Water Resour Manage (2016) 30:2143–2160
DOI 10.1007/s11269-016-1278-x

Parameters Estimation for the New Four-Parameter Nonlinear Muskingum Model Using the Particle Swarm Optimization

A. Moghaddam [1] **· J. Behmanesh** [1] **· A. Farsijani** [1]

Received: 18 June 2015 / Accepted: 29 February 2016 /
Published online: 5 March 2016

Abstract The Muskingum model is a popular method for flood routing in river engineering. This model has several parameters, which should be estimated. Most of the techniques have applied to estimate these parameters to reduce the distance between observed flow and estimated flows. In this paper, for the first time, the parameters of a novel form of the nonlinear Muskingum model are estimated by the Particle Swarm Optimization (PSO) algorithm. The new Muskingum model, which have four parameters, is applied for three benchmark examples and one real case in Iran. The sum of the squared (SSQ) or absolute (SAD) deviations between the observed and estimated outflows was considered as objective functions. The results showed that although the new Muskingum model became more complex but this model by using PSO technique can improve the fit to observed flow especially in multiple-peak hydrographs.

Keywords Four parameters Muskingum model · Flood routing · Particle swarm optimization · Hydrologic model

1 Introduction

Flood routing is one of the important topics to design water structures in water resources engineering. There are two main approaches in flood routing: the hydrological and hydraulic approaches. In the hydrological approaches, the flow rate calculated as a function of time. These methods are based on storage-continuity equation in the specific location. In the hydraulic approaches the flow rate determines as a function of both time and location. These methods are based on the Saint-Venant equations. There are other approaches which need more data and parameters for solving their models. One of the popular hydrological methods is

✉ A. Moghaddam
alireza.moghaddam@yahoo.com; a.moghadam@urmia.ac.ir

[1] Department of Water Engineering, Urmia University, Urmia, Iran

Muskingum method, which frequently used in flood routing that first, the U.S. Army Corps of Engineers developed this model.

The linear Muskingum model is based on two main equations: continuity equation and storage equation (Gill 1978; Tung 1985; Yoon and Padmanabhan 1993; Mohan 1997; Kim et al. 2001; Geem 2006; Al-Humoud and Esen 2006):

$$\frac{ds_t}{dt} = I_t - O_t \tag{1}$$

$$S_t = K[xI_t + (1-x)O_t] \tag{2}$$

Where, S_t, I_t and O_t are storage, inflow and outflow magnitude, respectively at time t. K is storage-time constant that may be considered as an approximation of travel time through the reach; x is a weighting factor which is usually between 0 and 0.5 for reservoirs storage and between 0 and 0.3 for stream channel (Mays 2010). The dimensions of O and I are (L^3T^{-1}) similar; x is dimensionless, S is L^3 and K is T. In the linear Muskingum model, the parameters K and x usually estimate graphically by a trial and error procedure. First x assumed and the values of $[xI_t+(1-x)O_t]$ compute by using observed data and plotted against S_t. If the value of x can reduce width of the plotted loop then the value of x right selected. K is equal to the slope of the straight fitted line through the loop (Barati 2011). K and x expect to capture the flood propagating characteristics of the reach in its entirety, and no additional topography information of the channel is required for flood routing (Yoon and Padmanabhan 1993). As said before, the linear Muskingum is based on graphical methods, which can make errors in visual judgments. Nevertheless, in some reaches of rivers the linear Muskingum routing is not suitable. As a result, the performance of nonlinear of Muskingum model is more appropriate (Mohan 1997). Two forms of nonlinear Muskingum models have been recommended [Eqs. (3)-(5)] in the previous researches to improve the fitness of nonlinear relationships (Gill 1978; Mohan 1997; Luo and Xie 2010).

$$S_t = K[xI_t + (1-x)O_t]^m \tag{3}$$

$$S_t = K[xI_t^m + (1-x)O_t^m] \tag{4}$$

These models have an additional parameter (m), which used as power exponents to improve the relation between accumulated storage and weighted flow. The linear model considers a special case of nonlinear models when m is equal to one. Unlike the linear model, in the nonlinear models K has the dimension of $[L^{3(1-m)}\,T^m]$ and does not describe the travel time of flood wave. Also, x does not to be the same as in the linear model. A graphical method that is used from historical inflow and outflow hydrographs is inappropriate for nonlinear Muskingum models. Hence, the determination of correct values of K and x is difficult and needs advanced methods (Gill 1978; Tung 1985; Yoon and Padmanabhan 1993; Mohan 1997; Kim et al. 2001; Das 2004, 2007; Geem 2006; Chu 2009; Chu and Chang 2009; Luo and Xie 2010; Barati 2011, 2013; Xu et al. 2011; Karahan et al. 2012).

Several previous researches on parameter estimation of the Muskingum method have been applied to finding the values of the parameters in the nonlinear Muskingum model. The mathematical techniques include segmented least squares method (S-LSM) (Gill 1978), nonlinear least squares (NONLR) (Yoon and Padmanabhan 1993), Lagrange multiplier (LMM) (Das 2004), the Broyden-Fletcher-Goldfarb-Shanno method (BFGS) (Geem 2006), Nelder-Mead simplex (NMS) method (Barati 2011), and the generalized reduced gradient (GRG) (Barati 2011) were used to solve the nonlinear forms of the Muskingum models.

In addition, various heuristic algorithms such as genetic algorithm (GA) (Mohan 1997), harmony search (HS) (Kim et al. 2001), immune clonal selection algorithm (ICSA) (Luo and Xie 2010), differential evolution (DE) (Xu et al. 2011), parameter setting free harmony search (PSF-HS) algorithm (Geem 2010), particle swarm optimization (PSO) (Chu and Chang 2009), simulated annealing (SA) algorithm, and shuffled frog leaping algorithm (SFLA) (Orouji et al. 2013) have been developed in order to search optimum parameters in nonlinear models.

Das (2007) suggested a chance-constrained optimization-based model for estimating the Muskingum model parameters. This model determined the parameters of the Muskingum model using minimizing the SSQ of difference between the observed and estimated outflows. Also, the new model is very complex and requires massive computation to estimate parameters of the Muskingum model (Luo and Xie 2010). Chu (2009) determined the parameters of the Muskingum model by a Neuro-Fuzzy approach, which has no physical basis. The mentioned method can be an alternative in the application of the Muskingum model (Barati 2013). Niazkar and Afzali (2014) applied the modified honey bee mating optimization (MHBMO) algorithm along the modified routing procedure to estimate the Muskingum parameters using two different objective functions. The results demonstrated that the MHBMO algorithm not only converged faster, but it also captured the best optimal parameters values. Latt (2015) examined the application of artificial neural network (ANN) approach based on the Muskingum equation, and compared the feedforward multilayer perceptron (FMLP) models to other reported methods. The FMLP model showed a clear-cut superiority over other methods in flood routing of well-known benchmark data.

The previous studies showed that the particle swarm optimization (PSO) was used to determine the three parameters in nonlinear Muskingum models, but in this paper the PSO algorithm is applied to estimate the parameters of a new nonlinear Muskingum model which consist of four parameters and was developed by Easa (2011). The PSO algorithm is a population-base evolutionary algorithm and already applied in Civil Engineering and Water Resources Engineering optimization problems such as reservoir operation (Nagesh Kumar and Janga Reddy 2007), water quality management (Afshar et al. 2011; Lu et al. 2002; Chau 2005), Basin wide water resources management (Shourian et al. 2008), flood control management (Meraji 2004). To check the performance of the new method, four real examples have assessed. The results showed the PSO is a simple and clear algorithm for understanding. Also, it does not need an initial guess for its correspondent parameters unlike other previous methods. Although the new four-parameter nonlinear Muskingum model is more complicated, but it can improve the routed flow better than the three-parameter nonlinear Muskingum, particularly in the multiple-peak hydrograph.

2 Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is a meta-heuristic optimization technique, which first introduced by Eberhart and Kennedy (1995). This algorithm is a member of the category of swarm intelligence that inspired from the flocking and schooling of birds and fishes. All possible solutions for a problem in PSO are in a search framework that called solution space. Each individual in this swarm is called a particle. Each particle represents a solution for the problem in optimization in each iteration, which flies in the location space. In each iteration, the result of three vectors defines moving each particle. One of them is velocity vector that is a randomly vector and other vectors are produced based on the best position of particles are found to now (Kuo et al. 2010). Each particle keeps track of its position vector, ***pbest***,which has obtained best fitness function, so far. The position vector, ***gbest***,which is the best value of fitness function achieved so far considering all the particles is also memorized. The position vectors of the particles update from velocity vector. These updated positions in PSO algorithm evaluate with an objective or fitness function in the each iteration and ***pbest*** and ***gbest*** are updated. The particle velocities V_{ij}^{t} and position X_{ij}^{t} calculate in the iteration number t using following equations (De Moura Meneses et al. 2009),

$$V_{ij}^{t+1} = W^t V_{ij}{}^{t} + c_1 r_1^t \left(\boldsymbol{pbest}(i,j) - X_{ij}{}^{t}\right) + c_2 r_2^t \left(\boldsymbol{gbest}(j) - X_{ij}{}^{t}\right) \tag{5}$$

$$X_{ij}^{t+1} = X_{ij}{}^{t} + V_{ij}{}^{t} \tag{6}$$

Where, $i=[1,2,\ldots,P]$ and $J=[1,2,\ldots,n]$. The c_1 and c_2 are acceleration constant that change between [2, 4]. The r_1, r_2 are random numbers between [0,1]. The ***gbest*** and ***pbest*** call global and particle best position, respectively. The W is an initial weight which represent the exploration and exploitation properties of algorithm and changes between [0.4–0.9]. The characteristics of exploration algorithm will increase if W closes to 0.9, otherwise, its exploitation properties increases. In this stage, a new modified factor which is called W_{damp} used in the each iteration based on Eq. (7) to increase exploration characteristics in the final steps. (Hossein Zaji and Bonakdari 2014). The value of W_{damp} factor used in this paper was 0.998, which reduces the W and particle movement in the each iteration.

$$W = W \times W_{damp} \tag{7}$$

3 New Non-Linear Muskingum Model

In this paper, the new non-linear Muskingum model has been used, which introduced first by Easa (2013). The new method considers the four parameters to determine. The new four-parameter non-linear Muskingum model is given by the following equation:

$$S_t = K\left[xI_t^a + (1-x)O_t{}^a\right]^m \tag{8}$$

In fact, the new model is similar to the Eq. (3). However, in the proposed model m corresponds to the non-linear form of the storage equation, while in the Eq.(3) m is related to the assumed linear form of it (Easa 2013).

The Eq. (8) is a general model, which can cover all of previous developed models e.g. (Eqs. 2–4) by considering $a=m=1$ in linear model (Eq. 2) and $a=1$ or $m=1$ in non-linear models (Eqs. 3–4). The dimension of K in the proposed model is not the same as the linear model. Although, the new proposed model is more complex than previous model, but the optimization models such as PSO can compute the best values for its parameters at less time. The extra parameter in new model causes to improve fitting between routed and observed outflow especially in multi-peak hydrograph examples. The proof and details of the derivation of Eq. (8) presented by Easa (2013).

3.1 Routing Procedure of the Nonlinear Muskingum Model

By rearranging Eq. (8), the routed outflow can be expressed as,

$$O_t = \left[\left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{1/m} - \left(\frac{x}{1-x}\right)I_t^a\right]^{1/a} \tag{9}$$

The first term in bracket of Eq. (9) should be greater than the second term so that the routed outflow not be negative value. By considering mentioned condition, the relation between the four parameters can be expressed as,

$$\frac{1}{m} > \frac{\ln(xI^a)}{\ln\left(S_t/K\right)} \tag{10}$$

By substituting Eq. (9) in Eq. (1), the time rate of changes volume the storage can be obtained as,

$$\frac{\Delta S_t}{\Delta t} = I_t - \left[\left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{1/m} - \left(\frac{x}{1-x}\right)I_t^a\right]^{1/a} \tag{11}$$

As well, the next accumulated storage can be rewritten as,

$$S_{t+1} = S_t + \Delta S_t \tag{12}$$

The routing procedure for the nonlinear model includes the following steps (Tung 1985; Kim et al. 2001; Geem 2006, 2010; Chu and Chang 2009; Karahan et al. 2012; Barati 2013):

1. Determination the hydrologic parameters (K, x, a, m) using the PSO algorithm. PSO select randomly the initial values of parameters based on the specified bounds.
2. Calculation the time rate of change of the storage volume by using Eq. (11). Here in the first time step initial outflow is the same as the initial inflow
3. Estimation the next accumulated storage using Eq. (12)
4. Calculation the outflow in the next time using Eq. (9)
5. Repetition steps 2–4 for all time steps

This procedure should be repeated with different parameters values several times so that the best fit between routed and observed outflow achieve.

4 The Objective Functions

The objective function of the optimization model minimizes the sum of the squared deviations between the observed and estimated outflows,

$$Minimize\ \ SSQ = \sum_{t=1}^{n} \left(O_t - O_t\right)^2 \tag{13}$$

Where, *SSQ* is the sum of the squared deviations between the observed and estimated outflows; O_t is the observed outflow at time *t*; $_t$ is the estimated outflow at time *t* that calculated by Eq. (9) and *n* is the number of time steps in flood routing. Also, the other objective function can be introduced to minimize the sum of absolute deviation between the observed and estimated outflows,

$$Minimize\ \ SAD = \sum_{t=1}^{n} \left|O_t - O_t\right| \tag{14}$$

Where, *SAD* is the sum of absolute deviation between the observed and estimated outflows and |.| denotes = absolute value.

4.1 Error of Peak Discharge

The error of peak discharge (EQ_p) provides a deviation of peak of observed and routed outflows (Latt 2015). EQ_p is given as,

$$EQ_p = \frac{\left|O_{observed}^{peak} - O_{routed}^{peak}\right|}{O_{observed}^{peak}} \tag{15}$$

Where, $O_{observed}^{peak}$ is the peak of observed outflow and O_{routed}^{peak} is the peak of routed outflow. A lower absolute value of EQ_p implies a more accurate model.

4.2 Error of Time to Peak

The error of time to peak (ET_p) provides a deviations of peak time of observed and routed outflows (Latt 2015). ET_p is given as,

$$ET_p = \left|T_{observed}^{peak} - T_{routed}^{peak}\right| \tag{16}$$

Where, $T_{observed}^{peak}$ is the time of peak of observed outflow and T_{routed}^{peak} is the time of peak of routed outflow. A smaller value of ET_p implies a more accurate prediction of occurrence of peak outflow.

4.3 Mean Absolute Relative Error Consideration

The mean absolute relative error (*MARE*) between the observed and routed outflows is considered as the error mode (Toprak 2009). *MARE* is given as,

$$MARE = \frac{1}{N}\sum_{i=1}^{N} \frac{\left|O_{observed}^{t} - O_{routed}^{t}\right|}{O_{observed}^{t}} \tag{17}$$

4.4 Variance Explained

The closeness of shape and size of the hydrograph can be measured using the criterion of variance explained as advocated by Je and Sutcliffe (1970) and recommended by the ASCE Task Committee on Difinition of Criteria for Evaluation of Watershed Models of the Watershed Management Committee, Irigation and Drainage Division (1993) (McCuen et al. 2006; Perumal and Sahoo 2007; Barati 2013, Niazkar and Afzali, 2014). The variance explained in percentage (*VarexQ*) is given as

$$VarexQ = \left[1 - \frac{\sum_{i=1}^{N} \left(O^t_{observed} - O^t_{routed}\right)^2}{\sum_{i=1}^{N} \left(O^t_{observed} - O^{mean}_{observed}\right)^2} \right] \times 100 \quad (18)$$

Where, $O^{mean}_{observed}$ = mean of observed outflows.

5 Application of the Routing Techniques

5.1 Numerical Experiment 1

In this example, the inflow–outflow hydrograph data of Wilson (1974) have been employed. The previous researches have applied the various optimization methods to estimate the Muskingum parameters by using these data. Table 1 shows the inflow and routed outflow of Wilson (1974), which calculated by new four-parameter Muskingum model and other methods.

In this paper, the objective functions SSQ and SAD have utilized to estimate the parameters. According to Table 2, the PSO method calculated the SSQ and SAD values 8.82 and 9.77, respectively, which were much smaller than another model. As well, SSQ and SAD values that presented by Easa were 7.67 and 10.31, respectively which used the generalized reduced gradient method with Solver associated in Excel.

Additionally, at the Table 2 the parameters EQ_p, ET_p, *MARE* and *VarexQ* compared between the previous methods and the presented method. Since the occurrence of the predicted peak outflow happen at the same time with that of the observed maximum outflow, ET_p values are zero for all methods exept the NONLR method. The EQ_p obtained 0.0001, which was less than all of applied methods except LMM (Das 2004) . Although, Easa used the four-parameter non-linear model, but the EQ_p value achieved 0.0037 and as a result the proposed method can make 96.7 % improvement in this term. Also the PSO algorithm can produce as well results as the most accurate procedures in terms of both *MARE* and *VarexQ*.

Figure 1 shows the observed and estimated outflow of the best five methods presented in the Table 2 respect to the proposed method. Moreover, Fig. 2 illustrates a comparison between the observed and the routed outflows.

The optimal parameters of the proposed model are K = 0.1659, x =0.2981, m =3.6830 and a = 0.4689. The optimization processes performed in 200 iteration that after several times of trial and error the best value of the PSO algorithm parameters was selected so that initial population = 100, $c_1 = c_2$ =2.05, W =0.4 and W_{damp} =0.998.

Table 1 Comparison of the observed and best-routed outflows for Wilson data

Time (h)	Inflow (cms)	Observed outflow (cms)	S-LSM	HJ + CG	HJ + DFP	NONLR	GA	HS	LMM	BFGS	ICSA	NMS	PSF-HS	DE	HS-BFGS	MHBMO	GRG (new Muskingum)	PSO (new Muskingum)
0	22	22	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00
6	23	21	22.00	22.00	22.00	22.60	22.00	22.00	22.00	22.06	22.00	22.00	22.00	22.00	22.00	21.74	22.00	22.00
12	35	21	22.80	22.40	22.40	23.00	22.40	22.40	22.80	23.02	22.42	22.42	22.40	22.40	22.40	23.91	22.34	22.32
18	71	26	29.60	26.80	26.70	24.20	26.30	26.60	29.60	25.35	26.59	26.61	26.60	26.60	26.60	28.66	25.68	25.48
24	103	34	39.10	34.90	34.80	33.20	34.20	34.40	38.70	32.74	34.44	34.46	34.50	34.50	34.50	34.31	33.14	32.73
30	111	44	47.60	44.50	44.70	47.10	44.20	44.10	47.00	45.99	44.20	44.17	44.20	44.20	44.20	44.00	43.63	43.27
36	109	55	58.00	56.70	56.90	56.80	56.90	56.80	57.80	58.13	56.91	56.85	56.90	56.90	56.90	56.44	55.77	55.68
42	100	66	67.10	67.30	67.70	66.20	68.20	68.10	67.60	68.76	68.12	68.06	68.10	68.10	68.10	67.84	66.48	66.64
48	86	75	74.80	75.90	76.30	75.00	77.10	77.10	75.80	77.48	77.12	77.07	77.10	77.10	77.10	77.24	75.25	75.58
54	71	82	80.40	81.90	82.20	80.70	83.20	83.30	81.90	83.08	83.35	83.32	83.30	83.30	83.30	83.55	81.62	82.00
60	59	**85**	**83.20**	**84.50**	**84.70**	83.50	**85.70**	**85.90**	**85.00**	**84.71**	**85.90**	**85.90**	**85.90**	**85.90**	**85.90**	**85.77**	**84.69**	**84.99**
66	47	84	82.80	83.40	83.50	**84.30**	84.20	84.50	84.40	84.01	84.51	84.54	84.50	84.50	84.50	84.44	83.77	83.91
72	39	80	80.10	79.90	79.80	79.90	80.20	80.60	81.40	79.83	80.53	80.58	80.60	80.60	80.60	80.00	80.15	80.10
78	32	73	74.50	73.60	73.30	74.30	73.30	73.70	75.20	73.85	73.64	73.71	73.70	73.70	73.70	73.03	73.02	72.84
84	28	64	67.20	65.80	65.50	65.30	65.00	65.40	67.10	65.68	65.34	65.41	65.40	65.40	65.40	64.47	64.27	64.06
90	24	54	58.10	56.90	56.50	55.90	55.80	56.00	57.20	56.90	55.94	56.00	56.00	56.00	56.00	55.16	54.20	54.07
96	22	44	48.10	47.80	47.50	45.10	46.70	46.70	46.50	47.11	46.63	46.67	46.70	46.70	46.60	45.86	44.62	44.63
102	21	36	37.60	38.90	38.70	35.40	38.00	37.80	35.50	37.72	37.75	37.76	37.80	37.80	37.70	37.24	35.82	36.00
108	20	30	28.20	31.50	31.40	28.70	30.90	30.90	26.40	29.96	30.50	30.47	30.50	30.50	30.40	30.28	29.16	29.43
114	19	25	21.90	25.80	25.90	24.30	25.70	25.30	21.10	24.31	25.27	25.23	25.20	25.20	25.20	25.19	24.68	24.97
120	19	22	19.10	22.00	22.10	20.90	22.10	21.80	19.00	20.85	21.78	21.74	21.70	21.70	21.70	21.88	21.72	21.97
126	18	19	19.00	20.10	20.20	20.40	20.20	22.00	19.00	19.43	20.02	20.00	20.00	20.00	20.00	19.96	20.13	20.30

Bold number is maximum outflow

Table 2 Comparison of the routing results of the different methods

Method	K	x	m	a	SSQ	SAD	EQ_p	ET_p	$MARE$	$VarexQ$
S-LSM	0.0100	0.2500	2.3470	-----	143.600	46.40	0.0216	0	0.056	98.83
HJ + CG	0.0669	0.2685	1.9291	-----	49.640	25.20	0.0059	0	0.030	99.59
HJ + DFP	0.0764	0.2677	1.8978	-----	45.540	24.80	0.0035	0	0.030	99.63
NONLR	0.0600	0.2700	2.3600	-----	41.280	25.20	0.0083	1	0.033	99.66
GA	0.1033	0.2873	1.8282	-----	38.230	23.00	0.0082	0	0.025	99.70
HS	0.0883	0.2873	1.8630	-----	36.780	23.40	0.0107	0	0.031	99.63
LMM	0.0753	0.2769	2.2932	-----	130.487	43.20	0.0000	0	0.055	98.94
BFGS	0.0863	0.2869	1.8679	-----	36.768	23.46	0.0106	0	0.026	99.69
ICSA	0.0884	0.2862	1.8624	-----	36.801	23.40	0.0105	0	0.025	99.69
NMS	0.0862	0.2869	1.8681	-----	36.765	23.46	0.0105	0	0.025	99.70
PSF-HS	0.0864	0.2869	1.8677	-----	36.768	23.70	0.0105	0	0.026	99.69
DE	0.5175	0.2869	1.8680	-----	36.770	23.46	0.0105	0	0.026	99.69
HS-BFGS	0.0862	0.2869	1.8681	-----	36.768	23.40	0.0105	0	0.025	99.70
MHBMO	0.6304	0.3386	1.8533	-----	37.451	21.22	0.0090	0	0.026	99.69
GRG (4 Parameters Muskingum)	0.8340	0.2690	4.0790	0.4330	7.670	10.31	0.0037	0	0.015	99.94
PSO (4 Parameters Muskingum)	0.1659	0.2981	3.6830	0.4689	8.820	9.77	0.0001	0	0.015	99.93

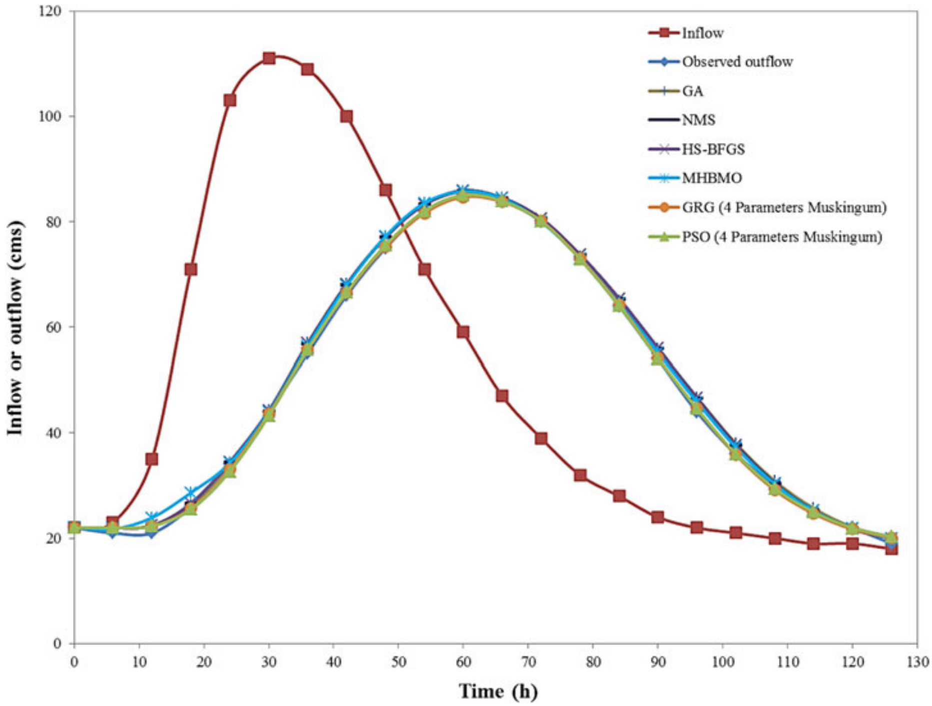

Fig. 1 Inflow, observed and computed outflows hydrographs with different methods by using the Wilson data

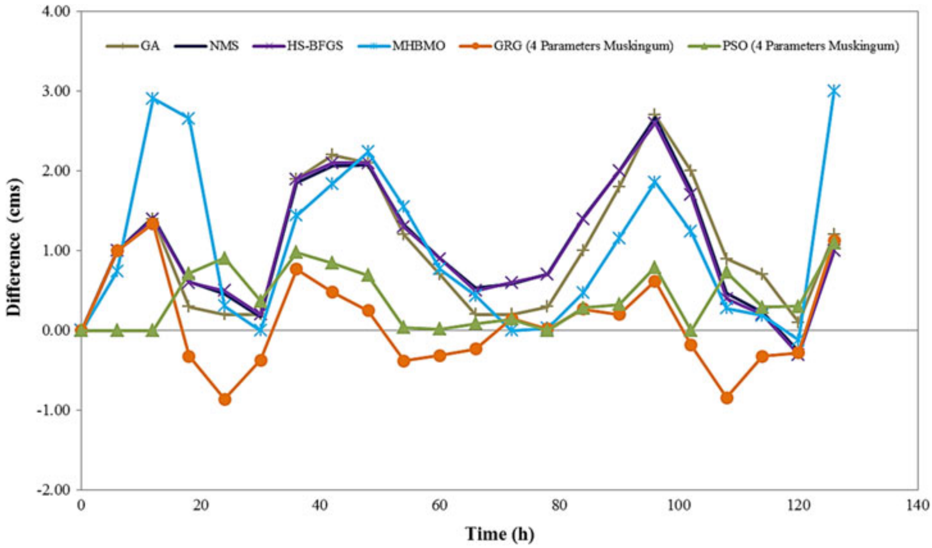

Fig. 2 Comparison the difference between the routed and observed outflows with various methods by using the Wilson data

5.2 Numerical Experiment 2

The data of this example firstly have recorded by Karahan et al. (2012) from the 1960 flood of the River Wye in the United Kingdom. The 69–75 km stretch of the River Wye from Everwood to Belmont has no tributaries and very small lateral flow (NERC 1975). It is, thus, an excellent example to demonstrate the use of flood-routing techniques (Bajracharya and Barry 1997). The inflow data, estimated outflow by O'Donnell (O'DONNELL 1985), HS-BFGS (Karahan et al. 2012), GRG (Easa 2013) and proposed method presented in the Table 3.

As shown in Table 3, by PSO algorithm the value of SSQ and SAD computed 31,099.52, 695.77, respectively which is better than all of methods. Also the PSO can obtained the less value than O'Donnell and HS-BFGS method in term of EQ_p. Moreover, the PSO can determinated 0.09 and 98.12 % for *MARE* and *VarexQ*, respectively, which are in acceptable range rather to other methods in the Table 3. The Fig. 3 described comparison between the observed and computed hydrographs by applying methods. In addition, Fig. 4 shows a comparison of the difference between the observed and the estimated outflows by applied methods. It can be understand from Fig. 4 the low swing in the PSO model chart.

The estimated Muskingum parameters by proposed model are K =0.612, x =0.401, a =1.133 and m =1.363. These values achieved by PSO optimization processes after only 30 iterations. The initial population, W, W_{damp} were selected 200, 0.5, 0.998 respectively and c_1 = c_2 =2.05.

5.3 Numerical Experiment 3

In this example, the Viessman and Lewis (2003) data have been applied to estimate the new Muskingum parameters. This example is one of multi-peak hydrographs, which inflow and observed outflow data shown in the Table 4. The PSO algorithm computed the SSQ =74,812.30 that indicate about 2 % reduction in comparison with the Easa method

Table 3 Comparison of the observed and best-routed outflows by using the River Wye data

	Time (h)	Inflow (cms)	Observed outflow (cms)	HS-BFGS	O'Donnell method	GRG (4 Parameters Muskingum)	PSO (4 Parameters Muskingum)
	0	154	102	154	102	154	102.0
	6	150	140	154	116	154	154.0
	12	219	169	152	120	152	152.1
	18	182	190	181	147	184	179.4
	24	182	209	191	158	192	190.9
	30	192	218	185	165	186	185.4
	36	165	210	187	176	187	186.9
	42	150	194	179	178	179	180.2
	48	128	172	162	176	162	164.1
	54	168	149	141	164	141	143.7
	60	260	136	154	160	155	152.8
	66	471	228	198	167	203	196.3
	72	717	303	264	218	281	267.3
	78	1092	366	344	303	363	351.4
	84	1145	456	416	484	443	431.8
	90	600	615	599	690	624	617.4
	96	365	830	871	700	893	881.5
	102	277	969	834	642	849	836.7
	108	227	665	689	572	709	696.2
	114	187	519	535	505	560	549.1
	120	161	444	397	442	424	416.8
	126	143	321	283	386	307	305.0
	132	126	208	202	338	219	221.4
	138	115	176	152	296	160	164.9
	144	102	148	124	260	127	131.2
	150	93	125	106	228	107	110.0
	156	88	114	94	201	94	96.4
	162	82	106	88	179	88	89.2
	168	76	97	82	160	82	82.7
	174	73	89	75	144	75	76.3
	180	70	81	73	130	73	73.0
	186	67	76	69	118	69	69.8
	192	63	71	66	109	66	66.7
	198	59	66	62	100	62	62.4
SSQ				37,944.14	251,802	32,299.2	31,099.52
SAD				829	2162	743.32	695.77
EQ_p				0.101	0.278	0.078	0.090
ET_p				6	6	6	6
MARE				0.11	0.33	0.10	0.09
VarexQ				97.71	84.78	98.05	98.12

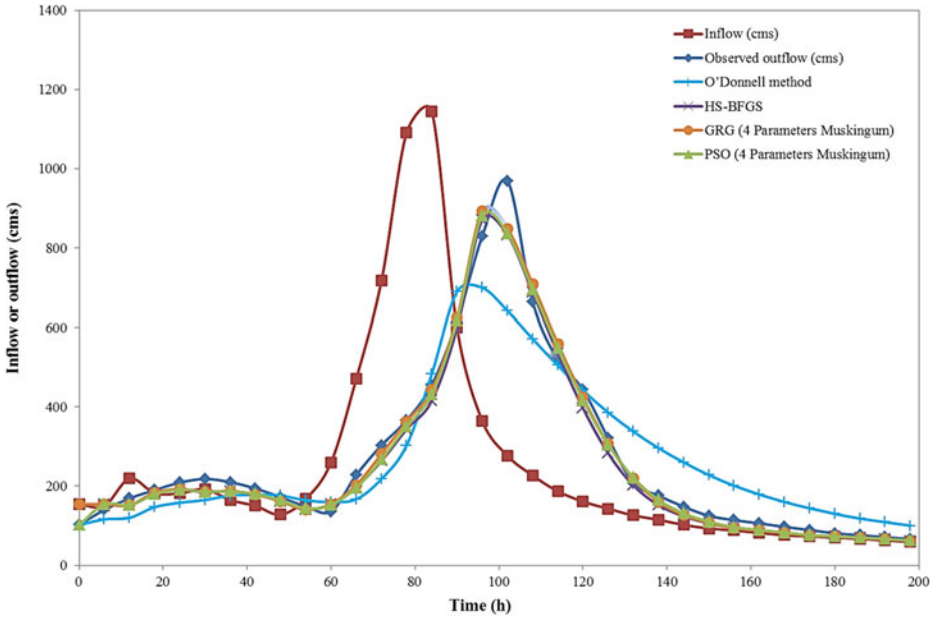

Fig. 3 Inflow, observed and computed outflows hydrographs with different methods by using the River Wye data

(SSQ = 76,785). In addition, in this example, the EQ_p, ET_p, *MARE* and *VarexQ* was determinated 0.0358, zero, 0.067 and 98.27 %, respectively by PSO method. In this example there are not the more datails of routed flow by Easa method to compare the results in terms of EQ_p, ET_p, *MARE* and *VarexQ*. The inflow, observed and estimated hydrograph by the PSO

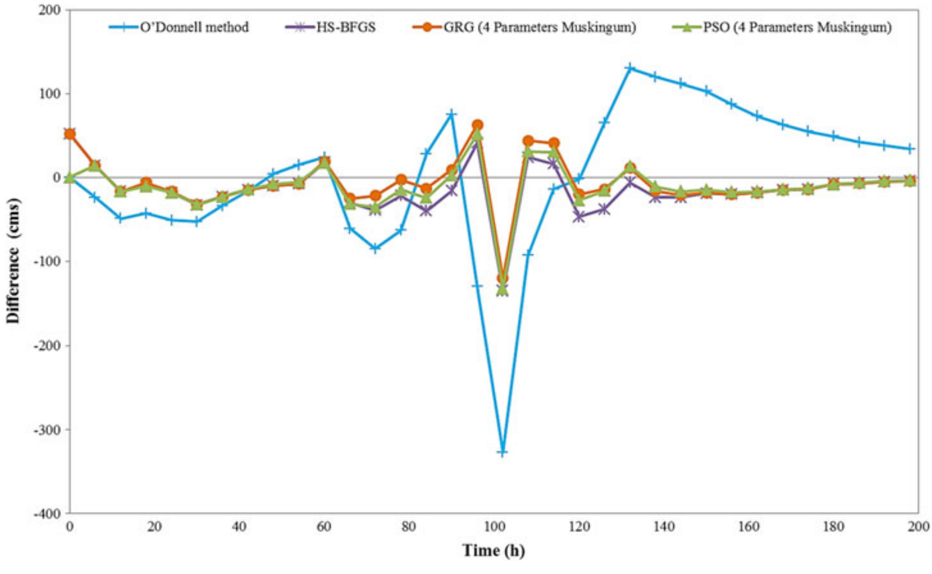

Fig. 4 Comparison the difference between the routed and observed outflows with various methods by using the River Wye data

Table 4 Comparison of the observed and best-routed outflows for Viessman and Lewis

Time (h)	Inflow (cms)	Observed outflow (cms)	PSO (4 Parameters Muskingum)
0	166.2	118.4	118.4
1	263.6	197.4	166.2
2	365.3	214.1	265.3161
3	580.5	402.1	349.5366
4	594.7	518.2	509.3545
5	662.6	523.9	565.471
6	920.3	603.1	623.6715
7	1568.80	829.7	780.4317
8	1775.50	1124.20	1124.2
9	1489.50	1379.00	1390.918
10	1223.30	1509.30	1455.157
11	713.6	1379.00	1378.246
12	645.6	1050.60	1107.706
13	1166.70	1013.70	865.7737
14	1427.20	1013.70	982.9366
15	1282.80	1013.70	1173.765
16	1098.70	1209.10	1234.349
17	764.6	1248.80	1186.476
18	458.7	1002.40	1003.865
19	351.1	713.6	713.4868
20	288.8	464.4	470.8424
21	228.8	325.6	325.6
22	170.2	265.6	235.3666
23	143	222.6	163.1713

algorithm shown in Fig. 5. It is obvious that the new applied method yields a considerable improvement in fitting multi-peak hydrographs. The optimal values of K, x, a and m are equal to 0.0605, 0.1310, 0.3213 and 4.574 respectively, which obtained after 500 iterations. The PSO parameters were obtained similar to Example 2.

5.4 Numerical Experiment 4

This example is about the Karoon River in Iran, which firstly have been gathered by Orouji et al. (2013). Karoon River is one of the great rivers in the southwest of Iran that supply urban, agricultural and industrial demand in the area. The applied data recorded by two hydrometric stations, Godar and Gotvand (Orouji et al. 2013). They applied SFLA and SA optimization techniques based on non-linear Muskingum (3 parameters) and reported the best values of SSQ and SAD 130,928.6 and 1835.6, respectively. The proposed method computed these parameters 68,790.84 and 1067.102, respectively, which indicate 47 % and 42 % improvement, respectively.

Table 5 and Fig. 6 present the results of estimated outflow respect to inflow and observed outflow by using proposed method and previous methods. Also, the PSO can calculated the EQ_p, *MARE* and *VarexQ* equal to 0.0132, 0.03 and 98.05 %, respectively, which have the more improvement than SA and SFLA methods.

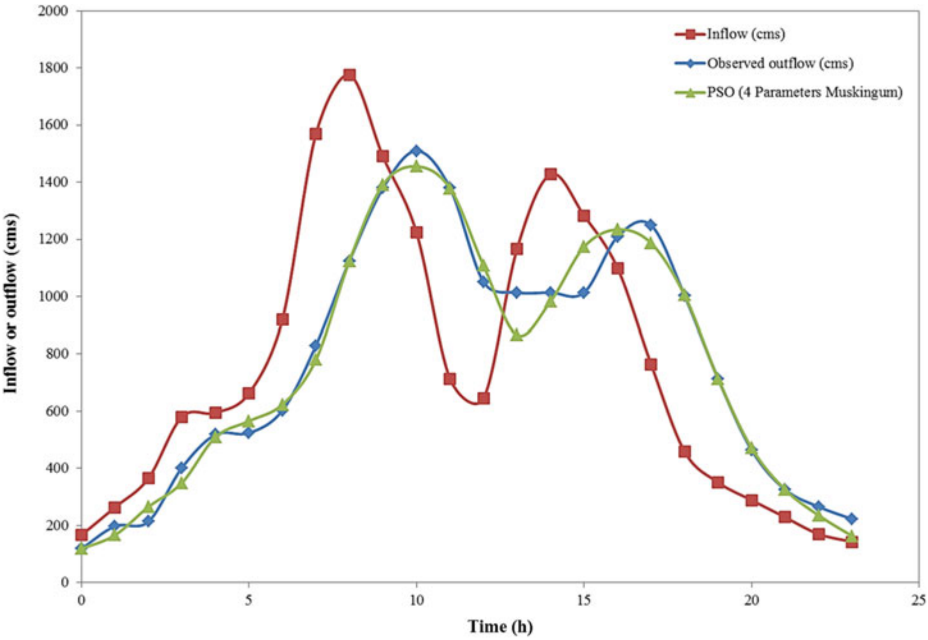

Fig. 5 Inflow, observed and computed outflows hydrographs for Viessman and Lewis

The optimal values of K, x, a and m computed 7740.68, −0.0527, 0.1786 and 0.9911, respectively. In addition, the maximum number of iterations was 300. In this study, only the initial population set to 500 and other parameters did not change.

6 Conclusions

Most of the researcher in the past only focused on robust optimization algorithms to improve the fitting performance of the nonlinear Muskingum model, which considered as an exponent parameter of the weighted storage. However, the resulted achievements in this field were not very considerable. In this paper, the four new non-linear Muskingum model that has introduced firstly by Easa (2013) to compute the routed outflow at a reach of river has used. The new Muskingum model which was based on power function of weighted storage was more suitable than linear function. The new Muskingum model considers more complex than previous models. But, it is not a non solver problem with recent advances in modern optimization algorithms which can find the optimal parameters for all of highly non-linear and non-convex problems.

In this research, the PSO algorithm has employed to estimate the parameters in the new non-linear Muskingum model. The performance of these algorithms compared with other techniques in literature based on minimizing the two objective functions, SSQ and SAD. In this regards, three benchmark examples and one real case in Iran were investigated. The results showed the PSO algorithm which uses the four-parameter in the Muskingum model, can compute the best values of SAD factor rather than all used other techniques in the examples. The SSQ factor improved in all studied cases except the Wilson example. On the other hand, the SAD, EQ_p, ET_p, *MARE* and *VarexQ* were acceptable in Wilson example.

Table 5 Comparison of the observed and best-routed outflows for Karoon River

Time (h)	Inflow (cms)	Observed outflow (cms)	SA	SFLA	PSO (4 Parameters Muskingum)
0	380	380.00	380.00	380.00	380.00
2	430	383.50	380.00	380.00	383.50
4	445	387.00	387.29	386.51	383.50
6	460	393.00	402.58	401.33	389.86
8	475	399.00	417.41	415.88	397.11
10	490	408.50	431.93	430.24	405.20
12	505	418.00	446.27	444.49	414.10
14	520	436.50	460.48	458.66	423.75
16	540	455.00	474.62	472.79	434.12
18	560	470.00	489.18	487.32	445.88
20	595	485.00	504.89	503.00	458.77
22	630	496.00	522.60	520.58	474.93
24	770	507.00	544.22	542.07	493.64
26	910	523.50	574.93	572.13	530.96
28	995	540.00	628.20	624.92	583.12
30	1080	558.00	694.42	691.19	641.80
32	1095	576.00	760.32	757.47	709.07
34	1110	702.50	826.08	823.81	770.97
36	1125	829.00	879.56	877.90	829.00
38	1140	938.50	924.16	923.04	882.62
40	1170	1048.00	962.14	961.50	931.59
42	1200	1061.50	994.84	994.62	979.48
44	1250	1075.00	1026.37	1026.55	1025.61
46	1300	1082.00	1056.48	1057.07	1075.02
48	1255	1089.00	1089.37	1090.39	1126.43
50	1210	1103.00	1127.60	1128.80	1154.07
52	1180	1117.00	1151.20	1152.35	1165.31
54	1150	1149.50	1162.38	1163.39	1167.58
56	1125	1182.00	1166.44	1167.22	1162.35
58	1100	1153.00	1164.43	1164.95	1152.78
60	1070	1124.00	1158.38	1158.64	1139.74
62	1040	1099.50	1149.10	1149.09	1122.79
64	995	1075.00	1136.05	1135.78	1103.12
66	950	1061.50	1120.20	1119.65	1077.57
68	885	1048.00	1098.77	1097.97	1048.30
70	820	1011.00	1072.96	1071.89	1011.34
72	800	974.00	1039.00	1037.73	969.68
74	780	935.50	997.51	996.20	935.50
76	760	897.00	959.23	957.99	904.95
78	740	872.50	923.71	922.61	877.14
80	730	848.00	890.53	889.63	851.40
82	720	829.50	859.43	858.76	829.47
84	720	811.00	832.20	831.75	810.05

Table 5 (continued)

	Time (h)	Inflow (cms)	Observed outflow (cms)	SA	SFLA	PSO (4 Parameters Muskingum)
	86	720	797.00	808.34	808.09	794.72
	88	730	783.00	789.34	789.24	782.18
	90	740	774.50	774.50	774.50	773.96
	92	750	766.00	765.02	765.06	768.80
SSQ				135,809.42	130,928.65	68,790.84
SAD				1873.71	1835.63	1067.10
EQ_p				0.0132	0.0125	0.0122
ET_p				0	0	2
MARE				0.06	0.05	0.03
VarexQ				96.06	96.20	98.05

The results confirmed that the PSO algorithm estimated the four parameters in the new Muskingum model with high accuracy along with a fast rate of convergence. In addition, the PSO method does not necessary assumption of initial values of the model parameters and no derivative. PSO can be achieved the nearly global optimum solution in lowest iterations by setting right the its parameters. The other experiences of PSO applications have shown that the main and sensitive parameters in this algorithm are c_1 and c_2 which can set $c_1 = c_2 = 2.05$ presented the best answer (Carvalho and Ludermir 2006; Prakash and Sydulu 2007; Li et al. 2010). These values are similar to the results of the present research. Consequently, by using the PSO with new four-parameters Muskingum models improved the outflow forecasting, significantly.

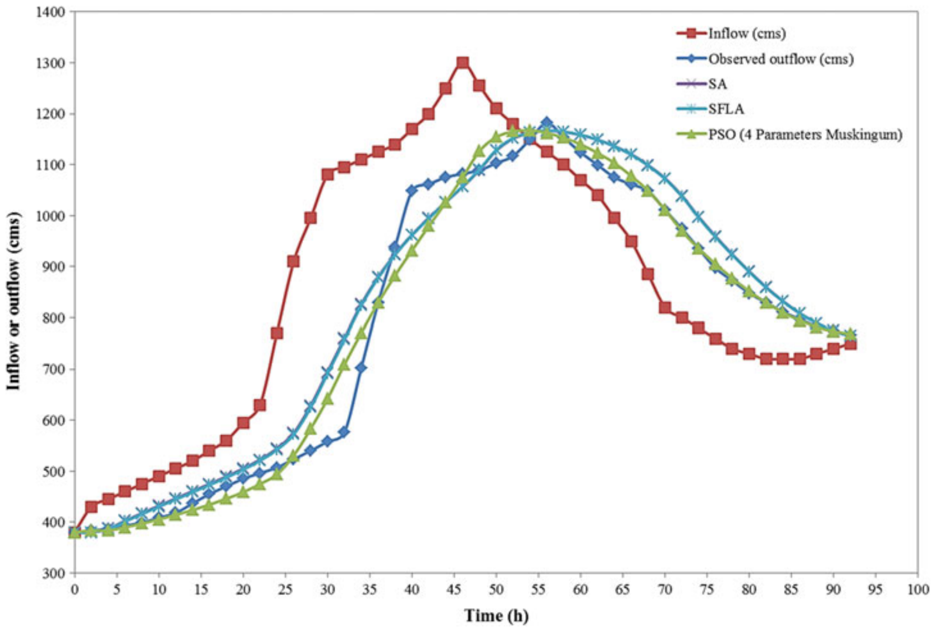

Fig. 6 Inflow, observed and computed outflows hydrographs for Karoon river

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

References

Afshar A, Kazemi H, Saadatpour M et al. (2011) Particle swarm optimization for automatic calibration of large scale water quality model (CE-QUAL-W2): application to Karkheh Reservoir, Iran. Water Resour Manag 2613–2632. doi: 10.1007/s11269-011-9829-7

Al-Humoud JM, Esen II (2006) Approximate methods for the estimation of Muskingum flood routing parameters. Water Resour Manag 20:979–990. doi:10.1007/s11269-006-9018-2

AT Committee, others (1993) The ASCE task committee on definition of criteria for evaluation of watershed models of the watershed management committee Irrigation and Drainage Division, Criteria for evaluation of watershed models. J Irri Drain Engng, ASCE 119:429–442

Bajracharya K, Barry DA (1997) Accuracy criteria for linearised diffusion wave flood routing. J Hydrol 195: 200–217

Barati R (2011) Parameter estimation of nonlinear Muskingum models using Nelder-Mead simplex algorithm. J Hydrol Eng 365–367. doi: 10.1061/(ASCE)HE.1943-5584.0000367

Barati R (2013) Application of Excel Solver for parameter estimation of the nonlinear Muskingum models

Carvalho M, Ludermir T (2006) An analysis Of PSO hybrid algorithms for feed-forward neural networks training. 2006 Ninth Brazil Symposium Neural Networks (SBRN'06) 2–2. doi: 10.1109/SBRN.2006.10.

Chau K (2005) A split-step PSO algorithm in prediction of water quality pollution. 1034–1039

Chu H-J (2009) The Muskingum flood routing model using a neuro-fuzzy approach. KSCE J Civ Eng 13:371–376. doi:10.1007/s12205-009-0371-6

Chu H, Chang L (2009) Applying particle swarm optimization to parameter estimation of the nonlinear Muskingum model. J Hydrol Eng 17:1139–1148. doi:10.1007/s12205-013-0037-2

Das A (2004) Parameter estimation for Muskingum models. J Irrig Drain Eng 0:140–147

Das A (2007) Chance-constrained optimization-based parameter estimation for Muskingum models. J Irrig Drain Eng 133:487–494

De Moura Meneses AA, Machado MD, Schirru R (2009) Particle swarm optimization applied to the nuclear reload problem of a pressurized water reactor. Prog Nucl Energy 51:319–326

Easa S (2013) New and improved four-parameter non-linear Muskingum model. Proc ICE-Water Manag 1–3.

Eberhart R, Kennedy J (1995) A new optimizer using particle swarm theory. MHS'95 Proc Sixth Int Symposium Micro Mach Human Sci. doi: 10.1109/MHS.1995.494215

Geem Z (2006) Parameter estimation for the nonlinear Muskingum model using the BFGS technique. J Irrigation Drain Eng 1270–1279

Geem Z (2010) Parameter estimation of the nonlinear Muskingum model using parameter-setting-free harmony search. J Hydrol Eng 684–688. doi: 10.1061/(ASCE)HE.1943-5584.0000352

Gill MA (1978) Flood routing by the Muskingum method. J Hydrol 36:353–363

Je N, Sutcliffe JV (1970) River flow forecasting through conceptual models part I—a discussion of principles. J Hydrol 10:282–290

Karahan H, Gurarslan G, Geem Z et al. (2012) Parameter estimation of the nonlinear Muskingum flood-routing model using a hybrid harmony search algorithm. J Hydrol … 352–360. doi: 10.1061/(ASCE)HE.1943-5584.0000608.

Kim JH, Geem ZW, Kim ES (2001) Parameter estimation of the nonlinear Muskingum model using harmony search. J Am Water Resour Assoc 37:1131–1138. doi:10.1111/j.1752-1688.2001.tb03627.x

Kuo RJ, Hong SY, Huang YC (2010) Integration of particle swarm optimization-based fuzzy neural network and artificial neural network for supplier selection. Appl Math Model 34:3976–3990. doi:10.1016/j.apm.2010.03.033

Latt ZZ (2015) Application of feedforward artificial neural network in Muskingum flood routing : A Black-Box Forecasting Approach Nat River Syst. doi: 10.1007/s11269-015-1100-1

Li Y, Bai B, Zhang Y (2010) Improved particle swarm optimization algorithm for fuzzy multi-class SVM. Syst Eng Electron, J 21:509–513

Lu WZ, Fan HY, Leung AYT, Wong JCK (2002) Analysis of pollutant levels in central Hong Kong applying neural network method with particle swarm optimization. Environ Monit Assess 79:217–230

Luo J, Xie J (2010) Parameter estimation for nonlinear muskingum model based on immune clonal selection algorithm. J Hydrol Eng 15:844–851. doi:10.1061/(ASCE)HE.1943-5584.0000244

Mays LW (2010) Water resources engineering. John Wiley & Sons
McCuen RH, Knight Z, Cutter AG et al. (2006) Evaluation of the Nash–Sutcliffe efficiency index
Meraji SH (2004) Optimum design of flood control systems by particle swarm optimization algorithm. M. Sc. thesis, Iran University of Science and Technology
Mohan S (1997) Parameter estimation of nonlinear Muskingum models using genetic algorithm. J Hydraul Eng 123:137–142
Nagesh Kumar D, Janga Reddy M (2007) Multipurpose reservoir operation using particle swarm optimization. J Water Resour Plan Manag 133:192–201
Natural Environment Research Council (NERC) (1975) Flood studies report. Institute of Hydrology, Wallingford, U.K.
Niazkar M, Afzali SH (2014) Assessment of modified honey bee mating optimization for parameter estimation of nonlinear muskingum models. 1–13. doi: 10.1061/(ASCE)HE.1943-5584
O'DONNELL T (1985) A direct three-parameter Muskingum procedure incorporating lateral inflow. Hydrol Sci J 30:479–496
Orouji H, Haddad OB, Fallah-Mehdipour E, Mariño MA (2013) Estimation of Muskingum parameter by meta-heuristic algorithms. Proc Inst Civil Eng: Water Manag 166(6):315–324. doi:10.1680/wama.11.00068
Prakash K, Sydulu M (2007) Particle swarm optimization based capacitor placement on radial distribution systems. In: Power Engineering Society General Meeting, 2007. IEEE. pp 1–5
Toprak ZF (2009) Flow discharge modeling in open canals using a new fuzzy modeling technique (SMRGT). CLEAN–Soil, Air, Water 37:742–752
Tung Y-K (1985) River flood routing by nonlinear Muskingum method. J Hydraul Eng 111:1447–1460
Wilson EM (1974) Engineering hydrology.Macmillan Education, Hampshire, U.K
Xu D-M, Qiu L, Chen S-Y (2011) Estimation of nonlinear Muskingum model parameter using differential evolution. J Hydrol Eng 17:348–353
Yoon J, Padmanabhan G (1993) Parameter estimation of linear and nonlinear Muskingum models. J Water Resour Plan Manag 119:600–610
Zaji AH, Bonakdari H (2014) Performance evaluation of two different neural network and particle swarm optimization methods for prediction of discharge capacity of modified triangular side weirs. Flow Meas Instrum 40:149–156