

Optimizing Multiple-Pollutant Waste Load Allocation in Rivers: An Interval Parameter Game Theoretic Model

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Received: 31 March 2016 / Accepted: 20 June 2016 /
Published online: 11 July 2016
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Abstract In this paper, a new methodology is developed for optimal multiple-pollutant waste load allocation (MPWLA) in rivers considering the main existing uncertainties. An interval optimization method is used to solve the MPWLA problem. Different possible scenarios for treatment of pollution loads are defined and corresponding treatment costs are taken into account in an interval parameter optimization model. A QUAL2Kw-based water quality simulation model is developed and calibrated to estimate the concentration of the water quality variables along the river. Two non-cooperative and cooperative multiple-pollutant scenario-based models are proposed for determining waste load allocation policies in rivers. Finally, a new fuzzy interval solution concept for cooperative games, namely, Fuzzy Boundary Interval Variable Least Core (FIVLC), is developed for reallocating the total fuzzy benefit obtained from discharge permit trading among waste load dischargers. The results of applying the proposed methodology to the Zarjub River in Iran illustrate its effectiveness and applicability in multiple-pollutant waste load allocation in rivers.

Keywords Interval optimization · Multiple-pollutant · Waste load allocation · Game theory · The Zarjub river

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1 Introduction

Waste load allocation (WLA), which refers to the determination of the allowable amount of waste load discharged to a river by different pollution sources, is known as an effective approach for river water quality management (Mahjouri and Bizhani-Manzar 2013).

Uncertainty analysis plays an important role in WLA problems. Since uncertainties come from different sources, various methods have been developed to address different types of uncertainties. For example, when vagueness or imprecision in the existing information is the source of uncertainties or when very limited data is available, fuzzy set theory is considered as a useful tool for handling uncertainties. WLA problems with respect to parameters imprecision have been addressed in a number of studies such as Mesbah et al. (2010) and Nikoo et al. (2013). Also, Ghosh and Mujumdar (2010), Du et al. (2013) and Mahjouri and Abbasi (2015) have addressed dual uncertainties in some WLA problems due to both randomness and fuzziness.

There are additional uncertainties that arise not necessarily from randomness or fuzziness of the variables but the related partial ignorance. In the past years, several interval parameter optimization techniques were developed to incorporate uncertainties, where knowing the parameters' probability distribution or fuzzy membership functions are not required. In this approach, only knowing the bounds of uncertain coefficients or inputs would suffice. Karmakar and Mujumdar (2006a) proposed a grey fuzzy waste load allocation model (GFWLAM) which addressed uncertainties due to fixing membership functions for different goals of pollution control agency and dischargers. They incorporated the membership parameters as interval grey numbers, which had known lower and upper bounds and unknown distribution information. They determined the optimal fractional removal levels of pollutants in the form of interval grey numbers. Therefore, the decision maker is provided with a range of optimal solutions to choose from, based on economic and technical considerations. Additionally, Rehana and Mujumdar (2009) developed an Imprecise Fuzzy Waste Load Allocation Model (IFWLAM) in which the imprecise fuzzy risk of low water quality was calculated considering the uncertainty due to partial ignorance of the parameters. Uncertainties in several other researches have been incorporated using interval programming (e.g. Karmakar and Mujumdar 2006b, 2007; Tan et al. 2010; Nikoo et al. 2012a, b; Tavakoli et al. 2014, 2015; Zhang et al. 2015; Soltani et al. 2016).

In a limited number of WLA studies, the interactions between multiple pollutants have been considered. Lence et al. (1988) discussed cost efficiency of transferable discharge permit markets for controlling multiple pollutants. In their work, three water quality variables of biochemical oxygen demand (BOD), phosphorus, and nitrogen were considered. Lence (1991) presented a weighted sum transferable discharge permit program for controlling BOD and phosphorus discharges. Ng and Eheart (2005) investigated effects of trading discharge permits on reliability of maintaining water quality. Hung and Shaw (2005) developed a new system of trading discharge permits namely trading-ratio system (TRS). They showed that the trading-ratio system (TRS) can consider the effect of the location of pollution sources and achieve the predetermined standards of environmental quality at a minimum total abatement cost. Ning and Chang (2007) proposed a dynamic discharge permit trading program to present an integrated simulation

and optimization analysis for generating spatially varied trading ratios. In their paper, two water quality variables of BOD and ammonia-nitrogen (NH₃-N) were taken into account.

Sarang et al. (2008) presented a program based on Transferable Discharge Permit (TDP) in which some water quality variables were considered. They proposed two types of TDP program 1) separate permits that manage each pollutant individually in separate markets, while each permit was assigned based on the quantity of the pollutant or its environmental effects, and 2) weighted-sum permits that aggregate some pollutants as a single commodity to be traded in a single market. Also, they performed a mathematical analysis of TDP programs for multiple pollutants and showed the practicality of the proposed approach for cost-efficient maintenance of river water quality.

In this paper, an interval parameter scenario-based multiple-pollutant waste load allocation methodology is developed for river water quality management. The proposed methodology can be considered as an extended version of the methodology developed by Mesbah et al. (2009) by adding some ideas from game theory and interval optimization. Some water quality variables and several scenarios for controlling pollution loads are considered in the methodology. Two optimization models are developed for determining non-cooperative and cooperative (cooperative means the possibility of trading pollution discharge permits by exchanging treatment scenarios among waste load dischargers) waste load allocation policies in rivers. In the non-cooperative WLA model, the initial discharge permits are allocated to dischargers considering their locations on the river and the river assimilative capacity and then the best combination of treatment scenarios for dischargers is obtained considering the objectives of minimizing the treatment cost of each discharger as well as maintaining river water quality standards. In the cooperative WLA model, dischargers are allowed to form a grand coalition and exchange treatment scenarios to reduce their total treatment cost considering multiple-pollutant discharge permit trading among dischargers. In this model, discharge permit trading is done by exchanging treatment scenarios among waste load dischargers. A new fuzzy interval solution concept for cooperative games, namely, Fuzzy Boundary Interval Variable Least Core (FIVLC), is also developed for reallocating fuzzy benefits of discharge permit trading among dischargers. One of the important financial factors that affect the willingness to trade is the current and needed waste load discharge credits of each discharger and their treatment costs.

The results of the developed methodology can motivate waste load dischargers to participate in coalitions and trade their pollution loads and reduce their treatment costs considering the main uncertainties. As multiple pollutants are considered in the methodology, comparing to conventional waste load allocation models, the results of the proposed method could be more acceptable to pollution load dischargers. The applicability and effectiveness of the methodology are examined by applying it to the Zarjub River in the northern part of Iran.

2 Methodology

The general framework of the proposed methodology for interval-parameter scenario-based multiple-pollutant waste load allocation in rivers is presented in Fig. 1. In this framework, it is assumed that the pollution loads of dischargers are quantified based

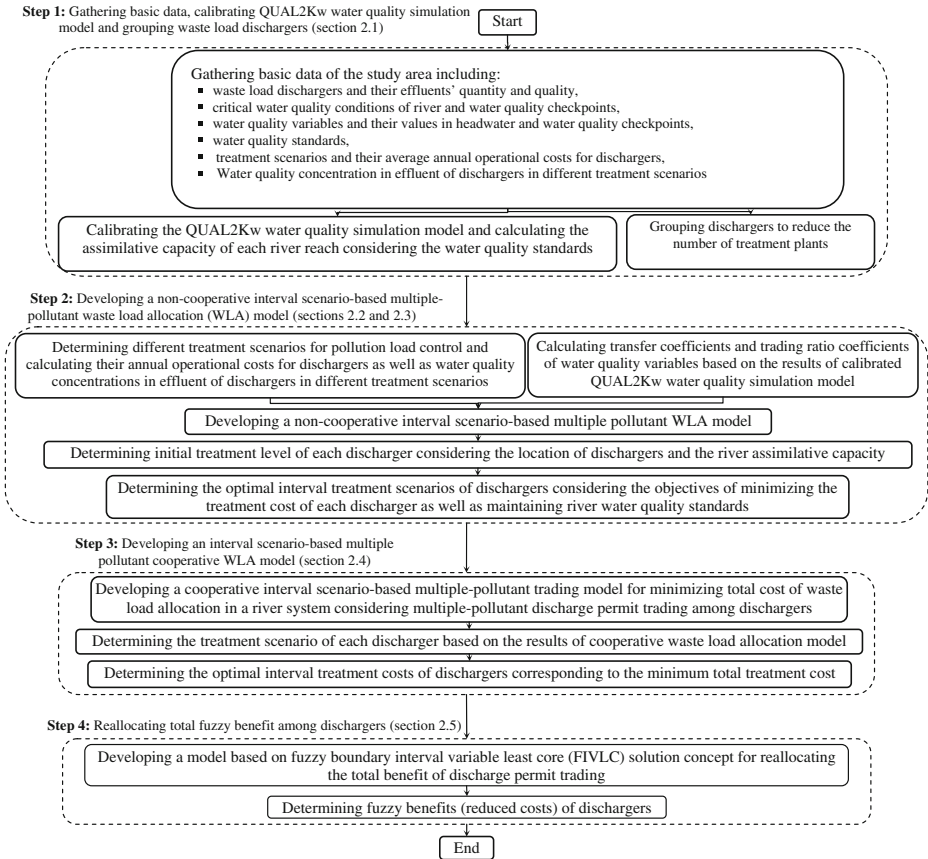


Fig. 1 A flowchart of the proposed methodology for interval-parameter multiple-pollutant waste load allocation in rivers

on the concentration of BOD and total nitrogen (TN) in their effluents and dissolved oxygen (DO), BOD and TN are considered as river water quality indicators. This methodology can be extended to consider more water quality variables.

The inputs of the proposed methodology are waste loads of dischargers (single or grouped point loads), physical characteristics of river in a critical condition (i.e., low flow), locations of water quality checkpoints, water quality variables (e.g., DO, TN and BOD in this study) and their values in headwater, standard concentrations for water quality variables, treatment scenarios of each discharger and their corresponding annual operational costs. Transfer coefficients and trading ratios, which are determined based on the results of a calibrated water quality simulation model (QUAL2Kw in this study), are mainly dependent on the physical characteristics of river and bio-chemical characteristics of pollution loads.

The methodology can incorporate key existing uncertainties in parameters and inputs (e.g., transfer coefficients, trading ratios and effluent loads in different treatment scenarios) using interval parameter programming. The outputs of the methodology are the treatment scenarios of dischargers in both cooperative and non-cooperative conditions. In the cooperative condition, dischargers participate in coalitions to reduce their treatment costs by exchanging their

treatment scenarios. A cooperative game theoretic model is also utilized for reallocating benefit gained in a coalition.

The main steps of the proposed methodology are explained in the following subsections:

2.1 Gathering Basic Data, Calibrating QUAL2Kw Simulation Model and Grouping Waste Load Dischargers

As shown in Fig. 1, at first, some basic data related to main pollution sources and their discharges, headwater quantity and quality, and construction and operational costs of different treatment scenarios are gathered. Basic data regarding water quality that should be prepared include the concentration of river water quality variables that violate the river water quality standards, low flow and the most critical water quality conditions as well as the uncertainties related to the headwater quality, pollution loads in different treatment scenarios, transfer coefficients and trading ratio.

In addition, to reduce the number of required treatment plants, dischargers are grouped so that one treatment plant can be used for each group of dischargers. Then, the QUAL2Kw water quality simulation model is calibrated using the existing data and the assimilative capacity of every river reach is calculated considering the maximum (or minimum) standard concentration of water quality indicators. The automatic calibration module of the QUAL2Kw is used for calibration of parameters. The values of the calibrated parameters using the developed QUAL2Kw model and its validation results for determining the water quality variables in the Zarjub river are presented in the section 4.

2.2 Estimating Trading Ratios Using Multiple-Pollutant Extended Trading Ratio System (ETRS)

Trading Ratio System (TRS) determines the amount of pollution discharge permits traded between dischargers in a river system and it has been designed for a single water quality variable. In the Extended Trading Ration System (ETRS), developed by Mesbah et al. (2009), only discharge permits in terms of BOD loads are traded while DO is considered as the river water quality indicator. The ETRS can be modified to consider more water quality variables by defining wastewater treatment scenarios for each pollution source.

In this paper, the policies for trading BOD and TN discharge permits are obtained using a modified version of ETRS considering the assimilative capacity of river based on the concentration of DO, BOD and TN. In the modified ETRS, BOD and TN discharge permits are traded while DO, BOD and TN are considered as the river water quality indicators. The river is divided into n different zones each containing a discharger and the BOD (or TN) transfer coefficients between zones i and j are defined as follows:

- r_{ij}^{BOD-DO} variation of the concentration of DO in zone j (mg/L) as a result of 1 kg increase in BOD load of the discharger in zone i .
- r_{ij}^{TN-DO} variation of the concentration of DO in zone j (mg/L) as a result of 1 kg increase in TN load of the discharger in zone i .
- $r_{ij}^{BOD-BOD}$ variation of the concentration of BOD in zone j (mg/L) as a result of 1 kg increase in BOD load of the discharger in zone i .
- r_{ij}^{TN-TN} variation of the concentration of TN in zone j (mg/L) as a result of 1 kg increase in TN load of the discharger in zone i .

The amount of BOD (or TN) discharge permit that two dischargers i and j can trade defines how much BOD (or TN) load they can trade while the DO, BOD and TN concentrations in downstream river do not violate the river water quality standards. The trading ratios between zone i and j for BOD (t_{ij}^{BOD}) and TN (t_{ij}^{TN}) loads are calculated as follows:

$$t_{ij}^{TN} = \min \left\{ \min \left\{ \frac{r_{ik}^{TN-DO}}{r_{jk}^{TN-DO}} \right\} \quad i < j \quad k = \{j, \dots, n\}; r_{ik}^{TN-TN} \right\} \quad (1)$$

$$t_{ij}^{BOD} = \min \left\{ \min \left\{ \frac{r_{ik}^{BOD-DO}}{r_{jk}^{BOD-DO}} \right\} \quad i < j \quad k = \{j, \dots, n\}; r_{ik}^{BOD-BOD} \right\} \quad (2)$$

where, n is the total number of zones (dischargers). The primary BOD and TN discharge permits for each discharger (zone) i is calculated as follows:

$$\frac{\bar{T}_i^{BOD}}{T_i^{BOD}} = \min \left[E_j^{BOD} - \sum_{k=1}^{j-1} r_{kj}^{BOD-BOB} \bar{T}_k^{BOD}; \min \left\{ \frac{E_j^{DO} - \sum_{k=1}^{j-1} r_{kj}^{BOD-DO} \times \bar{T}_k^{BOD}}{r_{ij}^{BOD-DO}} \right\} \quad j = i, \dots, n \quad k = \{1, 2, \dots, i-1\} \right] \quad (3)$$

$$\frac{\bar{T}_i^{TN}}{T_i^{TN}} = \min \left[E_j^{TN} - \sum_{k=1}^{j-1} r_{kj}^{TN-TN} \bar{T}_k^{TN}; \min \left\{ \frac{E_j^{DO} - \sum_{k=1}^{j-1} r_{kj}^{TN-DO} \times \bar{T}_k^{TN}}{r_{ij}^{TN-DO}} \right\} \quad j = i, \dots, n \quad k = \{1, 2, \dots, i-1\} \right] \quad (4)$$

where \bar{T}_i is the initial permit of discharger (zone) i (kg), and E_j^{DO} , E_j^{BOD} and E_j^{TN} are the total assimilative capacity of zone j based on water quality indicators of DO, BOD and TN, respectively (mg/L). It should be noted that $\bar{T}_1^{BOD} = E_1^{BOD}$ and $\bar{T}_1^{TN} = E_1^{TN}$. To estimate the total assimilative capacity of a zone and to modify the negative values of discharge permits, the reader is referred to Hung and Shaw (2005) and Mesbah et al. (2009).

2.3 Developing a Non-Cooperative Interval Scenario-Based Multiple-Pollutant WLA Model

In the non-cooperative WLA model, the initial discharge permits are allocated to dischargers considering their locations on the river and the river assimilative capacity using Eqs. 3 and 4. In this type of WLA the best combination of treatment scenarios for dischargers is obtained considering the objectives of minimizing the treatment cost of each discharger as well as maintaining river water quality standards. This model,

which should be run for each discharger, is formulated as follows:

$$\text{Min } Z_{nc,i}^{\pm} = C^{\pm}(s_i^{\pm nc}) \quad i = 1, 2, \dots, n \quad s_i^{\pm nc} \in S_i \tag{5}$$

$$e'_{BOD}{}^{\pm}(s_i^{\pm nc}) \leq \bar{T}_i^{\pm BOD} \quad i = 1, 2, \dots, m \tag{6}$$

$$e'_{TN}{}^{\pm}(s_i^{\pm nc}) \leq \bar{T}_i^{\pm TN} \quad i = 1, 2, \dots, m \tag{7}$$

where, \pm shows an interval number representing a bounded set of real numbers. $Z_{nc,i}$ is total treatment cost of discharger i in a non-cooperative condition. $C(s_i^{nc})$ denotes the treatment cost of discharger i by selecting non-cooperative treatment scenario s_i^{nc} . S_i is a set of possible treatment scenarios for discharger i . $e'_{BOD}{}^{\pm}(s_i^{\pm nc})$ and $e'_{TN}{}^{\pm}(s_i^{\pm nc})$ are respectively discharged BOD and TN loads of discharger i corresponding to non-cooperative treatment scenario s_i^{nc} .

2.4 Developing a Cooperative Interval Scenario-Based Multiple-Pollutant WLA Model

In this section, dischargers are allowed to form a grand coalition to reduce their total treatment cost. Therefore, a cooperative interval scenario-based multiple-pollutant WLA model is developed for reducing total treatment cost considering multiple-pollutant discharge permit trading among dischargers. In this model, discharge permit trading is done by exchanging treatment scenarios among waste load dischargers. A treatment scenario shows the treatment level required for each pollutant in wastewater of a discharger. Treatment scenarios should be defined based on all possible combinations of primary, secondary and tertiary wastewater treatment processes for the treatment plants of the study area.

The main difference between non-cooperative and cooperative WLA is in the possibility of exchanging treatment scenarios among waste load dischargers. In cooperative WLA, some dischargers can form a coalition and discharge permits can be optimally reallocated to the dischargers participating in the coalition. In the optimization model, which is presented in this step, the best discharge permit trading strategy is developed in a way that it provides the minimum total treatment cost and meets water quality standards. In the trading process, an upstream discharger can sell discharge permit to a downstream discharger to reduce total treatment cost of the system. The cooperative optimization model is formulated as follows:

$$\text{Min } Z_{trading,i}^{\pm} = \sum_{i=1}^n C^{\pm}(s_i^{\pm trading}) \quad s_i^{\pm trading} \in S_i \tag{8}$$

Subject to:

$$e'_{BOD}{}^{\pm}(s_i^{\pm trading}) \leq \bar{T}_i^{\pm BOD} \quad i = 1, 2, \dots, m \tag{9}$$

$$e'_{TN}{}^{\pm}(s_i^{\pm trading}) \leq \bar{T}_i^{\pm TN} \quad i = 1, 2, \dots, m \tag{10}$$

where, $C^{\pm}(s_i^{trading})$ denotes the upper and lower bounds of treatment cost of discharger i by selecting treatment scenario $s_i^{trading}$.

2.5 Developing a Fuzzy Boundary Interval Variable Least Core Game

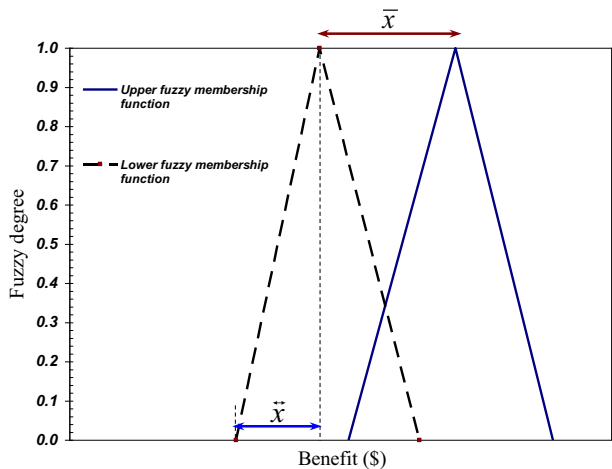
In the developed cooperative WLA model, waste load dischargers have the opportunity to form coalitions to reduce the total treatment cost of the system. Although cooperative cost allocation would be attractive, but the fairness in treatment cost allocation should be also taken into account to encourage waste load dischargers to cooperate. Fairness in treatment cost allocation means that share of each discharger participating in a coalition from the total reduced cost gained in the coalition should be proportional to the benefit they produce in the coalition. Cooperative game theory can be utilized for fair reallocation of treatment costs to waste load dischargers in a coalition. Sadegh and Kerachian (2011) and Jafarzadegan et al. (2013) used different fuzzy versions of Least Core solution concept of cooperative games for reallocating benefits of water allocation in which a fuzzy method was used for considering uncertainties. In this paper, a solution concept, namely FIVLC, is developed and used for reallocating fuzzy benefit of trading discharge permits among waste load dischargers.

In this step, to equitably and fairly reallocate treatment cost to waste load dischargers, possible coalitions among them are formed and treatment costs of the selected treatment scenario are reallocated to waste load discharges in every coalition using the FIVLC model.

In the FIVLC formulation, a triangular fuzzy boundary interval number function, \hat{x} , is shown by $\hat{x} = (\bar{x}, \vec{x})$ in which \bar{x} is the interval center of the fuzzy membership function (with membership degree equal to 1). \vec{x} denotes the left and right fuzziness of interval parameter \bar{x} (Fig. 2). The fuzzy boundary interval costs of players 1 and 2 acting individually are denoted by \hat{c}_1 and \hat{c}_2 , respectively. The fuzzy boundary interval cost of bilateral coalition $\{1, 2\}$ is denoted by \hat{c}_{12} .

The final fuzzy boundary interval cost allocated to players participating in a coalition are denoted by $\hat{\phi}_1$ and $\hat{\phi}_2$. For determining fuzzy boundary interval costs $\hat{\phi}_1$ and $\hat{\phi}_2$, two lower and upper bound models each having a two-step optimization procedure are used. In this

Fig. 2 A triangular fuzzy boundary interval membership function



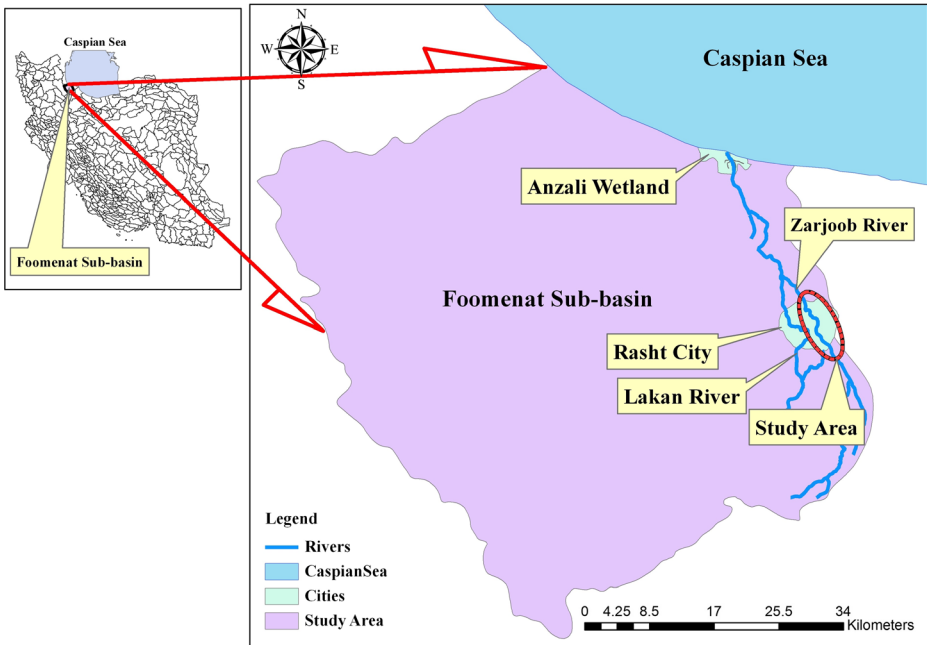


Fig. 3 A GIS view of the Zarjub River in Iran

model, in the first step, the lower and upper bounds of central points of fuzzy boundary interval costs $\hat{\phi}_1$ and $\hat{\phi}_2$ are determined using an interval least core bilateral game:

$$Max \bar{\epsilon} \tag{11}$$

$$\bar{\phi}_i + \bar{\epsilon} \leq \bar{c}_i \quad \forall i = 1, 2. \tag{12}$$

Table 1 General characteristics of each group of dischargers

Zone number	Wastewater discharge (m^3/day)		BOD concentration (mg/L)	Total nitrogen concentration (mg/L)	Corresponding population (person)
	LB	UB			
1	5737	8605	100	129	39,840
2	28,616	42,924	46	64	198,720
3	14,999	22,499	98	92	104,160
4	10,299	15,448	180	106	71,520

LB lower bound, UB upper bound

Table 2 Upper and lower bounds of headwater quality in low flow conditions (mg/L)

Water quality variable	Lower bound	Upper bound
DO	4.2	7.8
BOD	3.5	6.5
TN	2.6	4.8

$$\sum_{i=1}^2 \bar{\phi}_i = \bar{c}_{12} \tag{13}$$

In the above linear interval optimization model, the objective is to determine the maximum amount of imposed interval excess ($\bar{\varepsilon}$) in which two constraints 12 and 13 are satisfied. In the second step, the fuzzy boundaries of the costs $\hat{\phi}_1$ and $\hat{\phi}_2$ are determined based on the lower and upper bounds of the derived interval using Eqs. 11 to 13 and the following equations:

$$Max \bar{\varepsilon} - \bar{\varepsilon} \tag{14}$$

$$\bar{\phi}_i + \bar{\varepsilon} + \bar{\varepsilon} \leq \bar{\phi}_i - (\bar{c}_i - \bar{c}_i) \quad \forall i = 1, 2. \tag{15}$$

Table 3 Definition of different treatment scenarios and their average annual operational cost

Scenario	Only primary treatment (%)	Primary and secondary treatment (activated sludge) (%)	Primary, secondary and tertiary treatment (Nitrogen removal) (%)	Average annual operational cost (US \$ 100)			
				Discharger 1	Discharger 2	Discharger 3	Discharger 4
1	100	0	0	75.62	96.71	108	244.34
2	80	20	0	84.94	107.13	119.63	274.47
3	60	40	0	94.26	117.55	131.27	304.59
4	40	60	0	103.58	127.98	142.91	334.71
5	20	80	0	124.82	158.05	176.49	403.33
6	0	100	0	133.62	168.38	188.03	431.78
7	0	90	10	142.43	178.71	199.56	460.23
8	0	80	20	151.23	189.04	211.09	488.68
9	0	70	30	160.04	199.37	222.63	517.13
10	0	60	40	168.84	209.7	234.16	545.58
11	0	50	50	177.65	220.03	245.7	574.03
12	0	40	60	186.45	230.36	257.23	602.48
13	0	30	70	206.13	255.77	285.61	666.08
14	0	20	80	217.01	268.66	300	701.22
15	0	10	90	227.88	281.55	314.4	736.37

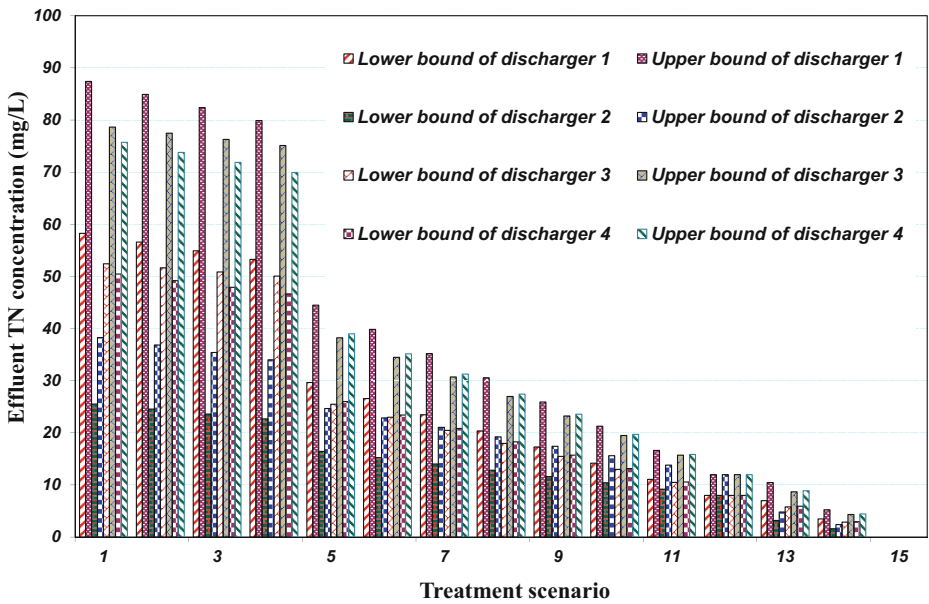


Fig. 4 The lower and upper bounds of total nitrogen (TN) concentration in effluent of dischargers in different treatment scenarios (The TN concentration in effluent of treatment plants using 15th scenario is zero)

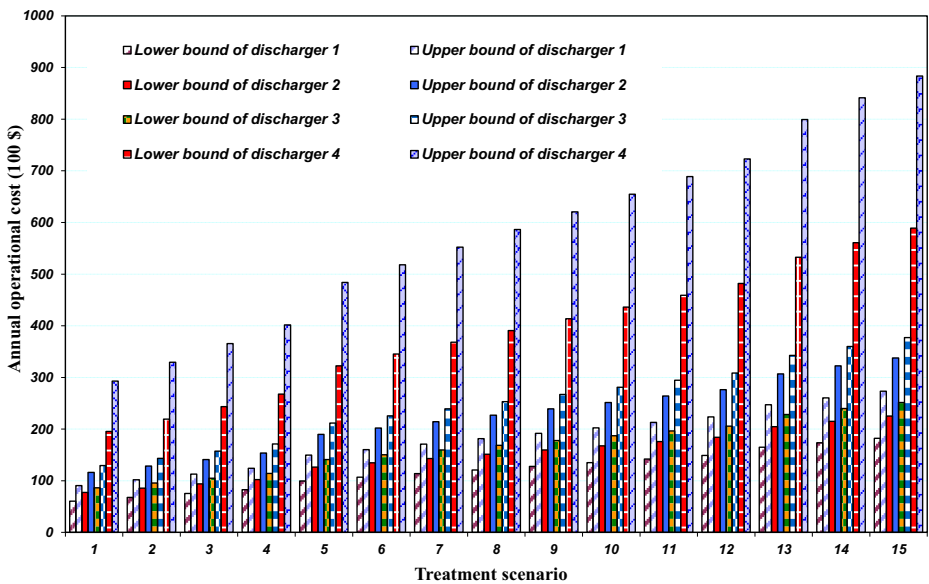


Fig. 5 The lower and upper bounds of annual operational treatment cost of the treatment scenarios for the dischargers (100 \$)

Table 4 The calibrated values for the parameters of QUAL2Kw model

Parameters	Unit	Value
O ₂ reaeration model	-	O'Connor-Dobbins (1958)
Slow CBOD oxidation rate	/d	0.385
Organic Nitrogen:		
Organic Nitrogen hydrolysis	/d	3.85
Organic Nitrogen settling velocity	m/d	0.2
Organic Phosphorus:		
Organic Phosphorus hydrolysis	/d	2.45
Organic Phosphorus settling velocity	m/d	0.86
Ammonium nitrification	/d	0.1
Nitrate:		
Nitrate denitrification	/d	1.8
Sediment denitrification transfer coefficient	m/d	0.81
Inorganic Phosphorus:		
Inorganic Phosphorus settling velocity	m/d	0.28
Sediment Phosphorus oxygen attenuation half sat constant	mgO ₂ /L	0.46

$$-\vec{\phi}_i + \vec{\epsilon} - \vec{\epsilon} \leq \vec{\phi}_i - (\vec{c}_i + \vec{c}_i) \quad \forall i = 1, 2. \tag{16}$$

$$\sum_{i=1}^2 \vec{\phi}_i = \vec{c}_{12} \tag{17}$$

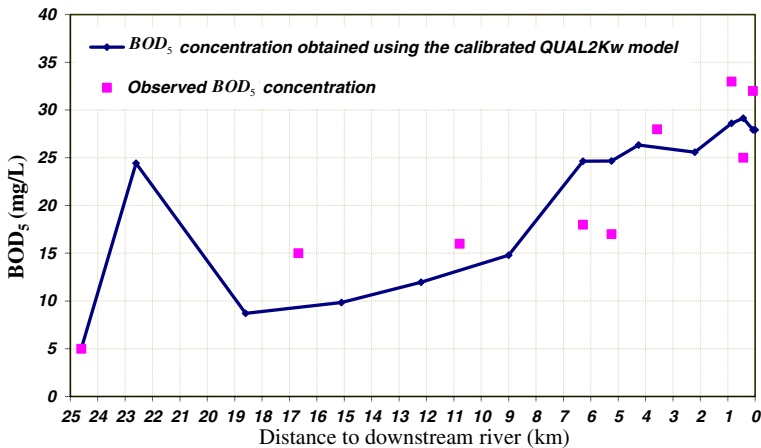


Fig. 6 The observed and simulated concentrations of BOD₅ (using the calibrated QUAL2Kw model) at different check points along the Zarjub river (mg/L) (September 2004)

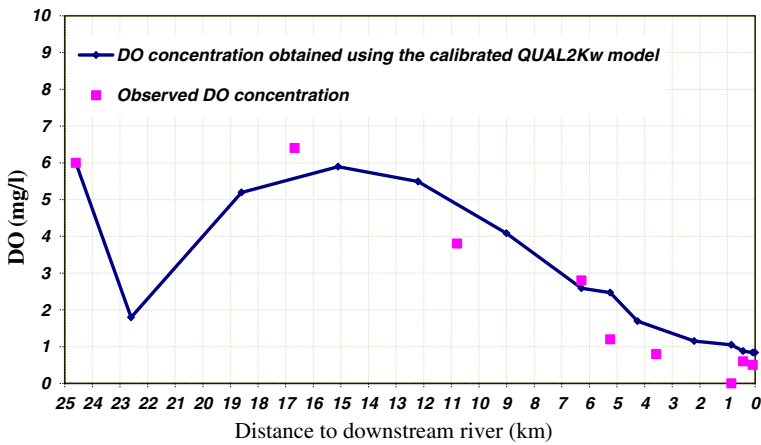


Fig. 7 The observed and simulated concentrations of DO (using the calibrated QUAL2Kw model) at different checkpoints along the Zarjub river (mg/L) (September 2004)

$$\vec{\phi}_i \leq \bar{\phi}_i \quad \forall i = 1, 2. \tag{18}$$

$$\vec{\phi}_1, \vec{\phi}_2, \vec{\varepsilon} \tag{19}$$

In the mentioned interval optimization models, the objectives are maximizing the value of imposed interval excess ($\vec{\varepsilon}$) as well as minimizing its fuzzy boundary ($\vec{\varepsilon}$). More details about fuzzy variable least core game are presented in Sadegh and Kerachian (2011) and Jafarzadegan et al. (2013).

3 Case Study

The study area is a 24 km reach of the Zarjub River located in the Rasht region in northern part of Iran (Fig. 3). This river is one of the main resources for supplying water demands of 54,000 ha of agricultural lands which discharge their return flows into the Zarjub River and Anzali Wetland. Municipal wastewater is the main pollution source of this river. According to the studies of Iranian Department of Environment (IDOE), the Zarjub River has a very critical

Table 5 Transfer coefficient intervals for BOD (mg/L DO/100 kg BOD, when the TN concentrations in wastewater of dischargers 1 to 4 are 130, 75, 61 and 100 (mg/L), respectively)

Zone number	1	2	3	4
1	[0.287, 0.351]	[0.018, 0.022]	[0.007, 0.009]	[0.001, 0.00011]
2	0	[0.117, 0.143]	[0.076, 0.092]	[0.012, 0.014]
3	0	0	[0.052, 0.064]	[0.024, 0.03]
4	0	0	0	[0.016, 0.02]

Table 6 Trading ratio intervals for TN (A trading matrix, when the TN concentrations in wastewater of dischargers 1 to 4 are 130, 75, 61 and 100 (mg/L), respectively)

Zone number	1	2	3	4
1	1	[0.09, 0.11]	[0.045, 0.055]	[0.063, 0.077]
2	0	1	[0.414, 0.506]	[0.63, 0.77]
3	0	0	1	[1.386, 1.694]
4	0	0	0	1

water quality condition that in some zones its water quality is almost similar to municipal wastewater (Mahjouri and Bizhani-Manzar 2013).

According to existing water quality data, the concentration of some water quality variables such as BOD, DO, TN and Coliform bacteria in this river violate the river water quality standards. As pathogens are fully removed through the disinfection process of wastewater treatment, in this paper, pathogens are not considered as water quality indicators. Since the concentration of DO in this river is very low, BOD and DO are selected as river water quality indicators for simulating and controlling DO concentration along the river. There is also the Anzali wetland downstream of the Zarjub River, which is very prone to eutrophication. Therefore, TN is also chosen as the third river water quality indicator. More details about the Zarjub river in the study area can be found in Niksokhan et al. (2009a,b) and Nikoo et al. (2012c). Since constructing a treatment plant for each individual waste load discharger is not economically efficient, 11 main municipal wastewater dischargers along the river are grouped into four major point sources. It is also assumed that the wastewater of each group of dischargers is transferred to its respective municipal wastewater treatment plant. General characteristics of each group of dischargers and headwater quality are given in Tables 1 and 2, respectively. As seen in these tables, the uncertainties related to the headwater quality and pollution loads in different treatment scenarios as well as transfer coefficients and trading ratios are incorporated in the interval parameter optimization technique.

Water quantity and quality data of the Zarjub River measured by IDOE have been collected and analyzed by Iran Water Resources Management Company (IWRMC) (2010). IWRMC showed that low flow and the most critical water quality conditions usually occur in September. This is due to high water temperature, high pollution loads and low river flow in this month. It was also shown that the minimum 7-day flow that would be expected to occur every 10 years varies between 0.094 and 0.140 m³/s (IWRMC 2010). The range of variations of headwater quality is given in Table 2.

Table 7 Trading ratio intervals for TN (A trading matrix, when the BOD concentrations in wastewater of dischargers 1 to 4 are 100, 46, 98 and 180 (mg/L), respectively)

Zone number	1	2	3	4
1	1	[0.153, 0.187]	[0.142, 0.175]	[0.135, 0.162]
2	0	1	[0.9, 1.1]	[0.88, 0.97]
3	0	0	1	[0.75, 0.89]
4	0	0	0	1

Table 8 Non-cooperative optimal intervals for treatment scenarios and treatment costs of dischargers

Discharger	1	2	3	4	Total non-cooperative treatment cost (US \$ 100)
Non-cooperative treatment scenario ^a	[7, 10]	[9, 10]	[10, 11]	[9, 11]	-
Non-cooperative treatment cost (US \$ 100)	[142.4, 168.8]	[199.4, 209.7]	[234.2, 245.7]	[517.1, 547]	[1093.1, 1198.3]

^a Treatment scenarios are obtained based on non-cooperative WLA model (Eqs. 5–7)

4 Results and Discussion

In order to find the optimum treatment scenario for each group of dischargers, 15 treatment scenarios are taken into account. These scenarios have been proposed to consider the most possible combinations of primary, secondary and tertiary wastewater treatment processes for the treatment plants of the study area. Definition of different treatment scenarios and their average annual operational costs are presented in Table 3. For example, in scenario 8, 80 % of the daily volume of the raw wastewater is treated using primary and secondary treatment units and discharged into the river. In this scenario, 20 % of the daily volume of raw wastewater is fully treated using primary, secondary and tertiary treatment units.

In defining these scenarios, the existing treatment technologies in Iran are taken into account. For instance, Fig. 4 shows the lower and upper bounds of TN concentration in effluent of dischargers in different treatment scenarios. These bounds have been selected using engineering judgment considering the existing wastewater treatment technologies in Iran. Also, the lower and upper bounds of annual operational treatment cost of the treatment scenarios are presented in Fig. 5.

In this paper, a QUAL2Kw-based water quality simulation model is developed and calibrated using the existing water quality data. The estimated values for the calibrated parameters of QUAL2Kw model are presented in Table 4. As an example, the concentrations of BOD₅ and DO at different checkpoints obtained using the calibrated QUAL2Kw and the related observed values in September 2004 are presented in Figs. 6 and 7, respectively. These figures illustrate the acceptable accuracy of the calibrated QUAL2Kw model.

The ranges for the values of transfer coefficients and trading ratios are calculated considering the uncertainties in headwater quality and waste load discharges in different river zones and based on the results of the calibrated water quality simulation model. As some examples, the transfer coefficient and trading ratio intervals for BOD as well as TN are presented in Tables 5, 6, and 7, respectively.

Table 9 Cooperative interval wastewater treatment scenarios of dischargers

Discharger	1	2	3	4
Cooperative treatment scenarios	[7, 10]	[9, 10]	[10, 11]	[9, 11]
Total cooperative treatment cost (US \$ 100)	[983.8, 1033.6]			

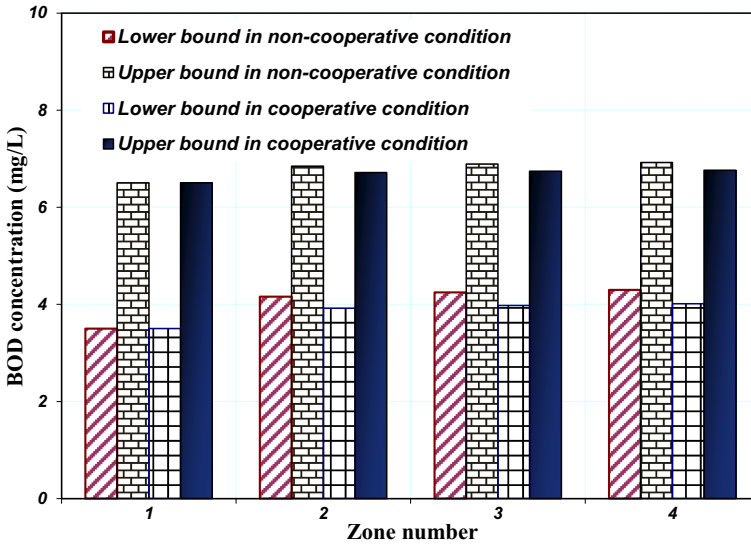


Fig. 8 The lower and upper bounds of BOD concentration in different river zones in non-cooperative and cooperative conditions

The non-cooperative treatment scenarios of dischargers are determined as interval numbers using the proposed methodology so that the water quality indicators meet the water quality standards (Table 8).

Using the cooperative interval scenario-based multiple-pollutant WLA model, the minimum value of the total treatment cost of the system and the treatment scenarios after trading discharge permits are determined (Table 9).

As presented in Table 9, not only the total cost is reduced by trading discharge permits between dischargers but also the river quality standards are met (Figs. 8, 9, and 10 and Tables 8 and 9). The

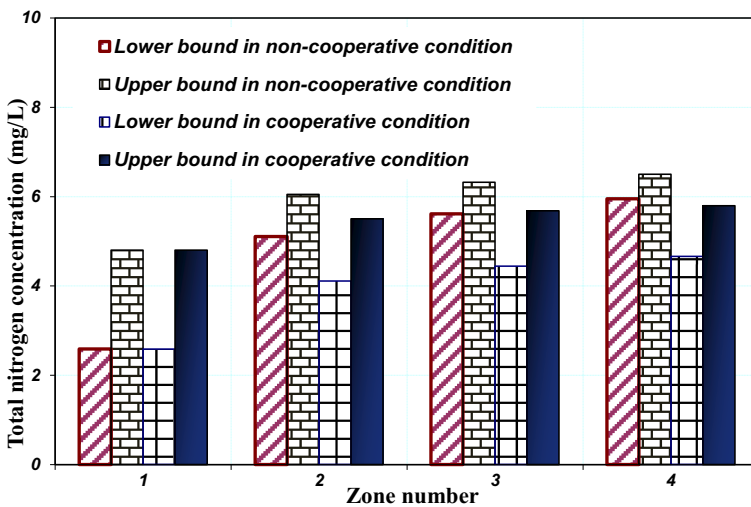


Fig. 9 The lower and upper bounds of TN concentration in different river zones in non-cooperative and cooperative conditions

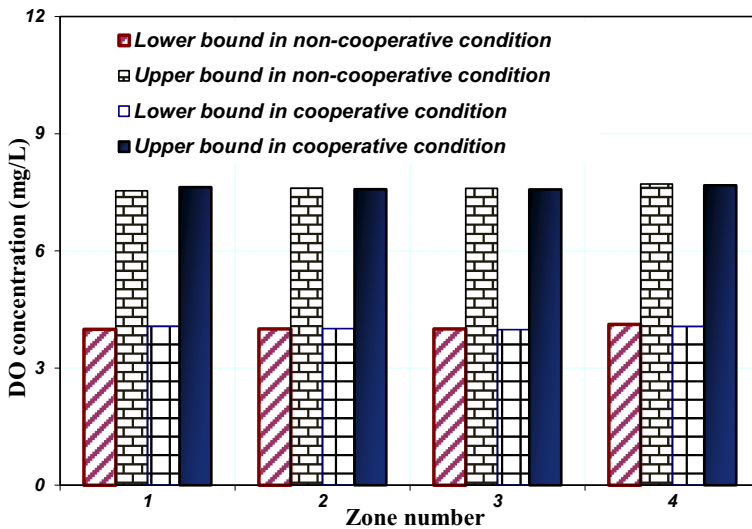


Fig. 10 The lower and upper bounds of DO concentration in different river zones in non-cooperative and cooperative conditions

lower and upper bounds of BOD and TN concentrations in different river zones in non-cooperative and cooperative conditions are presented in Figs. 8 and 9, respectively. As shown in these figures, in all models, the concentrations of BOD and TN in the river satisfy the water quality standards (i.e., 30 mg/L for BOD and 10 mg/L for TN).

As it is shown in Fig. 10, the DO concentration does not violate the minimum acceptable level (3 mg/L) in both non-cooperative and cooperative conditions. Several sequential bilateral discharge permit trading between waste load dischargers are carried out (Hung and Shaw 2005). Results of the lower and upper bounds of trading results, in different rounds, are presented in Tables 10 and 11, respectively. In both lower and upper bound models, the best trading states are (4, 3) and (3, 1) in the first and second trading rounds, respectively.

As presented in Tables 10 and 11, in the first round of trading, by changing its treatment scenario from 10 to 13 (or 11 to 15), discharger 3 provides discharger 4 with the possibility of changing his treatment level from scenario 9 to 5 (or 11 to 5). Therefore, operational cost decreases from $[1093.1, 1198.3] \times 10^2$ to $[1030.7, 1096.3] \times 10^2$ Dollars.

Table 10 The lower bounds of treatment scenarios and treatment costs based on the results of the cooperative model

Discharger	Non-cooperative treatment scenario	Treatment scenarios in different trading rounds			Best trading state (buyer, seller)			Cost reduction (US \$ 100)		
		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
1	7	7	9	10	(4, 3)	(3, 1)	(2, 1)	62.3	45.4	1.5
2	9	9	9	8						
3	10	13	9	9						
4	9	5	5	5						
Cost (US \$ 100)	1093.1	1030.7	985.4	983.8						

Table 11 The upper bounds of treatment scenarios and treatment costs based on the results of the cooperative model

Discharger	Non-cooperative treatment scenario	Treatment scenarios in different trading rounds			Best trading state (buyer, seller)			Cost reduction (US \$ 100)		
		1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
1	10	10	12	-	(4, 3)	(3, 1)	-	102	62.6	-
2	10	10	10	-						
3	11	15	10	-						
4	11	5	5	-						
Cost (US \$ 100)	1198.3	1096.3	1033.6	-						

Finally, FIVLC game model is used for reallocating the fuzzy benefit (reduced cost) of discharge permit trading among waste load dischargers.

Also, as an example, the lower and upper bounds of initial and reallocated fuzzy costs in two different trading rounds are presented in Table 12. The results show that FIVLC game is able to incorporate the fuzzy boundary interval costs in determining fuzzy benefit (reduced cost) shares of dischargers participating in a trading process.

5 Summary and Conclusion

In this paper, a new methodology based on the interval optimization and game theory was developed for optimal multiple-pollutant waste load allocation (MPWLA) in rivers considering the existing uncertainties (i.e., headwater quality, pollution loads, transfer coefficients and trading ratio). A QUAL2Kw-based water quality simulation model was developed and calibrated to estimate the concentration of the three water quality variables along the river regarding their interactions. Also, two multiple-pollutant non-cooperative and cooperative models were developed for determining waste load allocation policies in rivers. In addition, a new fuzzy interval solution concept for cooperative games was proposed for reallocating the total fuzzy benefit of discharge permit trading among waste load dischargers. The results of applying the proposed methodology to the Zarjub River illustrate its cost effectiveness and fairness in reallocating treatment costs to dischargers. As shown in the section 4, some

Table 12 Lower and upper bounds of non-cooperative and cooperative (reallocated) fuzzy costs in two different trading rounds

Trading rounds	Discharger	Initial cost (US \$ 100)		Reallocated cost (100 \$)	
		Lower	Upper	Lower	Upper
1st (trading between 4 and 3)	3	(234.2, 46.8)	(245.7, 49.1)	(184.7, 36.9)	(194.8, 39)
	4	(517.1, 103.4)	(574.0, 114.8)	(467.6, 93.5)	(523.1, 104.7)
2nd (trading between 3 and 1)	3	(234.2, 46.8)	(245.7, 49.1)	(163.4, 32.7)	(180.4, 36.1)
	1	(142.4, 28.5)	(168.8, 33.8)	(86.5, 17.4)	(88.6, 17.8)

dischargers have economic motivations to participate in coalitions and trade discharge permits because it can significantly reduce their treatment costs.

In future works, the interval MPWLA methodology can be extended for considering non-point source pollution sources. In addition, real-time waste load allocation rules can be developed using soft computing techniques based on the results of MPWLA model.

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