



Optimal Location of Pump as Turbines (PATs) in Water Distribution Networks to Recover Energy and Reduce Leakage

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Abstract Water distribution networks are high energy and low efficiency systems, where water pressure is frequently reduced by dissipation valves to limit leakage. The dissipation produced by the valves can be converted to energy production to increase the efficiency and reduce the energy impact of networks. If valves are replaced by turbines or pumps as turbines (PATs), they can both reduce pressure and produce energy. This study focuses on the optimal location of PATs within a water distribution network in order to both produce energy and reduce leakage. A new optimization model is developed consisting of several linear and non-linear constraints and a newly proposed objective function, where the turbine installation costs as well as the energy production and the economic saving due to the reduction of leakage can be accounted all together. The case study shows that the application of the mathematical model to a synthetic network ensures better results, in terms of both energy production and water saving, in comparison to other procedures.

Keywords Energy saving · Water Leakage · Water losses · Pump as Turbines (PATs) · Energy production · Water distribution networks

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1 Introduction

The supply of water to end users is energy demanding, due to the requirements of treatments and the need for pumping. Due to these energy demands and processes involved, the greenhouse gas emissions related to water supply are high (McNabola et al. 2014b). As an example, in the UK, water distribution has been estimated to be the fourth most energy intensive sector, with a total amount of 5 million tons of $C O_2$ per year (Ainger et al. 2009). In the US, the total amount of energy associated with water can be accounted as nearly 4 billion dollars per year (Zilberman et al. 2008). Water supply systems are generally characterized by low efficiency. A large amount of energy used to pump water is lost because most of the water supply and distribution systems are affected by water leakage (Olsson 2012).

Because the maintenance, rehabilitation or replacement of old and damaged pipes are expensive, water leakage is often kept under control by reducing the working pressure of the network through dissipation valves (Karadirek et al. 2012). This is conducted to face the variations in ground elevation and to set optimal pressure values within district metered areas (Alvisi and Franchini 2014). A preliminary study by Cabrera et al. (2010) showed that 40–60% of energy used to distribute water is lost due to leakage and head losses, while 30–60% of energy is effectively used to supply the end users.

Awareness about the scarcity of energy sources and impact of human activities on the environment has been increasing, while the cost of energy has been raising for the last decades. These issues suggest that the optimal management of a water system should be focused on the reduction of both energy consumption and greenhouse gas emissions, in order to increase efficiency and reduce the carbon footprint.

1.1 Energy Recovery in Water Networks

In this context, every effort should be made to reduce both leakage and energy dissipation in water networks, and several authors suggest the replacement of pressure regulating valves with turbines to reduce pressure and also recover energy (Ramos et al. 2010; McNabola et al. 2014a; Gallagher et al. 2015). The use of Pump As Turbines (PATs) has been recently demonstrated to be a viable and economical solution due to the large availability and the lower cost of pumps when compared to classical turbines (Carravetta et al. 2011; Fecarotta et al. 2015). Several works have been presented to show their hydropower potential in water distribution (Ramos et al. 2005; Fontana et al. 2012; McNabola et al. 2014b). Some authors propose different technical solutions for the regulation (Carravetta et al. 2014a; Fontana et al. 2016). Some others investigate the behaviour of PATs (Fecarotta et al. 2016; Carravetta et al. 2014b; Pugliese et al. 2016). Finally, some studies analyze the mutual behaviour of water network and hydropower devices (Arriaga 2010; De Marchis et al. 2016; Fecarotta et al. [accepted](#)). However, detailed methodologies on the optimal locations of turbines—and their optimal number—to be placed in a given network, are still lacking.

1.2 Optimal Location of Turbines

Few investigations analyze the problem of the optimal location of a hydropower device within a water network. The complexity of the problem lies in the nature of the network. The placement of a concentrated head loss, i.e. a turbine, along a pipe of a closed loop network affects the distribution of discharge among the branches of the network, as well as the available power. Moreover, the equations that are used to model fluid motion and

water losses are non-linear. Furthermore, the location problem is a mixed integer problem, since it involves continuous variables (e.g. discharge, pressure) and integer variables (the presence of the turbine within a branch). Finally, the number of variables can be very large, depending on the size of the network and on the type of the simulation (steady, quasi-steady, transient). Fecarotta et al. (2015) showed a simplified two steps method where first the problem of optimal location of PRVs was solved and then some valves were replaced by PATs, with a good economic viability. Giugni et al. (2014) showed that two different objective functions can be selected with the purpose of optimizing either pressure reduction or power output potential, with consequent differing results. In the latter case, the author introduced a penalty factor within the objective function to avoid low values of service pressure. A genetic algorithm was used to find the optimal location of a fixed number of turbines under steady conditions, while an external solver was used to model the hydraulic behaviour of the network. A second run of the optimization model was used to set the optimal values of head losses within the turbines under daily variable conditions, assuming the location to be known. Corcoran et al. (2015) performed a two steps optimization: in the first step, the optimal location of a fixed number of turbines was found to maximize the power production under steady average conditions then, the turbines were regulated according to the daily demand. A mixed integer non-linear solver was used to find the optimal solution and the hydraulic equations were written as equality constraints of the optimization problem. In both researches, the turbines were simulated as simple head losses, while Samora et al. (2016) used the affinity equations to better simulate the behaviour of real turbines within the network. A simulated annealing technique was performed to assess the best location of a fixed number of turbines, to optimize the energy production within the real water network in Lausanne. The size of the turbine of each branch was assigned while the presence of the turbine was a result of the optimization. The produced power was calculated with reference to the yearly variability of the monthly-averaged daily pattern.

1.3 Aim of the Paper

The novelty of this paper lies in the investigation of an objective function not considered by previous authors, and one which provides a more holistic assessment of the benefits of PATs in water networks. Despite the encouraging results of previous investigations, none of these have considered optimising both the income due to energy production and savings due to reduced water losses. In addition, in all of the previous works, the total number of turbines installed in a network was assigned as a constraint.

A new single objective function, i.e. the net present value (NPV) of the turbine investment, has been proposed here to pursue maximum energy production and minimum water losses. NPV takes into account the incomes due to the energy production and the economic saving due to the reduction of water losses and the outcomes due to the cost of installation of the turbines at the same time. The total number of turbines has not been constrained. It is a result of the new optimization procedure. The complete set of constraints, which comprise both the hydraulic equations to simulate the network behaviour and several operating constraints, have been written and a Mixed Integer Non-Linear Programming (MINLP) algorithm has been applied to solve the problem. The model has been applied to a literature synthetic network (Jowitt and Xu 1990) both considering the average demand and the whole daily pattern. Different scenarios have been investigated to study the variability of the solution with the design parameters. The solutions are compared with previous studies on the same network to show the improvements introduced.

2 Optimization Model

2.1 Variables

Given a network of n nodes and l links, the aim of the optimization model is to determine the best number and location of turbines to reduce the pressure and produce energy. The k th pipe of the network can be equipped or not with a turbine, producing at t time the $H_k^T(t)$ head-loss. Thus, a binary variable that indicates the presence of the turbine, I_k should be introduced for each link of the network. The total number of turbines can then be calculated as the sum of I_k . The operations of the turbine affect the hydraulic behaviour of the network, thus the discharge $Q_k(t)$ flowing through the k th pipe and the head $H_i(t)$ at the i th node are variables of the problem as well. If the node is a reservoir, its head is a boundary condition, while the inflow or outflow discharge is unknown. The network can be modeled as a directed graph, so that each link of the network has a direction and the discharge is positive if it has the same direction of the pipe and negative otherwise. The network is modeled in quasi-steady conditions, meaning that the hydraulic transient due to the demand variation are neglected. The demand of each node $q_i^d(t)$ is time dependent and can be calculated as:

$$q_i^d(t) = c_d(t) \cdot \overline{q_i^d} \tag{1}$$

where $\overline{q_i^d}$ is the average demand of the i th node and $c_d(t)$ is the demand coefficient which depends on time but is independent of the network nodes. If the daily pattern is divided in n_t timesteps of Δt_i hours, then the number of variables can be accounted as:

$$\begin{cases} I_k & l \\ H_k^T(t) & l \cdot n_t \\ Q_k(t) & l \cdot n_t \\ H_i(t) & n \cdot n_t \end{cases} \tag{2}$$

The demand coefficient can assume the same values at two or more different time steps, so that the solution is the same and does not need to be calculated. More efficiently, if the frequency distribution of the demand coefficient is evaluated, the daily pattern can be divided into a small number of n_d ranges of the demand coefficient, with an associated duration of Δt_d . Thus, if $n_d < n_t$, the number of variables can be significantly reduced as follows:

$$\begin{cases} I_k & l \\ H_k^T(c_d) & l \cdot n_d \\ Q_k(c_d) & l \cdot n_d \\ H_i(c_d) & n \cdot n_d \end{cases} \tag{3}$$

2.2 Objective Function

The objective function has been chosen as the NPV of the investment:

$$\text{maximize} \left(NPV = \sum_{y=0}^Y \frac{C_y^{in} - C_y^{out}}{(1+r)^y} \right) \tag{4}$$

where Y is the number of y years, C_y^{in} and C_y^{out} are the cash inflow and outflow, respectively, at the $y - th$ year and r is the discount rate, set equal to 0.05.

The cash outflow only occurs when $y = 0$ and is considered as the sum of the turbines cost, c^T . It can be calculated as:

$$c^T = c_P P_r^T + c_z + c_{inst} \tag{5}$$

where c^T is the total cost of the turbine in euros, P_r^T is the rated power of the turbine in kW and c_P and c_z are coefficient specific to the cost of PATs, predetermined in previous studies, set to 220 €/kW and 450 € respectively (Ramos et al. 2009). Y has been selected as 10 years, so that the maintenance cost of the turbines can be neglected. The sum c_{inst} of the piping cost and the grid connection has been assigned as 2500 € (Fecarotta et al. 2015).

The cash inflow is the sum of the incomes due to the produced energy and water savings. The energy income E_y^p at the $y - th$ year has been calculated as:

$$E_y^p = c_e \sum_{k=1}^l 365 \int_{day} \gamma I_k H_k^T(t) Q_k(t) \eta^T dt \tag{6}$$

where γ is the specific weight of water (9806 N/m³), H_k^T is the turbined head of the k th pipe, η^T is the turbine efficiency and t is the daily time. The turbine efficiency should vary depending on the discharge, according to the efficiency curve of the selected machine. At this stage, in order to simplify the model, a conservative constant value equal to 0.65 has been considered. The energy unit selling price c_e has been considered constant and equal to 0.1 €/kWh. Equation 6 can be modified with reference to the discrete distribution of the demand:

$$E_y^p = c_e \sum_{k=1}^l 365 \sum_{d=1}^{n_d} \gamma I_k H_k^T(c_d) Q_k(c_d) \eta^T \Delta t_d \tag{7}$$

The annual water saving W_y^s is calculated as the difference between the leaked water volume without the pressure control V_l^0 and the volume leaked by the network with turbines V_l^T . A water unit cost c_w of 0.3 €/kWh has been considered (Fecarotta et al. 2015). Thus,

$$W_y^s = c_w (V_l^0 - V_l^T) = c_w \left(\int_{day} Q_l^0 dt - \int_{day} Q_l^T dt \right) \tag{8}$$

where Q_l is the total leakage through the all pipes of the network. Equation 8 can be modified as follows:

$$W_y^s = c_w \left(\sum_{d=1}^{n_d} Q_l^0(c_d) \Delta t_d - \sum_{d=1}^{n_d} Q_l^T(c_d) \Delta t_d \right) \tag{9}$$

As suggested by Araujo et al. (2006), the leakage can be allocated in the nodes of the network as a pressure-driven demand, so that:

$$Q_l = \sum_{i=1}^n q_i^l \tag{10}$$

where n is the number of nodes and q_i^l is the leaked discharge through the i th node, that can be assessed as:

$$q_i^l = f_i \cdot p_i^\beta \tag{11}$$

where p_i is the pressure head (in meters) of the i th node, β an exponent depending on the material of the pipe and on the shape of the orifice (Greyvenstein and van Zyl 2007). For the sake of simplicity and in order to compare this study to some literature results, such

exponent is kept constant and equal to 1.18 (Araujo et al. 2006). The leakage coefficient, f_i , is constant for each node and is estimated as:

$$f_i = c \sum_{j=1}^{K_i} 0.5L_{i,j} \tag{12}$$

where c is a coefficient equal to $0.00001 \text{ l}/(\text{s} \cdot \text{m}^{1+\beta})$ (Araujo 2005) and K_i is the number of pipes approaching the i th node and connecting the nodes i and j , whose lengths are $L_{i,j}$.

2.3 Constraints

The optimization problem is subject to a certain number of linear and non-linear constraints, representing the physical modelling of the hydraulic network and the technical or practical limit of the variables related to the behaviour of the network.

2.3.1 Hydraulic Constraints

There are two kinds of hydraulic constraint: the mass continuity equations within the nodes and the momentum balance along the pipes. The continuity equation can be written as:

$$\sum_{k \in K_i} Q_k^{in} - \sum_{k \in K_i} Q_k^{out} - f_i v_i^\beta = q_i^d \tag{13}$$

where Q_k is the discharge flowing through the k th link contained in the set K_i of links connected to the i th node and the superscript in and out indicate either the discharge is flowing into or out of the node. A continuity equation can be written for each node and each Δt_d , so that they are $n \cdot n_d$. The momentum balance equation can be written as:

$$H_i - H_j - r_k L_k - I_k \cdot H_k^T \cdot \text{sign}(Q_k) = 0 \tag{14}$$

where k is the pipe that connects the i th and j th nodes and r_k are the unit head loss along the pipe. H_k^T is always positive and is multiplied by the sign of Q_k to ensure that the head loss produced by the turbine has the same direction as the flow. The unit head loss has been calculated by the Hazen-Williams formula:

$$r_k = \frac{10.67}{C_k^{1.852}} \frac{|Q_k|^{1.852}}{D_k^{4.8704}} \cdot \text{sign}(Q_k) \tag{15}$$

where C_k and D_k are the roughness coefficient and the diameter of the k th pipe. There is a momentum balance equation for each pipe and for each demand interval, so that $l \cdot n_d$ equations can be written.

2.3.2 Linear Constraints

The linear constraints can be further divided in boundary constraints and proper linear constraints. The boundary constraints are used to force the pressure head at the nodes within an allowable range. These two boundaries, p_{min} and p_{max} have been set to 25 m and 100 m respectively. Such boundary constraint can be written as:

$$p_{min} \leq H_i - z_i \leq p_{max} \tag{16}$$

where z_i is the elevation of the i th node. Such constraints can be written for each node, excluding the reservoirs, where the head is constant. For each demand interval, so that 2 ·

$(n - s)$ inequalities can be written, with a total number of $2 \cdot (n - s) \cdot n_d$. The head loss within the turbines should be always positive, according to Eq. 14, thus:

$$H_k^T \geq 0 \tag{17}$$

This inequality ensures that the head loss has always the same direction as the discharge. This means that, if the discharge reverses due to the demand change, the head loss reverses too, but this is generally not admissible for a turbine unless it is placed in a complex hydraulic circuit. Thus, the consistency of the solution of the optimization problem should be further analysed with reference to such a limitation of the model (i.e. the flow cannot reverse through a PAT and still produce energy).

Finally, two linear constraints have been introduced, as follows:

$$\overline{H_k^T} \geq \overline{H_{k \min}^T} \cdot I_k \tag{18}$$

$$H_k^T(c_d) \leq H_{k \max}^T \cdot I_k \tag{19}$$

where $\overline{H_k^T}$ is the time average head loss of the k th turbine and $H_{k \min}^T$ is a fixed value fixed to 0.5 m. The first inequality forbids the algorithm to select turbines with an average head loss smaller than $\overline{H_{k \min}^T}$ and can be written for each node of the network (l inequalities). The second inequality forces the value of H_k^T to be zero if I_k is zero and to be less than $H_{k \max}^T$ if I_k is one. $H_{k \max}^T$ has been calculated as the difference between the maximum and the minimum allowable head within the network. Figure 1 shows the feasible region on the plane $(H_k^T(c_d), I_k)$ for the continuous relaxed problem and the allowable values of $H_k^T(c_d)$ in the mixed integer problem.

2.3.3 Tolerances

Most of the equations that are used to create the mathematical model are strongly non-linear, so that the resolution of the optimization can be very difficult and time demanding.

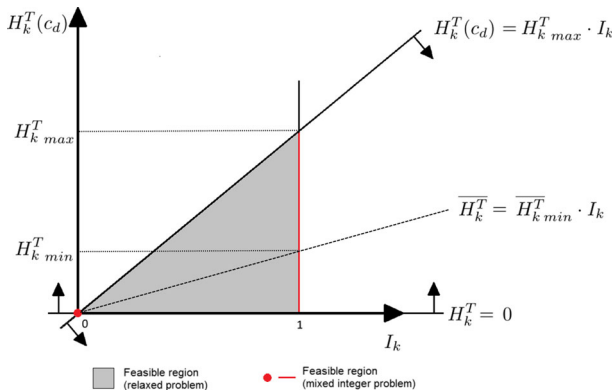


Fig. 1 Graphical description of the feasible region of $(H_k^T(c_d), I_k)$

In order to simplify the problem the two Eqs. 13 and 14 have been substituted with the two following inequalities, where small tolerances (tol_Q and tol_H) are admitted:

$$\left| \sum_{k \in K_i} Q_k^{in} - \sum_{k \in K_i} Q_k^{out} - f_i p_i^\beta - q_i^d \right| \leq tol_i^Q \tag{20}$$

$$\left| H_i - H_j - r_k L_k - I_k \cdot H_k^T \cdot sign(Q_k) \right| \leq tol_k^H \tag{21}$$

Such modification increases the dimension of the feasible region and help the algorithm to find the optimal solution in a reasonable time. The value of tol_i^Q has been calculated for each node and for each demand as follows:

$$tol_i^Q = \varepsilon_Q \left(q_i^d + \sum_{j \in J_i} q_j^d \right) \tag{22}$$

where J_i is the set of the j nodes connected to the i th node by a single pipe, while tol_i^H has been calculated as follows:

$$tol_k^H = \varepsilon_H H_k^T_{max} \tag{23}$$

ε_Q and ε_H have constant values equal to 0.01. The two tolerances introduce errors in the evaluation of the best solution and in the hydraulic modelling of the network. The introduction of such errors, which are small due to the small values of ε_Q and ε_H , can be acceptable if the uncertainties in the determination of the demand and in the results of the application of the Hazen-Williams equation (15), are taken into account.

2.4 Optimization Algorithm

The optimization model, as described above, can be written as follows:

$$\left\{ \begin{array}{l} \text{maximize} \\ I_k, H_k^T \end{array} \quad NPV = \sum_{k=1}^l -c_k^T \cdot I_k + \sum_{y=1}^Y \frac{E_y^P + W_y^S}{(1+r)^y} \right. \tag{24}$$

$$\left. \begin{array}{l} \text{subject to} \\ \left| \sum_{k \in K_i} Q_k^{in} - \sum_{k \in K_i} Q_k^{out} - f_i p_i^\beta - q_i^d \right| \leq tol_i^Q \\ \left| H_i - H_j - r_k L_k - I_k \cdot H_k^T \cdot sign(Q_k) \right| \leq tol_k^H \\ p_{min} \leq (H_i - z_i) \leq p_{max} \\ \frac{H_k^T}{H_k} \geq 0 \\ \frac{H_k^T}{H_k} \geq \frac{H_k^T}{H_k^{min}} \cdot I_k \\ H_k^T(c_d) \leq H_k^T_{max} \cdot I_k \\ 0 \leq I_k \leq 1 \\ I_k \in \mathbb{Z}, \quad H_k \in \mathbb{R}, \quad Q_k \in \mathbb{R}, \quad H_i \in \mathbb{R} \\ \forall i, j = 1 \dots n, \quad \forall k = 1 \dots n_l, \quad \forall d = 1 \dots n_d \end{array} \right.$$

In the second constraint, as well as in the objective function (for the calculation of E^P , as expressed by Eqs. 6 and 7), the product between H_k^T and I_k increase the non-linearity of the model and is apparently redundant, since the fourth and the fifth constraints force H_k^T to be 0 if I_k is zero. Nevertheless, several simulations that have been carried out showed that

such configuration of the model gives the best results in terms of both optimal solution and computational time.

The Basic Open-source Nonlinear Mixed INteger programming (BONMIN) (Belotti et al. 2013) code has been chosen to solve the optimization. A branch-and-bound based algorithm has been selected (Bonami et al. 2008) to investigate all the different solution. The code used the Interior Point OPTimizer (IPOPT) (Wächter and Biegler 2006) to solve the continuous relaxed problem and the Coin-or branch and cut (Cbc) algorithm (Schoenemann 2010) to solve the mixed integer problem. Even if the algorithm is designed for convex problems, it can retrieve heuristic solutions in case of non-convex problems and several options allow improvement in the quality of the solutions (Bonami and Lee 2009). Due to the complexity of the problem described by Eq. 24, the convexity has not been proven and all the options for the resolution of non-convex problems have been selected.

For the resolution of the network without turbines, the MATLAB FMINCON optimization algorithm (Byrd et al. 2000) for continuous problems has been used. The mathematical model of the network without turbines can be obtained from Eq. 24 by setting I_k and H_k^T to zero. Due to the simplification of the problem, even tol_i^Q and tol_k^H can be set to zero. The resulting optimizing function is null as well, so that the algorithm searches for the solutions which satisfies the constraints.

3 Case Study: a Synthetic Network

The optimization procedure has been applied to the literature synthetic ‘anytown’ network first proposed by Sterling and Bargiela (1984) and detailed in Figure ORF1 (Online Resource) and table ORT1 (Online Resource). The network had 25 nodes and 37 pipes. This means that there are more than 137.4 billion combinations of turbines within the network. The network is fed by gravity by three water reservoirs whose level was kept constant for the sake of simplicity. The same network was used in many studies about optimal valve location (Jowitt and Xu 1990) and was also used by Corcoran et al. (2015) and Giugni et al. (2014) in their research about the optimal location of turbines. The users’ demand is concentrated in the nodes of the network and the daily variability is modeled by introducing the demand coefficient c_d , as expressed by Eq. 1. The variability of c_d during the day is shown in Figure ORF2 (Online Resource) Leakage are concentrated in the nodes of the network as well, according to Eq. 11. Two kinds of simulations have been carried out, investigating only the average flow conditions as well as the whole daily pattern. For each of the two conditions, three scenarios have been considered: in the first one, the network was simulated in

Table 1 Summary of the different conditions and scenarios

	Scenario		
	1	2	3
Average condition	$c_d = 1$ $f_i = 0$ $c_w = 0$	$c_d = 1$ $f_i \neq 0$ $c_w = 0$	$c_d = 1$ $f_i \neq 0$ $c_w \neq 0$
Daily pattern	$c_d = c_d(t)$ $f_i = 0$ $c_w = 0$	$c_d = c_d(t)$ $f_i \neq 0$ $c_w = 0$	$c_d = c_d(t)$ $f_i \neq 0$ $c_w \neq 0$

the theoretical situation of zero leakage, while in the second and in the third scenarios the leakage are taken into account. The economic value of water is taken into account only in the third scenario. Table 1 illustrates the difference among the scenarios.

3.1 Average Conditions

The optimization procedure was first tested with the average demand (see [Online Resource ORF2](#)). In this case, the problem is smaller because $n_d = 1$ and thus the number of variables significantly decreases. The results for the three scenarios, in terms of installation pipes, produced power and node pressure are shown in Fig. 2. In the first scenario, the model set the optimal location of four turbines at links 1, 5, 8 and 11 with a produced power of 1.47, 1.91, 0.58, and 8.85 kW respectively (total = 12.81 kW). The pressure plot shows that the gain in terms of pressure reduction is relevant, if the node pressure is compared with the

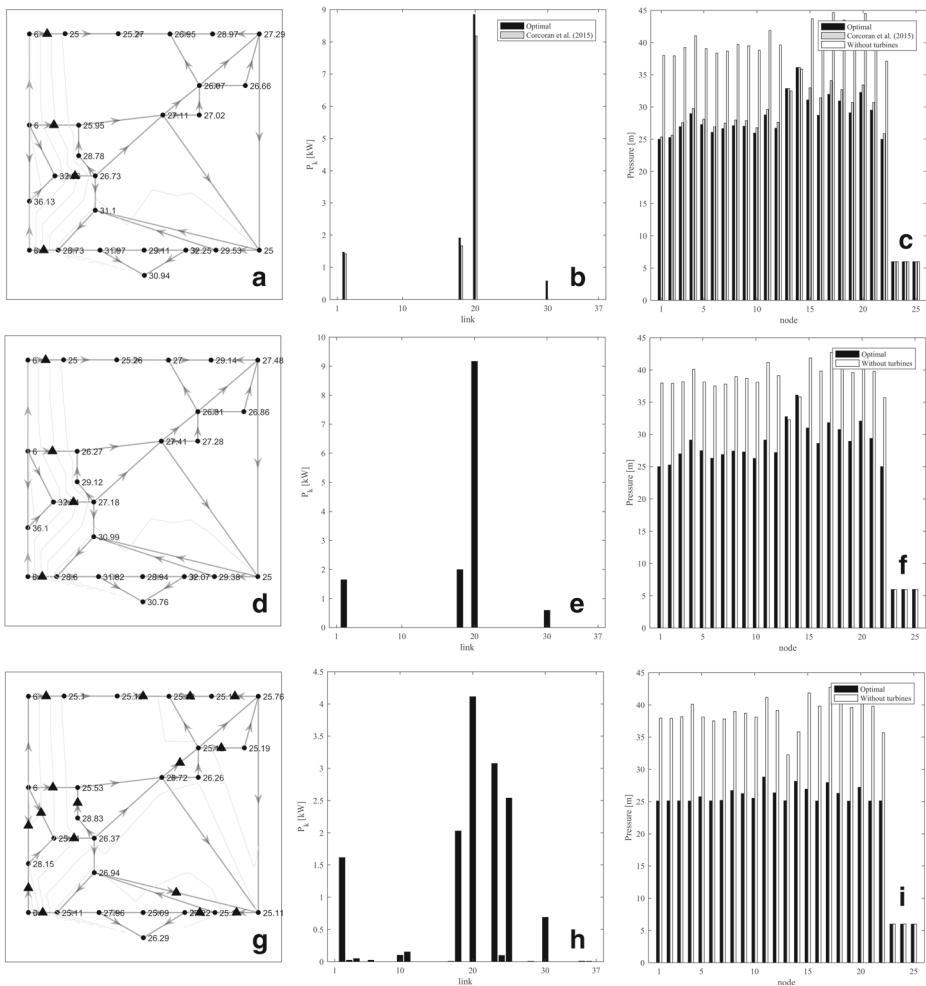


Fig. 2 Results of the optimization model for scenario 1 (a, b, c), scenario 2 (d, e, f) and scenario 3 (g, h, i) in average conditions: turbines locations, turbines power and nodes pressure

Table 2 Summary of the results for the different scenarios for the average condition

	Corcoran et al. (2015)	Proposed optimization		
		1	2	3
NPV [€]	64 915	72 051	75 936	833 740
No. of turbines	3	4	4	16 / 6
Average power [kW]	11.27	12.81	13.4	14.53 / 14.06
Produced energy [kWh/day]	270.5	307.4	321.6	348.7 / 337.4
Leakage reduction [m ³ /day]	–	–	755.8	929.4

no-dissipation condition (in white). Both the power plot and the pressure plot also show the solution proposed by Corcoran et al. (2015). The new optimization ensures better results in terms of both power production and pressure reduction. The NPV has been calculated both for the new model and for the solution proposed by Corcoran et al. (2015), and resulted as 72'051 and 64'915 € respectively, with a benefit increase of 11%.

For the second scenario, the model proposed the same location of turbines with an increase in the energy production, accounted as 1.65, 2.00, 0.59 and 9.17 kW for pipes 1, 5, 8 and 11 respectively, (total = 13.4 kW). The total amount of saved water has been accounted as 275'870 m³ per year while the NPV resulted as 75'936 €.

The results of the third scenario are quite different, because, in this case, the economic value of lost water is taken into account. The high income due to the water saving pushes the algorithm to select more dissipation points because the increase in NPV due to water saving is higher than the decrease due to the installation of a new turbine. The pressure plot shows that the pressure is slightly different from its minimum value in the whole network and the water saving can be accounted as 339'240 m³ per year. The calculated NPV was 833'740 €. The power plot shows that the produced power of some of the 16 proposed turbines is very low and for them the installation is probably not realistic. The total produced power is 14.53 kW. The introduction of a minimum allowable power constraint could avoid the selection of low power turbines but, due to its non-linear nature it is not suitable for this algorithm. All the attempts that have been made in this direction resulted in failures. Regardless, the selection of the turbine can be based on technical and practical considerations, such as the real installation costs, the location of the pipe, the ease of grid connection. In this study, a minimum power constraint is set a-posteriori, with a minimum value of 0.5 kW per turbine, and only 6 turbines have been selected among the solution, with a total power of 14.06 kW. For all the other dissipation points, pressure reducing valves can be placed, in order to pursue the same benefit in pressure reduction and water saving.

All the results are summarized in Table 2.

3.2 Daily Pattern

The optimization model (24) has been designed to deal with the daily pattern of user demand. Thus, a simulation has been run with the pattern shown in the Figure ORF2 (Online Resource). The main results, obtained for the proposed scenarios, are reported in Table 3, where the results obtained by Giugni et al. (2014) are also shown for comparison. Note that, Giugni et al. (2014), in their work, proposed six different solutions, with two different objective functions (OF^1 and OF^2 respectively) and one, two or three installed turbines (a, b and c respectively). Figure 3 shows the daily pattern of the turbine characteristics for the

Table 3 Summary of the results for the different scenarios for the daily variable demand, and comparison with Giugni et al. (2014)

	Proposed optimization					
	1			2		3
NPV [€]	66 638			72 290		790 320
No. of turbines	5			4		20/6
Average power [kW]	12.50			12.89		13.43 / 12.63
Produced energy [kWh/day]	299.9			309.4		322.5 / 303.1
Leakage reduction [m ³ /day]	–			744.8		901.2
	Giugni et al. (2014)					
	OF^1			OF^2		
	a	b	c	a	b	c
No. of turbines	1	2	3	1	2	3
Average power [kW]	5.15	8.28	8.62	5.39	11.47	13.62
Produced energy [kWh/day]	123.5	198.6	206.9	129.3	275.3	327.0
Leakage reduction [m ³ /day]	568.8	689.8	732.5	529.1	604.1	709.5

three scenarios, in terms of turbined head, turbined discharge and produced power, while Fig. 4 shows the optimal location of turbines for scenario 1, 2 and 3 respectively.

Again, in scenario 3, the number of installed turbines is quite high, due to the nature of the optimization function. If only the devices with an average power greater than 0.5 kW are selected (where all the others can be replaced with PRVs), then the total power reduces to 10.87 kW, while the total number of installed turbines reduces to 6.

4 Discussion

The results obtained by the application of the new algorithm are quite promising. Comparing the results obtained by Corcoran et al. (2015) with the results of the first scenario for average conditions, an increase in the produced energy and in the resulting NPV is observed. The new algorithm obtains very good results even if the leakage is taken into account, and Table 2 shows that the produced energy further increases when the water losses are modeled and the water cost is considered in the evaluation of the objective function. Therefore, even though Corcoran et al. (2015) set an objective function to maximize power production for average conditions, using the NPV objective function proposed here resulted in 11% higher power production.

The comparison between the results of the daily pattern simulations and the results obtained by Giugni et al. (2014) shows that a single objective function, which contains both the energy selling price and the economic saving due to leakage reduction, allows an optimal solution to both reduce leakage and produce power: Table 3 shows that the energy production obtained by the simulations of scenario 2 and 3 is close to the energy production obtained by Giugni et al. (2014) in OF^2 , and is much larger than the value obtained for OF^1 . Furthermore, the values of the water saving obtained with the new algorithm, both for scenario 2 and 3, are larger than the values obtained by Giugni et al. (2014), even if OF^1 was specifically designed to reduce the pressure within the network.

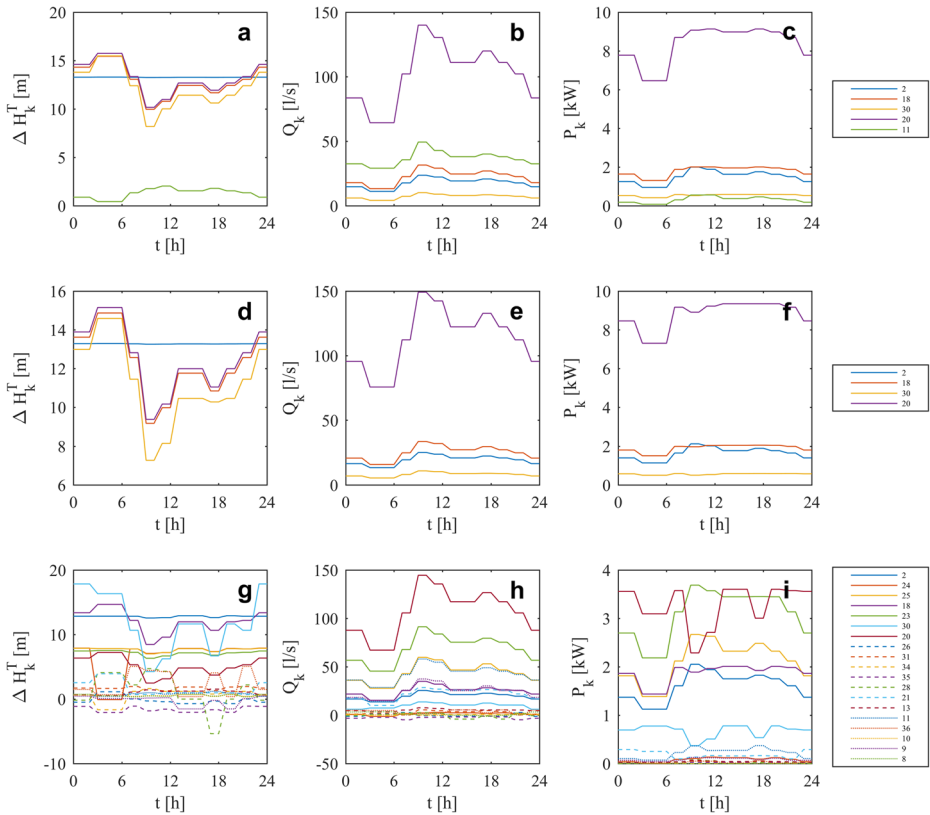


Fig. 3 Results of the optimization model for scenario 1 (a, b, c), scenario 2 (d, e, f) and scenario 3 (g, h, i) in daily variable conditions: turbined head, turbined discharge and turbines power during the day

Although it contains promising results, the new model presents some weakness. First of all, the PATs are not simulated as real machines, but only as head losses along the pipes, and a conservative value of 0.65 has been chosen as average turbine efficiency in the calculation of the produced power. After the algorithm has selected the best PATs locations and operations, a machine should be selected and installed, for example by the application of the Variable Operating Strategy (Carravetta et al. 2014a), in order to maximize the energy production.

In the second instance, when the cost of water is taken into account in the objective function, the high economic saving due to the reduction of leakage pushes the algorithm to select a high number of PATs. The comparison of the results of scenario 2 and 3 shows that the installation of more PATs in scenario 3 reduces the pressure at the nodes, and the income produced by saving water is larger than the outcome due to the installation of more PATs. Furthermore, the additional PATs of scenario 3 present very little power and the installation of such low power devices is probably not viable. A minimum power constraint would avoid this problem but it is not suitable due to its non-linearity. Thus, in this paper, an empirical approach is proposed: after the algorithm identifies the optimal location, only the PATs that exceed a certain power are chosen for the installation, while the low power PATs are replaced by valves, in order to achieve the same results in terms of water saving and reduce the installation costs.

List of symbols

β	Exponent in the relation between leaking discharge and pressure
γ	Specific weight of water
Δt_d	Sum of timesteps with the same the demand coefficient
Δt_i	Timestep
ϵ_H	Coefficient in the evaluation of tol_H
ϵ_Q	Coefficient in the evaluation of tol_i^Q
η^T	Efficiency of the turbine
c	Coefficient in the evaluation of f_i
c^T	Total turbine cost
c_d	Demand coefficient
c_{inst}	Installation cost of the turbine (piping and grid connection)
c_p, c_z	Coefficients for the evaluation of the turbine total cost
c_w	Cost of water
C_k	Roughness coefficient of the $k - th$ pipe of the Hazen-Williams formula
C_y^{in}	Cash inflow at the $y - th$ year
C_y^{out}	Cash outflow at the $y - th$ year
D_k	Diameter of the k -th pipe
E_y^p	Energy production during the $y - th$ year
f_i	Coefficient in the evaluation of leakage through the i th node
H_i	Head at the i th node
H_k^T	Head loss within the turbine installed along the k th pipe
$\overline{H_k^T}$	Average head loss within the turbine
$H_k^{T,max}$	Maximum head loss within the turbine
$\overline{H_k^{T,min}}$	Minimum average head loss within the turbine
i, j	Indexes for nodes
I_k	Binary variable representing the presence of a turbine along the k th pipe
k	Index for pipes
K_i	Number of pipes approaching the i th node
l	Number of pipes of the network
$L_{i,j}$	Length of the pipe connecting nodes i and j
L_k	Length of the k th pipe
n	Number of nodes of the network
n_d	Number of ranges in the frequency distribution of the demand coefficient
n_t	Number of timesteps
NPV	Net present value
p_{max}	Maximum allowable pressure
p_{min}	Minimum allowable pressure
p_i	Pressure of the i th node
P_r^T	Rated power of the turbine
q_i^d	Instantaneous i th node demand
$\overline{q_i^d}$	Average i th node demand
q_i^l	Leaking discharge through the i th node
Q_k	Discharge through the k th pipe
Q_k^{in}	Discharge flowing through k th pipe into the i th node
Q_k^{out}	Discharge flowing through k th pipe out of the i th node

Q_l^0	Leaking discharge without turbines
Q_l^T	Leaking discharge with turbines
r	Discount rate
r_k	Resistance coefficient of the k th pipe calculated by Hazen-Williams formula
s	Number of reservoirs
t	Time
tol_i^H	Allowable tolerance in the momentum equation
tol_i^Q	Allowable tolerance in the continuity equation
V_l^0	Leaked volume of water without turbines
V_l^T	Leaked volume of water with turbines
W_y^s	Water saving at the $y - th$ year
Y	Number of years considered for the NPV calculation
y	Year
z_i	Elevation of the i th node

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