility.
2. The researches of LID facility optimization are almost in certain condition, and few studies focus on LID facility optimization under uncertainty. The novelty of the study is developing a multiple uncertainty model to tackle the uncertainties existed in LID optimization, and the model could quantify the impact of the uncertainty on the LID facility sizing and the cost of the LID optimization distribution. The researches of LID facility optimization are almost in certain condition, and few studies focus on LID facility optimization under uncertainty. The novelty of the study is developing a multiple uncertainty model to tackle the uncertainties existed in LID optimization, and the model could quantify the impact of the uncertainty on the LID facility sizing and the cost of the LID optimization distribution.
3. In the study, uncertainty planning includes interval planning, stochastic planning, fuzzy planning, scenario analysis and CCP, among them, the interval planning, the stochastic planning and the fuzzy planning handles the interval number, stochastic number and fuzzy number, respectively, in the model, and the scenario analysis addresses the different rainfall standard, and the CCP addresses the probability violation of the model. When the multiple uncertainties exist together in the actual optimal allocation problem, the uncertainties is effectively handled with the integration of different uncertainty planning.
4. The uncertainty results of the model are represented in the form of interval numbers, and the range of the optimal results could be represented under the uncertainty. The possible influence of the uncertainty could be forecasted using this method, and the adverse impact of the uncertainty could be avoided.

5. Uncertainty can occur due to several reasons, and the uncertainties of the study arise from the hydrological/hydraulic factors and the economic factor, as well as the different geology, climate, project construction and planning would cause different uncertainties in different forms, and these uncertainties should be individually addressed.
6. The method of the study is universal, and the method could be extended to other researches for LID facility optimization to forecast the influence of uncertainty on LID facility planning cost and capacity.

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### Compliance with Ethical Standards

**Conflict of Interest** We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled "The Impact of Uncertainty Factors on the Optimization Allocation of Best Management Practices and Low-impact Development".

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