Water Resour Manage (2016) 30:3535–3552
DOI 10.1007/s11269-016-1369-8

Comparison of Surrogate Measures for the Reliability and Redundancy of Water Distribution Systems

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Published online: 31 May 2016

Abstract An investigation into the effectiveness of surrogate measures for the hydraulic reliability and/or redundancy of water distribution systems is presented. The measures considered are statistical flow entropy, resilience index, network resilience and surplus power factor. Looped network designs that are maximally noncommittal to the surrogate reliability measures were considered. In other words, the networks were designed by multi-objective evolutionary optimization free of any influence from the surrogate measures. The designs were then assessed using each surrogate measure and two accurate but computationally intensive measures namely hydraulic reliability and pipe-failure tolerance. The results indicate that by utilising statistical flow entropy, the reliability of the network can be reasonably approximated, with substantial savings in computational effort. The results for the other surrogate measures were often inconsistent. Two networks in the literature were considered. One example involved a range of alternative network topologies. In the other example, based on whole-life cost accounting, alternative design and upgrading schemes for a 20-year design horizon were considered. Pressure-dependent hydraulic modelling was used to simulate pipe failures for the reliability calculations.

Keywords Pressure-dependent analysis · Hydraulic reliability and redundancy · Resilience index · Surplus power factor · Statistical flow entropy · Water distribution system

1 Introduction

The construction and subsequent rehabilitation and upgrading of a water distribution system represents a major economic infrastructure investment that necessitates an optimal long-term

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strategy. In the design of water distribution systems hydraulic reliability and failure tolerance are considered measures of robustness that determine the ability of the network to satisfy demands under both normal and abnormal operating conditions. However, reliability measures are particularly difficult to evaluate (Wagner et al. 1988). The traditional approach to achieving low-cost designs involves minimising the construction costs while satisfying the pressure requirements throughout the network. These designs aim to utilise the smallest pipe sizes with little or no spare capacity built-in to mitigate the effects of component failures or any increase in the demands. To improve the long-term performance of water distribution systems a more balanced approach that includes some measures of hydraulic reliability in the design process has been advocated (Templeman 1982). Various reliability measures for water distribution systems have been proposed previously. Measures such as hydraulic reliability and failure tolerance are so computationally demanding that their direct incorporation in optimization algorithms is often impractical. Several surrogate measures which sacrifice accuracy but offer a substantial reduction in the computational effort have therefore been suggested e.g. flow entropy, resilience index and surplus power factor.

For over two decades, flow entropy has been considered in the design of water distribution networks as it seems that an increase in the entropy corresponds generally to improvements in the hydraulic reliability (Awumah et al. 1991). It has been used also for layout optimization as it has the advantage that it reflects the arrangement of the paths in a distribution network (Tanyimboh and Sheahan 2002; Saleh and Tanyimboh 2014). The available evidence suggests that higher entropy values increase the uniformity of the pipe diameters along with the reliability (Tanyimboh and Templeman 1993b; Tanyimboh and Setiadi 2008). Strong positive correlation between flow entropy and both hydraulic reliability and failure tolerance has been reported (Tanyimboh and Templeman 2000; Tanyimboh et al. 2011; Gheisi and Naser 2015). Atkinson et al. (2014) observed that an increase in entropy promotes an increase in capacity that is more globally distributed throughout the distribution network. Recently, Liu et al. (2014) proposed an extension to the flow entropy function called diameter-sensitive flow entropy. Singh and Oh (2015) incorporated the Awumah et al. (1991) entropy formulation in a variant known as Tsallis entropy (Tsallis 1988). However Tanyimboh and Templeman (1993a, b, c, d) showed that the underlying entropy models by Awumah et al. (1991) do not satisfy certain axiomatic properties of probability schemes. Raad et al. (2010) also suggested an extension to the flow entropy function that includes the resilience index (Todini 2000).

One flow entropy value does not correspond to a unique design of a water distribution network due to the invariance of the entropy function; i.e. re-ordering the probabilities in a finite probability scheme leaves the statistical entropy value unchanged (Shannon 1948). Thus statistical entropy may be considered a dependable surrogate reliability measure only if different designs with similar entropy values possess comparable characteristics. It was discovered previously that in general designs with the same maximum entropy value are very similar in terms of hydraulic reliability and construction cost (Tanyimboh and Sheahan 2002; Tanyimboh and Setiadi 2008; Tanyimboh et al. 2011).

Comparisons of surrogate reliability measures include Prasad and Park (2004), Raad et al. (2010), Baños et al. (2011)), Tanyimboh et al. (2011), Wu et al. (2011), Greco et al. (2012), Atkinson et al. (2014), Liu et al. (2014) and Gheisi and Naser (2015). While some of the studies simulated operating conditions with insufficient pressure realistically with pressure-dependent modelling (Liu et al. 2014; Gheisi and Naser 2015), others did not (e.g. Greco et al. 2012; Atkinson et al. 2014). It is well known that the demand-driven analysis approach often yields misleading results when applied to operating conditions with insufficient pressure (Tanyimboh et al. 1999, 2003). Tanyimboh et al. (1999) stated: "*Compared to [demand-driven analysis] the results obtained by [head-driven analysis] are superior in the sense that,* *in addition to identifying precisely the nodes with inadequate flow/pressure the actual outflows at the nodes in question are determined. By contrast, the above does not hold true for [demand-driven analysis] (Tanyimboh and Tabesh 1997). In fact [demand-driven analysis] results can sometimes be both infeasible and very misleading.*"

Some of the studies (e.g. Prasad and Park 2004; Wu et al. 2011; Raad et al. 2010) did not include accurate measures of hydraulic reliability/redundancy. For example the pipe failure analysis in Raad et al. (2010) did not consider a probabilistic pipe failure model. Instead, it seems that pipe failures were taken as equally likely. However, it is generally accepted that large pipes fail less frequently than small pipes (Su et al. 1987; Cullinane et al. 1992; Khomsi et al. 1996). Also some studies (e.g. Raad et al. 2010) carried out comparisons on a consistent basis by optimizing each surrogate measure whereas other studies (e.g. Liu et al. 2014) optimized some but not all.

Water undertakings in the UK have statutory duties such as maintaining a satisfactory and continuous supply, and planned and unplanned interruptions are considered key performance indicators (see e.g. Twort et al. 2000, pp. 88 and 615–6). Consequently the performance of a water distribution network when one or more pipes are unavailable is an important design consideration. It has been suggested (see e.g. Tanyimboh and Templeman 1998; Kalungi and Tanyimboh 2003; Tanyimboh and Kalungi 2009) that the *failure tolerance* be considered in addition to the hydraulic reliability, as their mutual relationship is not monotonic. The reason is that water distribution networks are exceedingly complex systems. Investigations into surrogate reliability measures (e.g. Liu et al. 2014, Raad et al.) and the reliability of water distribution systems in general often do not address failure tolerance explicitly. It is worth clarifying that tolerance to component unavailability relates, in the first instance, primarily to the prescribed statutory demands rather than demand uncertainty.

The aim of this paper is to investigate the similarity of designs of water distribution networks whose maximum entropy values are equal, based on energy dissipation. If flow entropy is consistent, then such designs should have similar underlying characteristics. The paper also investigates the effectiveness of the resilience index, network resilience, surplus power factor and flow entropy, as surrogate measures of the reliability/redundancy of water distribution systems. Surrogate reliability measures are often maximised when used to design water distribution systems (Prasad and Park 2004; Baños et al. 2011; Wu et al. 2011; Raad et al. 2010; Farmani and Butler 2014; Liu et al. 2014; Saleh and Tanyimboh 2014). However, any surrogate measure that is optimized in the design process can be expected to have an unfair advantage. This is addressed by comparing the surrogate measures on an equal basis in this article. The influence of the network topology is considered also, together with the effects of pipe failures. Two networks in the literature are analysed and discussed.

2 Hydraulic Reliability Measures

Hydraulic reliability represents the probabilistic expectation of the ratio of the flow delivered to the flow required (Tanyimboh and Templeman 1995, 1998; Gargano and Pianese 2000; Tanyimboh et al. 2001). The failure tolerance, similarly, represents the probabilistic expectation of the ratio of the flow delivered to the flow required when one or more components of the distribution system are unavailable (Tanyimboh and Templeman 1995, 1998; Tanyimboh et al. 2001). The evaluation of the hydraulic reliability requires multiple analyses of the distribution system under both normal and subnormal operating conditions. This requires pressure-dependent modelling (Bhave 1981;

Germanopoulos 1985; Gupta and Bhave 1996; Ackley et al. 2001). Pressure-dependent modelling differs from demand-driven modelling in that pressure-dependent modelling treats the available flow at a demand node as a function of the nodal pressure (e.g. Shirzad et al. 2013; Kovalenko et al.et al. 2014; Ciaponi et al. 2015; Abdy Sayyed et al. 2015) whereas demand-driven modelling equates the available nodal flow to the demand.

Pressure-dependent modelling was used here to simulate the post-failure pipe closures. Pipe closures reduce the flow carrying capacity of a water distribution system. Pressure-dependent modelling is therefore essential to ensure the simulations are realistic. Pressure- dependent modelling was carried out with the hydraulic models PRAAWDS (Tanyimboh et al. 2003; Tanyimboh and Templeman 2010) and EPANET-PDX (Siew and Tanyimboh 2012a; Seyoum et al. 2013; Seyoum and Tanyimboh 2014). It was assumed the pipes can be closed individually; in practice the actual locations of isolation valves would be considered. Only failures involving one pipe at a time were considered due to the low probability of multiple-pipe failures (Su et al. 1987; Tanyimboh and Templeman 1998; Gargano and Pianese 2000). We investigated several pipe failure models including: (a) Su et al. 1987; (b) Fujiwara and Tung 1991; (c) Cullinane et al. 1992; (d) Khomsi et al. 1996; and (e) Tabesh et al. 2009. Cullinane et al. (1992) and Khomsi et al. (1996) proved the most consistent. Both models gave very similar results and so only the results based on Cullinane et al. (1992) are included here.

2.1 Statistical Entropy

Informational entropy is a measure of the amount of uncertainty that a probability distribution represents (Shannon 1948). The flow entropy of a water distribution system is a measure of the relative uniformity of the pipe flow rates (Tanyimboh and Templeman 1993a, b, c, d).

$$\frac{S}{K} = -\sum_{1=1}^{I} \frac{Q_i}{T}\ln\left(\frac{Q_i}{T}\right) - \frac{1}{T}\sum_{j=1}^{J} T_j\left[\frac{Q_j}{T_j}\ln\left(\frac{Q_j}{T_j}\right) + \sum_{ij\in N_j}\frac{q_{ij}}{T_j}\ln\left(\frac{q_{ij}}{T_j}\right)\right] \tag{1}$$

where S/K is the entropy with K an arbitrary positive constant; T is the total supply; T_j is the total flow reaching node j; Q_i represents the inflow at a supply node; Q_j represents the demand at a demand node; q_{ij} is the flow rate in pipe ij; I represents the number of supply nodes; J represents the number of demand nodes; and N_j represents all the pipe flows from node j.

2.2 Hydraulic Reliability

Hydraulic reliability may be characterised as the extent to which the distribution system fulfills the demands at adequate pressure, considering both normal and abnormal operating conditions (Tanyimboh and Templeman 1995, 1998, 2000).

$$\begin{aligned} R = &\frac{1}{T}\left(p(0)T(0) + \sum_{m=1}^{M} p(m)T(m) + \sum_{m=1}^{M-1}\sum_{n=m+1}^{M} p(m,n)T(m,n) + \ldots\right) \\ &+ \frac{1}{2}\left(1 - p(0) - \sum_{m=1}^{M} p(m) - \sum_{m=1}^{M-1}\sum_{n=m+1}^{M} p(m,n) - \ldots\right) \end{aligned} \tag{2}$$

R represents the hydraulic reliability that takes values from 0 to 1; M is the number of links (e.g. pipes, valves and pumps) in the network; *p(0)* is the probability that *no link* is out of service; *p(m)* is the probability that *only link m* is not in service; *p(m,n)* is the probability that *only links m and n* are not in service; *T(0), T(m)* and *T(m,n)* are, respectively, the total flow supplied with all links in service, *only link m* unavailable and *only links m and n* unavailable; *T* is the sum of the nodal demands. Equation 2 is clearly truncated; it includes additional terms involving three or more simultaneous link failures i.e. *p(l,m,n), p(k,l,m,n)* and *T(l,m,n), T(k,l,m,n), etc.* with analogous definitions.

Equation 2 has two major components. The first component (that starts with $^1/_T$) corresponds to the reliability defined as the probabilistic expectation of the fraction of the demand satisfied. The second component (that starts with ½) is a correction factor that compensates for the omission of some operating conditions that involve the failure of multiple pipes simultaneously (Tanyimboh and Sheahan 2002). The correction factor is a function of the pipe failure probabilities only – its evaluation does not require a hydraulic simulation model. Therefore, the correction factor can provide guidance on the number of simultaneous failures worth considering.

2.3 Failure Tolerance

The failure tolerance represents the probabilistic expectation of the fraction of the required flow that the distribution system satisfies at adequate pressure when one or more components are not in service (Tanyimboh and Templeman 1995, 1998; Tanyimboh et al. 2001).

$$FT = \frac{R - p(0)T(0)/T}{1 - p(0)} \tag{3}$$

FT is the failure tolerance that takes values from 0 to 1. The effect of the *FT* function in Eq. 3 is to modify the reliability by removing the contribution of the fully connected network condition, during which all pipes are in service. The contributions of all the states or conditions that correspond to single-link failures could be removed also, to obtain the failure tolerance when two links or more are out of service. The numerator and denominator in Eq. 3 would then have an additional term each, i.e. $-\sum_m p(m)T(m)/\mathrm{T}$ and $-\sum_m p(m)$, respectively. Clearly, the values of all the terms used in the *FT* function are available in Eq. 2; thus more terms of higher order could be included in theory. In practice the computational demands often outweigh any benefits (Su et al. 1987; Tanyimboh 1993, page 221). An extended discussion with additional examples and illustrations is available in Gheisi and Naser (2013, 2015). For the individual demand nodes Eqs. 2 and 3 would remain essentially the same and the respective nodal demands and available flows simply replace the total network demand and available flow. The formal derivation of Eq. 3 is available in Tanyimboh and Templeman (1998).

2.4 Resilience Index

Todini (2000) proposed the resilience index as a possible measure of redundancy defined as

$$RI = \frac{\sum_{j=1}^{n_d} Q_j \left(H_j - H_j^{req} \right)}{\sum_{r=1}^{n_r} Q_r H_r + \sum_{i=1}^{n_p} \frac{P_i}{\gamma} - \sum_{j=1}^{n_d} Q_j H_j^{req}} \tag{4}$$

RI is the resilience index; for demand node j, Q_j is the demand, H_j is the head and H_j^{req} is the head above which the demand is satisfied in full; Q_r and H_r are the water supply flow rate and head at reservoir r, respectively; P_i is the power input at pump i; γ is the specific weight of water; n_d, n_r and n_p are the number of demand nodes, reservoirs and pumps, respectively. The total power input to the network may be accounted for by the energy losses in the flow through the network and the power that is available at the demand nodes. Equation 4 shows the resilience index is a measure of the available surplus power that, potentially, could be further dissipated by the flow through network in the event that extra stresses arise (e.g. due to an increase in demand).

2.5 Network Resilience

Prasad and Park (2004) modified the resilience index to reflect the relative sizes of the pipes meeting at a demand node. For demand node j, the uniformity coefficient u_j was defined as the ratio of the mean diameter to the largest diameter. The network resilience NR is thus

$$NR = \frac{\sum_{j=1}^{n_d} u_j Q_j \left(H_j - H_j^{req}\right)}{\sum_{r=1}^{n_r} Q_r H_r + \sum_{i=1}^{n_p} \frac{P_i}{\gamma} - \sum_{j=1}^{n_d} Q_j H_j^{req}} \tag{5}$$

2.6 Surplus Power Factor

Vaabel et al. (2006) introduced the surplus power factor as a measure of the spare hydraulic capacity in a pipe. It is based on the pipe flow rate and takes values between 0 and 1 (Eq. 6).

$$s_{ij} = \frac{P_{ij}^{\max} - P_{ij}}{P_{ij}^{\max}}; \quad \forall ij \tag{6}$$

For pipe ij, s_{ij} is the surplus power factor; $P_{ij}^{\max}$ and P_{ij} are the maximum and available hydraulic power at the downstream end of the pipe, respectively. Both the minimum s_{min} and mean $E(s_{ij})$ of the surplus power have been reported previously (e.g. Wu et al. 2011).

$$s_{\min} = \min\{s_{ij}, \ \forall ij\}; \quad E\left(s_{ij}\right) = \frac{1}{N_{IJ}} \sum\nolimits_{ij} s_{ij} \tag{7}$$

where N_{IJ} is the number of pipes.

2.7 Energy Dissipation

Rowell and Barnes (1982) suggested that flow in a pipe could be characterized with reference to the energy dissipation rate. For a water distribution network the energy dissipation rate is $E = \rho g \sum_{ij \in IJ} q_{ij} h_{ij}$ (9)

E is the energy dissipation rate; ρ is the density of water; g is the acceleration due to gravity; for pipe ij, q_{ij} is the flow rate and h_{ij} is the head loss; and IJ includes all pipes in the network.

3 Results and Discussion

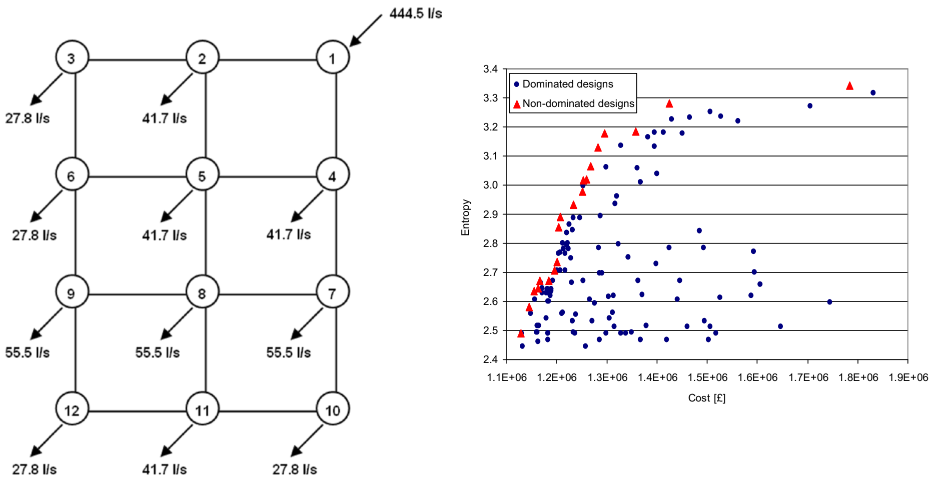
3.1 Example 1

This example involves 137 minimum-cost maximum-entropy designs based on 65 layouts for the network in Fig. 1 introduced by Awumah et al. (1991). Initially Tanyimboh and Sheahan (2002) and Tanyimboh and Setiadi (2008) generated the designs to investigate flow entropy. Subsequently Tanyimboh et al. (2011) used the designs to investigate resilience index. The head at the supply node is 100 m; demand node elevations are 0 m; required residual head at demand nodes is 30 m; pipes are 1000 m long; Hazen-Williams roughness coefficient is 130; pipe diameters are continuous in the range 100 to 600 mm.

There are 60 designs with a unique maximum entropy value and 77 designs that have the same maximum entropy value with one or more designs. The 77 designs with the same maximum entropy value as one or more designs belong to 29 maximum-entropy groups (or sets) with two to six designs. The coefficient of variation of the energy dissipation rate was used to assess the similarity of the designs *within* each maximum-entropy set. The results were compared as follows.

- Group A: The set comprising the 77 designs with non-unique maximum entropy values.
- Group B: The full set of 137 designs.
- Group C: The set comprising the 60 designs with unique maximum entropy values.

Based on the number of members in the different maximum entropy sets, the weighted mean of the coefficient of variation was 0.0092. The weighted mean represents the average similarity within individual maximum entropy sets. The arithmetic mean was 0.0090; while the weighted mean considers the number of members in each set, the arithmetic mean does not. The smallest coefficient of variation was 0.0002 and the largest was 0.0326. The coefficient of variation for Group A, B and C were: 0.0282, 0.1496 and 0.2022, respectively. As percentages,



(a) Parent network (b) Maximum entropy vs. cost

Fig. 1 Maximum entropy designs investigated in Example 1. **a** Parent network. **b** Maximum entropy vs. cost the ratio of the coefficient of variation for Group A, B and C to the *weighted mean* of the maximum entropy sets are 307%, 1,624%, and 2,196%, respectively. These results are consistent with the make-up of Group A, B and C. Group A consists of 29 clusters. Group C has no such clusters. Group B is a simple combination of Group A and C and is thus intermediate. The logical inference is that the designs *within* the maximum entropy sets have similar physical characteristics. This is also consistent with previous results (Tanyimboh and Sheahan 2002; Tanyimboh and Setiadi 2008; Tanyimboh et al. 2011) and supports the hypothesis that generally flow entropy is reasonably consistent.

Figure 1b shows that 20 designs are non-dominated based on cost minimization and entropy maximization. The investigation of the surrogate reliability measures that follows is based on the 20 non-dominated designs. The hydraulic simulations were carried out with PRAAWDS (Program for Realistic Analysis of Availability of Water in Distribution Systems) (Tanyimboh et al. 2003; Tanyimboh and Templeman 2010). The pipe failure rates were estimated as in Cullinane et al. (1992). Table 1 shows positive correlation between flow entropy and both hydraulic reliability and failure tolerance. The correlation here based on the non-dominated solutions is greater than reported previously in Tanyimboh et al. (2011) for the full set of 137 solutions.

Table 1 also suggests there is no reasonable correlation between resilience index and both hydraulic reliability and failure tolerance. Queries about the resilience index have been discussed previously (e.g. Reca et al. 2008, Tanyimboh et al. 2008, 2011 and Farmani and Butler 2014). The network resilience, similarly, shows no clear trend. The modified resilience index by Jayaram and Srinivasan (2008) was not investigated as it is essentially the same as the resilience index for single-source networks (Tanyimboh et al. 2011). There is positive correlation for the average surplus power factor. As with flow entropy, the correlation is stronger with failure tolerance than reliability.

Table 1 Relationship between reliability and surrogate reliability measures

Surrogate reliability measures	Coefficient of determination R^2					
	Example 1[a]		Example 2		Example 3	
	Reliability	Failure tolerance	Reliability	Failure tolerance	Reliability	Failure tolerance
Flow entropy	0.64 *(0.522)*	0.83 *(0.803)*	0.358	0.548	0.549	0.352
Resilience index	0.000 *(0.077)*	0.000 *(−0.002)*	−0.019	−0.290	0.008	0.017
Network resilience	−0.003 *(−0.002)*	−0.017 *(−0.172)*	−0.130	−0.501	U[b]	U
Mean surplus power factor	0.388[c] *(U[b])*	0.578[c] *(U)*	−0.125	−0.038	U	U
Failure tolerance	0.73 *(0.707)*	–	0.559	–	0.866	–

[a] Corresponding values in Tanyimboh et al. (2011) are shown italicised in parentheses

[b] Unavailable

[c] For the minimum surplus power factor, R^2 values were −0.0158 and −0.0596 for reliability and failure tolerance, respectively

3.2 Example 2

Surrogate reliability measures are often employed as objectives in the design optimization of water distribution systems. A query that arises is that, with a particular surrogate measure involved in the design process, the designs achieved may favour that particular measure. Therefore, the designs investigated next were optimized without reference to any surrogate measure. The assessment involves 32 minimum-cost designs for *32 new layouts* for the network in Fig. 1. Continuous pipe diameters were adopted to avoid any surplus flow capacity in the network that may be unavoidable with discrete pipe sizes. This further ensures that subsequent comparisons of the designs and surrogate measures are robust and fair. The two parent layouts shown in Fig. 2 have a combined total of 32 fully-looped layouts that have not been investigated previously. Both Layout A and B will remain fully-looped if pipe 2–5 is removed or closed. Therefore, with four such "non-essential" pipes the total number of fully-looped layouts is 32 or $2 \times \sum_{i=0}^{i=4} C_i$.

The *32 new minimum-cost designs* were generated with a multiobjective genetic algorithm NSGA II (Deb et al. 2002). Two objectives that were both minimized were used namely the construction cost and sum of the demand node pressure deficits. The minimum node pressure constraints were thus satisfied. EPANET 2 (Rossman 2002) was used for the hydraulic simulations that ensured the conservation of mass and energy constraints were satisfied. The decision variables were the pipe diameters. The cheapest feasible designs achieved are shown in Table 2. The costs vary from £1,062,034 to £1,373,739 and the maximum surplus head is 10 mm. Any small surplus in the head at the critical node resulted mainly from rounding off the diameters. The critical node is the node that has the smallest residual head. The optimization was executed 10 times for each layout. Table 1 shows that only flow entropy has a reasonable correlation with hydraulic reliability and failure tolerance. These results may be indicative of a fundamental relationship between flow entropy and hydraulic reliability/redundancy as the surrogate measures had no role in the pipe sizing. Pressure-dependent modelling in PRAAWDS was used for the reliability calculations with pipe failure rates estimated as in Cullinane et al. (1992).

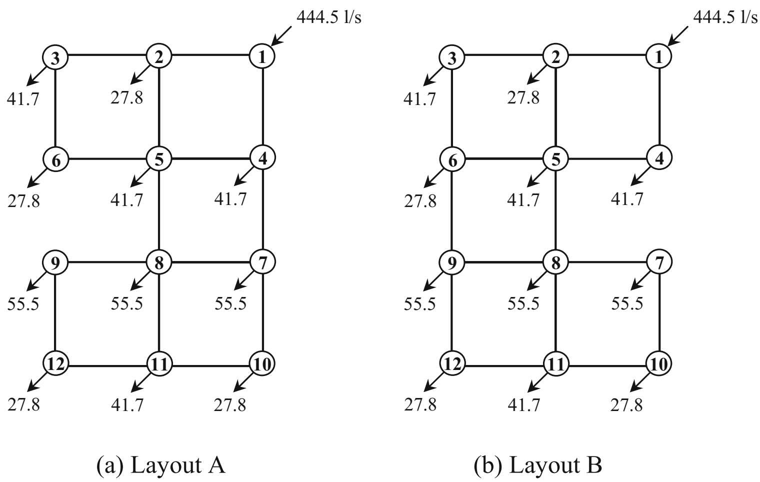


Fig. 2 Parent layouts for Example 2. **a** Layout A. **b** Layout B

Table 2 Pipe diameters for the least-cost designs achieved

Design	Pipe diameters for the pipes indicated (mm)																
	1–2	2–3	1–4	2–5	3–6	4–5	5–6	4–7	6–9	5–8	7–8	8–9	7–10	8–11	9–12	10–11	11–12
1	210	172	410	100	105	172	119	362	–	100	304	236	143	159	167	100	100
2	196	162	411	0	101	189	127	358	–	100	313	231	135	170	165	100	102
3	255	180	381	164	128	–	104	363	–	100	319	235	144	165	164	100	100
4	192	151	415	100	100	351	125	191	–	328	–	230	133	197	149	100	102
5	199	169	410	100	103	184	125	360	–	100	233	208	245	–	100	219	165
6	220	192	410	–	147	–	100	389	–	165	345	187	139	203	100	100	166
7	182	159	410	–	100	358	125	191	–	317	–	193	135	225	100	100	166
8	186	169	408	–	100	189	129	367	–	100	273	237	208	–	167	180	100
9	398	147	235	358	100	–	117	205	–	320	–	221	142	192	150	100	115
10	259	160	376	190	100	–	135	361	–	100	240	208	245	–	100	221	167
11	205	154	406	100	100	295	123	292	–	247	–	206	249	–	100	224	166
12	350	156	309	299	101	–	116	285	–	252	–	213	251	–	101	223	169
13	179	163	412	–	100	295	128	283	–	252	–	207	247	–	100	224	172
14	216	201	403	–	149	–	100	384	–	168	290	198	247	–	103	213	162
15	263	241	392	–	208	–	176	374	–	100	–	226	349	285	100	341	157
16	359	350	307	–	326	–	313	289	–	285	–	230	254	–	100	232	177
17	428	284	329	284	223	301	296	–	369	175	135	283	204	231	228	307	219
18	421	361	305		360	263	373	–	198	395	110	349	203	184	302	274	254
19	276	100	304	595	523	319	–	–	510	599	465	228	460	303	500	397	518
20	374	332	350	340	247	294	320	–	392	162	296	–	144	308	323	105	340
21	390	305	315	247	197	253	340	–	374	224	358	360	292	–	103	288	272
22	345	320	359	–	257	367	–	–	248	331	103	296	203	178	312	249	308
23	387	317	285	–	321	266	376	–	100	423	362	–	350	230	350	340	312
24	433	349	313	–	360	249	436	–	215	372	128	367	215	–	314	266	326

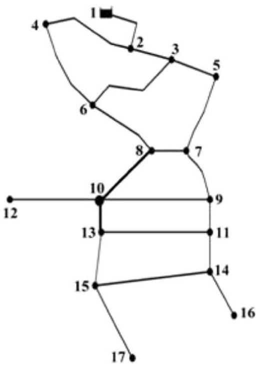
Table 2 (continued)

Design	Pipe diameters for the pipes indicated (mm)																
	1–2	2–3	1–4	2–5	3–6	4–5	5–6	4–7	6–9	5–8	7–8	8–9	7–10	8–11	9–12	10–11	11–12
25	269	100	281	542	309	357	–	–	382	583	430	–	407	289	482	394	480
26	253	449	421	340	373	371	–	–	413	188	328	461	313	–	103	259	199
27	388	270	320	279	300	275	343	–	405	114	233	–	299	–	369	344	353
28	285	100	304	397	245	331	–	–	284	490	474	–	417	–	362	398	385
29	397	455	303	–	304	277	375	–	111	390	437	–	308	–	236	326	339
30	346	325	370	–	288	326	–	–	264	344	114	305	217	–	359	250	284
31	100	285	597	–	409	597	–	–	460	598	503	–	462	325	526	397	561
32	249	250	432	–	179	399	–	–	198	381	350	–	314	–	198	265	231

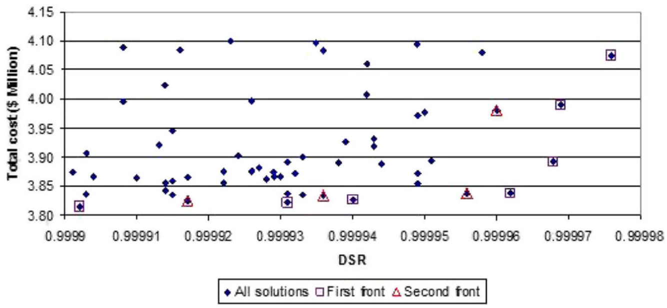
3.3 Example 3

The skeletonised network consists of a reservoir, 17 nodes and 21 pipes serving a population of 10,640 (Fig. 3a) (Tanyimboh and Kalungi 2008, 2009). The design optimization problem considers the deterioration over time of the structural integrity and hydraulic capacity of the pipes. The whole-life cost accounting method considers the discount and inflation rates. The costs include construction; repairs; and pipe failures and associated third party costs. The aim of the design is to minimize the total cost while satisfying the demands for a design horizon of 20 years. The decision variables are the pipe diameters, the rehabilitation and upgrading options i.e. paralleling and/or replacement, and the time of upgrading. A two-phase design with eight alternative Phase I durations of 7 to 14 years was considered. A complete description of the optimization problem is available in Tanyimboh and Kalungi (2008, 2009).

A penalty-free multi-objective evolutionary algorithm (PF-MOEA) was used to solve the optimization problem (Siew et al. 2014). The algorithm uses binary coding, single-bit mutation, single-point crossover and binary tournament selection for crossover. Ten optimization runs with random initial populations were carried out with 160,000 hydraulic simulations per optimization run. The population size was 100. The probability of crossover and mutation were 1.0 and 0.005, respectively. The minimum node pressure requirements were addressed



(a) Network layout



(b) Cost-effectiveness chart. DSR i.e. demand satisfaction ratio is the fraction of the demand that is satisfied

Fig. 3 Minimum-cost designs investigated in Example 3. **a** Network layout. **b** Cost-effectiveness chart. DSR i.e. demand satisfaction ratio is the fraction of the demand that is satisfied with pressure-dependent modelling in EPANET-PDX (pressure-dependent extension) (Siew and Tanyimboh 2012a, 2012b; Seyoum and Tanyimboh 2014). A desktop computer (with CPU of 3.2 Hz and RAM of 2 GB) was used. The average CPU time per optimization run was 2.58 h. Details of the methodology and algorithm are available in Siew et al. (2014) and Siew and Tanyimboh (2012a, b). The optimization algorithm (PF-MOEA) has consistently achieved good results previously. EPANET-PDX was used for the pipe failure simulations in the reliability calculations and pipe failure rates were estimated as in Cullinane et al. (1992).

Table 3 presents the total costs for the solutions achieved by the genetic algorithm. The linear programming (LP) solutions by Tanyimboh and Kalungi (2008, 2009) are included for comparison purposes. All the solutions shown satisfy the demands in full. The reliability values considered here relate to the network and operating conditions at the end of the design horizon i.e. Year 20. The cheapest solution achieved by the evolutionary algorithm (EA) costs $3,814,298, for a Phase I duration of 9 years. The cheapest linear programming (LP) solution costs $3,953,663, for a Phase I duration of 11 years. It can be observed that apart from the solution with a Phase I duration of 14 years, the EA solutions are cheaper than the corresponding LP solutions. The main reason is that the LP solutions are maximum-entropy designs that are inherently more expensive (Tanyimboh and Templeman 2000).

Each execution of the evolutionary algorithm generates designs for eight alternative Phase I duration of 7 to 14 years. Selecting the cheapest feasible solution from each Phase I duration provides in total 80 solutions from 10 optimization runs. In practice and as might be expected there are fewer than 80 solutions as some of the solutions feature in multiple runs. The overall best solutions were identified as shown in Fig. 3b based on the cost effectiveness of the solutions. All the solutions are feasible; however the maximum difference in cost between the solutions is approximately $300,000. Solutions in the two leading non-dominated fronts in Fig. 3b were selected for further analyses; two fronts were necessary as the first front has only seven solutions. Then the correlation analysis was carried out for resilience index and flow entropy that are used the most. Table 1 shows that the correlation between flow entropy and both reliability and failure tolerance is reasonable while the relationship between resilience index and both reliability and failure tolerance is much weaker.

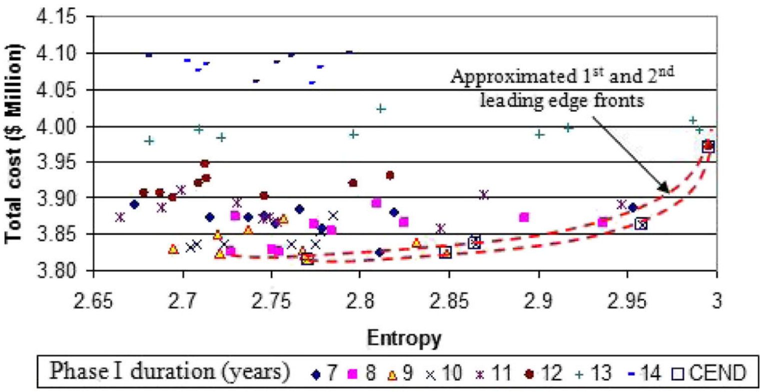
Table 3 Optimal costs for the design and capacity expansion schemes achieved

Phase I Durations	Phase II Durations	PF–MOEA Costs ($ Million)			[a]LP Costs ($ Million)		
(years)	(years)	Phase I	Phase II	Total	Phase I	Phase II	Total
7	13	2.862	0.998	3.860	2.907	1.386	4.293
8	12	2.950	0.877	3.827	3.006	1.071	4.077
9	11	3.047	0.768	**3.814**[b]	3.084	0.966	4.050
10	10	3.148	0.689	3.837	3.200	0.789	3.989
11	9	3.281	0.593	3.873	3.315	0.639	**3.954**[b]
12	8	3.399	0.504	3.902	3.414	0.544	3.958
13	7	3.565	0.415	3.980	3.523	0.461	3.984
14	6	3.725	0.334	4.059	3.631	0.409	4.040

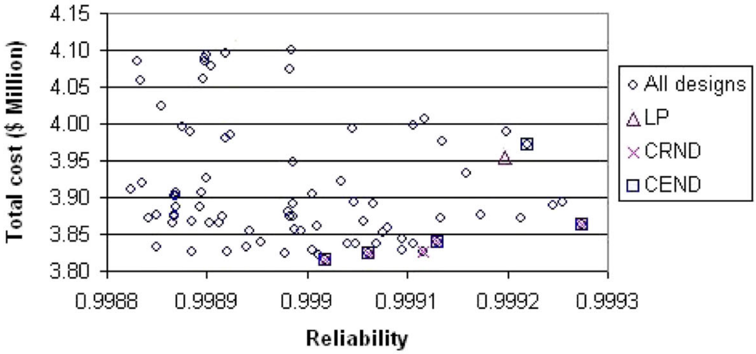
[a] The linear programming (LP) costs are from Tanyimboh and Kalungi (2008)

[b] The cheapest solution for each approach is highlighted in bold

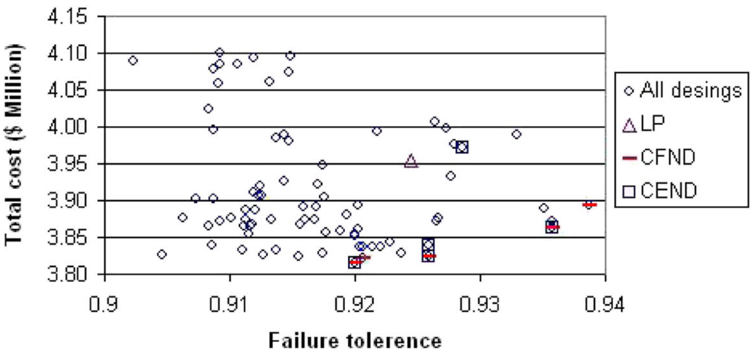
To demonstrate further the advantages of entropy as a surrogate performance measure, Fig. 4a shows a plot of the cost against entropy. The range of entropy values is narrow and the



(a) Flow entropy



(b) Reliability



(c) Failure tolerance

Fig. 4 Reliability and failure tolerance cross-checks for the cost-entropy nondominated solutions. CEND, CFND and CRND are respectively: cost-vs-entropy, cost-vs-failure tolerance and cost-vs-reliability nondominated solutions. **a** Flow entropy. **b** Reliability. **c** Failure tolerance reason is that entropy was not considered in the design optimization. Due to the narrow range of entropy values, there are only five non-dominated solutions in the cost-entropy space. Also, it can be seen that the solutions with Phase I durations of 12 to 14 years tend to be uncompetitive. Figure 4b and c show plots of cost vs. reliability and cost vs. failure tolerance, respectively. The best linear programming solution is shown also. The aim here is to illustrate the efficiency and robustness of screening the solutions to be considered further using entropy. Figure 4b shows that five solutions are non-dominated in the cost-reliability space of which four are also non-dominated in the entropy-cost space. Figure 4c shows that five solutions are non-dominated in the cost-failure tolerance space of which three are also non-dominated in the entropy-cost space. Therefore, it can be inferred that a solution that is Pareto-optimal in the entropy-cost space will likely perform well based on reliable and/or failure tolerance (Figure 4b and c). Also, it can be seen that the Pareto-optimal front in the entropy-cost space also approximates well the Pareto-optimal fronts in the reliability-cost and failure tolerance-cost spaces. It is worth noting that the solutions in the reliability-cost and failure tolerance-cost fronts that are not in the first entropy-cost front are in fact in the second entropy-cost front.

Considering cost, hydraulic reliability and failure tolerance, three solutions are nondominated. These solutions can be identified in Fig. 4c as the three nondominated solutions that are also nondominated in the entropy-cost space. The pipe diameters and other details of the solutions are available in Siew et al. (2014). Multiobjective evolutionary algorithms often yield vast populations of candidate solutions. These results show that it is highly effective to use entropy to select the solutions that merit further consideration.

4 Conclusions

A comparison of surrogate reliability and redundancy measures for water distribution systems has been carried out. The measures considered were flow entropy, resilience index, network resilience and surplus power factor. These measures were assessed considering single-pipe failures for two networks in the literature. Both pre-existing maximum entropy designs in the literature and new least cost designs were investigated. The main conclusion is that only flow entropy seems to have the properties of a reasonable measure of hydraulic reliability and redundancy for water distribution systems. Three cases were considered and only flow entropy had a positive correlation with both hydraulic reliability and redundancy consistently. The other measures often had negative or very little correlation. It was also shown that generally designs with the same maximum entropy value tend to have comparable characteristics. These results seem to reinforce the hypothesis that the relationship between flow entropy and hydraulic reliability/redundancy is fundamental.

Some limitations of the study are that the flow entropy model investigated was developed for a single operating condition. It would be desirable to investigate possible extensions to multiple operating conditions (Czajkowska and Tanyimboh 2013). For example, if only one operating condition based on the peak flow is used to optimize the design then in general the system may be infeasible for other critical operating conditions (Alperovits and Shamir 1977). More research is thus indicated including comparisons based on large networks.

Acknowledgments This research was funded in part by the UK Engineering and Physical Sciences Research Council (EPSRC Grant Reference EP/G055564/1), the Government of Libya, the UK Government (Universities

UK Overseas Research Students' Award Scheme) and the University of Strathclyde. We acknowledge with thanks the contribution of Douglas Bale (EPSRC Vacation Bursaries, 2010; EPSRC Reference EP/P502764/1).
All supporting data are included in the manuscript.

Compliance with Ethical Standards

Conflict of Interest There is no conflict of interest.

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