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Non Stationary Analysis of Extreme Events

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Abstract Traditional approaches to the analysis of extreme hydrological series are based on the stationarity assumption for the underlying processes, namely that the probability distribution of the hydrological variable does not change with time. Over the last decade however, a growing interest has arisen both from a scientific as well as engineering point of view, toward the development of tools able to cope with the apparent non stationary features (either natural or anthropogenic) observed in many hydrological processes. Though most of the works deal with extreme precipitation and floods, less attention has been devoted to modeling droughts under non stationarity paradigm. In the paper, a brief review of the available tools for modeling non stationary series is presented. An extension of such methodologies to drought lenght modeling is developed, taking into account the non stationary nature of the underlying series and/or of the threshold level used for drought definition. An example of application of the developed methods to four precipitation series in Sicily, Italy, exhibiting different degrees of trends is also presented.

Keywords Non stationarity · Probabilistic analysis · Extreme values · Droughts

1 Introduction

Traditional probabilistic methods applied in hydrology and water resources planning and management studies assume that extreme hydrologic series are stationary and broadly independent in time. The stationarity paradigm for hydrometeorological processes has been questioned however under the push of several factors among which:

- evidence of long term variability in climate;
- changes in climate due to anthropogenic factors;

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– need to account for changes in hydrological response of watersheds (modification of land use, hydrographic network, development of diversion structures, etc.);

Furthermore, there are many problems water engineers have to face for which non stationarity arises regardless of the stationary or non stationary features of hydrometeorological processes. For example, assessment of drought risk for a water supply systems should take into account the planned infrastructural changes of the system, as well as the variability with time of water demands, due to expected changes in population, degree of agricultural and/or industrial development, increasing awareness toward ecological flows.

In general, attention has been focussed by hydrologists and water resources engineers to non-stationary flood frequency analysis, as clear violation of stationarity can be observed in many river basins mainly due to human activities, which eventually affect flood-related hydrologic variables, such as river discharge and stage, water storage capacity and runoff coefficient, among others, leading to an increase in exposure to floods (Kundzewicz, 2011).

Several approaches have been proposed in literature to model extreme variables, such as extreme floods exhibiting some kind of nonstationary feature. Usually they include probabilistic and stochastic models whose parameters may vary according to some time dependence structure, such as gradual or abrupt changes in the mean, variance or higher order moments (e.g. Strupczewski et al. 2001; Sveinsson et al. 2005; Khaliq et al. 2006; El Adlouni et al. 2007; Villarini et al. 2009; Cooley 2013; Katz 2013; Razmi et al. 2017; Agilan and Umamahesh 2017). In addition, a covariate analysis can be carried out by modelling the parameters of the probability distribution of non-stationary variables as a functios of any physical factors that are recognized to exert an influence on the variables of interest, such as climatic indices (i.e. AMO; NAO, etc.), anthropogenic factors or meteorological variables (Coles et al. 2001; Griffis and Stedinger 2007; Villarini et al. 2010; López and Francés 2013; Xiong et al. 2015).

While most works deal with extreme precipitation and floods, less attention has been devoted to modeling droughts, in particular multiyear droughts, assuming non-stationarity either with reference to the climatic forcings or to the adopted threshold level that takes into account, implicitly, a demand level that may vary with time (Wang et al. 2015). Climatic drivers, such as the ongoing warming of the atmosphere, can be considered as the leading causes of increase in the intensity and occurrence of drought events in many regions of the world (Solomon et al. 2007; Field et al. 2013). Recently non-stationary return period of droughts, in terms of low-flow or drought index series, has been investigated by using projections of climatic models, driven by various climatic scenarios (Du et al. 2015; Mondal and Mujumdar 2015). Unfortunately, there are still too many sources of uncertainty in projections, so that no uniform pattern of changes in drought events is observed across the projections from different climate models; in addition existing climatic models are often unsuitable to reproduce extremes at small spatial scales due to coarse spatial resolution.

Regardless of the causes of nonstationarity, either hydrological or climatic (due to natural variability or to global warming), the need for the development of new probabilistic models to deal with non-stationarity is largely recognized by the scientific community. Furthermore, in a non-stationary context, the traditional concepts of return period and risk commonly applied for operational purposes are no longer valid and need to be reformulated (Cooley 2013; Salas and Obeysekera 2014; Serinaldi 2015). The paper presents methods for non stationary frequency analysis of extreme events, focussing in particular on droughts. More specifically, concepts such as return period and risk are discussed within a non stationary framework. Analytical approximations for moments, probability distributions and return

period of drought characteristics are derived as a function of the underlying non stationary distribution of hydrometeorological series. Finally, an application to annual precipitation series in Sicily is presented.

2 Frequency Analysis for Non Stationary Processes

2.1 General

From a time series analysis point of view, the term stationary refers to a process whose probabilistic features are not dependent with time. For instance, with reference to a time series X_t, where t is time, strict stationarity implies that the vector $(X_1, X_2, \ldots, X_n)$ has the same multivariate distribution of $(X_{1+h}, X_{2+h}, \ldots, X_{n+h})$ for all integers h and $n > 0$ (Brockwell and Davis 2002). In a broad sense, strict stationarity means that the probability distribution of X_t does not depend with time. Stationarity can also be defined in weak sense by limiting the condition to the first and second order moments. Then, X_t is said to be weakly stationary if its expected value and covariance function do not depend with time. Obviously, strict stationarity implies weak stationarity but not vice-versa.

Relaxing the stationarity assumption for a time series affects several aspects, including the modeling of the non stationary component, of the probability distribution as well as the need to revise traditional concept such as return period and risk (Salas and Obeysekera 2014).

Most of the proposed approaches for non stationary modeling of probability distributions are based on assuming a parametric Probability Density Function (pdf), whose parameters vary with time according to prespecified parametric funtional forms (Strupczewski et al. 2001; Delgado et al. 2010; Delgado et al. 2014; Stedinger and Griffis 2011). Alternatively, trends in the moments of the distribution are assumed (Strupczewski et al. 2001) In some cases, climatic indices such as El Nino Southern Oscillation (ENSO), North Atlantic Oscillation (NAO), etc., or other hydrometeorological data are adopted as covariates for the parameters in order to take into account interannual or interdecadal variability of climate (Li et al. 2015; Prosdocimi et al. 2014). Also sometimes other covariates that reflect changes due to urbanization and/or land use in the watershed are employed to model non stationary streamflow series.

Then, fitting a non stationary probability distribution to observed (nonstationary) data involves estimating the parameters of the functional forms expressing the variability with the covariates (either time or others) of the pdf. Alternative and more traditional approaches include the removal of a deterministic trend from the time series, fitting a pdf to the resulting stationary residuals and adding back the trend component to the estimated quantiles of the residuals. Sometimes, parametric trends are incorporated in the first few moments of the distributions (Strupczewski et al. 2001).

Regardless of the way past apparent non stationarities observed in hydrometeorological series are incorporated in the modelling of the pdf, yet the problem with time varying distribution parameters is that they implicitly assume that they would continue to change in the future according to the same pattern observed in the past. Thus, some approaches are based on employing only a limited set of recent observed data (e.g. 30 years) for frequency analysis, implicitly assuming that hydrometeorological variables are representative of a given climate state (Raff et al. 2009).

In what follows, some of the most widely adopted methods for modelling probability distribution, return period and risk in a non stationary context are illustrated in some details.

2.2 Modeling Probability Distributions

In general terms, incorporating time variability in a probability distribution involves assuming a time varying form for one or more of its parameters. With reference to a generic cumulative distribution function with parameter vector Θ this entails assuming the latter as a function of time t, thus the distribution of the random variable X_t becomes $F_{X_t}(x; \Theta_t)$.

Most of the proposed applications in hydrology deal with floods and precipitation, and therefore several methods have been developed with reference to extreme value distributions. On the other hand, less attention has been devoted to other distributions, generally employed for other types of extreme events such as droughts.

As an example, with reference to a three parameters Generalized Extreme Value (GEV) distribution with cumulative distribution function:

$$F_{X_t}(x; \mu, \alpha, \kappa) = exp \left\{ -\left[1 - \kappa \frac{(x - \mu)}{\alpha} \right]^{1/\kappa} \right\} \tag{1}$$

by modeling the parameters μ, α, κ as a function of time, different types of non stationarities can be taken into account. For instance, a linear trend in the location parameter μ can be implemented by assuming $\mu(t) = \mu_0 + \mu_1 t$. Sometimes non linear dependence with time is considered for the scale parameter α as $\ln \alpha(t) = \alpha_0 + \alpha_1 t$ (Šraj et al. 2016). Shape parameter is generally assumed to be constant as its value is not easy to be estimated reliably (Coles et al. 2001; Salas and Obeysekera 2014).

Russo et al. (2013) adopted a non stationary gamma distribution with fixed shape parameter r and a time varying scale parameter β_t to analyze changes in probability of future precipitation over Europe:

$$f_{X_t}(x) = \frac{1}{\beta_t \Gamma(r)} \exp \left(\frac{x}{\beta_t} \right) \left(\frac{x}{\beta_t} \right)^{r-1} \tag{2}$$

where and a linear trend with time was assumed for $\beta_t = \alpha_0 + \alpha_1 t$. Note that since the expected value of X_t is $E[X_t] = r\beta_t$ while the variance is $Var[X_t] = r\beta_t^2$ this is equivalent to assume a linear trend in the mean and a constant coefficient of variation.

Several methods have been proposed to estimate the non stationary parameters among which the Maximum likelihood (MLE) method is the one generally employed (Katz et al. 2002; Salas and Obeysekera 2014). Alternative methods include generalized maximum likelihood method (GMLE) (El Adlouni et al. 2007; Gül et al. 2014) or Markov chain Montecarlo approach (Šraj et al. 2016). A detailed illustration of the above methods can be found in Coles et al. (2001).

Furthermore, many software packages implementing non stationary estimation of extreme values distributions have been developed (for a review see Gilleland et al. (2013)).

2.3 Modeling Return Period and Risk

Return period (Fuller 1914) and risk concepts have been commonly employed in hydrological practice assuming an underlying stationary distribution for the extreme events. Under the hypothesis of an indipendent and identically distributed annual hydrometeorological variable X, the return period of a given x_0 value can be computed as the expected value of the interarrival time between two occurrences $X > x_0$. Since under the i.i.d. hypothesis

the interarrival time T will be distributed as a geometric random variable with parameter $p = P[X > x_0]$, the return period will be the expected value of T, namely (Mood et al. 1974):

$$T = \frac{1}{p} = \frac{1}{P[X > x_0]} \tag{3}$$

Recently Volpi et al. (2015) showed that the Eq. 3 is valid also in the case of dependent or serially correlated values, but not in the case of non-stationary series.

Extensions of Eq. 3 to the non stationary case have been proposed by Salas and Obeysekera (2014), Cooley (2013). More specifically, assuming the exceedence probability $p_t = P[X_t > x_0]$ changing with time t, the return period of T becomes:

$$T = 1 + \sum_{x=1}^{\infty} \prod_{t=1}^{x} (1 - p_t) \tag{4}$$

Risk is defined as the probability of observing at least one event $X > x_0$ in an n-year period. For instance, risk expresses the probability that a given hydraulic structure may fail at least once during its n-year lifetime. Under the i.i.d. hypothesis for X, risk can be easily computed by observing that the number Z of occurrences $X > x_0$ in n-year period is distributed according to a binomial distribution with parameters (n, p) and therefore the probability of observing at least one occurrence will be:

$$R = P[Z \geq 1] = 1 - P[Z = 0] = 1 - \binom{N}{0} p^0 (1 - p)^n - 0 = 1 - (1 - P[X > x_0])^n \tag{5}$$

Salas and Obeysekera (2014) proposed an extension of the above equation for the non stationary case as:

$$R = 1 - \prod_{t=1}^{n} (1 - p_t) \tag{6}$$

In Eqs. 4 and 6, time varying exceedence probabilities can be modeled by means of a non stationary probability distribution.

3 Modeling Drought in a Non Stationary Context

3.1 General

Probabilistic characterization of droughts has been the subject of a significant amount of research in the last 30 years (e.g. Rossi et al. 1992; Sharma 1997; Tsakiris et al. 2007; Nalbantis and Tsakiris 2009; Vangelis et al. 2010; Cancelliere and Salas 2010; Bonaccorso et al. 2013, 2015; Tsakiris et al. 2016), also with reference to water supply system operational aspects (e.g. Bowles et al. 1987; Rossi and Cancelliere 2013; Tsakiris et al. 2013; Haro et al. 2014). While most works deal with stationary conditions, less attention has been devoted to modeling droughts assuming non stationarity either with reference to the climatic forcings or to the implicit demand levels (Duan and Mei 2014; Zargar et al. 2014; Mohammed et al. 2017).

In general terms, non stationarity in drought modeling may arise because there is a change in time of the probabilities related to the underlying hydrological variables (due to anthropogenic or natural causes) or because the demand level, usually represented by a threshold, changes in time, which in turn will cause non stationarity in the corresponding deficits. To better clarify this concept, in Fig. 1a the time series of a non stationary fictitious
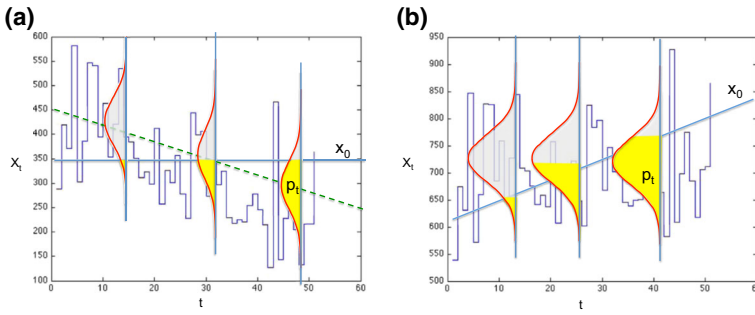
Fig. 1 Changes of probabilities of deficits in a non stationary series with a constant threshold x_0 (*left*) and in a stationary series with a non constant threshold x_0. The continuous line represents the threshold, the dashed line (*left*) is the time varying mean

hydrometeorological variable is plotted where for illustrative purposes, a very strong trend in the mean has been assumed. In the same plot, the continuous horizontal line represents the threshold value, assumed equal to the long term mean, while the dashed line represents the time varying mean. Furthermore, the non stationary probability distribution functions at three time instants are also plotted, along with the corresponding probabilities of deficit p_t (yellow areas), namely the probability of observing a value less than the threshold. As clearly indicated by the figure, as a consequence of the time variability of the mean, the probability of deficits also varies with time, and in particular it increases since the mean decreases with time.

In Fig. 1b, a similar plot is shown, where this time a stationary series is considered but a increasing threshold with time is assumed. Such time varying threshold may correspond to a time varying demand level. For instance, with reference to rainfed agriculture and a stationary precipitation series, non stationarity in temperature, e.g. increasing with time, may lead to increase in potential evapotranspiration, thus leading to an increase of water demand, despite the stationary nature of the precipitation series. Also in this case, the time variability of the threshold level will lead to time varying probabilities of deficits, which in turn will lead to non stationary probabilities of drought occurrence.

Regardless of the causes of nonstationarity in the deficit series, and without loss of generality, in what follows we will assume that the probability of a deficit, namely that the hydrological variable is lower than a fixed threshold, is time dependent:

$$p_t = P[X_t \leq x_{0t}] \tag{7}$$

where again the time dependent structure may be due to changes of the underlying probability function of the hydrological variable X_t or to modifications in the threshold level x_{0t}.

In what follows, analytical expressions for the probability of drought length will be derived, based on assuming a non stationary probability distribution for the underlying series.

3.2 Drought Length Modelling

With reference to a generic time interval t, the interest lies here in determining the probability of observing a drought beginning at time t and lasting l intervals. By definition, such

event will be a sequence of l deficits $X_{t+\tau} \leq x_{0t+\tau}, \tau = 0, 2, , l - 1$ preceeded and followed by the two surpluses $X_{t-1} > x_{0t-1}$ and $X_{t+l} > x_{0t+l}$. The corresponding probability will be:

$$P[X_{t-1} > x_{0t-1}, X_t \leq x_{0t}, X_{t+1} \leq x_{0t+1}, ..., X_{t+l-1} \leq x_{0t+l-1}, X_{t+l} > x_{0t+l}] \quad (8)$$

The above probability can be interpreted as the joint probability of the two events "a drought starts at time t" and "it has length l". Alternatively, one may be interested in the event "a drought has length l given it started at time t". In the latter case, since in order for a drought to start a surplus must be preceded by a deficit (namely the event $\{X_{t-1} > x_{0t-1}, X_t \leq x_{0t}\}$, the probability becomes:

$$P[X_{t+1} \leq x_{0t+1}, ..., X_{t+l-1} \leq x_{0t+l-1}, X_{t+L} > x_{0t+l}|X_{t-1} > x_{0t-1}, X_t \leq x_{0t}] = f_{L_t}(l) \quad (9)$$

and it represents the pdf $f_{L_t}(l)$ of the length $L_t = l$ of a drought that already started at time t (Cancelliere and Salas 2004). If we assume serial independence for the underlying hydrological variable X_t, the joint probability in Eq. 9 can be split into the product of the probabilities, and therefore the the pdf of drought length given by Eq. 9 becomes:

$$f_{L_t}(l) = P[X_{t+1} \leq x_{t+1}] \ldots P[X_{t+l-1} \leq x_{0t+l-1}]P[X_{t+l} > x_{0t+l}] = (1 - p_{t+l}) \prod_{\tau=1}^{l-1} p_{t+\tau} \quad (10)$$

where the conditioning clearly disappears since serial independence is assumed.

The above (10) enables to compute the probability of a drought of length l starting at time t under the assumption that the probability of observing a deficit p_t varies with time t.

Equation 10 simplifies somewhat if we assume that p_t is a linear function of time t, e.g.:

$$p_{t+\tau} = p_t + \delta\tau \quad (11)$$

Substitution of Eqs. 11 into 10 yields:

$$f_{L_t}(l) = (1 - p_t - \delta l) \prod_{\tau=1}^{l-1} (p_t + \delta\tau) \quad (12)$$

Rearranging terms, letting $\alpha = \frac{p_t}{\delta}$ one can write:

$$f_{L_t}(l) = (1 - p_t - \delta l)\delta^{l-1} \prod_{\tau=1}^{l-1} [\alpha + \tau] \quad (13)$$

Note that by letting $\delta = 0$ in Eq. 11, stationary conditions are assumed for X_t and Eq. 13 reduces to the well known geometric distribution for drought length (Llamas and Siddiqui 1969)

$$f_{L_t}(l) = (1 - p_t)p_t^{l-1} \quad (14)$$

Equation 13 can be further modified observing that:

$$\prod_{\tau=1}^{l-1} (\alpha + \tau) = \frac{\Gamma(\alpha + l)}{\alpha\Gamma(\alpha)} \quad (15)$$

where in usual notation $\Gamma()$ is the complete gamma function (Abramowitz and Stegun 1965). Substituting:

$$f_{L_t}(l) = (1 - p_t - \delta l)\delta^{l-1} \frac{\Gamma(\alpha + l)}{\alpha\Gamma(\alpha)} \quad (16)$$

The above (16) enables to compute the probability of a drought of length l starting at time t under the assumption of a linear dependence of p_t with time.

The expected value of drought length can be computed making use of the pdf expressed by Eq. 10:

$$\mathrm{E}\left[L_t\right] = \sum_{l=1}^{\infty} l f_{L_t}(l) = \sum_{l=1}^{\infty} l(1 - p_{t+l}) \prod_{\tau=1}^{l-1} p_{t+\tau} \tag{17}$$

which in the case of linear dependence of p_t with time specialises into:

$$\mathrm{E}\left[L_t\right] = \sum_{l=1}^{\infty} l f_{L_t}(l) = \sum_{l=1}^{\infty} l(1 - p_t - \delta l) \delta^{l-1} \frac{\Gamma(\alpha + l)}{\alpha \Gamma(\alpha)} \tag{18}$$

Deriving closed form solutions for Eqs. 17 and 18 may be combersome and therefore it is preferable to resort to numerical solutions. Alternatively, in order to compute an approximate expression for the mean drought length, one can assume that during the drought the probability of deficit (and conversely of surplus) do not change with time. This may be a reasonable assumption if the rate of change of p_t with time is moderate, considering also that in the infinite sums in Eqs. 17 and 18, the pdf generally exhibit a fast decay with length. Then, an analytical approximation to the expected value of drought length is:

$$\mathrm{E}\left[L_t\right] = \frac{1}{p_{t+1}} \tag{19}$$

In practice, as shown in Section 2.2, non stationary conditions are generally assumed for the parameters of the distribution of X_t, which are assumed to vary with time according to prespecified functional forms, either linear or not linear. Even in this case, the closed form expression for drought lenght pdf given by Eq. 16, based on assuming that p_t evolves linearly with time (11), may still find practical application. Indeed, let us assume that X_t is distributed according to a generic non stationary cumulative density function $F_{X_t}(x; \Theta_t)$ where Θ_t is a vector of time varying parameters. It follows that $p_t = F_{X_t}(x_0; \Theta_t)$. Then, expanding in Taylor series $p_{t+\tau}$ around p_t, it follows:

$$p_{t+tau} \approx p_t + \frac{d F_{X_t}(x_0; \Theta_t)}{dt}(t + \tau) \tag{20}$$

Comparing (20) with Eq. 11, it follows that, as an approximation, we can assume $\delta = \frac{d F_{X_t}(x_0; \Theta_t)}{dt}$.

In order to better illustrate the above point, let's assume X_t distributed according to a non stationary log-normal distribution with a linear dependence of the location parameter with time (Aissaoui-Fqayeh et al. 2009):

$$f_{X_t}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}(\frac{logx - \mu_0 - \mu_1 t}{\sigma})^2} \tag{21}$$

where σ, μ_0 and μ_1 are parameters.

It follows:

$$\delta = \frac{d}{dt} \int_{-\infty}^{\frac{logx_0 - \mu_0 - \mu_1 t}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}y^2} dy \tag{22}$$

The derivative with respect to t in the above equation can be computed recalling Leibnitz rule as:

$$\delta = -\mu_1 \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2}(\frac{logx_0 - \mu_0 - \mu_1 t}{\sigma})^2} \tag{23}$$

Following a similar line of reasoning, the parameter δ can be derived on the basis of any underlying non stationary pdf for the X_t.

4 Application

In order to better illustrate the above mathematical development, an application to annual precipitation series exhibiting different degrees of trend has been carried out. In particular, four precipitation stations in Sicily have been selected namely Trapani, Agrigento, Petralia Sottana and Caltanissetta, for which relatively long series of monthly precipitation data are available.

Figure 2 shows the time series of annual precipitation series at four rain gauges in Sicily (Italy), namely Trapani, Agrigento, Petralia Sottana and Caltanissetta along with the corresponding linear trend lines.

Preliminarly, the significance of the trends has been verified for all the four series, by means of the Student-t test (tS) and the Mann-Kendall test (MK).

In Table 1 the results of the application of both tests are reported, as well as the rate of change in precipitation series assuming a linear trend. From the table it can be inferred that three of the investigated rain gauges exhibit trends, statistically significant at 5% level. In particular, the test statistics agree in detecting a significant negative trend for Trapani and Caltanissetta and a significant positive trend for Petralia Sottana, whereas no trend is inferred for Agrigento. In addition, the rate of change in mean precipitation ranges from about -1 mm per year for Trapani to 1 mm per year for Petralia Sottana. In order to model the probability of deficit, a non stationary approach has been adopted for fitting probability distributions to the series. More specifically a log normal distribution with a linear dependence of the location parameter with time has been selected for all the stations (see Eq. 21).

Fitting of the parameters σ, μ_0 and μ_1 has been carried out through Maximum Likelihood Method.
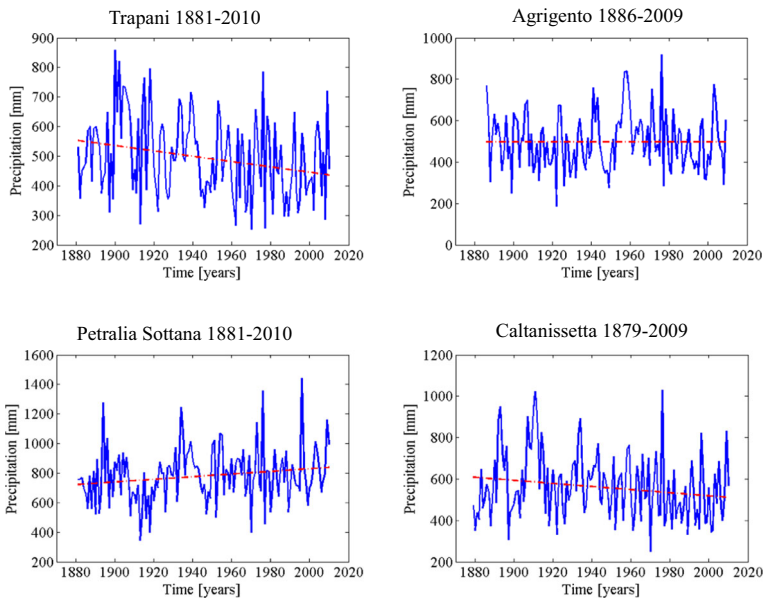
Fig. 2 Time series of investigated annual precipitation series (*solid line*) and corresponding linear trend (*dashed line*)

Table 1 Trend test results

rain gauge	Observation period	t_S	MK	Variation [mm/year]
Trapani	1881-2010	−2.931(0.004)	−2.733(0.006)	−0.903
Agrigento	1886-2009	0.008(0.994)	−0.389(0.698)	0.003
Petralia Sottana	1881-2010	2.115(0.036)	1.972(0.05)	0.900
Caltanissetta	1879-2010	−2.074(0.040)	−2.060(0.040)	−0.744

Significant test statistic values (at 5% significance level) and related p-values in bracket are marked in italic

Then, the probability of deficit at time t has been computed as the non exceedence probability of the long term mean, assumed as a constant threshold, by means of the non-stationary lognormal pdf in Eq. 21.

Furthermore the probabilities of drought of given length l have been computed using Eq. 4. The results are shown in Fig. 3, where for each rain gauge, the probabilities of deficit p_t and the probabilities of drought length $l = 1$, $l = 3$ and $l = 5$ years are plotted as a function of time.

Inspection of Fig. 3 reveals that, as expected, the probability of deficit increases in the cases of decreasing trends of precipitation (Trapani and Caltanissetta), decreases for increasing trend (Petralia Sottana) while is constant for Agrigento, which does not exhibit trend. This is consistent with the fact that as the series tends to exhibit smaller values, the probability of observing values below a fixed threshold increases, vice versa for the opposite case. From the figure it can also be inferred that in the case of decreasing trend (Trapani and Caltanissetta), the probability of drought length l = 1 exhibits a decreasing pattern with time,
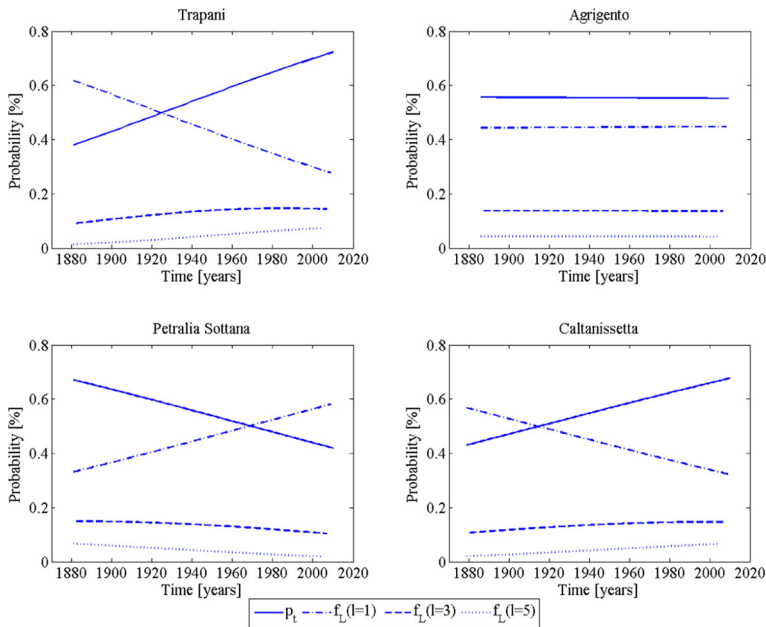
Fig. 3 Probabilities of single deficit p_t and probabilities of a drought length equal to 1 year ($f_L(1)$), 3 years ($f_L(3)$) and 5 years ($f_L(5)$) vs. time for the 4 investigated series
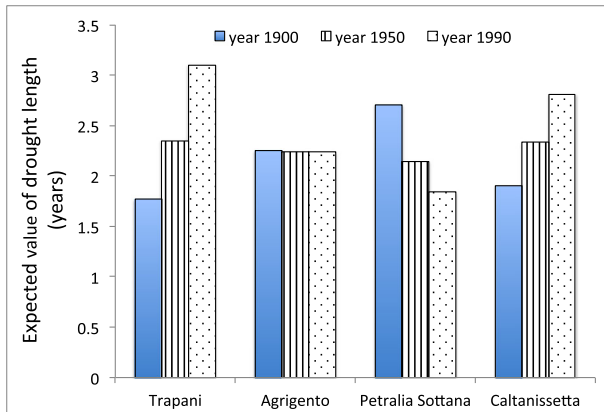
Fig. 4 Non-stationary expected values E[L] of drought length (years) computed with reference to different years for the 4 investigated series

whereas the probabilities of longer droughts (l = 3 and l = 5), show an increasing shape. Such apparent contrasting behaviour finds an explanation in the fact that as the values tend to be smaller, short droughts tends to be less frequent, while more longer droughts are to be expected.

Figure 3 also indicates a fairly linear behaviour of deficits p_t with time. This confirms the validity of the proposed linearization approach expressed in Eq. 20, which enables to make use of the closed form Eq. 16 to derive the non stationary pdf of drought length.

Finally the expected value of drought length $E[L]$ has been computed with reference to different years, namely 1900, 1950 and 1990 by means of Eq. 17. The results are reported in Fig. 4, from which it can be inferred that, as expected, the mean value of drought length tends to increase with time for series exhibiting decreasing trend (Trapani and Caltanissetta). Conversely, series with increasing trend (Petralia Sottana), tends to exhibit shorter droughts as time progresses. On the other hand, Agrigento (no trend) does not exhibit any significant changes in mean drought length.

5 Conclusive Remarks

Probabilistic characterization of extreme events in a non stationary setting requires the developments of new tools, able to overcome the limitations of more traditional stationary approaches. Besides variability of climate (either natural or anthropogenic), non stationarity may arise in many water resources problems, due to anthropogenic effects on hydrological cycle, modification in water demand levels as well as infrastructural changes. Thus there is a growing need of new methods able to incorporate and take into account non stationarities in hydrometeorological variables and/or other factors.

The bulk of recent literature on the subject reveals that most of the attention has been devoted to the analysis of floods and extreme precipitation in a non stationary context, generally adopting a time varying structure for the underlying probability distribution, whose variability may also be linked to external covariates representing physical factors exerting an influence on the variables of interest. Within such a framework, traditional concepts widely applied in engineering practice such as return period and risk have been revised

to take into account the non stationary nature of the underlying variables. However less attention has been devoted to the non stationary analysis of droughts, which still present several challenges. Indeed, the multiyear nature of the drought phenomenon, the fact that non stationarity may arise because of changes either of the underlying variable, of the demand level or both, as well as the need to take into account jointly several characteristics (length, severity, areal extension, etc.) poses several problems, some of which still are not solved.

The methodology presented here for characterizing drought length assuming non stationarity either in the hydrological variable or in the demand level (threshold) enables to compute the probability of a drought of length l starting at time t under the assumption that the probability of observing a deficit p_t varies with time t. Furthermore, the expected value of the length of a drought starting at a given time t has also been derived.

Application of the methodology to four long annual precipitation series in Sicily exhibiting different degrees of trend in the mean has highlighted the feasibility of the derived expression to characterize drought length in the presence of non stationarity. Further, the derived methodology is flexible enough to accomodate for virtually any type of stationarity in the series, provided it is modeled adequately.

Ongoing research is oriented to extend the results to other drought characteristics (e.g. severity, intensity), as well as to better take into account the inevitable uncertainty related to the assessment of non stationarity in hydrological series.

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