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Regionalization of Rainfall Intensity-Duration-Frequency using a Simple Scaling Model

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Abstract Design storm is one of the most important tools to design hydraulic structures, hydrologic system and watershed management, mostly extracted by intensity- duration - frequency (IDF) curves for a given specific duration and return period. As for conventional methods to calculate IDF curves, the precipitation should be recorded for different durations so that foregoing curves can be extracted. Such data can be collected from rain gauge stations. In many areas, just daily precipitation data are available by which IDF curves cannot be extracted as per conventional methods. The aim of this research is to make IDF curves for short-term durations according to time scaling model as well as daily rainfalls. The relationships of this method are characterized with three variables including mean (μ_{24}) and standard deviation (σ_{24}) of daily rainfall intensity, and scaling exponent (H) by which all IDF curves might be drawn. The method used in present paper entails for less computational steps than conventional methods and by far has low parameters considerably than others in turn increases reliability. Scaling method is used to extract the IDF curves in rain-gauge stations in Khuzestan province located in southwest Iran and results proved the efficiency and robustness of the scaling method. Also ability of scaling concept method was examined in constructing of regional IDF.

Keywords IDF curves · Design storm · Scaling properties · Scaling model · Khuzestan · Iran

1 Introduction

To determine IDFs, rainfall intensity is required in different return periods in many models of hydrological and water quality and quantity calculations processes are needed. As a whole the main objective to determine the amount and intensity of rainfall in short term, is the estimation of the flooding caused by heavy rainfall. Such floods are utilized in the design of hydraulic

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structures (dams, bridges, embankments, etc.), networks for collection and disposal of urban runoff and watershed management and flood control operations among others. Intensity - duration - frequency (IDF) curve can be used to measure rainfall intensity and return period appropriate to achieve the project objectives. Local IDF equations and curve are often estimated using frequency analysis of records of intensities abstracted from rainfall depths of different durations, observed at a given recording rainfall gauging station. Such a method cannot be used in areas with no records of rainfall intensity data (Ghahraman and Abkhezr 2004). If some rainfall recording gauges be installed in area, by some conventional interpolation methods, data needed for no gauge areas can be extracted, but generally such value are full with error and uncertainty (Diaconis and Efron 1983).to deal with such uncertainties, the researchers were presented intensity -duration - frequency empirical equations.

Sherman (1931) proposed the following empirical relation:

$$I = \frac{KT^a}{(t+c)^b} \tag{1}$$

Where t represents rainfall duration in minute, T is return period and K, a, b and c are constants vary upon geographical position.

This equation is one of the most common relationships to calculate the IDF that is still widely used.

Bernard (1932) introduced an empirical equation as follows:

$$I_t^T = \frac{a_0 T^{a_1}}{t^{a_2}} \tag{2}$$

Where I_t^T is rainfall intensity for duration t (minute) and return period T (year), a_0, a_1 anda_2 are constants related to geographical position.

Bell (1969) obtained eq. 3 for the frequency range 2 to 100 years and duration 5 to 120 min, according to the United States data.

$$R_t^T = [0.21\ln(T) + 0.52].\left[0.54t^{0.25}-0.5\right].R_{60}^{10} \tag{3}$$

Where R_t^T is rainfall depth (mm) in frequency T (years) and duration t (minute). He achieved the standard error of between 5 to 7% and recommended using the relation (3) for the rest of the world, including Australia, South Africa, Hawaii, Alaska and Puerto Rico among others. Chen (1983) showed that rainfall of 1 h duration and 10-year return period in equation Bell (1969) cannot well reflect the diversity of geographical conditions. Koutsoyiannis et al. (1998) reported that IDF is a mathematical relationship between rainfall intensity (I) with duration (D) and return period (T). Sivapalan and Blöschl (1998) purposed a methodology for estimating catchment IDF curves which utilizes the spatial correlation structure of rainfall. Nhat et al. (2006) specified IDF regional relations for the Red River Delta in Vietnam based on the rainfall depth and return period of IDF curves. In Iran, the first comprehensive study of rainfall intensity –duration-frequency was performed by Ghahramn and Sepaskhah (1990) for 34 rain gauge stations. They accepted proposed Bell equation (eq. 3) and modified its value for local conditions. Rainfall-duration and rainfall-frequency ratios were found to be similar to other points of the world. Ghahraman (1995) showed that contrary to Bell, rainfall depth-return period ratio is a function of rainfall duration and rainfall depth-duration ratio is a function of the return period as well. So R_t^T / R_{60}^{10} cannot consider as multiplied R_t^T / R_{60}^T by R_t^T / R_t^{10}. He offered the following equation for Iran IDF relations:

$$R_t^T / R_{60}^{10} = \alpha t^{\beta} T^{\gamma} \tag{4}$$

In addition to intensity-duration-frequency, rainfall in 1 h duration and 10 years return period is needed. Therefore Ghahramn and Sepaskhah (1990) offered a relation for entire Iran based on Vaziri (1984):

$$R_{60}^{10} = e^{0/8153} \left(R_{1440}^{2}\right)^{1/1374} (MAP)^{-0/3072} \tag{5}$$

Where R_{1440}^2 denotes mean maximum daily rainfall and MAP is maximum annual precipitation. Vaziri (1992) based data to 1991 classified Iran to seven regions and to determine the relationships between rainfall, duration and different return periods with height of rain gauge stations, different correlation relationships was fitted on mentioned variables and proposed the relation 6 for each area:

$$R_{60}^{10} = \left(a - \beta Ln\left(R_{1440}^{2}\right)\right) R_{1440}^{2} \tag{6}$$

Ghahraman and Abkhezr (2004) investigated the latest data of IDF curves from 66 stations in Iran reported by Meteorological Organization in 1995 and obtained comprehensive relations on intensity- duration –frequency for this information.

The results showed significant changes compared with previous study in Iran and this could be attributed to change the parameters of the probability distribution function due to increase the length of rainfall intensity data time series. The equations for estimating rainfall at 1 h duration and 10 years return period were offered based on parameters such as average annual rainfall and average maximum daily rainfall. Ghahraman et al. (2010) according to the theory of linear moments analyzed the short rainfalls duration in the Khorasan province using 24 recording rain gauge stations. The study area was divided into two homogeneous regions, and for each region a given IDF was obtained. The final equations were function of duration, frequency, R_{60}^{10} (one hour rainfall with 10 years return period) and MAP (mean annual rainfall) that can be taken to estimate rainfall intensity in 5 to 60 min duration and 2 to 100 years return periods.

Ghahraman (1998) to generalized point rainfall to regional rainfall, established relationship between DAD and IDF curves. In order to incorporate those DAD and IDF curves, the spatial distribution of rainfall is determined using Horton's equation. He dated 6.6.1992 for heavy rainfall in Mashhad, a new category called IDFA curves (intensity-duration-frequency-area) for that region provided. These curves at the same time offer the characteristics of precipitation such as rain intensity and duration in a given area and specific return period that is required by designers. However, due to differences in rainfall patterns, these curves cannot be transferred to other areas.

As a whole, most common equations for IDF curves are as follow:

Talbot equation:

$$\mathrm{i} = \frac{\mathrm{a}}{\mathrm{d} + \mathrm{b}} \tag{7}$$

Bernard equation:

$$\mathrm{i} = \frac{\mathrm{a}}{d^e} \tag{8}$$

Kymyjama equation:

$$\mathrm{i} = \frac{\mathrm{a}}{d^e + b} \tag{9}$$

Sherman equation:

$$\mathrm{i} = \frac{\mathrm{a}}{(d+b)^e} \tag{10}$$

In these equations, i represents rainfall intensity (mm/h), d is rainfall duration (minute) and a,b,c are constants depending on hydrometeorologic conditions.

Koutsoyiannis et al. (1998) offered an equation for IDF curves

$$\mathrm{i} = \frac{\mathrm{w}}{(d+\theta)^{\eta}} \tag{11}$$

Where, i is rainfall intensity, d is rainfall duration, and w, θ and η denote on non-negative constants. Given the studies, both θ and η are independent on return period however, W is a function of return period so that Where if $\{w_1\ \theta_1\ \eta_1\}$ and $\{w_2\ \theta_2\ \eta_2\}$ are values for parameters $\{w, \theta\ , \eta\ \}$ for return periods T1 and T2 respectively, then for $T_1 > T_2$:

$$\theta_1 = \theta_2 = \theta \geq 0; 0 < \eta_1 = \eta_2 = \eta < 1; \ldots\ldots w_1 > w_2 > 0 \tag{12}$$

As a whole, given the IDF equations, following relation can be rewritten (Koutsoyiannis et al. 1998):

$$\mathrm{i} = \frac{\mathrm{a(T)}}{\mathrm{b(d)}} \tag{13}$$

Here, a (T) and b (d) are functions of return period and rainfall duration respectively. in eq. 13, $b(d) = (d+\theta)^{\eta}$ with $\theta > 0$ and $0 < \eta < 1$,while, a(T) is defined by the probability distribution of maximum rainfall intensity and eq. 13 is fitted to most empirical relations of IDF on many sites. In recent years, research has focused on the mathematical representation of rainfall fields both in time and space, including the development of scaling invariance models to derive short duration rainfall intensity-frequency relations from daily data. Gupta and Waymire (1990) studied rainfall spatial variability by introducing the concepts of simple and multiple scaling to characterize the probabilistic structure of the precipitation processes. Burlando and Rosso (1996) showed that both the simple scaling and multiscaling lognormal models can be used to derive Depth Duration Frequency (DDF) curves of point precipitation. Nguyen et al. (2002) developed a GEV distribution model to estimate of regional short duration extreme rainfall based on the scaling theory. Pegram et al. (1999) developed a simple scaling methodology to use daily rainfall statistics to infer the IDF curves for rainfall duration less than one day.

Yu et al. (2004) developed regional Intensity–duration–frequency (IDF) formulas for non-recording sites based on the scaling theory. Forty-six recording rain gauges over northern Taiwan provide the data set for analysis. Three scaling homogeneous regions were classified by different scaling regimes and regional IDF scaling formulas were developed in each region

Noori Gheidari (2009) using characteristics of rainfall time scale and daily rainfall data, extracted IDF curves in Zanjan station. These studies indicated the IDF curves calculated using characteristics of rainfall time scale had the best coincidence to recorded rainfall intensity data and show that the ability of the this method to calculating of IDF curves.

Elsebaie (2012) developed IDF relationship of rainfall using Gumbel and Log Pearson Type III (LPT) distributions, with chi-square goodness of fit test to choose the best probability distribution for two regions in Saudi Arabia. This research showed that the Gumbel distribution is slightly better than the LPT III distribution. Also the results showed that in all the cases the correlation coefficient between observed and estimated IDFs is very high and the goodness of fit of the formulae to estimate IDF curves in the region is very well performing.

Mirhosseini et al. (2013) assessed the effect of climate change on IDF curves in Alabama. They creating IDF curves fitting different statistical distributions to annual maximum series of short time rainfall depth based on different tests. They concluded that the GEV distribution was the best probability distribution for short time rainfall depth in different durations.

Rasel and Hossain (2015) developed empirical models based on Indian Meteorological Department (IMD) empirical reduction formula to estimate the short duration rainfall intensity for annual maximum daily rainfall in Seven Divisions of Bangladesh. They applied Gumbel distribution to estimate the rainfall intensity for different duration and return periods. They showed a good correlation between the rainfall intensity computed by the method and the observations.

Zope et al. (2016) developed IDF curves based on Kothyari and Garde (1992) empirical equation for Mumbai City, India. They showed that using maximum daily rainfall with the same return period instead of 2-years return period rainfall (as used in Kothyari and Garde equation) will also give good results.

Blanchet et al. (2016) introduced a regional GEV scale-invariant framework for Intensity–Duration–Frequency analysis and applied it to the Mediterranean region of Cévennes-Vivarais, France. They assumed extreme daily rainfall is GEV-distributed and the extremes of aggregated daily rainfall follow simple-scaling relationships. Based on these assumptions, they developed a GEV simple-scaling model for extremes of aggregated daily rainfall over different rainfall durations where scaling applies. Their model showed a good agreement for durations higher than 8 h but it shows limitations for shorter (than 8 h) durations even after correcting for measurement frequency.

Ghanmi et al. (2016), applied simple scaling invariance assumption, Gumbel distribution and PWM parameter estimation methods for maximum daily rainfall and calculated regional rainfall intensity, duration frequency curves in northern Tunisia. Their results showed that estimated IDF using scale invariance approach is very close to the empirical IDF.

Kumar et al. (2016) compared scaling behavior in rainfall IDF relationships that are derived using conventional simple moments and Linear Probability Weighted Moments (LPWMs) for four stations in three urban cities that belong in different climatic zones of India.The IDF curves derived based on LPWMs show a good agreement with observations and it is accordingly concluded that LPWMs provide a more reliable tool for investigating scaling in sequences of observed rainfall corresponding to various durations.

Because daily precipitation data is the most accessible and abundant source of rainfall information, it seems natural, at least for the regions where data at higher time resolution are scarce, to develop and apply methods to derive the IDF characteristics of short-duration events from daily rainfall statistics.

On the other hand, most of the previous studies done in this field focused on derived IDF curves of point rainfall. Since the hydrological models in watershed scale to determining the flood of storms, need to the regional short duration rainfall therefore it is necessary developed regional IDF relations.

Scaling method was used to extract the IDF curves in rain-gauge station in Khuzestan province located in southwest Iran and results proved the efficiency and robustness of the scaling method. Also ability of scaling concept method was examined in constructing of regional IDF. The significance of this method is laid in determination of rainfall intensity in any duration and return period. Regional approach gives better data where there are not available for a specific location or limited data.

2 Materials and Methods

IDF curves are provided based on scaling model as follow, the random variable I_d represents annual maximum rainfall intensity with duration d defined as:

$$I_d = \max\left[\frac{1}{d}\int_{1-d/2}^{1+d/2} x(\varepsilon)d(\varepsilon)\right] \tag{14}$$

Where $x(\varepsilon)$ represents continuous function of rainfall intensity and d is rainfall duration.

$x(\varepsilon)I_d$ random variable is defined as annual maximum rainfall intensity with moving average maximum value $x(\varepsilon)$ in duration d and it has cumulative probability distribution:

$$\Pr(I_d \le i) = F_d(i) = 1 - \frac{1}{T(i)} \tag{15}$$

Based on Burlando and Rosso (1996) and Pegram et al. (1999), the variable I_d is included in following relation when had simple scale characteristics.

$$I_d = \left(\frac{d}{D}\right)^{-H}.I_D \tag{16}$$

Where, I_d is annual maximum rainfall intensity with duration D, H denotes scaling exponent. The above equation shows that the rainfall probability distribution for various durations has the same probability distribution.

If ratio $\frac{D}{d}$ is represented with λ (scale factor), the foregoing equation can be rewritten as follow

$$I_d = \lambda^H.I_{\lambda d} \tag{17}$$

This equation may be rewritten in terms of the moments of order q about the origin, denoted by $E\left(I_d^q\right)$; in these terms, the resulting expression is:

$$E\left(I_d^q\right) = \lambda^{H_q}.E\left(I_{\lambda d}^q\right) \tag{18}$$

In above equation, H_q is scale exponent for q order moment. If we take the logarithm of the above equation, it can be said that H_q for given q, is the regression line slope, (log $E\left(I_{\lambda d}^q\right)$) against logarithm scale parameter (log (λ)).

H_q variations in relation to q indicate simple scale characteristics or multi scaling. So that if H_q variations to q is constant, H_q is linear function of q and indicating rainfall time scale invariance, suggesting scale simplicity. However, variations of H_q to q is not constant, Hq is a non-linear function of q, and representing rainfall multi scaling (Gupta and Waymire 1990).

Figure 1 shows H_q variations to q along with its different conditions. Pegram et al. (1999) showed, in case scale invariance is incorporated in precipitation data, then

$$\mu_{\mathrm{d}} = \lambda^{-H} \mu_{\lambda d} \tag{19}$$

$$\sigma_{\mathrm{d}} = \lambda^{-H} \sigma_{\lambda d} \tag{20}$$

Where μ_{d} and σ_{d} are mean and standard deviation of rainfall intensity in duration d respectively. If rainfall intensity data is characterized with cumulative distribution function F, then rainfall intensity in duration d and return period T can be defined with Chow (1964):

$$\mathrm{i}_{\mathrm{d,T}} = \mu_{\mathrm{d}} + \mathrm{K}_{\mathrm{T}} \sigma_{\mathrm{d}} = \mu_{\mathrm{d}} + \sigma_{\mathrm{d}} \mathrm{F}^{-1}\left(1 - \frac{1}{\mathrm{T}}\right) \tag{21}$$

By substituting relations (19) and (20) in above and dividing to d^{-H}, we have:

$$i_{\mathrm{d,T}} = \frac{\mu_{\lambda \mathrm{d}} (d\lambda)^{-H} + \sigma_{\lambda d} (d\lambda)^{-H} F^{-1}\left(1 - \frac{1}{T}\right)}{d^{-H}} \tag{22}$$

Where $\lambda^{-H} \mu_{\lambda d}$and $\lambda^{-H} \sigma_{\lambda d}$ are constants coefficients. Comparing (13) and (22) shows that $\eta = -H, \theta = 0, w = \mu_{\lambda \mathrm{d}} (d\lambda)^{-H} + \sigma_{\lambda d} (d\lambda)^{-H} F^{-1}\left(1 - \frac{1}{T}\right)$

As it was stated previously, w is a function of return period T. If $d\lambda$ is assumed equals with 24, then eq. 22 can be simplified as follow

$$i_{\mathrm{d,T}} = \frac{\mu_{24} (24)^{-H} + \sigma_{24} (24)^{-H} F^{-1}\left(1 - \frac{1}{T}\right)}{d^{-H}} \tag{23}$$

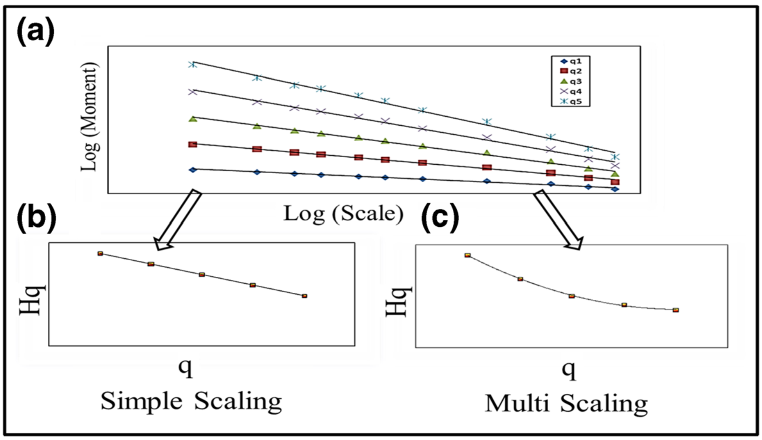


Fig. 1 Simple and multi scaling in term of statistical moments. (**a**) Moments of different orders q are plotted as function of scale in a log-log plot. (**b**) Linear relation H_q and q process is simple scaling, c) nonlinear relation H_q and q process is multi scaling

Where$i_{\mathrm{d},\mathrm{T}}$, rainfall intensity with duration d and return period T, is a function of 24 h rainfall characteristics ($\mu_{24}\ \sigma_{24}$).

Thus, using the above equation, which result the theory of rainfall invariance time scales, from daily rainfall data, IDF curves are constructed. In eq. 23, F is Cumulative distribution function rainfall intensity. Distributions of extreme rainfall events are usually modeled by one of the extreme value (EV) distributions, called type I (Gumbel distribution), II, or III, or by the generalized extreme value (GEV) distribution.

In this research, based on the dominant distribution of 24 h rainfall intensity of area study, Gumbel distribution was selected. Accordingly, the relation 23 can be rewritten as follows in relation to the IDF scope of the study area:

$$i_{\mathrm{d,T}} = \frac{\mu_{24}(24)^{-H} - \sigma_{24}(24)^{-H} Ln\left(-Ln\left(1-\frac{1}{T}\right)\right)}{d^{-H}} \tag{24}$$

As for eq. 24, three variables average (μ_{24}), standard deviation of daily rainfall (σ_{24}) and scale exponent (H) can be found. Given these three parameters in one area we can calculate rainfall intensity (IDF curves) in any duration and return period on both point and regional status. Regionalization contains two main objectives. One of the places where their data is not available, the analysis is done based on regional data and other places where short record data are available, the combined use of data at site and regional data from other stations in the area, more complete information for the probability distribution are concluded.

3 Study Area

In this research, data provided from 20 recording rain gauge stations located in Khuzestan province (installed by the Ministry of Energy). Data were collected in the form of 15-min intervals rainfall from 20 recording rain gauges. Longest recorded data were related to Ahvaz and Sad-e Shohada stations with 38 years and lowest for Pay-e Pol station with 12 years (Table 1). These data was frequency analyzed and rainfall intensity in different duration and return periods calculated and IDF curves derived for all 20 recording rain gauge stations. Location of Khuzestan province and stations is shown in Fig. 2.

4 Determination of IDF Using Simple Scaling Model

To determine IDF using the time and scale invariance properties, following steps were performed:

1- The time scale properties of data should be determined that is simple or multiple. According to recorded data, the annual maximum intensity of rainfall series in different durations (15 to 1440 min) extracted and then, moments (about the origin) of order 1 to 5 for all 20 stations were calculated.
2- The average of the moment orders 1 to 5 for different durations in all stations were determined.
3- The average of the moment orders 1 to 5 for different durations versus rainfall durations (15 to 1440 min) was plotted on a logarithmic chart. The scaling exponent or scaling

Table 1 Location, coordinates, elevation and record length of stations

Station	Latitude	Longitude	Height (m)	Record length(Years)
Dolab	33°-02’	49°-24’	500	18
Tange Panj	32°-56’	48°-46’	540	23
Sade Dez	32°-33’	48°-27’	525	28
Paye pol	32°-25’	48°-09’	90	12
Sade Tanzimi	32°-25’	48°-27’	142	24
Sade Gotvand	32°-15’	48°-49’	75	27
Pole lali	32°-04’	49°-36’	150	22
Sade Abbaspoor	32°-04’	49°-36’	820	28
Susan	31°-59’	49°-52’	600	26
Abdolkhan	31°-50’	48°-23’	40	27
Arab Hasan	31°-51’	48°-53’	33	25
Izeh	31°-49’	49°-51’	764	28
Pole Shalu	31°-45’	50°-08’	700	26
Bagh Malek	31°-33’	49°-52’	675	35
Ahwaz	31°-20’	48°-41’	20	38
Machin	31°-23’	49°-43’	380	25
Idanak	30°-57’	50°-25’	560	25
Kamp Jarahi	30°-43’	49°-11’	8	21
Sade Shohada	30°-40’	50°-17’	333	38
Dehmolla	30°-30’	49°-40’	32	28

index H_q, estimated from the slope of linear regression relationship between the moment of order q and rainfall durations. For example, for Ahvaz station data presented in Fig. 3.

4- Scaling exponent diagram, H_q against order of moment q (1 to 5) were determined separately for each station. Thus, the graph is plotted on the x-axis order of moments (1 to 5) and the y-axis scaling exponent related to the order of moments. If the correlation between H_q and q is linear, indicating invariance timescale in rainfall data. Fig. 4 indicates the relationship between scaling exponent H_q with order of moment q in Ahvaz station provides an example.

5- The slope of linear regression relationship between scaling exponent H_q and order of moment q, is H parameter in Eq. 24 extracted using regression relationships for all stations.

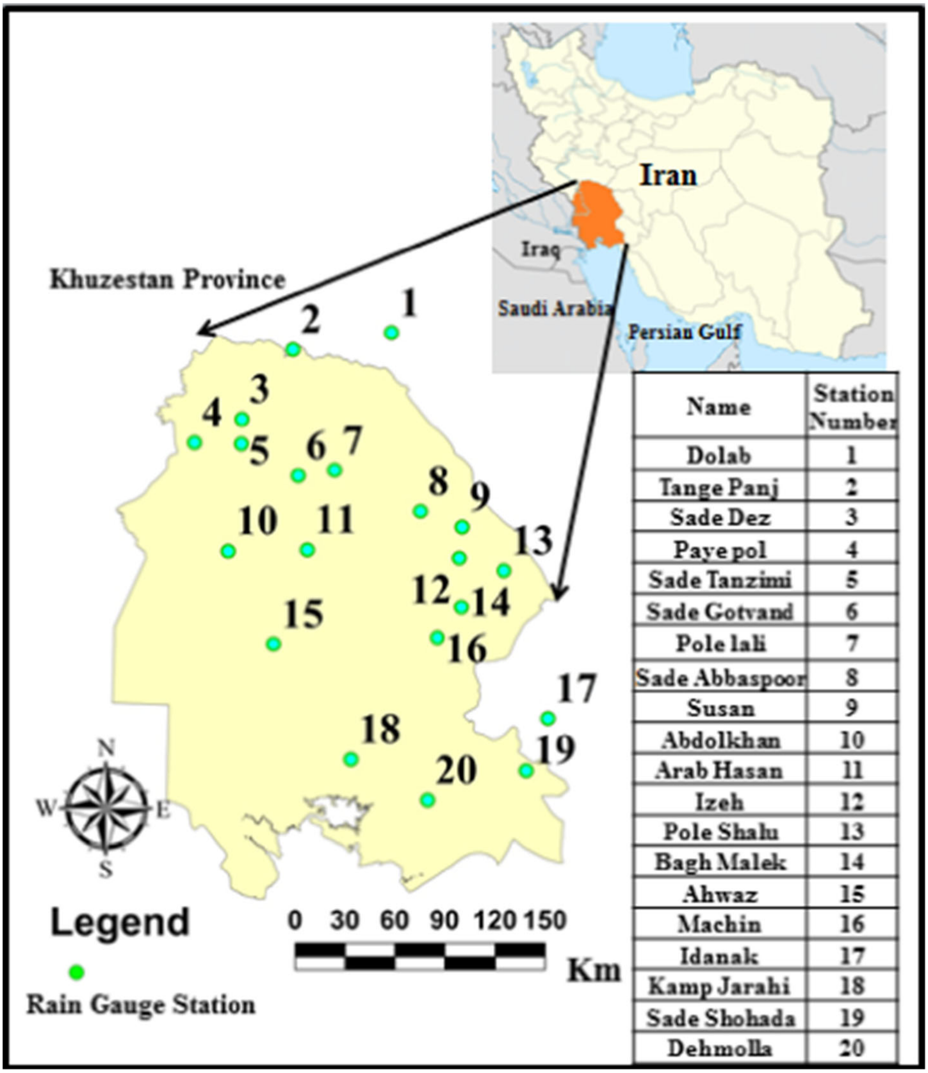
6- The average (μ_{24}) and standard deviation (μ_{24}) of maximum daily precipitation intensity for all stations using the daily rainfall data and extracted annual maximum daily precipitation series were calculated.

7- Identifying the variables average daily rainfall intensity (μ_{24}), standard deviation of daily rainfall intensity (σ_{24}) and H (scaling exponent) for each station (steps 5 and 6), constant values were determined for Eq. 24 in all stations. For example, in Ahvaz station the relationship as below:

$$i_{\mathrm{d,T}} = \frac{14.66 - 6.77 Ln\left(-Ln\left(1-\frac{1}{T}\right)\right)}{d^{0.705}} \tag{25}$$

8- Using Eq. 24 for each station, rainfall intensity in different durations and return periods calculated and IDF curves were plotted.

In order to validate the scale model, IDF curves obtained using this method were compared with IDF curves extracted from frequency analysis of rainfall intensity data in recorded rain gauge stations.



Name	Station Number
Dolab	1
Tange Panj	2
Sade Dez	3
Paye pol	4
Sade Tanzimi	5
Sade Gotvand	6
Pole lali	7
Sade Abbaspoor	8
Susan	9
Abdolkhan	10
Arab Hasan	11
Izeh	12
Pole Shalu	13
Bagh Malek	14
Ahwaz	15
Machin	16
Idanak	17
Kamp Jarahi	18
Sade Shohada	19
Dehmolla	20

Fig. 2 The location of stations in the Khuzestan province

In most previous studies, the graphical method has used to compare the IDF curves obtained from the scale model and the IDF curves extracted from the rain gauge stations. For example, in Fig. 5, the curves obtained using scale model and data of Ahvaz station are compared for 25 and 100-year return periods.

In this study, in addition to more accurate graphical method, the accuracy of the method was calculated numerically were compared to determine the error of eq. 26:

$$E(\%) = \left[\frac{\left(I_t^T\right)_o - \left(I_t^T\right)_e}{\left(I_t^T\right)_o}\right] * 100 \tag{26}$$

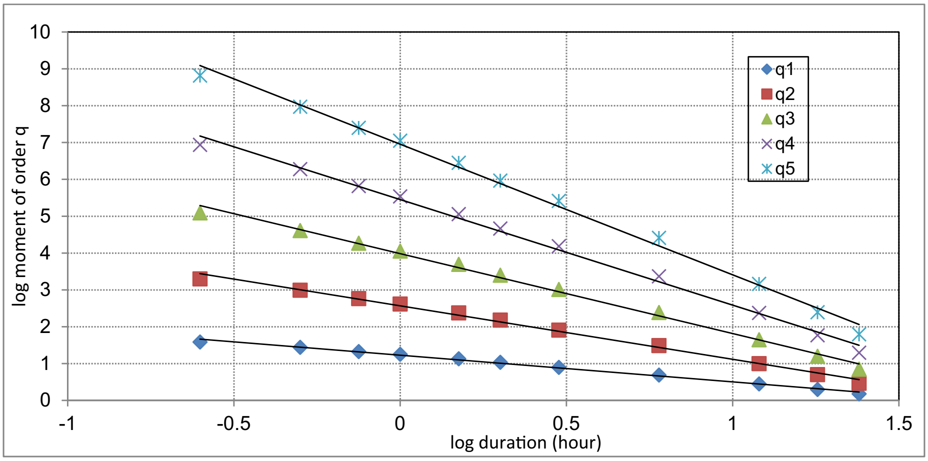


Fig. 3 Relationship between moments order q precipitation with rainfall duration in Ahvaz station

Where E (%) indicates error percent, $\left(I_t^T\right)_o$ is rainfall intensity with frequency t and return period T from gauge and $\left(I_t^T\right)_e$ is rainfall intensity with frequency t and return period T obtained from scaling model. The accuracy of this method for estimating the intensity of rainfall to sustain 15 min to 24 h, In return periods of 2 to 100, is calculated separately for all stations. Thus, the accuracy of the method is both graphically and numerically evaluated.

5 Results

According to recorded storm data, maximum of annual rainfall intensity were extracted in different durations (15 to 1440 min) and moments of 1 to 5 order about coordinate axis estimated. Then

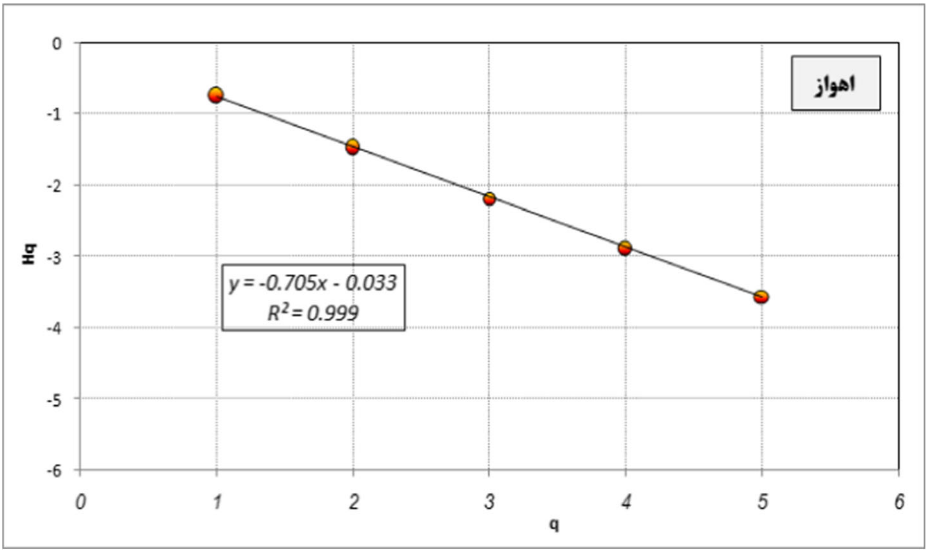


Fig. 4 Relationship between scaling exponents (Hq) with moments order of q in Ahvaz station

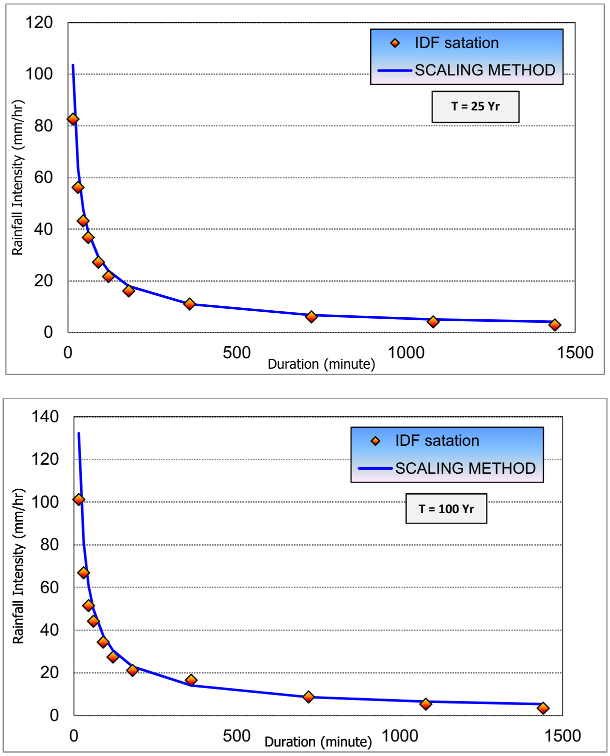


Fig. 5 Comparison of IDF curves derived from rain gauge stations and scaling theory

average of these moments determined for all stations and logarithmic graph of moments drown against different durations. The slope of the regression line drawn between moment of q order and duration is the exponent of scale or H_q. relations between q order moment (1 to 5) and rainfall duration is presented in Table 2. In all stations, this relationship is linear and has a coefficient of determination of 0.95 to 0.99 and a correlation coefficient of 0.975 to 0.995.

H_q for each moment order from one to five is the coefficient of the regression equation (x) in Table 3. H_q is drawn against moments for all stations. Relationship between H_q and q is linear in all stations and has coefficient of determination of 0.9997 to 1 and correlation coefficient of 0.9998 to 1. This linear relationship indicates that there is no variability in temporal scale of rainfall and confirms using this method to estimate short time intensities from 24-h rainfall.

By extracting annual maximum of daily rainfall, average of daily rainfall intensity (μ_{24}) and standard deviation of daily rainfall intensity (σ_{24}) are estimated for all stations (Table 3). Frequency analysis of 24-h rainfall intensity demonstrated that Gumbel is the most suitable distribution fitted to 24-h rainfall intensity in all stations. Therefore, by determining three parameters including μ_{24}, σ_{24} and H and given that in all stations Gumbel distribution was fitted to rainfall intensity, eq. 24 defined in stations. Constant coefficients of equations (μ_{24}, σ_{24} and H) are presented in Table 3. For example, relationship between IDF curves in Ahvaz station is presented in eq. 25. Then using these equations, rainfall intensity- duration- frequency values were estimated and IDF curves were drawn.

In order to validate and determine error of scaling model, IDF curves achieved from this method are compared with IDF curves of recorded data and error criteria such as root mean square error (RMSE), R- squared (R^2), index of agreement (IoAD) and Nash- Sutcliffe (CE)

Table 2 Relations between q order moment and rainfall duration

Station	q_1	q_2	q_3	q_4	q_5
Abbaspoor	$y = -0.6321\times + 1.2997$ $R^2 = 0.9876$	$y = -1.2381\times + 2.6694$ $R^2 = 0.9924$	$y = -1.8467\times + 4.0982$ $R^2 = 0.9936$	$y = -2.4608\times + 5.5742$ $R^2 = 0.9935$	$y = -3.0812\times + 7.0864$ $R^2 = 0.993$
Abdolkhan	$y = -0.7484\times + 1.1641$ $R^2 = 0.9874$	$y = -1.4261\times + 2.4462$ $R^2 = 0.9913$	$y = -2.0915\times + 3.8137$ $R^2 = 0.99$	$y = -2.7477\times + 5.2403$ $R^2 = 0.9875$	$y = -3.3978\times + 6.7061$ $R^2 = 0.9849$
Ahwaz	$y = -0.723\times + 1.2281$ $R^2 = 0.9946$	$y = -1.4524\times + 2.5684$ $R^2 = 0.9951$	$y = -2.1681\times + 3.9833$ $R^2 = 0.9954$	$y = -2.8647\times + 5.4507$ $R^2 = 0.9954$	$y = -3.5454\times + 6.9559$ $R^2 = 0.9952$
Arab Hasan	$y = -0.7936\times + 1.197$ $R^2 = 0.9669$	$y = -1.4883\times + 2.4679$ $R^2 = 0.9741$	$y = -2.1628\times + 3.7933$ $R^2 = 0.9752$	$y = -2.8268\times + 5.1614$ $R^2 = 0.975$	$y = -3.4879\times + 6.563$ $R^2 = 0.9745$
Baghmalek	$y = -0.6035\times + 1.2014$ $R^2 = 0.9931$	$y = -1.1871\times + 2.5027$ $R^2 = 0.9975$	$y = -1.7784\times + 3.8822$ $R^2 = 0.9982$	$y = -2.3733\times + 5.3256$ $R^2 = 0.9972$	$y = -2.9696\times + 6.8183$ $R^2 = 0.9954$
Sad-e Shohada	$y = -0.6399\times + 1.1864$ $R^2 = 0.9895$	$y = -1.2675\times + 2.4597$ $R^2 = 0.994$	$y = -1.9143\times + 3.7965$ $R^2 = 0.9953$	$y = -2.5769\times + 5.1833$ $R^2 = 0.9955$	$y = -3.252\times + 6.6099$ $R^2 = 0.9951$
Dehmolla	$y = -0.7443\times + 1.1931$ $R^2 = 0.9941$	$y = -1.5045\times + 2.5258$ $R^2 = 0.9948$	$y = -2.2993\times + 3.967$ $R^2 = 0.9943$	$y = -3.1277\times + 5.4976$ $R^2 = 0.9931$	$y = -3.9759\times + 7.0901$ $R^2 = 0.9919$
Doolab(Endica)	$y = -0.5416\times + 1.2067$ $R^2 = 0.9801$	$y = -1.0476\times + 2.5307$ $R^2 = 0.9732$	$y = -1.5369\times + 3.9456$ $R^2 = 0.9653$	$y = -2.0184\times + 5.4231$ $R^2 = 0.9587$	$y = -2.4989\times + 6.9397$ $R^2 = 0.9544$
Gotvand	$y = -0.6386\times + 1.2772$ $R^2 = 0.9963$	$y = -1.2697\times + 2.627$ $R^2 = 0.997$	$y = -1.892\times + 4.0349$ $R^2 = 0.9973$	$y = -2.5073\times + 5.4891$ $R^2 = 0.9971$	$y = -3.119\times + 6.9793$ $R^2 = 0.9967$
Idanak	$y = -0.5152\times + 1.3014$ $R^2 = 0.9847$	$y = -1.0288\times + 2.7236$ $R^2 = 0.9697$	$y = -1.5364\times + 4.2698$ $R^2 = 0.9433$	$y = -2.0345\times + 5.9083$ $R^2 = 0.916$	$y = -2.5243\times + 7.6013$ $R^2 = 0.8954$
Station	q_1	q_2	q_3	q_4	q_5
Izeh	$y = -0.5724\times + 1.2662$ $R^2 = 0.9964$	$y = -1.14\times + 2.5874$ $R^2 = 0.9976$	$y = -1.7072\times + 3.9551$ $R^2 = 0.9985$	$y = -2.2796\times + 5.3621$ $R^2 = 0.9991$	$y = -2.8598\times + 6.8011$ $R^2 = 0.9994$
Kamp-e Jarrahi	$y = -0.7398\times + 1.2564$ $R^2 = 0.9879$	$y = -1.488\times + 2.6055$ $R^2 = 0.9875$	$y = -2.2319\times + 4.0147$ $R^2 = 0.9874$	$y = -2.968\times + 5.4648$ $R^2 = 0.9873$	$y = -3.6968\times + 6.9439$ $R^2 = 0.9873$
Lali	$y = -0.6068\times + 1.2489$ $R^2 = 0.9968$	$y = -1.2025\times + 2.5512$ $R^2 = 0.9961$	$y = -1.7984\times + 3.8943$ $R^2 = 0.9953$	$y = -2.3974\times + 5.2692$ $R^2 = 0.9944$	$y = -3.0005\times + 6.6693$ $R^2 = 0.9934$
Mashin	$y = -0.6544\times + 1.2344$ $R^2 = 0.9858$	$y = -1.249\times + 2.548$ $R^2 = 0.9921$	$y = -1.826\times + 3.9177$ $R^2 = 0.9946$	$y = -2.3938\times + 5.3267$ $R^2 = 0.9959$	$y = -2.957\times + 6.7631$ $R^2 = 0.9967$
Pay-e Pol	$y = -0.6871\times + 1.3891$ $R^2 = 0.9944$	$y = -1.3649\times + 2.8303$ $R^2 = 0.9951$	$y = -2.0312\times + 4.3039$ $R^2 = 0.9958$	$y = -2.6879\times + 5.799$ $R^2 = 0.9963$	$y = -3.3389\times + 7.3097$ $R^2 = 0.9968$
Pol-e Shaloo	$y = -0.4306\times + 1.0703$ $R^2 = 0.9936$	$y = -0.8336\times + 2.1978$ $R^2 = 0.9985$	$y = -1.2371\times + 3.3662$ $R^2 = 0.9983$	$y = -1.6416\times + 4.5664$ $R^2 = 0.9967$	$y = -2.0455\times + 5.7918$ $R^2 = 0.9948$

Table 2 (continued)

Station	q_1	q_2	q_3	q_4	q_5
Sad-e Dez	$y = -0.6095\times + 1.2865$ $R^2 = 0.9948$	$y = -1.2492\times + 2.6853$ $R^2 = 0.995$	$y = -1.914\times + 4.1773$ $R^2 = 0.9937$	$y = -2.5916\times + 5.736$ $R^2 = 0.9914$	$y = -3.2723\times + 7.3377$ $R^2 = 0.9885$
Sad-e Tanzimi	$y = -0.7906\times + 1.2746$ $R^2 = 0.9537$	$y = -1.4967\times + 2.6648$ $R^2 = 0.9668$	$y = -2.2245\times + 4.1688$ $R^2 = 0.9645$	$y = -2.9746\times + 5.7657$ $R^2 = 0.958$	$y = -3.738\times + 7.4205$ $R^2 = 0.9516$
Soosan	$y = -0.5119\times + 1.2131$ $R^2 = 0.999$	$y = -1.0176\times + 2.4982$ $R^2 = 0.9992$	$y = -1.5268\times + 3.8452$ $R^2 = 0.9992$	$y = -2.0404\times + 5.2418$ $R^2 = 0.999$	$y = -2.557\times + 6.6748$ $R^2 = 0.9982$
Tang-e Panj	$y = -0.5024\times + 1.4295$ $R^2 = 0.9987$	$y = -1.0167\times + 2.939$ $R^2 = 0.9985$	$y = -1.5492\times + 4.5121$ $R^2 = 0.9975$	$y = -2.097\times + 6.1351$ $R^2 = 0.9959$	$y = -2.6563\times + 7.7961$ $R^2 = 0.9938$

Table 3 Mean, Standard deviation and Scale exponent of Khuzestan Province stations

Station	μ_{24}	σ_{24}	H	Station	μ_{24}	σ_{24}	H
Abbaspoor	2.60	0.74	−0.612	Izeh	2.80	1.04	−0.571
Abdolkhan	1.69	0.74	−0.661	Kamp-e Jarrahi	1.41	0.63	−0.739
Ahwaz	1.56	0.72	−0.705	Lali	2.73	0.78	−0.598
Arab Hasan	1.68	0.60	−0.672	Mashin	2.45	1.05	−0.575
Baghmalek	2.49	1.30	−0.553	Pay-e Pol	2.43	0.91	−0.662
Sad-e Shohada	1.98	0.83	−0.617	Shaloo Pol-e	3.12	1.14	−0.403
Dehmolla	1.62	0.62	−0.699	Sad-e Dez	2.7	0.68	−0.666
Doolab(Endica)	2.32	1.55	−0.488	Sad-e Tanzimi	2	0.81	−0.632
Gotvand	2.24	1.08	−0.62	Soosan	3.34	0.98	−0.511
Idanak	2.88	1.26	−0.502	Tang-e Panj	4.94	1.64	−0.538

estimated. In all stations, values of these criteria indicated low error from this method and confirmed good coincidence of two IDFs. Table 4 values presents for all station, while in Table 5 values presents for different return periods.

5.1 Regional Equations for IDFs

The eq. 24, which provided for the preparation of IDF curves is based on the theory of non-timescale invariance, is obtained. This relation is a function of three parameters including mean daily precipitation intensity (μ_{24}), deviation of daily precipitation (σ_{24}) and index scale (H) as follows:

$$I_D^T = f(H, \mu, \sigma) \tag{27}$$

Where I indicate rainfall intensity, T is return period and D is rainfall frequency. IDF curves are based on the formula above three parameters are obtained. After determining mentioned

Table 4 values of different error criteria for all stations

Station	RMSE	R^2	IoAD	CE
Abbaspoor	8.690386	0.995657	0.966457	0.853
Abdolkhan	8.149143	0.980529	0.964186	0.803
Ahwaz	6.362086	0.989886	0.982771	0.943
Arab Hasan	5.146657	0.974757	0.9783	0.888
Baghmalek	10.01411	0.994157	0.946486	0.500
Sad-e Shohada	8.819057	0.993429	0.951357	0.717
Dehmolla	23.22474	0.972314	0.852229	0.750
Doolab(Endica)	3.5027	0.981314	0.987	0.940
Gotvand	4.968243	0.995143	0.988486	0.947
Idanak	7.425171	0.912986	0.959771	0.300
Izeh	8.538186	0.995129	0.956971	0.550
Kamp-e Jarrahi	7.274457	0.957914	0.9755	0.873
Lali	6.938514	0.991871	0.962343	0.772
Mashin	8.332057	0.994729	0.958457	0.762
Pay-e Pol	11.75774	0.989629	0.958214	0.747
Pol-e Shaloo	7.000186	0.981157	0.902157	0.250
Sad-e Dez	14.35087	0.974786	0.9289	0.545
Sad-e Tanzimi	16.55419	0.955129	0.904129	0.720
Soosan	8.549843	0.994514	0.946171	0.703
Tang-e Panj	15.99396	0.980586	0.928371	0.561

Table 5 Values of different error criteria for different return periods

Return period (yr)	2	5	10	20	25	50	100
RMSE	3.551425	5.78769	7.776905	10.00112	10.76239	13.25044	15.92734
R^2	0.98723	0.988155	0.98635	0.98247	0.980735	0.973595	0.96343
IoAD	0.969785	0.965735	0.958545	0.949545	0.946365	0.93565	0.923765
CE	0.81746	0.79886	0.75203	0.68799	0.663825	0.57812	0.475235

parameters (μ_{24},σ_{24} and H), three spatial distribution maps of these parameters are prepared using interpolation techniques in Arc- GIS the study area. Thus, by using the obtained values for each parameter in the rain gauge stations located in Khuzestan, spatial distribution maps of these three parameters were determined using the interpolation in Arc- GIS. Figures 6, 7, 8 represent the distributed maps forμ_{24},σ_{24} and H, respectively. Based on these maps, values ofμ_{24},σ_{24} and H estimate for any point or area with no recorded rainfall data and then, by using these parameters, constant coefficients of eq. 24 including $\mu_{24}(24)^{-H}$ and $\sigma_{24}(24)^{-H}$ are determined. By determining the amount of these two coefficients, for each point or area, IDF curves relationships obtain with only two unknown variables (d and T). So IDFs can be defined in given point or region only by entering different return periods (in terms of year) and duration (in term of hour) in mentioned equation.

If μ_{24}, σ_{24} and H extract for a specific point, IDF achieved from equation is a point. But if these variables use as weighted average of a basin or a region, IDF curves derived from them are regional IDFs that are of great importance and application in hydrological studies.

6 Discussion and Conclusion

In present study, the efficiency and accuracy of the scaling theory in IDF curves extraction are evaluated by using daily rainfall and comparing estimated IDFs based on scaling model and recorded data in the southwestern Iran as the first study of its type. The Khuzestan province included both mountainous and plain regions that are compared for their scale invariance IDF properties which were compared in this study as one of its new findings. In addition, the parameters of scaling characteristics are shown in a spatial view which is also one of new representation to the literature. As the Khuzestan province is a flood-prone area with heavy rainfall, this study also brings deeper insights for flood analysis in future across the study region.

From technical points of view, we can conclude that most of the empirical equations using to estimate IDF curves have regional coefficients which determined for specific climatic conditions and cannot be used for other areas. Moreover, these methods present IDF curves just for a specific point and are not capable to determine regional IDF curves that are of particular importance in hydrological studies. On the other hand, most of previous methods in addition to complexity, number of parameters, and length of relationship, short term rainfall (e.g. 1- h 10- year rainfall) is used which is only able to be estimated in recording rain gauge stations and in the absence of this information, it is impossible to identify IDF curves.

The advantage of this approach over previous methods is that experimental methods have regional coefficients which have been determined for specific climatic conditions and cannot be used for other areas. While, in scaling invariance method constant coefficients of equations is derived using rainfall data from stations located in a certain region. Moreover, in scaling method IDF curves are drawn just by using three parameters, while in common methods for

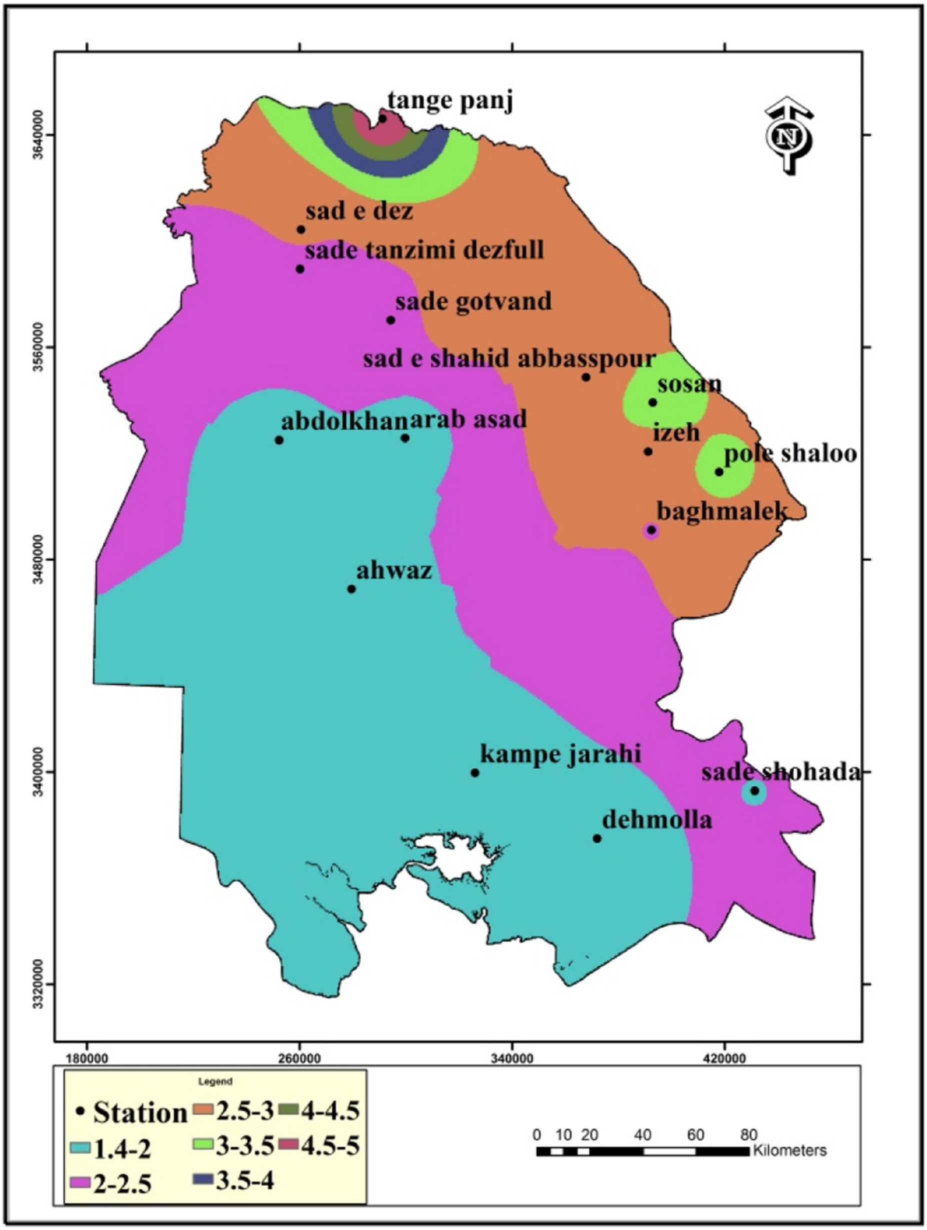


Fig. 6 Spatial distribution map of μ_{24}

IDF estimation for each curve more parameters are needed. Since reduction in parameters increase the reliability of the results, applied method has more confidence. These methods mainly have regional coefficient and they cannot be used for other areas. While in scaling model, constant coefficients of equations estimate using rainfall data of stations locate in a certain region. So in scaling theory weaknesses in previous approaches have been met.

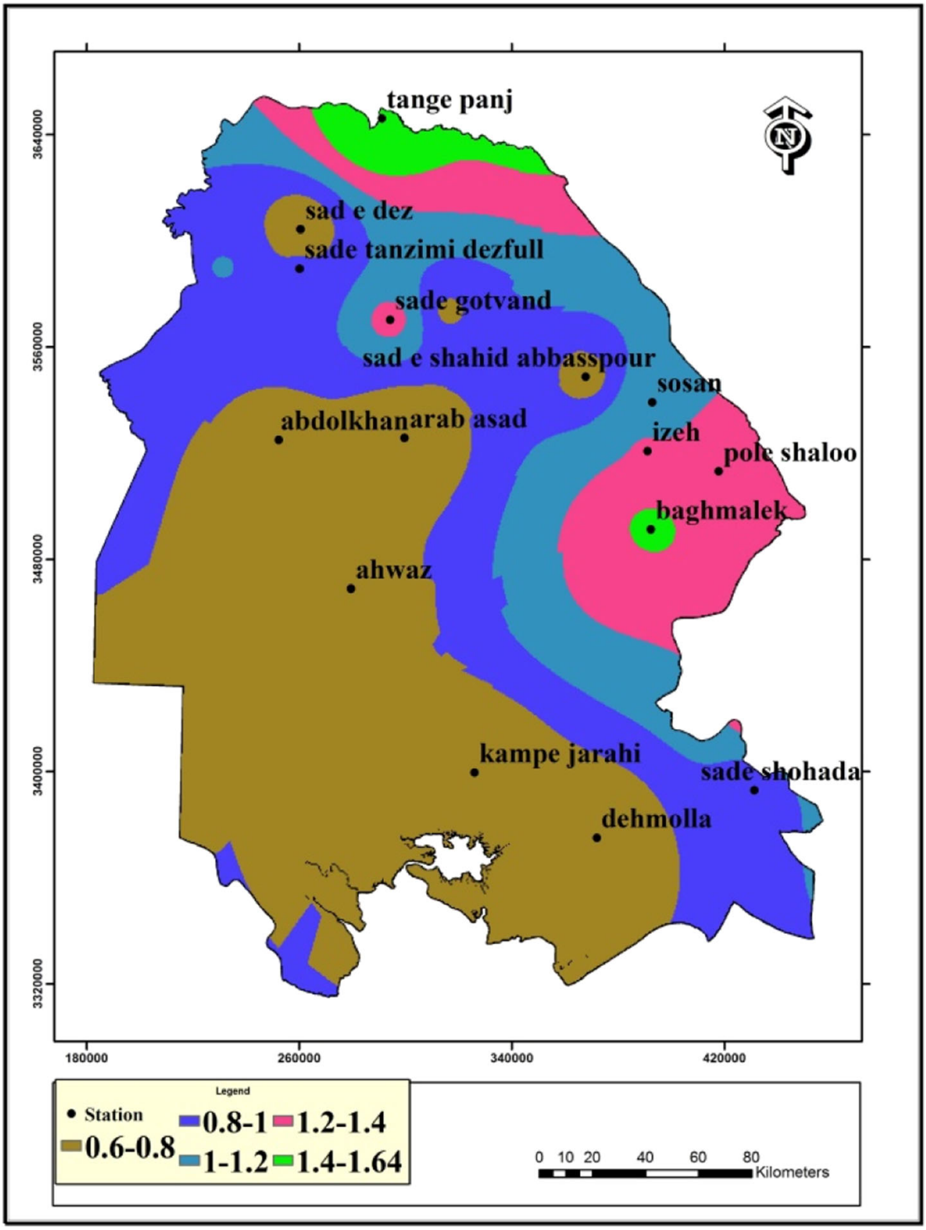


Fig. 7 Spatial distribution map of σ_{24}

Since the daily precipitation data are more accessible and abundant than other meteorological parameters in the Basin, the scaling method can extract short duration rainfall in most of regions with availability of daily rainfall.

Scaling theory can be used if there is no time scale precipitation variability that easily examined by investigating short term rainfall in limited rain gauge stations of a region. Variations of H_q related to q indicate whether rainfall is simple or multi scaling. According

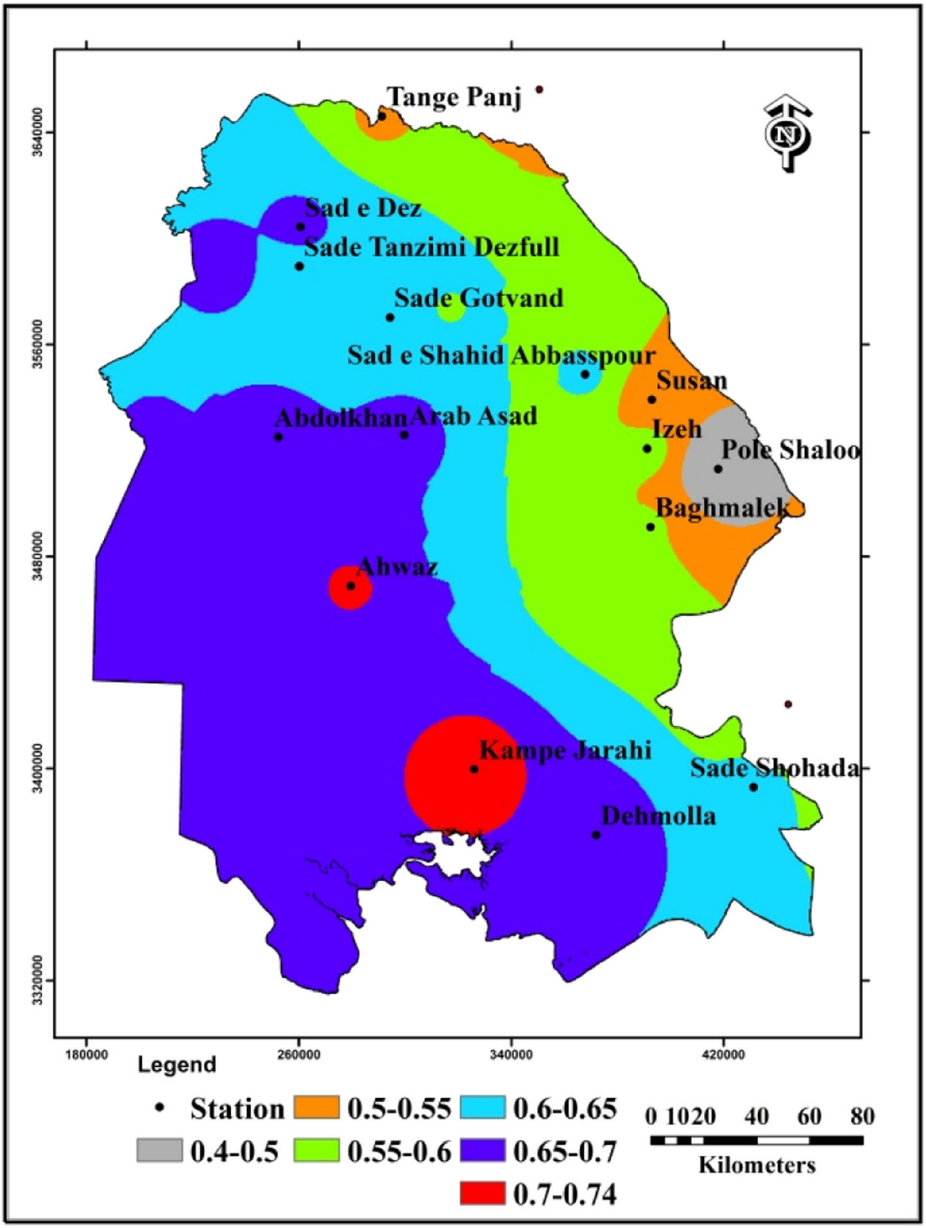


Fig. 8 Spatial distribution map of scale exponent (H)

to linear relation between H_q and q in all stations, it is found that Variations of H_q related to q is constant and it means there is no time scale variability in study area. Therefore, it is possible to use scaling theory equations for IDF curves extraction.

IDF curves achieved using scaling theory is compared with those obtained from recording gauge stations data. Results demonstrate that in all stations, there is good coincidence between two groups of IDF curves. Moreover, values of error criterion indicate good accuracy of the method.

Mean absolute error for presented equations mainly is under 25% and only in few cases error is higher than 25%. Error higher than 25% mainly is seen in durations greater than 6 h (e.g. 12, 18 and 24 h durations). So it can be said that the estimated values of rainfall intensity in different durations and return periods present better results for rainfall with duration up to 6 h. Comparison of error criteria related to two IDF curves groups show that for rainfalls with duration up to 6 h, minimum of mean error differ from 8% in Ahvaz station to 13.7% in Izeh station. Likewise, maximum of mean error is different from 9% in Dehmolla to 35.5% in Baghmalek station. Moreover, results indicate that among 20 recording rain gauge stations, in 13 stations average of estimated error in different durations (15 min to 6 h) is lower than 20%. These stations are mostly located in northern and central parts of Khuzestan province. In other words, scaling method is more accurate in mentioned parts and can be used with high sureness in these areas. While, error rate of scaling model is high in Pol-e shalu, Idanak and Izeh and increase in higher return periods. It may relate to microclimate conditions of these stations or rainfall characteristics. However, more detailed studies are needed to determine the reasons for error rates. It is also worth noting that error value is consistent with the range reported in the literature. For example, Pagliara and Viti (1993) suggested the range of ±20% for errors in Italy. Kothyari and Garde (1992) estimated the error rate of ±30% for India. Likewise, Ghahraman et al. (2010) provided a maximum error of about 19% for Khorasan Province in Iran.

Error in the mountainous area is more than flat ones. Data availability for longer period, lead to better coincidence and more accurate results (e.g. Ahvaz and Abbaspoor stations). Error rate increases by raising return periods, since increased uncertainty in interpolation. Rainfall intensity values obtained from the scaling model for different durations and return periods are in 95% confidence level result from storm frequency analysis of rain gauge stations. Therefore, estimated error for scaling model seems reasonable.

One of the most important capabilities of scaling model is IDF curves extraction for regions with no data. To determine IDF relations for these regions or define regional IDFs, equations can be provided with the help of three parameters including average of daily rainfall intensity (μ_{24}), standard deviation of daily rainfall (σ_{24}) and scale exponent (H) in any point and so just by entering intended duration and return period, corresponding rainfall intensity is achieved.

By using regional values of parameters μ24, σ24 and H and presented equation for each station, IDF curves are determined for any point or area. Previous methods provide IDF curves as a point and they are not able to define regional curves which are of great importance in hydrological studies. While, in scaling method IDF curves can be provided in a regional or point way based on the purpose of study.

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