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Particle Filters to Estimate Properties of Confined Aquifers

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Abstract Involving a limited resource, the assessment of groundwater aquifers is of utmost importance. A key component of any such assessment is the determination of key properties that permit water resource managers to estimate aquifer drawdown and safe yield. This paper presents a particle filtering approach to estimate aquifer properties from transient data sets, leveraging recently published analytically-derived models for confined aquifers and using sample-based approximations of underlying probability distributions. The approach is examined experimentally through validation against three common aquifer testing problems: determination of (i) transmissivity and storage coefficient from non-leaky confined aquifer performance tests, (ii) transmissivity, storage coefficient, and vertical hydraulic conductivity from leaky confined aquifer performance tests, and (iii) transmissivity and storage coefficient from non-leaky confined aquifer performance tests with noisy data and boundary effects. On the first two well-addressed problems, the results using the particle filter approach compare favorably to those obtained by other published methods. The results to the third problem, which the particle filter approach can tackle more naturally than the previously-published methods, underscore the flexibility of particle filtering and, in turn, the promise such methods offer for a myriad of other geoscience problems.

Keywords Water resources · Assessment · Groundwater hydrology · Particle filter · Aquifer testing

1 Introduction

Groundwater is a key component of the worldwide water supply. In the USA, the National Groundwater Association estimates that up to 33 % of all water used is from a groundwater

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source (www.ngwa.org). Similar uses occur in many countries around the globe. Unfortunately, groundwater aquifers are being depleted worldwide at an alarming rate (Qiu 2010; Konikow 2013). Therefore, the assessment of remaining groundwater resources is of critical importance. Groundwater aquifers used for water supply or irrigation purposes are primarily either unconfined, water-table aquifers or deeper confined aquifers. Confined aquifers are typically preferred by water resource managers, owing to their isolation from possible pollution sources due their protective confining layers. Ongoing assessment of shrinking groundwater resources usually includes the determination of aquifer properties and the development of yield estimates from the studied aquifer based upon acquired field data.

Using transient monitoring data for the purposes of determining aquifer properties is a common technique in the groundwater industry. The most common method is the aquifer performance test, where a pumping well is used to stress the aquifer by removing water at a high rate and causing drawdown of pressure levels in the aquifer. The drawdown is measured at one or more observation wells placed at different radii from the pumping well. The aquifer test itself results in a transient condition within the aquifer, where drawdown is a function of time and space as well as various boundary conditions. In the most basic model for this process, water is pumped from a homogeneous and isotropic confined aquifer of infinite extent with no effect from boundary conditions. This so-called unsteady non-leaky confined aquifer test, Theis (1935), has been studied extensively by a host of researchers with the intent being to estimate the two primary aquifer parameters, the transmissivity (T) and the storage coefficient (S). Another well-studied model, the so-called unsteady leaky confined aquifer test, assumes that the well derives its pumped groundwater laterally from within the primary confined aquifer and from leakage either above or below the primary aquifer through a semi-pervious confining unit Hantush and Jacob (1955). In addition to parameters T and S, estimating the vertical hydraulic conductivity of the confining unit (K_v) is also of concern.

Numerous extensions to the two confined aquifer tests have been proposed and studied. Walton (1962) generalized the Theis solution and found graphical methods to estimate transmissivity and storativity using a "type curve" approach. Dagan (1985) and Dagan and Rubin (1988) have looked at flow in confined aquifers using a stochastic approach. Lebbe and Breuck (1995) used inverse numerical modeling to estimate aquifer parameters along with factors that materially affect the accuracy of the estimates themselves. Tumlinson et al. (2006) used numerical evaluations to develop estimates of aquifer parameters in laterally heterogeneous confined aquifers. Trinchero et al. (2008) have studied pumping tests in leaky-confined aquifers, where the solution reverts to a confined aquifer curve if leakance of the confining unit is very small. Veling and Maas (2010) re-evaluated the Theis and Hantush well functions used in type curve matching mentioned earlier. Singh (2010b) proposed an alternate approximate analytical solution to the unsteady leaky confined aquifer case. Yeh and Chang (2013) recently examined research advances regarding the modeling of well hydraulics including those for confined aquifers. Yang and Yeh (2012) developed a general semi-analytical solution for partially penetrating aquifer test wells in a confined aquifer. Brown (2013) used optimization routines in Microsoft Excel and Solver to estimate aquifer properties in both non-leaky and leaky confined models.

While these unsteady (non-leaky and leaky) confined aquifer tests lend themselves to well-developed methods for estimating aquifer properties from data, it is also well-known that they neglect many real-world issues. Thus, richer models and techniques for deriving estimates from such models remain of interest. For example, aquifer performance tests in the field are often subject to "signal noise" from another nearby well or from boundary effects, which can result in an erratic drawdown at the primary monitoring well that may be difficult to interpret within existing solution approaches. This paper describes a relatively new computational technique called "particle filtering" in combination with previously-published analytical solutions to efficiently estimate confined aquifer parameters from field drawdown data measured at one or more observation wells. To our knowledge, this technique has not been used to solve these problems previously, yet we find when applied properly that it is no less accurate than previously published methods but also more flexible in the sense that it readily extends to richer models not easily solved otherwise. The technique is demonstrated and validated using three aquifer testing scenarios, namely the aforementioned canonical non-leaky and leaky confined aquifer tests as well as a third test in which signal noise is introduced into the data. The results provide strong evidence that the particle filtering method provides accurate estimates in all cases, including the third case in which previously-published methods are not as applicable. Further uses of particle filtering in groundwater hydrology are suggested for future research.

2 Particle Filtering Approach

2.1 Technical Rationale

The use of particle filters in the geosciences is fairly new, but the related Kalman filter has been used to study various groundwater problems since the 1990s. The Kalman filter equations are derived assuming that the measurement model is linear and all noise sources are Gaussian, neither of which is necessarily the case in aquifer drawdown tests. Thus, its application to groundwater problems introduces numerous additional considerations, such as how to approximate the models before processing each measurement or how to correct the Kalman filter equations to maintain acceptable performance when involving a non-linear model. In one of the first geoscience applications of the Kalman filter, Ferraresi et al. (1996) estimated hydrogeological parameters for aquifers in Libya. Hantush (1997) looked at spatially varying aquifer parameters using a Kalman filtering approach. Yeh and Huang (2005) use a modified Kalman filtering approach to develop estimates for leaky-confined aquifer pumping tests. Yeh et al. (2007) compared global optimization methods to extended Kalman filter solutions for leaky-confined aquifer parameter problems. Singh (2010a) developed diagnostic curves for identifying leaky confined aquifer parameters using a Kalman filter among other techniques. Nan and Wu (2011) used an ensemble Kalman filter with localization to estimate hydrogeologic parameter fields in two dimensions and three dimensions. Zhou et al. (2011) proposed new approaches of handling limitations inherent in the ensemble Kalman filter. Xu et al. (2013) used an ensemble Kalman filter to evaluate hydraulic conductivity, using parallel computing to increase computational power and decrease computational time.

As the limitations regarding the linear assumptions underlying the Kalman filter were being characterized, other researchers were investigating alternate approaches to study groundwater hydrology problems. Shigidi and Garcia (2003) used artificial neural networks to estimate aquifer parameters. Camp and Walraevens (2009) used a sampling approach employing Latin hypercube parameter sampling to develop estimates of key aquifer parameters during field testing. They used numerical inversion of LaPlace space solutions using the well-known Stehfest algorithm to develop analytical solutions that were linked to the parameter sampling approach. Wang and Huang (2011) used a Monte-Carlo approach to study the effect of aquifer heterogeneity on flow and solute transport in two-dimensional isotropic porous media. Recently, new data assimilation techniques have been used to improve hydrologic and hydrogeologic predictions. Included in these new techniques are "Sequential Monte Carlo (SMC)" methods in statistics, which are closely related to particle filtering methods in the sense that both employ sampled-based approximations for the probability distributions from which estimates are derived. A particle filter, however, organizes its computations more akin to the Kalman filter for linear-Gaussian models, while placing no restriction on the underlying models as long as they can be efficiently implemented as a computer program to be invoked repeatedly within each step of the filter. Recent work along these lines in the geoscience literature includes Noh et al. (2011), studying surface water hydrologic problems, and Pasetto et al. (2012), comparing the performance of the ensemble Kalman filter and a particle filter for a synthetic hydrogeologic case. Particle filtering is especially popular for object tracking and robotic navigation problems in electrical engineering and computer science, where numerous survey papers are now available (Arulampalam et al. 2002; Doucet and Johansen 2011).

2.2 General Solution Methodology

Estimating parameters using a particle filter depends upon characterizing the unknown parameters and the available data within a general stochastic dynamic system model in state-space form. In such models, each stage $k = 0, 1, \dots$ is comprised of two equations that together characterize the evolution of a (latent) state vector $\mathbf{x}_k$ (representing the unknown values in stage k) as well as how the (observed) measurement vector $\mathbf{y}_k$ (representing the data received in stage k) depends upon that state:

$$\text{System Equation:} \qquad \mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{d}_k) \tag{1}$$

$$\text{Measurement Equation:} \qquad \mathbf{y}_k = h(\mathbf{x}_k, \mathbf{v}_k) \tag{2}$$

This is analogous to the setup underlying the Kalman filter except that (i) the functions f and h need not be linear and (ii) the random vectors $\mathbf{d}_k$ and $\mathbf{v}_k$, which model uncertainty in the state evolution and in the measurement process, respectively, need not be described by Gaussian distributions. Another component of such models is a given distribution for the initial state vector $\mathbf{x}_0$, which also need not be Gaussian as is assumed by the Kalman filter.

A particle filter begins with using the given initial state distribution to generate N equally-weighted samples, or *particles*, denoted by the collection $\{\mathbf{x}_0^i\}_{i=1}^N$. Then, upon receiving the initial measurement $\mathbf{y}_0$, the weights of all particles are reassigned by comparing their simulated measurements $\mathbf{y}_0^i = h(\mathbf{x}_0^i, \mathbf{v}_0)$ to the observed measurement $\mathbf{y}_0$, where particles in areas of the state space that produce simulated measurements close to what is actually observed become more highly weighted. These updated weights are then normalized so that they sum to unity and the collection of weighted particles $\{(\mathbf{x}_0^i, w_0^i)\}_{i=1}^N$ approximate the state distribution conditioned on the observed measurement. Specifically, the associated minimum-mean-square-error estimate $\hat{\mathbf{x}}_0$ is approximated by the weighted average of all the particles i.e.,

$$\hat{\mathbf{x}}_0 \approx \sum_{i=1}^{N} w_0^i \mathbf{x}_0^i \tag{3}$$

and, denoting $\mathbf{A}'$ as the transpose operation of a matrix $\mathbf{A}$, the associated error covariance is approximated by

$$\hat{\Sigma}_0 \approx \sum_{i=1}^{N} w_0^i \left(\mathbf{x}_0^i - \hat{\mathbf{x}}_0\right) \left(\mathbf{x}_0^i - \hat{\mathbf{x}}_0\right)'. \tag{4}$$

The particle filter then proceeds to the so-called resampling step, in which a new collection of N particles is generated in a manner that allows for the deletion of lowly-weighted particles in favor of the replication of highly-weighted particles. These resampled particles are then simulated through the system equation $\mathbf{x}_1^i = f(\mathbf{x}_0^i, \mathbf{d}_0)$, predicting the next state by a new collection of equally-weighted particles $\{\mathbf{x}_1^i\}$ in preparation for another reweighting by the subsequent measurement $\mathbf{y}_1$. This procedure continues for $k = 1, 2, 3, \ldots$ until the final measurement is processed, the sequence of estimates $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3, \ldots$ and the associated error covariances computed analogously to Eqs. 3 and 4 for the initial stage. In short, the algorithm sequentially evolves its solution according to a "survival of the fittest" process in which particles with unlikely parameter estimates are discarded and those whose estimates produce simulated measurements resembling the observations are retained. Key algorithmic considerations in the implementation of a particle filter include how many particles to use and what type of sampling/resampling procedures to invoke in each iteration, design choices which are application-dependent to the extent that they are entwined with properties of the model's functions f and h as well as the distributions characterizing the system disturbances $\mathbf{d}_k$, measurement noises $\mathbf{v}_k$ and initial state $\mathbf{x}_0$. The reader interested in more details is encouraged to consult available tutorial papers and texts (Arulampalam et al. 2002; Doucet and Johansen 2011). The following sections present a particle filter that solves the parameter estimation problems for both non-leaky and leaky unsteady aquifer cases.

3 Methodology Applied to Aquifer Parameter Estimation

In this section, the general particle filtering methodology described in Section 2 is specialized to the problem of aquifer parameter estimation. We start with the well-studied unsteady non-leaky confined aquifer scenario of Theis (1935), where the unknown aquifer parameters of interest are its transmissivity T and storativity S, while the observations are a sequence $y_0, y_1, y_2, \ldots$ of (scalar) drawdown measurements taken during an aquifer performance test. Our state-space model utilizes the well function defined by Theis (1935), which relates the transmissivity T and storativity S to synthetic drawdown s via the series approximation

$$s = \left(\frac{Q}{4\pi T}\right)\left[-0.5772 - \ln(u) + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} - \ldots\right] \tag{5}$$

$$u = \frac{r^2 S}{4Tt} \tag{6}$$

where Q denotes the known pumping rate, r denotes the known radius from the pump to the observation well and t denotes the known observation time. It should be noted that Eq. 5 is not the only way to approximate Theis' well function; for example, Abramowitz and Stegun (1964) provide efficient polynomial approximations instead of the series solution. While the results in this paper are based on using the series approximation, the particle filtering methodology applies equally well when using other approximations for the governing well function.

Armed with a well function, let the state vector $\mathbf{x}_k = [S \quad T]'$ contain the unknown aquifer parameters of interest. Then, the measurement equation h of our state space model in successive stages $k = 0, 1, 2, \ldots$, in correspondence with a sequence of observation times $t_0 < t_1 < t_2 < \ldots$ with which to evaluate Eqs. 5 and 6, can be expressed as

$$y_k = h(\mathbf{x}_k, v_k) = s_k + v_k, \quad k = 0, 1, 2, \ldots. \tag{7}$$

Here, random variable v_k captures drawdown measurement error as well as modeling errors arising within Theis' approximation, which we assume is described by a zero-mean Gaussian distribution with known standard deviation σ_v. This measurement equation is linear in synthetic drawdown s_k and measurement noise v_k, but it is worth noting that the former is a highly nonlinear function of the state vector $\mathbf{x}_k$, or the aquifer parameters S and T to be estimated.

It remains to specify the system equation f of our state-space model. Because the unknown parameters are assumed to have fixed values during the aquifer performance test, the static model $\mathbf{x}_{k+1} = \mathbf{x}_k$ is appropriate in principle. However, static models are problematic for a particle filtering approach because the dynamics of the system equation are the mechanism by which a particle filer judiciously explores the state space; that is, in the case of static state dynamics, the candidate state values are entirely determined by the stage-0 samples from the initial state distribution—only their weights, not their locations, are revised as observations are processed. Satisfactory performance with static models depends on luck that at least one initial particle location takes its value near the correct one, the chance of which can be increased only by increasing the number of particles used (and incurring the associated computational overhead). This phenomenon for static models is referred to as "particle impoverishment" and is a known limitation of the approach. The use of "artificial dynamics," introduced by Liu and West (2001), overcomes this limitation of static models by rather perturbing the state vector in each iteration e.g.,

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{d}_k) = \mathbf{x}_k + \mathbf{d}_k, \quad k = 0, 1, 2, \ldots, \tag{8}$$

where we assume that the disturbance $\mathbf{d}_k$ is a zero-mean Gaussian random vector with known covariance matrix Σ_d. The initial state distribution is taken to be jointly uniform over given lower and upper bounds on the two parameters, storativity S and tranmissivity T, based on knowledge of the region under test.

Algorithm 1 summarizes the particle filter implementation specified above for the described unsteady non-leaky confined aquifer scenario, while Fig. 1 visualizes its behavior at five selected iterations after initialization. The figure shows six scatter plots of all particle locations in the state space, the horizontal and vertical axes in each plot corresponding to storativity S and transmissivity T, respectively. Specifically, Fig. 1a visualizes the $N = 2000$ samples drawn from a given uniform initial state distribution across the bounded state space $[10^{-5}, 10^{-2}] \times [5, 1000]$, each such initial particle is assigned equal weight and thus the initial parameter estimate (the blue triangle marker) is simply the central value. Fig. 1b–f visualize the collection of weighted particles $\{\mathbf{x}_k^i, w_k^i\}_{i=1}^N$ as drawdown observations are sequentially processed, each subfigure showing (i) the particle locations occupying smaller and smaller portions of the state space, (ii) the particle weights coded relatively by color (with red and blue indicating high and low, respectively) and (iii) the implied parameter estimate (i.e., the sample mean and sample covariance via Eqs. 3 and 4, respectively) evolving to the upper-left region of the state space. This dataset has thirteen stages and the final state estimate $\hat{\mathbf{x}}_{12}$ after the thirteenth iteration is $[1.09 \times 10^{-3} \quad 708]'$, which compares well with previously-published answers of $[1.06 \times 10^{-3} \quad 712]'$ derived from graphical curve fitting methods (the black square marker).

Algorithm 1 Particle Filter for Estimating Aquifer Parameters

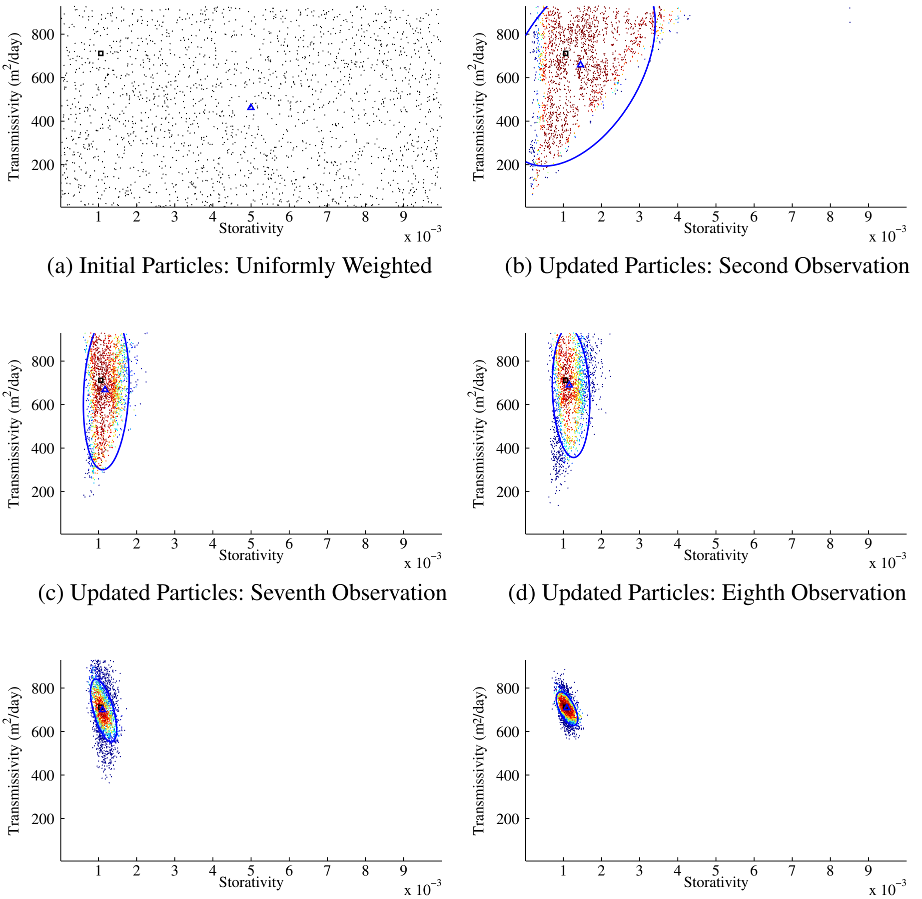
Step 0: Initialization
Iteration $k := 0$
for $i = 1 : N$ **do**
    Generate sample $\mathbf{x}_0^i$ drawn uniformly from the entire state space $[0, S_{\max}] \times [0, T_{\max}]$;
    Set initial weight $w_0^i := 1/N$
end

Step 1: Prediction Step
Upon receiving observation y_k (associated to time t_k of the aquifer performance test),
if $k > 0$ **then**
    for $i = 1 : N$ **do**
        Draw sample $\mathbf{d}_{k-1}$ from a zero-mean Gaussian distribution with covariance matrix Σ_d;
        Update particle location $\mathbf{x}_k^i := \mathbf{x}_{k-1}^i + \mathbf{d}_{k-1}$
    end
end

Step 2: Correction Step
for $i = 1 : N$ **do**
    Compute synthetic drawdown s_k^i via (5) and (6) with S and T taken from particle $\mathbf{x}_k^i$;
    Update particle weight $\tilde{w}_k^i := w_{k-1}^i \cdot \exp\left(-\frac{1}{2}(y_k - s_k^i)^2/\sigma_v^2\right)$
end
Upon computing total wieght $W_k := \sum_{i=1}^N \tilde{w}_k^i$,
for $i = 1 : N$ **do**
    Normalize weight $w_k^i := \tilde{w}_i^k / W_k$
end
Update second-order statistics via (3) and (4)

Step 3: Resample Decision
Upon quantifying degeneracy by calculating the effective sample size $N_{eff} = \left(\sum_{i=1}^N (w_k^i)^2\right)^{-1}$,
if $N_{eff} < \frac{N}{2}$ **then**
    Resample N particles form the current weighted set
    for $i = 1 : N$ **do**
        Re-initialize weight $w_k^i := 1/N$
    end
end
Iteration $k := k + 1$ and return to **Step 1**

Observe in Fig. 1 how the initial set of particles sparsely cover the entirety of the state space. After several observations, particle locations are updated such that coverage density about the likely area of the state space is increased. This desirable property occurs because of the artificial dynamics—the particle filter with a static state equation would not alter the initial locations and thus the density of particles in likely regions of the state space would never increase from that implied by Fig. 1a. This behavior, namely the concentration of computational resources to the most likely areas of the state space, is an important feature of particle filters, especially for models having higher dimensional state vectors. For example, the leaky confined aquifer scenario assumes flow during a performance test can also arise from vertical leakage through confining units from aquifers above or below the zone of interest, and thus introduces vertical hydraulic conductivity K_v as a third state variable. The above particle filter extends readily to this scenario, modifying the measurement equation h with formulas to efficiently estimate the Hantush well function (Hantush and Jacob (1955)). Specifically, letting drawdown s, rate Q, radius r and time t be defined as in the Theis model and defining m' as the thickness of the confining bed through which leakage occurs,



(a) Initial Particles: Uniformly Weighted
(b) Updated Particles: Second Observation
(c) Updated Particles: Seventh Observation
(d) Updated Particles: Eighth Observation
(e) Updated Particles: Eleventh Observation
(f) Updated Particles: Thirteenth Observation

Fig. 1 Visualization of five selected iterations after initialization of our particle filtering solution for the unsteady non-leaky confined aquifer scenario of Mays (2011). Each subfigure shows a collection of weighted particles (with *red* and *blue* indicating *high* and *low* weights, respectively) over the two-dimensional state space, the horizontal and vertical axes corresponding to storativity S and transmissivity T, respectively. Each subfigure also indicates the minimum-mean-square parameter estimate and its error covariance (the *blue triangle* and *two-sigma ellipse*) implied by the shown set of particles, which clearly converges to a previously-published answer (the *black square*) derived from graphical curve-fitting methods on the entire dataset

Veling and Maas (2010) propose a computationally efficient approximation in terms of the exponential integral E_1 and the modified Bessel Function K_0,

$$s = \left(\frac{Q}{4\pi T}\right) F(\rho, \tau) \tag{9}$$

where,

$$F(\rho, \tau) = \begin{cases} 2K_0(\rho) - J(\rho, \tau) & \tau > 0 \\ J(\rho, -\tau) & \tau \leq 0 \end{cases} \tag{10}$$

Table 1 Mays Particle Filter Parameters

$Q[m^3/min]$	$r[m]$	N	range	Σ	σ
1.89	61	2000	$\begin{bmatrix} 10^{-5} & 10^{-2} \\ 5 & 1000 \end{bmatrix}$	$\begin{bmatrix} 10^{-9} & 0 \\ 0 & 863 \end{bmatrix}$	0.03048

and

$$J(\rho,\tau) = \omega(\rho)E_1\left(\frac{\rho}{2}\exp(-\tau)\right) + (1-\omega(\rho))E_1(\rho\cosh(\tau)),$$

$$\omega(\rho) = \frac{E_1(\rho) - K_0(\rho)}{E_1(\rho) - E_1(\frac{\rho}{2})}, \qquad \rho = \frac{r}{\sqrt{Tc}}, \qquad \tau = \ln\left(\frac{2}{\rho}\frac{t}{Sc}\right), \qquad c = \frac{m'}{K_v}. \tag{11}$$

In turn, augmenting the state to $\mathbf{x}_k = [S \;\; T \;\; K_v]'$ and employing (9) in Step 2 of Algorithm 1 extends the particle filter solution to the leaky confined problem.

4 Results

4.1 Unsteady Non-Leaky Confined Aquifers

The solution methodology presented herein is first validated via two well known and previously published benchmark non-leaky confined aquifer parameter estimation problems. In the first, from Mays (2011), a test well screened in a confined aquifer is pumped at a rate of 31.5×10^{-3} m^3/sec for 4,000 minutes. Time-drawdown data was collected at an observation well located 61 m from the test well. After 4,000 minutes the maximum drawdown measured at the observation well was 2.13 m. Mays' graphical curve fitting solution provides an estimate of the aquifer properties as 1.06×10^{-3} and 712 m^2/day for S and T respectively.

When provided the input parameters collected in Table 1, the particle filter estimates an S of 1.08×10^{-3} and a T of 714 m^2/day. A visual representation of the output of Algorithm 1, after having sequentially processed all observations in the data set, is provided in Fig. 1f and reproduced in Fig. 2.

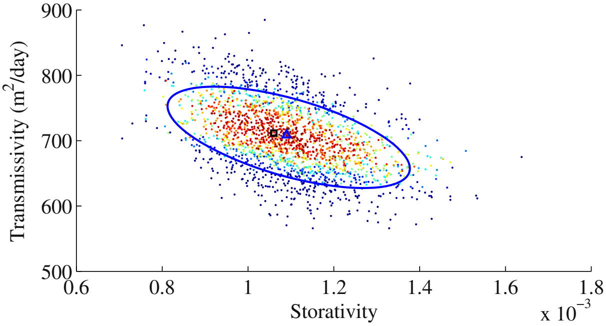


Fig. 2 Final particle cloud for the Mays (2011) aquifer test. Particles, colored in accordance to their respective likelihoods where red is more likely than blue, are presented relative to the previously published solution (*black square*) as well as the particle filter's estimate and error covariance (*blue triangle* and *ellipse*)

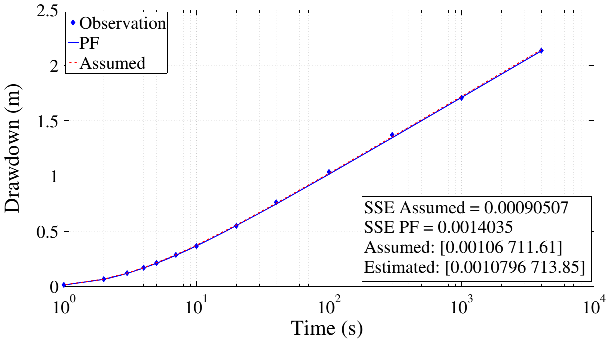


Fig. 3 Drawdown vs time plot of the Mays (2011) data set with the synthetic drawdown curves produced by the parameter estimates from both the observed results (*red*) and the particle filter estimator (*blue*) overlain

The time-drawdown data for this aquifer test, accompanied by synthetic drawdown curves produced by the parameter estimates of both the published Mays solution and the particle filtering solution, is presented in Fig. 3 with the sum of squared error metric for both sets of parameter estimates.

In the second problem, a data set collected from an aquifer test carried out in 1953 on a village well in Gridley, Illinois, in which the effects of a constant-rate excitation of $13.879 \times 10^{-3} m^3/sec$ located 251m from the observation well were recorded. Walton (1962) presents transmissivity and storativity parameter estimates for this dataset computed from the superposition of time-drawdown field data onto the non-leaky artesian type curve, arriving at S and T estimates of 2.2×10^{-5} and 125.45 m^2/day respectively. Additional parameter estimates were generated by the AQTESOLV pumping test software (HydroSOLVE, Inc.), producing 2.095×10^{-5} and 123 m^2/day.

When provided the input parameters similar to those of Table 1, the particle filtering methodology presented herein produces estimates of 2.12×10^{-5} and 122.6 m^2/day for S and T respectively.

4.2 Unsteady Leaky Confined Aquifers

Validation of the proposed solution methodology is continued through the use of two previously published benchmark leaky confined aquifer tests. The two necessary adaptations to Algorithm 1 are (i) an increase in the dimension of the latent state variable **x** in order to accommodate the vertical hydraulic conductivity term k_v, and (ii) the substitution of the measurement function in Step 2 of Algorithm 1 with that which is presented in Eq. 9.

First, a dataset presented by Walton (1962) originating from a controlled pump test made under leaky artesian conditions in glacial drift aquifers near Dieterich Illinois. The test well

Table 2 Walton - Dieterich Particle Filter Parameters

$Q[m^3/min]$	$r[m]$	$m'[m]$	N	range	Σ	σ
0.0944	96	4.27	2000	$\begin{bmatrix} 10^{-5} & 10^{-2} \\ 5 & 1000 \\ 10^{-7} & 1 \end{bmatrix}$	$\begin{bmatrix} 10^{-9} & 0 & 0 \\ 0 & 863 & 0 \\ 0 & 0 & 9 \times 10^{-6} \end{bmatrix}$	0.03048

was pumped at a constant rate of 135.9 m^3/day for 1,185 minutes while the effect of said excitation was observed in a well located 29.3 m from the test well. The thickness of the overlying confining unit was 4.27 m. After 1,185 minutes the maximum drawdown measured at the observation well was 1.96 m. Walton reports aquifer parameter estimates of 2.0×10^{-4} for S, a T of 18.754 m^2/day, and a K_v value of 4.482×10^{-3} m/day.

Parameter estimates found by the methodology presented in this paper are 1.74×10^{-4}, 21.54 m^2/day, and 3.28×10^{-3} m/day for S, T, and K_v, respectively. Input parameters to the modified Algorithm 1 are collected in Table 2 and a comparison of synthetic drawdown curves produced by the previously published estimation results and those found by the particle filter are presented in Fig. 4.

For the second benchmark estimation problem under leaky artesian conditions, a test well screened in a leaky confined aquifer Cooper (1963) is pumped at a rate of 5, 451 m^3/day for 1,000 minutes. The thickness of the overlying confining unit was 30.48 m. Time-drawdown data was collected at an observation well located 30.48 m from the test well. After 1,000 minutes the maximum drawdown measured at the observation well was 2.2 m. Lohman (1972) reports the aquifer properties as an S of 9.95×10^{-5}, a T of 1, 236 m^2/day, and a K_v value of 0.1 m/day.

Given similar input parameters to those found in Table 2, the herein presented methodology produces values of 1.08×10^{-4}, 1, 193 m^2/day, and 0.124 m/day for S, T, and Kv, respectively.

4.3 Unsteady Confined Aquifer with Noise and Boundary Effects

Synthetic validation data was developed using the three dimensional finite-difference groundwater modeling code MODFLOW McDonald and Harbaugh (1988). This validation experiment includes a pumping well withdrawing water from a non-leaky confined aquifer at a rate of 1, 136 m^3/day. The aquifer is 30 m thick with T value of 850 m^2/day and an S value of 5×10^{-4}. Aquifer drawdown is monitored in an observation well located 45.72 m directly east of the pumping well. For the sake of computational efficiency, the model grid is one quadrant with the pumping well located at the origin. The model grid size is unimodal with grid cells at 7.62 $m \times 7.62$ m. The MODFLOW model was validated against the Theis

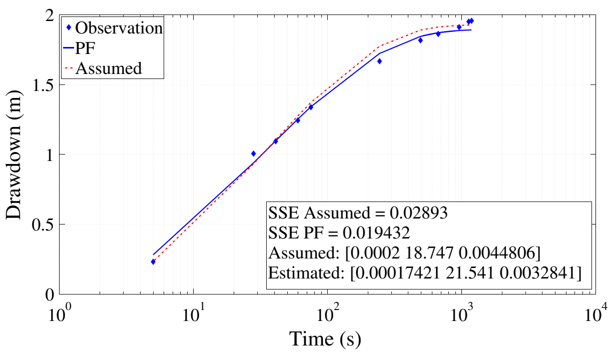


Fig. 4 Drawdown vs time plot of the Walton - Dieterich data set with the synthetic drawdown curves produced by the parameter estimates from both the previously published (*red*) and particle filter solutions (*blue*) overlain

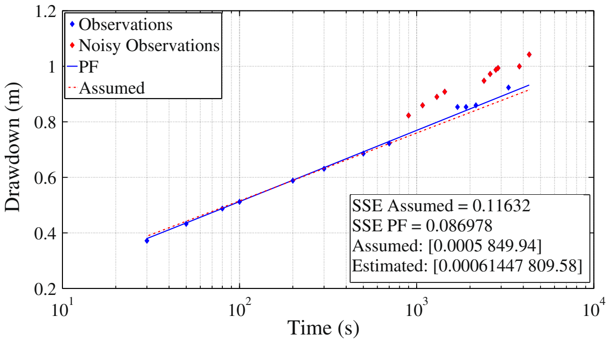


Fig. 5 An original time-drawdown data set with significant sinusoidal signal corruption due to a periodic secondary well with an unknown pump rate and at an unknown location. The known model parameters provide the (*red*) drawdown curve in the absence of the corrupting well, while the (*blue*) drawdown curve is produced by the particle filter's parameter estimates derived from the entire data set

analytical solution discussed above and matched the exact solution closely underestimating the exact drawdown by $0.015m$ on average. Once the initial validation was completed, the base model was modified to include a second pumping well located 140.5 m northeast from the model observation well. This well withdraws water from the non-leaky confined aquifer at a rate of 70.8 m^3/day, however, due to the model boundaries on the quadrant model, the effective drawdown is about double what an actual field drawdown would be in an infinite aquifer. This well also only pumps every 12 hours versus continuously for the first pumping well. Therefore, this second pumping well imparts a sinusoidal noise factor to the observation well.

In order to make this validation test totally blind, the solution methodology presented herein utilizes neither the "noise" well's position nor pumping rate. This simulation is comparable to real-world aquifer tests when a local irrigation well is known to exist nearby but the exact location cannot be established due to access restrictions. Therefore, in order to estimate the S and T values, the methodology contends with observation well data subject to three different types of noise simulating real-world issues including unidirectional low bias from the actual numerical model results; no-flow boundary effects that multiply the assigned model pumping rate; and, sinusoidal drawdowns due to temporal pumping rate without an exact location. Accuracy of the presented methodology largely depends on the ability to classify those data points which are corrupted by the secondary excitations and increase accordingly the parameter which encapsulates the filter's measurement uncertainty, σ_v, for those filter iterations.

Figure 5 presents the time-drawdown data for this problem as well as the drawdown curves produced by the known parameter values and particle filtering estimate when initialized with the input parameters collected in Table 3.

Table 3 Noise/Boundary Effects Particle Filter Parameters

$Q[m^3/min]$	$r[m]$	N	range	Σ	σ
1136	45.72	2000	$\begin{bmatrix} 10^{-5} & 10^{-2} \\ 5 & 2000 \end{bmatrix}$	$\begin{bmatrix} 10^{-9} & 0 \\ 0 & 863 \end{bmatrix}$	0.03048,0.3048

5 Conclusion and Future Work

This paper has applied the particle filtering methodology to estimate properties of confined aquifers using transient data from aquifer performance tests. Experimental results demonstrate (i) an accuracy that matches that of previously-published solution methods in numerous well-studied scenarios and (ii) an ability to generalize to scenarios not as easily addressed by previously-published methods. Particle filtering as a means to address measurement uncertainty is common practice in the sub-disciplines of electrical engineering and applied mathematics, but it is only beginning to find application in the water research community. The widespread familiarity of the first four data sets that we considered, two under non-leaky assumptions using the Theis well function and two under leaky assumptions using the Hantush well function, affords an accessible introduction of the particle filtering approach. Its true usefulness, however, becomes evident in the fifth data set that we considered, which injects noise into the drawdown data in a manner that challenges previously-published solution methods but is readily addressed by the particle filtering approach.

An interesting extension of the work presented here is to estimate properties of tidally responsive aquifers, as first described by Jacob (1950). The effect of earth tides on aquifer response at groundwater monitoring wells was further studied by Bredehoeft (1967), who introduced specific storage and porosity estimates. Of particular interest are the recent adaptations of Jacob's original models for coastal confined aquifers presented by Dong et al. (2012).

Recall the results in Fig. 1, which illustrate a particle filters ability to provide estimates (and the associated error intervals) sequentially during the performance test (in contrast with solutions that operate on the drawdown data in-batch after the test). A sequential algorithm presents the opportunity to avoid unnecessarily long performance tests if diminishing marginal improvement in estimation accuracy is observed or if it can otherwise be inferred that neither surface water bodies nor impervious boundaries will likely be reached by continuing the test. Quantifying the extent to which a particle filter can adequately inform an online decision process for when to terminate a performance test, potentially avoiding unnecessary expense, is another interesting direction for future work.

Finally, the particle filtering approach taken in this paper should be applicable to parameter estimation problems that arise within other hydrogeology applications. Solutions based on classical approaches (e.g., the Kalman filter, least-squares) carry the risk of oversimplifying the underlying models to satisfy the needed linear-Gaussian assumptions. That said, there are examples in the hydrogeology literature that address nonlinear estimation problems using the so-called extended Kalman filter, Huang and Yeh (2012), and its variations. Thus, a related line of inquiry could be to compare (both in estimation accuracy and in computational overhead) a particle filtering solution to previously-published nonlinear estimation techniques in the hydrogeology literature.

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