

A Time-Dependent Drought Index for Non-Stationary Precipitation Series

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Abstract In a rapidly changing environment, a greater concern about the establishment and improvement of drought indices is expected. The main goal of this study is to develop and apply a time-dependent Standardized Precipitation Index (SPI_t) that takes account of the possible non-stationary behaviors in precipitation records. Summer precipitation observations (1959 ~ 2011) from 21 raingauge stations in the Luanhe River basin are fitted with non-stationary Gamma distributions respectively by means of the Generalized Additive Models in Location, Scale and Shape (GAMLSS). The temporal variability of the distribution's parameter (related to the mean) is flexibly described by an optimized polynomial function. Based on the non-stationary distribution, the SPI_t is calculated and then employed to assess the spatio-temporal characteristics of summer drought in the basin. Results of the non-stationary modeling indicate an overall decreasing trend in the summer precipitation during 1959 ~ 2011, and especially a significant decrease in the period of 2000 to 2011. The SPI_t is found to be more robust and reliable compared with the traditional Standardized Precipitation Index (SPI). Moreover, remarkable difference is observed between the historical drought assessments of SPI_t and SPI in the Luanhe River basin, implying that the non-stationarity of hydrological time series cannot be ignored in drought analyses and forecasts. The proposed SPI_t method can be a feasible alternative for drought monitoring under non-stationary conditions, intended to provide a valuable reference for further studies.

Keywords Drought · Standardized precipitation index · Time-dependent standardized precipitation index · Non-stationarity · GAMLSS

1 Introduction

Drought is recognized as one of the most complex natural phenomenon with different temporal and spatial characteristics, affecting economy, society and environment significantly (Wilhite

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et al. 2007; Vicente-Serrano et al. 2012; Sol'áková et al. 2014). Due to the effects of climate change and anthropogenic stress, drought disasters become more frequent and extensive in recent decades and have caused huge economic damage and human suffering in many areas across the world (Villarini et al. 2011; Wen et al. 2011; Yoo et al. 2012; Li et al. 2013; Wang et al. 2015).

Up to now, the best and most widely used approach for characterizing droughts is to establish an appropriate indicator, which has the ability to identify drought characteristics and assess the effects of drought quantitatively (Mishra and Singh 2010; Li et al. 2013; Tabari et al. 2013; Duan and Mei 2014). Depending on research objectives, a wide range of drought indices have been developed in previous literatures, including the Palmer Drought Severity Index (Palmer 1965), the Crop Moisture Index (Palmer 1968), the Surface Water Supply Index (Shafer and Dezman 1982), the Standardized Precipitation Index (SPI) (McKee et al. 1993), the Standardized Runoff Index (Shukla and Wood 2008), the Standardized Precipitation Evapotranspiration Index (Vicente-Serrano et al. 2010) and the joint deficit index (Kao and Govindaraju 2010).

With the advantages of computational simplicity, the SPI has become the most accepted and robust index, arguably considered as an essential element for an efficient drought monitoring system (Vicente-Serrano 2006; Hayes et al. 2011; Pasho et al. 2011; Li et al. 2012). It not only can be calculated at various time scales, but also is capable of statistically comparing drought severity both in time and space (Bonaccorso et al. 2003; Vicente-Serrano 2006; Patel et al. 2007; Wen et al. 2011). Moreover, as a probability-based index, the SPI are consequently sensitive to the factors and assumptions that govern probabilistic hydrology (Hosking and Wallis 1997; Angelidis et al. 2012; Russo et al. 2013).

One of the fundamental assumptions in statistical inferences for hydrological time series is the assumption of stationarity (Salas 1993; Strupczewski et al. 2001). Stationary hydrological series keep its distributional properties invariant with time, implying free of trends and abrupt changes (Brillinger 2001; Serinaldi and Kilsby 2015). However, the stationary assumption has been widely questioned in the context of global warming and intensive man-induced disturbances (e.g., Held and Soden 2006; Vicente-Serrano and López-Moreno 2008; Hejazi and Markus 2009; Yang and Tian 2009; Wang et al. 2013; Chang et al. 2015; Zhang et al. 2012). In addition, several recent studies conducted in various regions of the world reveal clear violations of this assumption (e.g., Villarini et al. 2010; Wilson et al. 2010; Giraldo Osorio and García Galiano 2012; Wagesho et al. 2012; Ishak et al. 2013).

Computation of the traditional SPI involves fitting a stationary gamma distribution to given precipitation records, which has relied heavily on the assumption of stationarity. Hence, under a changing environment, non-stationary behaviors exhibited in precipitation observations would make the availability and validity of traditional SPI most likely diminished, and accordingly calls for alternative indicators that can furnish opportunities to directly identify droughts under non-stationary conditions. Finding out such an appropriate non-stationary drought index is of particular importance for implementing adequate adaptation and mitigation strategies. With this point of view, some researchers have attempted to establish new drought indicators which can account for the non-stationarity. For example, Türkeş and Tatlı (2009) proposed a modified SPI which considers the local-time means of the precipitation series, and suggested that the modified SPI methodology could be successfully applied in the regions where the precipitation series with high variability. Russo et al. (2013) developed the Standardized Nonstationary Precipitation Index (SnsPI) using a non-stationary Gamma

distribution with a linearly time varying mean, finding that the SnsPI is more robust than the common SPI for the projections of dryness and wetness under climate changes.

In order to model non-stationary time series, different techniques have been introduced in previous literatures (Coles 2001; Khaliq et al. 2006; El Adlouni et al. 2007; Strupczewski et al. 2009; Gilleland et al. 2013; Vasiliades et al. 2015; Xiong et al. 2015). The Generalized Additive Model in Location, Scale and Shape (GAMLSS) proposed by Rigby and Stasinopoulos (2005) has received significant attention. This model provides a high degree of flexibility to address non-stationary probabilistic modeling, and thus has been successfully applied for a variety of non-stationary analyses in hydrology (e.g., Villarini et al. 2009a; Giraldo Osorio and García Galiano 2012; López and Francés 2013). Recently, several researchers used GAMLSS for non-stationary modeling of long-term hydrologic records (such as rainfall and temperature), and found linear or nonlinear changes over time in the hydrometeorological variables (e.g., Villarini et al. 2009b; Villarini et al. 2010; Jiang and Xiong 2012). The Mann-Kendall trend test (Mann 1945; Kendall 1975), regarded as the most widely used method for trend detection in hydrology, could effectively verify the trend behaviors modeled by GAMLSS model. As to improving the existing drought indices, the GAMLSS is apparently expected to be an effective tool to account for the non-stationarity, although little attention has been paid to this aspect so far.

The Luanhe River basin located in North China serves as the principal water supply for the Tianjin city. In this basin, the summer precipitation (from June to August) that accounts for about 70 % of the annual precipitation plays a key role in the water budgets of the hydrological cycle. Combined with concentrated agricultural activities, summer is the most sensitive season for triggering drought episodes. Therefore, proper understanding and monitoring the evolution of summer drought has great significance to regional agricultural production and socio-economic development.

In this study, we proposed and applied a time-dependent Standardized Precipitation Index (SPI_t) that is sufficiently robust to monitor droughts under non-stationary conditions, expecting to provide a new perspective to establish more appropriate drought indices. Focusing on summer droughts in the Luanhe River basin, the GAMLSS was used to model summer precipitation records over the period of 1959 ~ 2011 with non-stationary Gamma distributions. Then the SPI_t was developed based on the non-stationary model so as to incorporate the possible non-stationarity of observed precipitation. The performances of the SPI_t and traditional SPI were compared in order to further test the robustness and reliability of SPI_t.

2 Study Area and Data

The study area, Luanhe River basin, is located in the north of the Haihe River basin, China, between 115°30'~119°15'E longitude and 39°10'~42°30'N latitude (Fig. 1). It covers a drainage area of 33,700 km², of which mountainous area accounts for nearly 98 % while plain area accounts for nearly 2 %. The elevation within the basin varies from more than 2200 m in the northwest, to less than 2 m in the southeast. This area belongs to the temperate continental monsoon climate, and receives mean annual precipitation of 400~700 mm with highly seasonal and interannual variability. About 70~80 % of annual precipitation is concentrated in the rainy months (June to September), especially in July and August. Its mean annual temperature and potential evapotranspiration are -0.3~11 °C and 950~1150 mm respectively.

The Luanhe River basin acts as the main source of water for economic, social and environmental activities in the Tianjin city, the largest opening coastal city of North China.

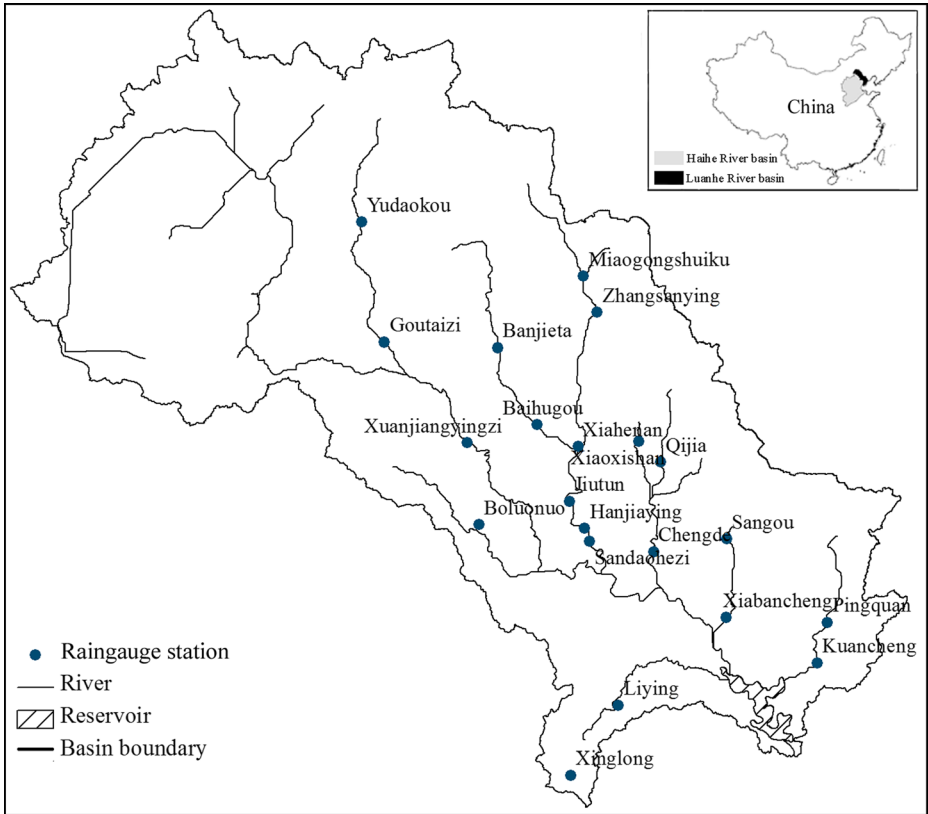


Fig. 1 Location of the Luanhe River basin and the 21 raingauge stations

However, in recent decades, the amount of water supplied to the city continuously reduced, mainly due to the consecutive droughts and soil and water conservations in the basin (Li and Feng 2007). Indeed, the Luanhe River basin has experienced much more frequent and extreme drought since the late 1990s (Wang et al. 2015). Several severe droughts for 1961, 1963, 1968, 1972, 2007 and 2009, and multi-year droughts for 1980 ~ 1984 and 1997 ~ 2005 have been reported for this region (Ma et al. 2013; Yang et al. 2013).

Historical precipitation records were obtained from 21 raingauge stations which measure precipitation only. The locations of the selected stations are presented in Fig. 1. The monthly precipitation data from all of these stations are available during the entire period 1959 ~ 2011. The data were originally provided by Hydrology and Water Resource Survey Bureau of Hebei Province.

3 Method

3.1 Generalized Additive Models in Location, Scale and Shape (GAMLSS)

Generalized Additive Models for Location, Scale and Shape (GAMLSS) introduced by Rigby and Stasinopoulos (2005) has been proved to be a valuable and flexible modeling framework for analyzing the time series with non-stationary behaviors (Villarini et al. 2009b; Villarini

et al. 2010). GAMLSS are (semi) parametric regression type models, in which a parametric distribution assumption is required for the response variable, and the selected distribution’s parameters can vary as a linear and/or nonlinear function of explanatory variables and/or random effects.

A brief introduction to the theory behind GAMLSS is provided here. For a more comprehensive discussion, refer to Rigby and Stasinopoulos (2005) and Stasinopoulos and Rigby (2007). In a GAMLSS model, observations y_t for $t=1,2,\dots,n$ are assumed to be independent and fitted to a distribution function $f(y_t|\theta^t)$, conditional on $\theta^t=(\theta_{t1},\theta_{t2},\dots,\theta_{tp})$, representing a vector of p distribution parameters at time t . A general distribution family, including highly skew and/or kurtotic continuous and discrete distributions, is supported by GAMLSS. The distribution parameters θ characterized as location, scale, and shape parameters are related to explanatory variables by monotonic link functions $g_k(\cdot)$, $k=1,2,\dots,p$, given by:

$$g_k(\theta_k) = \eta_k = \mathbf{X}_k\beta_k + \sum_{j=1}^{J_k} \mathbf{Z}_{jk}\gamma_{jk} \tag{1}$$

where θ_k and η_k are vectors of length n , $\theta_k=(\theta_{1k}, \theta_{2k}, \dots, \theta_{nk})^T$, $\beta_k=(\beta_{1k}, \beta_{2k}, \dots, \beta_{Jkk})^T$ is a parameter vector of length J_k , \mathbf{X}_k is a fixed known design matrix of order $n \times J_k$, \mathbf{Z}_{jk} is a fixed known $n \times q_{jk}$ design matrix, and γ_{jk} is a q_{jk} dimensional random variable. In Eq. (1), the η_k , for $k=1,\dots, p$, are comprised of a parametric component $\mathbf{X}_k\beta_k$ (functions of explanatory variables) and additive components $\mathbf{Z}_{jk}\gamma_{jk}$ (random effects). If $J_k=0$, the model is reduced to a fully parametric GAMLSS model.

In this study, GAMLSS were applied to non-stationary modeling of summer precipitation for the Luanhe River basin. Within the GAMLSS framework, the precipitation time series were assumed to be distributed by 2-parameter Gamma distribution with its location parameter μ (related to mean) linked to time. Since the observations might show non-linear behaviors over time, the temporal variability of the location parameter was described by an optimized polynomial function (Eq. (2)), instead of assuming that the parameter is a linear function of time.

$$g_1(\mu_t) = g_1[\mu(t)] = a_0 + a_1t + \dots + a_qt^q \tag{2}$$

where a_i is the polynomial coefficient, $i=0,1,\dots,q$, and q is the degree of polynomial.

The dependence of the parameter on time can be linear or smooth through the defined polynomial function. For modeling with a balance between accuracy and complexity, the degree of polynomial was optimized using the Akaike Information Criterion (AIC) (Akaike 1974) and the Schwarz Bayesian Criterion (SBC) (Schwarz 1978). The RS algorithm was used for parameters estimation in the GAMLSS framework, with the objective of maximizing the penalized likelihood function (Rigby and Stasinopoulos 1996a, b). To test the adequacy of fit, the residuals of each model were checked by analyzing the first four moments of their distribution (Dunn and Smyth 1996) and Filliben correlation (Filliben 1975), together with visual inspection of diagnostic plots of the residuals, such as residuals vs. response, Q-Q plots or worm plots (Stasinopoulos and Rigby 2007). The independence and normality of the residuals ensure that the residual information can be explained as random signal, indicating an adequate description of the systematic information provided by the model. The calculations related to GAMLSS modeling in this study were performed in R using the available gamlss package (Stasinopoulos and Rigby 2007).

3.2 Traditional Standardized Precipitation Index (SPI)

The Standardized Precipitation Index (SPI) originally proposed by McKee et al. (1993) is one of the most useful indices for monitoring and assessing drought conditions. The procedure of traditional SPI calculation involves fitting a 2-parameter Gamma distribution to a given time series of precipitation. The Gamma distribution has a probability density function (PDF) defined as follows:

$$f_X(x_k|\mu, \sigma) = \frac{1}{(\sigma^2\mu)^{1/\sigma^2}} \frac{x_k^{\frac{1}{\sigma^2}-1} \exp[-x_k/(\sigma^2\mu)]}{\Gamma(1/\sigma^2)}; x_k > 0, \mu > 0, \sigma > 0 \tag{3}$$

where μ and σ are usually characterized as location and scale parameters, x_k is the amount of precipitation over k consecutive months, and $\Gamma(\cdot)$ is the mathematical Gamma function.

The 2-parameter Gamma distribution is denoted as Gamma (μ, σ). The expectation and variance of the variable $X \sim$ Gamma (μ, σ) are

$$E(X) = \mu; \quad Var(X) = \sigma^2\mu^2 \tag{4}$$

According to the approximate conversion provided by Abramowitz and Stegun (1965), the cumulative probability of x_k is then transformed to a standard normal deviation with a zero mean and unit variance, which is the value of SPI. The values of SPI are climatologically consistent for any location because of the standardization relative to a specific period (Russo et al. 2013). Table 1 shows the range of SPI values along with their classifications (McKee et al. 1995).

3.3 Time-dependent Standardized Precipitation Index (SPIt)

The time-dependent Standardized Precipitation Index (SPIt) is defined analogously to SPI but based on a non-stationary Gamma distribution with its location parameter changing over time. This non-stationary distribution is developed by using GAMLSS as described in Section 3.1.

The SPIt is calculated in the following sequence. (1) The time scale of k months is specified depending on desired application, and the time series of k -month consecutive precipitation x_k are prepared. (2) Within the GAMLSS framework, a non-stationary model is developed by fitting the precipitation data x_k to a non-stationary Gamma distribution. The location parameter of this distribution is described as an optimized polynomial function of time (i.e., $g_1[\mu(t)] = a_0 + a_1t + \dots + a_qt^q$) which is selected by minimizing AIC and SBC. Thus the precipitation amount x_k at time t (represented as x_{kt}) is modeled as $x_{kt} \sim$ Gamma (μ_t, σ). (3) The cumulative probability of x_k formed from the non-stationary Gamma distribution is converted to a standard normal deviate (with zero mean and unit variance), which is calculated using the approximate conversion provided by Abramowitz and Stegun (1965). This standard normal value is the

Table 1 Drought classification of SPI and SPIt

Drought condition	Index value	Drought category
Normal	Great than 0	D0
Near normal	-1.0~0	D1
Moderate drought	-1.5~-1.0	D2
Severe drought	-2.0~-1.5	D3
Extreme drought	Less than -2.0	D4

SPI_t for the particular precipitation at time t . A schematic of the procedure for calculating the SPI_t is presented in Fig. 2.

A Positive SPI_t implies a wet condition, while negative values of SPI_t indicate dry conditions. Since the SPI_t is normalized (same as SPI), the classifications listed in Table 1 can be adopted for the SPI_t as well. As compared with the traditional SPI, the SPI_t has the ability to capture and model non-stationary in longer records, and therefore it is expected to be more reasonable and satisfactory for drought analyses.

3.4 Mann-Kendall Test

The nonparametric Mann-Kendall trend test was derived by Mann (1945) and Kendall (1975), and has been widely used to test trend in hydrology. According to this test, the null hypothesis H_0 states that the data (x_1, \dots, x_n) is a sample of n independent and identically distributed random variables. The test statistic Z is calculated by the following formula.

$$Z = \begin{cases} (S-1)/\sqrt{\text{Var}(S)} & \text{if } S > 0 \\ 0 & \text{if } S = 0 \\ (S+1)/\sqrt{\text{Var}(S)} & \text{if } S < 0 \end{cases} \quad (5)$$

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^n \text{sgn}(x_j - x_k), \quad \text{sgn}(x_j - x_k) = \begin{cases} +1 & \text{if } (x_j - x_k) > 0 \\ 0 & \text{if } (x_j - x_k) = 0 \\ -1 & \text{if } (x_j - x_k) < 0 \end{cases} \quad (6)$$

where x is the variable; n is the sample size; the statistical S is approximately distributed normally when $n \geq 10$, with its variance $\text{Var}(S) = [n(n-1)(2n+5)]/18$.

In a two-sided test for trend, H_0 should be accepted if $|Z| \leq Z_{1-\alpha/2}$ at the level of α significance. Alternatively, the presence of a trend is accepted if $Z < -Z_{1-\alpha/2}$ or $Z > Z_{1-\alpha/2}$, designating a decreasing trend or an increasing trend respectively.

4 Results

4.1 Modeling with GAMLSS

The non-stationary probabilistic models of summer precipitation for the 21 stations were built using 2-parameter Gamma distribution in the GAMLSS framework, with the distribution's

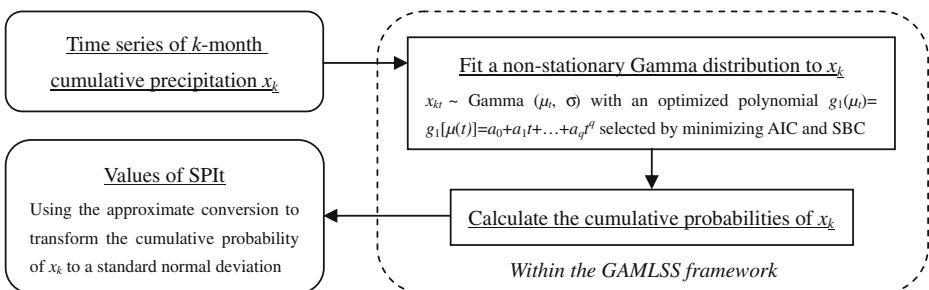


Fig. 2 Procedure for the calculation of the SPI_t

location parameter varying as a polynomial function of time. The degree of the polynomial (q) considered in this study was less than or equal to 3, so as to avoid an over-complicated function and provide enough flexibility to describe the variation. For $q=0$, the model simplified to a stationary model with constant parameters. The optimized time-varying model for each station was selected by minimizing AIC and SBC. Table 2 provides the parameter estimates of the selected models.

It can be seen in the Table 2 that in most sites (about 86 %) the non-stationary models offer the best overall results for fitting the summer precipitation in the Luanhe River basin. Among the 21 selected models, only 3 models are found to be stationary with time independent parameters, while 7 models characterize their location parameters as linear functions of time. Moreover, the parameter μ includes time dependence via a quadratic polynomial in 7 cases, as well as a cubic polynomial in 4 cases. These results imply that a polynomial function is effective and flexible enough to highlight linear or non-linear dependencies in precipitation properties over time.

In order to evaluate the quality of fit, the mean, variance, skewness, kurtosis, and Filliben correlation coefficient of the residuals for each selected model were computed and summarized in Table 2. Visual inspections of worm plot were also performed to check the residuals normality. Take Pingquan and Zhangsanying stations as examples (Fig. 3a and b). Similar results were found for the other stations.

For Pingquan station, the residuals have a mean of 0.00, a variance of 1.02, a coefficient of skewness of -0.01 , and a coefficient of kurtosis of 2.83, implying that the residuals are approximately normally distributed. The independence of the residuals is verified with a fairly acceptable Filliben correlation coefficient of 0.995 (for a sample size of 53, the critical value of the Filliben's coefficient is 0.978). The visual inspection of worm plot (Fig. 3a) also supports the inference that the model fit the data adequately. As to Zhangsanying station, the residuals with a mean of 0.00, a variance of 1.02, a coefficient of skewness of -0.05 , a coefficient of kurtosis of 2.59, and a Filliben correlation coefficient of 0.986, indicate that they are likely to be independently and identically distributed random noise. Furthermore, the residual plots shown in Fig. 3b do not highlight any significant departure from normality. Accordingly, these results demonstrate that the selected model tends to describe the variability exhibited by the data reasonably well.

4.2 Trend Analyses of Summer Precipitation

The statistic Z values of Mann-Kendall (MK) test focusing on the 53-year summer precipitation records of the 21 stations are summarized in Table 2. At a significance level of 0.05 ($\alpha=0.05$), a Z value of more than 1.64 indicates a significant upward trend, while a value of less than -1.64 indicates a significant downward trend. A value of $|Z|>1.28$ shows a significant upward/downward trend at a significance level of 0.1 ($\alpha=0.1$). According to the results of MK tests, 67 % and 33 % of the stations are identified with a significant decreasing trend of summer precipitation at $\alpha=0.1$ and $\alpha=0.05$ respectively.

The selected model with a time-varying μ_t was also used to detect time trends in the summer precipitation for each station. The estimated distributions as represented by five different percentiles (5th, 25th, 50th, 75th and 95th) and observed values for Qijia, Kuancheng, Pingquan and Zhangsanying stations, are illustrated in Fig. 4 as examples.

Table 2 Summary of parameter estimates and residual statistics for the optimized models, and the statistic Z values of Mann-Kendall tests

Stations	μ				σ	Residual					Z
	a_0	a_1	a_2	a_3		M	V	S	K	F	
Xiaoxishan	347.000	\	\	\	0.308	0.00	1.02	0.25	2.56	0.992	-0.73
Qijia	364.700	\	\	\	0.280	0.00	1.02	-0.03	2.52	0.987	-0.68
Baihugou	326.919	\	\	\	0.279	0.00	1.02	-0.15	2.63	0.994	-0.94
Sandaohezi	390.240	-1.528	\	\	0.281	0.00	1.02	0.13	3.15	0.985	-1.96**
Xiabancheng	444.394	-2.437	\	\	0.296	0.00	1.02	-0.04	2.75	0.996	-2.09**
Xuanjiangyingzi	409.796	-2.093	\	\	0.268	0.00	1.02	-0.22	2.77	0.988	-2.12**
Sangou	389.329	-1.517	\	\	0.290	0.00	1.02	-0.21	2.73	0.991	-1.33*
Kuancheng	525.112	-2.557	\	\	0.312	0.00	1.02	0.10	2.53	0.991	-1.30*
Goutaizi	366.137	-1.675	\	\	0.273	0.00	1.02	-0.02	2.53	0.990	-1.96**
Banjietata	337.624	-1.353	\	\	0.276	0.00	1.02	0.17	2.61	0.987	-1.53*
Jiutun	357.680	1.431	-0.052	\	0.264	0.00	1.02	-0.15	2.58	0.993	-1.19
Xinglong	363.370	2.241	-0.070	\	0.220	0.00	1.02	0.08	2.42	0.989	-1.47*
Pingquan	330.940	4.992	-0.115	\	0.292	0.00	1.02	-0.01	2.83	0.995	-1.10
Liyang	453.960	3.155	-0.091	\	0.263	0.00	1.02	-0.22	2.74	0.991	-1.31*
Chengde	389.360	-0.935	-0.017	\	0.281	0.00	1.02	-0.23	2.62	0.976	-1.91**
Boluonuo	363.940	0.765	-0.043	\	0.256	0.00	1.02	0.03	2.67	0.992	-1.79**
Miaogongshuiku	328.790	2.651	-0.064	\	0.240	0.00	1.02	0.02	2.56	0.987	-0.81
Zhangsanying	340.350	-4.474	0.329	-0.005	0.268	0.00	1.02	-0.05	2.59	0.986	-0.77
Xiahenan	404.480	-9.139	0.422	-0.006	0.275	0.00	1.02	-0.28	2.79	0.988	-1.51*
Hanjiaoyang	369.140	0.786	-0.124	0.002	0.248	0.00	1.02	-0.09	2.66	0.987	-2.19**
Yudaokou	309.820	-6.241	0.284	-0.004	0.235	0.00	1.02	-0.12	2.72	0.986	-1.37*

M, V, S, K and F in the table represent the mean, variance, skewness, kurtosis, and Filliben correlation coefficient of the residuals respectively. The symbol '*' means $\alpha=0.1$, while '**' means $\alpha=0.05$

Based on a stationary model, no significant temporal trend is observed in the summer precipitation of Qijia station (Fig. 4a), which is in agreement of the result of MK test. Similar behavior is found in Xiaoxishan and Baihugou stations. As to the stations with non-stationary models, the summer precipitation tends to decrease with time overall, while the detailed changing patterns are different from each other. For instance, a linear decreasing trend was found in Kuancheng station as shown in Fig. 4b. Whereas, in the case of Pingquan and Zhangsanying stations, the fitted models capture non-linear behaviors associated with the data, reflected in the patterns of quadratic and cubic polynomial curves respectively (Fig. 4c and d). It is demonstrated that the selected models are able to capture the large scatter and the temporal variability of summer precipitation.

In general, the non-stationary models reveal a pattern of decreasing trend in summer precipitation over the basin during 1959 ~ 2011, and especially a significant decrease in 2000 ~ 2011, which is in agreement with previous studies (Wei and Feng 2011; Zhang et al. 2011; Wang et al. 2013). It is indicated that the GAMLSS model could better describe the observed behavior over time, shown to be a useful complement to other statistical methods (such as MK trend test) for identifying non-stationarity in hydrological time series. In addition, these results also reinforce the questioning of the stationary assumption in the study area, as

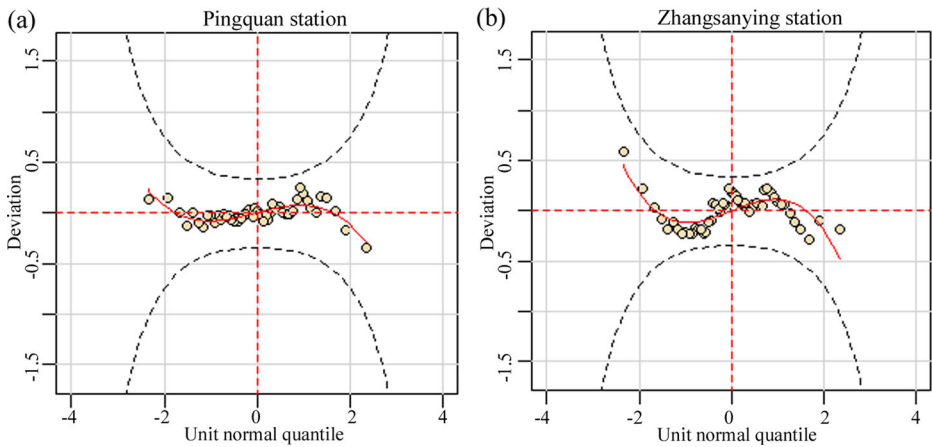


Fig. 3 Worm plots from the selected model for summer precipitation in Pingquan and Zhangsanying station. For a satisfactory fit, the data points should be within the two blank dotted lines (95 % confidence interval)

well as the urgent need for new drought indices that can take into account the nonstationarity of hydrologic records.

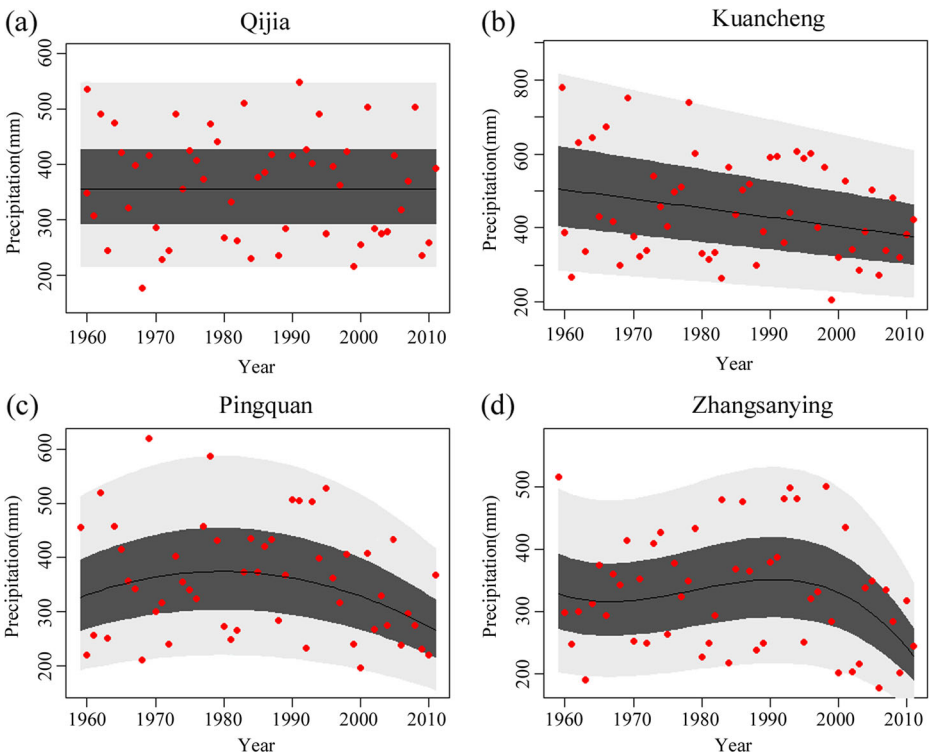


Fig. 4 GAMLSS modeling of the summer precipitation for Qijian, Kuancheng, Pingquan and Zhangsanying satations. The *light grey* region represents the area between the 0.05 and 0.95 quantiles; the *dark grey* region the area between the 0.25 and 0.75 quantiles, while the *blank line* the median (0.5 quantile). The red dots represent the observed values

4.3 SPIt and SPI

Both the proposed SPIt and the traditional SPI were applied to characterize the summer drought condition in the Luanhe River basin.

Figure 5, for example, presents the classifications of historical summer droughts using traditional SPI and SPIt methods at Xiahenan station during the period 1959 to 2011. These two methods provide similar drought classifications in most cases, meanwhile defining several differences. The summer with cumulative precipitation of 178.1 mm in 1981 is the lowest one in the record, which is classified in D3 class by traditional SPI, while in D4 class by SPIt. In 1972, the summer precipitation of 247.8 mm belongs to D1 class as categorized by traditional SPI, but it is identified as a more severe category (D2) when using SPIt. Based on the traditional SPI classification, the precipitation of 230 mm in summer represents moderate drought state. Nevertheless, corresponding to this precipitation amount, the drought state defined by SPIt changes from year to year, owing to the non-stationary nature of historical precipitation series. For instance, according to SPIt, the summer precipitation of 233.7 mm in 1984 is assigned to D2 class, whereas the precipitation with a similar amount (230.7 mm) in 2006 is assigned to D1 class. In addition, the cumulative precipitation in the summer of 1999 and 2009 are 194.1 and 191.2 mm respectively. Due to the slight difference in the amount of precipitation (2.9 mm), both of these summers are classified in D3 class by traditional SPI. However, SPIt considering the time dependence of precipitation records, classifies 1999 in D3 class but 2009 in D2 class. These results support the inference that the SPIt, which is capable of modeling the non-stationary time series, seems to be a more appropriate method to assess droughts under a changing condition as compared with the traditional SPI.

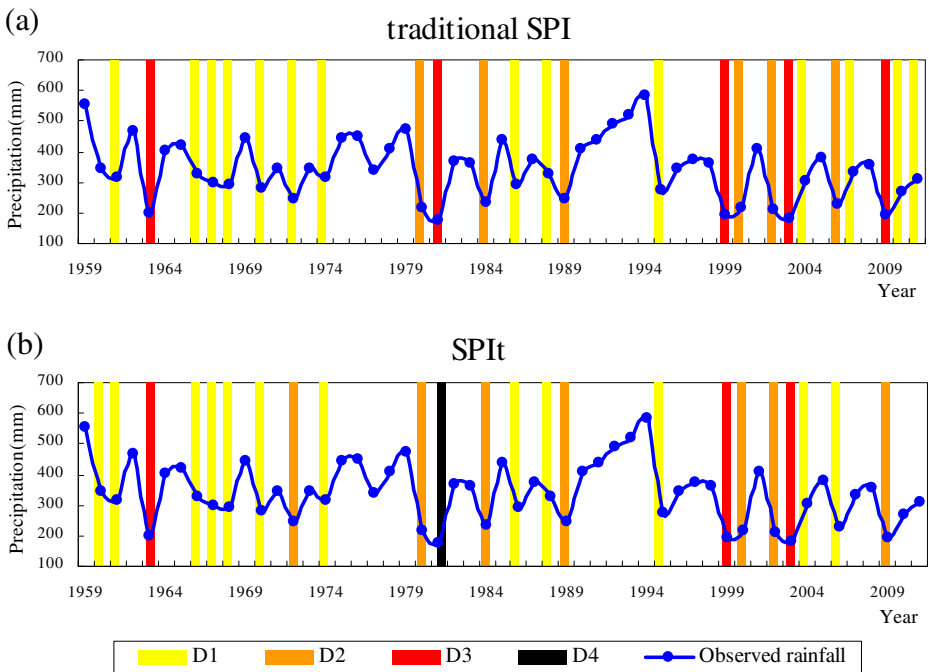


Fig. 5 Classifications of historical summer droughts at Xiahenan station during the period 1959–2011 using traditional SPI and SPIt methods

The spatial distribution of SPI and SPIt for two dry summers in 1963 and 2009 are illustrated in Fig. 6. As a whole, SPIt indicates a more dry summer than SPI does in the case of 1963 (Fig. 6a). Although SPI and SPIt classes are same for about 76 % of the study stations in 1963, some significant differences are found between the droughts identified by the two indices. As can be seen in Fig. 6b, the SPI and SPIt show different drought classes in considerable parts of the study area for the summer in 2009. Compared with the traditional SPI, the SPIt defined lower (i.e., more wet) drought classes for about 50 % of the stations. In 2009, the overall summer drought state evaluated by the SPIt is less severe than that evaluated by the SPI, which is in contrast to the pattern in 1963.

The ratio between the number of drought occurrences in each drought class and the length of the period considered was represented as the frequency of drought in this study. The frequency of summer drought was calculated using SPI and SPIt for each station in the study area, considering the periods of 1959 ~ 1969, 1970 ~ 1979, 1980 ~ 1989, 1990 ~ 1999, 2000 ~ 2011 and 1959 ~ 2011. As an average of the 21 stations, the frequencies related to the near normal, moderate, severe and extreme drought expressed in percent are given in Fig. 7. For the drought category of near normal (Fig. 7a), the average frequencies obtained from the traditional SPI and SPIt are similar for all the periods considered, and reach a minimum value of about 17 % during 1990 ~ 1999. When the frequencies for moderate drought are compared

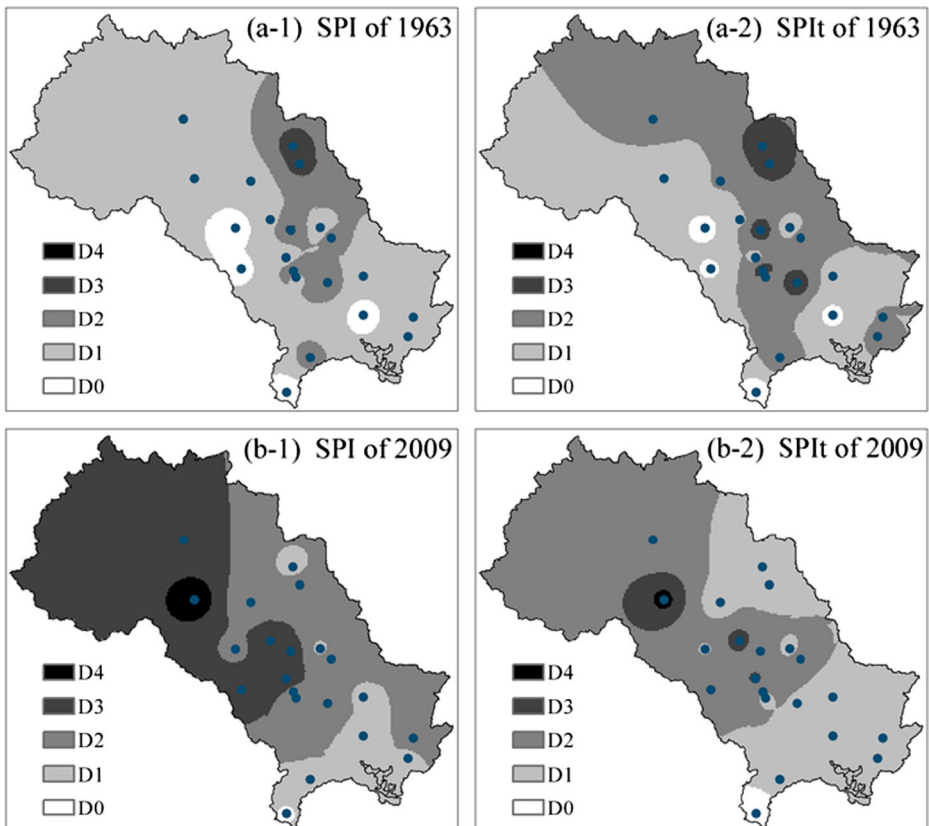


Fig. 6 Region illustration of drought condition based on SPI and SPIt for the summers in 1963 and 2009

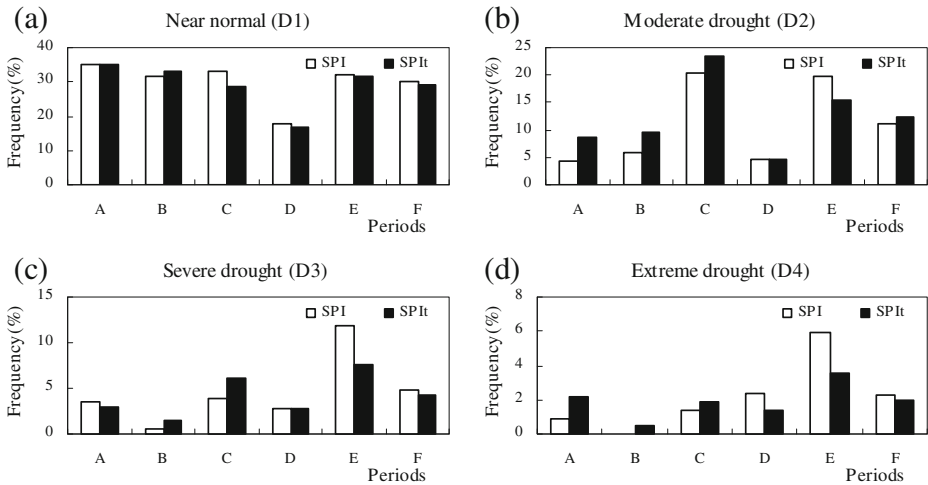


Fig. 7 Frequency related to different drought classes for different periods. **a**, **b**, **c**, **d**, **e**, and **f** represents the periods of 1959 ~ 1969, 1970 ~ 1979, 1980 ~ 1989, 1990 ~ 1999, 2000 ~ 2011 and 1959 ~ 2011 respectively

(Fig. 7b), it is found that the drought frequencies based on SPI are greater than that of SPIt for the periods before 2000, while the opposite is the case for the period 2000 ~ 2011. Associated with severe drought class (Fig. 7c), the frequencies obtained by both SPI and SPIt methods tend to increase generally over time, especially for the period 2000 ~ 2011 with the highest frequencies of 11.9 and 9.5 % respectively, whereas the magnitude of change in the frequency of SPI is apparently greater than that of SPIt. Similar results are observed for the frequency of being extreme drought (Fig. 7d), in which both SPI and SPIt detect the maximum rates during 2000 ~ 2011 (6.0 and 3.6 % respectively), revealing a deterioration of drought conditions in recent decade. With respect to the entire study period (1958 ~ 2011), the SPI and SPIt methods show almost similar responses to the frequency of drought in various classes.

5 Discussion

An appropriate drought index usually serves as an important base in regional drought monitoring. The traditional SPI is defined based on a stationary Gamma distribution, in which the precipitation data used should be treated as stationary time series. However, due to the significant influences of climate changes and human activities, the stationary assumption for longer precipitation records can no longer be taken for granted (Villarini et al. 2010; Russo et al. 2013), consequently diminishing the validity and accuracy of the traditional SPI method. Hence it is necessary to update the existing definition of the probability-based SPI to adapt to the uncertainties in a changing environment.

In this study, benefiting from the non-stationary models developed using GAMLSS, significant non-stationarities are identified in the summer precipitation of the Luanhe River basin. Under non-stationary conditions, the PDF of hydrological time series would cease to be invariant to temporal translation (Gunderlik and Burn 2003; Wagesho et al. 2012), with time varying statistical properties. Therefore, the proposed SPIt that can respond to the evolution of the PDF over time is shown to be more robust and suitable than the traditional SPI for drought assessments under a changing environmental condition.

However, the non-stationary model developed in this study, which only considers the time variation in the distribution's location parameter, could not concretely explain the non-stationary behaviors in precipitation. Due to the combined effect of multiple non-stationary factors (such as climate fluctuations and anthropogenic interventions), increasing uncertainties are expected in the continued progress of the non-stationarity. Accordingly, the time-dependent SPI_t might be constrained in the projection of droughts in the future. Future studies should establish new drought indices using the non-stationary model which incorporates climate indices and/or anthropogenic indices as covariates. Moreover, the evaluation of the robustness of the non-stationary index is further in focus.

As a standardized and multi-scale index, the traditional SPI allows objectively evaluating different kinds of drought. SPI with a shorter time scale (2–3 months) could be adequate for depicting agricultural droughts, while hydrological and water resources droughts can be replicated well by SPI on longer time scales (e.g., 12 months) (Szalai et al. 2000; Paulo and Pereira 2006; Mishra and Singh 2010). Based on the same concept of the SPI, the precipitation-based SPI_t can be calculated for a variety of time scales as well, and correspondingly monitor both short- and long-term drought. Hence, it is reasonable to assume that the correlations of SPI_t with different types of drought would be similar to those of SPI. In this study, the SPI_t calculated using summer precipitation is characterized by 3-month time scale. Since soil moisture conditions respond to precipitation anomalies on a relatively short scale, the 3-month SPI_t seems to be more appropriate for measuring drought severity affecting agricultural practices. Nevertheless, the performance of drought indices is region specific. The connection of SPI_t with different types of drought in the Luanhe River basin should be examined quantitatively in further studies.

6 Conclusions

In this study, summer precipitation records in the Luanhe River basin were modeled with a non-stationary Gamma distribution using GAMLSS. Based on the non-stationary distribution, the time-dependent Standardized Precipitation Index (SPI_t) was developed and then employed to characterize the spatio-temporal variations of summer drought in the basin. The main conclusions are summarized as follows.

- (1) The good fits of the non-stationary distributions imply that the GAMLSS is a flexible and appropriate tool to model the non-stationarities in hydrological time series. The GAMLSS models well reproduce the time dependence of the summer precipitation records in the Luanhe River basin, highlighting an overall decreasing trend in summer precipitation during 1958 ~ 2011, especially a significant decrease in the period of 2000 to 2011.
- (2) Based on the non-stationary Gamma distribution, the proposed SPI_t is capable of taking the non-stationarity of precipitation records into account, and thus it is to some extent found to be more robust and reliable than the traditional SPI. Differences between the historical drought assessments of SPI and SPI_t indicate that the presence of non-stationarity cannot be ignored in regional drought monitoring. The SPI_t method is proven to be a feasible alternative under non-stationary conditions, contributing to provide a new perspective for constructing appropriate drought indices in the future. It is of great importance for the development of mitigation and adaptation strategies.

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