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Coupling Forecast Methods of Multiple Rainfall–Runoff Models for Improving the Precision of Hydrological Forecasting

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Abstract With much more higher requirement for the precision of flood forecasting and length of forecast period in the hydrological operational predication, three coupling forecast methods which include real-time correction—combination forecast (RC-CF) method, combination forecast—real-time correction (CF-RC) method, and Integral Parameters Optimization (IPO) method are proposed in this paper for the purpose of improving the precision of flood forecasting. These coupling forecast methods are based on the real-time correction and combination forecast methods. Thereafter, two methods (method A & method B) are proposed for the purpose of prolonging the forecast period. Furthermore, indices of accuracy assessment which consist of mean absolute error, mean relative error, certainty coefficient and root-mean-square error are utilized to evaluate the forecast results of coupling forecast methods. Moreover, with a case study of Xiangjiaba station in the Jinsha River, advantages and disadvantages of these coupling forecast methods are obtained through the comparison of forecast results calculated by these methods, and they provide the basis for selection of coupling forecast methods. The result shows that the IPO method performs better than other two methods which behave undifferentiated. It is found that the IPO method combined with method B can be a viable alternative for flood forecasting of multiple hydrological models.

Keywords Real-time correction · Combination forecast · Coupling forecast methods · Forecast period prolonging · Accuracy assessment

Highlights of this Paper
(1) Real-time error correction combined with multi-model combination forecast
(2) Coupling forecast methods of multiple rainfall-runoff models
(3) Obvious forecast precision improvement and forecast period prolonging

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1 Introduction

Error appears each time when hydrological models are utilized to forecast rainfall, runoff or any other hydrological features, and it occurs for many reasons which includes error of model structures, error of observations of data, error of selection of parameters and error of initial states of systems (Elliott and Timmermann 2004; Hyndman and Koehler 2006), etc. In reaction to this phenomenon, real-time correction methods have been widely employed (Bauwens and Vandewiele 1989; Krstanovic and Singh 1993; Seo and Breidenbach 2002; Hsu et al. 2003). Applications of real-time correction methods make a great contribution to enhance forecast accuracy of hydrological models. Up to now, real-time correction methods that are commonly researched on can be summarized as follows: autoregressive error correction method, recursive least-squares method and Kalman filtering method. Autoregressive error correction method establishes a model to realize the function of correcting error series on its own correlation, and obtains results by calculating the difference of forecast series and corrected error series (Guang-Te and Singh 1994). Recursive least-squares method achieves the function of real-time correction by updating parameters of forecast error factors according to input hydrological data (Cattivelli et al. 2008). For the Kalman filtering method, through applying of modern stochastic estimation theory, estimation of minimum variance of system status without deviation is calculated. Thereafter, state variables are corrected by the way of weighting real-time forecast models according to the principle of minimum error covariance matrix, so that the real-time corrected results are obtained (Plett 2004). Xiong et al. (2001) had pointed out that the effect of autoregressive error correction method is no worse than any other complex forecasting methods, so the autoregressive error correction method is adopted for research in this paper.

However, it's of great difficulty in acquiring high forecast accuracy only depending on one single forecast model. Considering of the limitations and regional characteristics of hydrological models under different physical backgrounds, many scholars put forward the concept of combination forecast. Winkler and Makridakis (1983) proposed the concept of combination of forecasts first, they had compared forecast results of single hydrological model with two or more models' combination forecast results. Furthermore, the concept of combination forecast had been widely used in other research fields, including statistics, managements, economics and meteorology (Terregrossa 2005; Rapach et al. 2009), etc. Shamseldin et al. (1997) are the first to introduce the concept of combination forecast to rainfall-runoff forecast models. Since then, many scholars have researched on combination forecast of multiple hydrological models (Hersbach 2000; Bowler et al. 2008; Jiang et al. 2014).

In summary, Considering of the advantages of real-time correction and combination forecast methods, this paper proposes the coupling forecast methods for flood forecasting based on real-time correction and combination forecast methods.

2 Materials and Methods

2.1 Basic Concepts

2.1.1 Real-Time Correction Method

Auto-regressive (AR) error correction model is a kind of time series models which search the relationship of relevant factors to forecast target component only through its own historical observations (Zhang et al. 2011; Zhao and Chen 2015). Since the effects of totally same data fitted by different order numbers of AR models are different, we choose Akaike Information Criterion (AIC) index to select the best order of AR model to fit object series (Symonds and Moussalli 2011).

2.1.2 Multi-Model Combination Forecast Method

Multi-model combination forecast method assigns weights to all the hydrological models by taking account of the performances of each hydrological model in the same station, and then the final forecast results are calculated by summed up all the results of weighted hydrological models (Bowler et al. 2008; Jiang et al. 2014). The results of combination forecast are calculated as:

$$F_t = (F_1, F_2, \ldots, F_n) = \sum_{i=1}^{m} \omega_i \hat{Q}_t^i \tag{1}$$

Where $\hat{Q}_t^i$ denote the forecast result of i-th hydrological model, m denotes the number of hydrological model, n denotes the length of forecast series, ω_i denote the weight of i-th hydrological model, and they submit to:

$$\sum_{i=1}^{m} \omega_i = 1 \tag{2}$$

The expectation of variance of error series is marked as:

$$E(F_t - Q_t)^2 = E\left(\sum_{i=1}^{m} \omega_i \hat{Q}_t^i - Q_t\right)^2 \tag{3}$$

Where Q_t denotes the observed series.

For the purpose of minimizing the expectation of variance of error series, the problem of solving weights of all the hydrological models has transformed to solving the following linear programming problem:

$$\begin{cases} Min\ \ E\left(\sum_{i=1}^{m} \omega_i \hat{Q}_t^i - Q_t\right)^2 = Min\ \ E\left[\sum_{i=1}^{m} \omega_i \left(e_t^i + Q_t\right) - Q_t\right]^2 = Min\ \ E\left(\sum_{i=1}^{m} \omega_i e_t^i\right)^2 \\ s.t.\ \ \ \sum_{i=1}^{m} \omega_i = 1 \end{cases} \tag{4}$$

Where e_t^i denotes the error series of i-th forecast result.

By the introduction of Lagrange multiplier λ (Ioffe 1993), the objective function is constructed as following:

$$L(\omega_1, \omega_2, \ldots, \omega_m, \lambda) = E\left(\sum_{i=1}^{m} \omega_i e_t^i\right)^2 + \lambda\left(\sum_{i=1}^{m} \omega_i - 1\right) \tag{5}$$

After differentiating the objective function with respect to ω_1, ω_2,…, ω_m and λ respectively, the weights of all the hydrological models can be obtained by solving the difference equations.

2.2 Coupling Forecast Methods

According to the real-time correction method and combination forecast method mentioned above, the research in this paper is mainly focused on real-time correction—combination forecast (RC-CF) method, combination forecast—real-time correction (CF-RC) method and Integral Parameters Optimization (IPO) method.

2.2.1 RC-CF Method

Suppose the number of hydrological models is m, the observed series is marked as Q_t, and the forecast results of all the hydrological models are marked as $Q_t^1, Q_t^2, \ldots, Q_t^m$. Firstly, by the use of real-time correction method, the real-time corrected results of each hydrological model are calculated as $Q_t^{1*}, Q_t^{2*}, \ldots, Q_t^{m*}$. Then, according to multi-model combination forecast method, coupling weights $\omega_1, \omega_2, \ldots, \omega_m$ of each hydrological model are calculated to obtain final RC-CF results as:

$$F_t^{RC-CF} = \sum_{i=1}^{m} \omega_i Q_t^{i*} \tag{6}$$

2.2.2 CF-RC Method

Suppose the number of hydrological models is m. firstly, according to multi-model combination forecast method, coupling weights $\omega_1, \omega_2, \ldots, \omega_m$ of each hydrological model are calculated to obtain combination forecast results as:

$$F_t^* = \sum_{i=1}^{m} \omega_i Q_t^i \tag{7}$$

Then, by the use of real-time correction method, real-time corrected results of F_t^* are calculated as F_t^{CF-RC}.

2.2.3 IPO Method

Both of the RC-CF method and CF-RC method take the process of real-time correction and multi-model combination forecast as isolated parts. However, in the process of real-time correction and multi-model combination forecast, these two methods need to calculate a set of parameters under a given target, respectively. So, we consider to take the process of real-time correction and multi-model combination forecast as a whole part, and conform parameters of two parts under a same target.

Suppose the number of hydrological models is m, the observed series is marked as Q_t, and the forecast results of all the hydrological models are marked as $Q_t^1, Q_t^2, \ldots, Q_t^m$. The order of AR model should be confirmed firstly since the parameters of AR model are chosen as decision variable in the IPO model. The simulation effects vary indeed when different orders of AR model are tested on one series. The final simulation result of IPO model is determined by two parts (real-time correction & combination forecast). So a better result of real-time correction method doesn't equal a better result of IPO method. Also, considering the complexity of dealing with the constraints, we decide to adopt second order AR model here. But it need to be clarified that other order AR model is also accepted. The error series calculated by corrected series and observed series are obtained as:

$$\begin{aligned} F_t - Q_t &= \sum_{i=1}^{m} \omega_i \left(Q_t^i - e_t^i\right) - Q_t \\ &= \sum_{i=1}^{m} \omega_i Q_t^i - Q_t - \sum_{i=1}^{m} \omega_i e_t^i \\ &= \sum_{i=1}^{m} \omega_i Q_t^i - Q_t - \sum_{i=1}^{m} \omega_i \left(\alpha_i^1 e_{t-1}^i + \alpha_i^2 e_{t-2}^i\right) \end{aligned} \tag{8}$$

Where α_i^1, α_i^2 denote the parameters of AR model for i-th hydrological model, e_t^i denotes the error series of i-th hydrological model.

$$Min\ \ E(F_t - Q_t)^2 = Min\ \ E\left[\sum_{i=1}^{m} \omega_i Q_t^i - Q_t - \sum_{i=1}^{m} \omega_i \left(\alpha_i^1 e_{t-1}^i + \alpha_i^2 e_{t-2}^i\right)\right]^2 \tag{9}$$

(a) Constraints of parameters of AR model

The expression of AR(2) model is as:

$$x_t = c + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \varepsilon_t \tag{10}$$

Where x_t denotes the t-th point of series x, x_{t-1} denotes the t-1-th point of series x, x_{t-2} denotes the t-2-th point of series x, c denotes the constant term, ε_t denotes the stochastic term.

To make sure that AR(2) model is steady (Kapetanios et al. 2003), ϕ_1, ϕ_2 meet the condition as:

$$\begin{cases} \phi_2 \pm \phi_1 < 1 \\ |\phi_2| < 1 \end{cases} \tag{11}$$

The constraints of parameters of AR model can be expressed as:

$$\begin{cases} \alpha_i^2 \pm \alpha_i^1 < 1 \\ \left|\alpha_i^2\right| < 1 \end{cases} \quad (i = 1, 2, \cdots, m) \tag{12}$$

(b) Constraints of coupling weights

The constraints of coupling weights can be expressed as:

$$\sum_{i=1}^{m} \omega_i = 1 \tag{13}$$

Where $\omega_1, \omega_2, \ldots, \omega_m$ denote the coupling weights of combination forecast models. Specially, the coupling weights don't have to be positive.

Differential Evolution (DE) algorithm is firstly proposed by Storn and Price (1995, 1997). For the simplicity principle and convenience in computer programming, it has been applied for solving various problems (Vasan and Raju 2007; Shaheen et al. 2009; Eum and Simonovic 2010; Piotrowski and Napiorkowski 2012). One form of DE algorithm is the DE/rand/1/bin strategy. This format of DE contains three operators: mutation, crossover and selection. Considering of different actions and value ranges of parameters of real-time correction and combination forecast methods, an improved DE algorithm is proposed for the solution of IPO model. In the DE/rand/1/bin format of DE algorithm, it contains three operators: mutation, crossover and selection. In the process of mutation, it's the operations among particles, so it's not proper for actions of improvement that make difference between parameters of real-time correction and combination forecast methods. So we concentrate on the process of crossover and selection. In the process of crossover, since the value ranges and sensitivities of parameters of real-time correction and combination forecast methods are not exactly the same, different crossover parameters that include CR_{rc} and CR_{cf} will be applied to crossover procedure for parameters of real-time correction method and combination forecast method, respectively. While in the process of selection, the particle which obtain a better value of object function calculated by parameters of original particle and evolved particle is selected into next population. Since better results of object function don't mean better results of real-time correction, so the index of certainty coefficient (R^2) which is explained in Section 2.3.3 is adopted for the selection between original particle and evolved particle. The index of R^2 is calculated by parameters of real-time correction method for evaluating the effect of real-time corrected series of each hydrological model. The outline of solution to IPO model is illustrated in Fig. 1.

2.3 Materials for Case Study

2.3.1 Description of the Study Area and the Data Used

In this research, the Jinsha River watershed, which has been investigated intensively recently (Wang et al. 2011, 2014; Ouyang et al. 2014; Peng et al. 2014), is taken as the study area. Jinsha River, which flows through Qinghai, Tibet, Sichuan, and Yunnan provinces, is an important part of the upper Yangtze River. In the lower Jinsha River, it contains the most abundant hydropower resources of Jinsha River. Xiluodu and Xiangjiaba reservoirs locate in the lower reaches of Jinsha River, and these two reservoirs control an area of 458.8×10^3 km^2, accounting for 97 % of the watershed area. Approximate 10 years (1 June 2004 to 30 December 2013) of hydrological data, including hourly mean areal precipitation, potential evapotranspiration and stream flow, are available for model calibration.

2.3.2 Description of the Hydrological Models

In this paper, three classical models which include XinAnJiang (XAJ) model, Antecedent Precipitation Index (API) model, and Tank model are employed to help us with the research of verifying the effect of the coupling forecast methods proposed above.

XAJ model was first proposed by Zhao Ren-Jun (1992), which is one of the few world famous hydrological models in China. It has been researched for many years (Lu et al. 2008; Huan and Zhu 2009; Tian et al. 2013, 2014). XAJ model is a dispersion model, which can be used in humid areas and sub-humid regions of the wet season. The whole river basin is divided

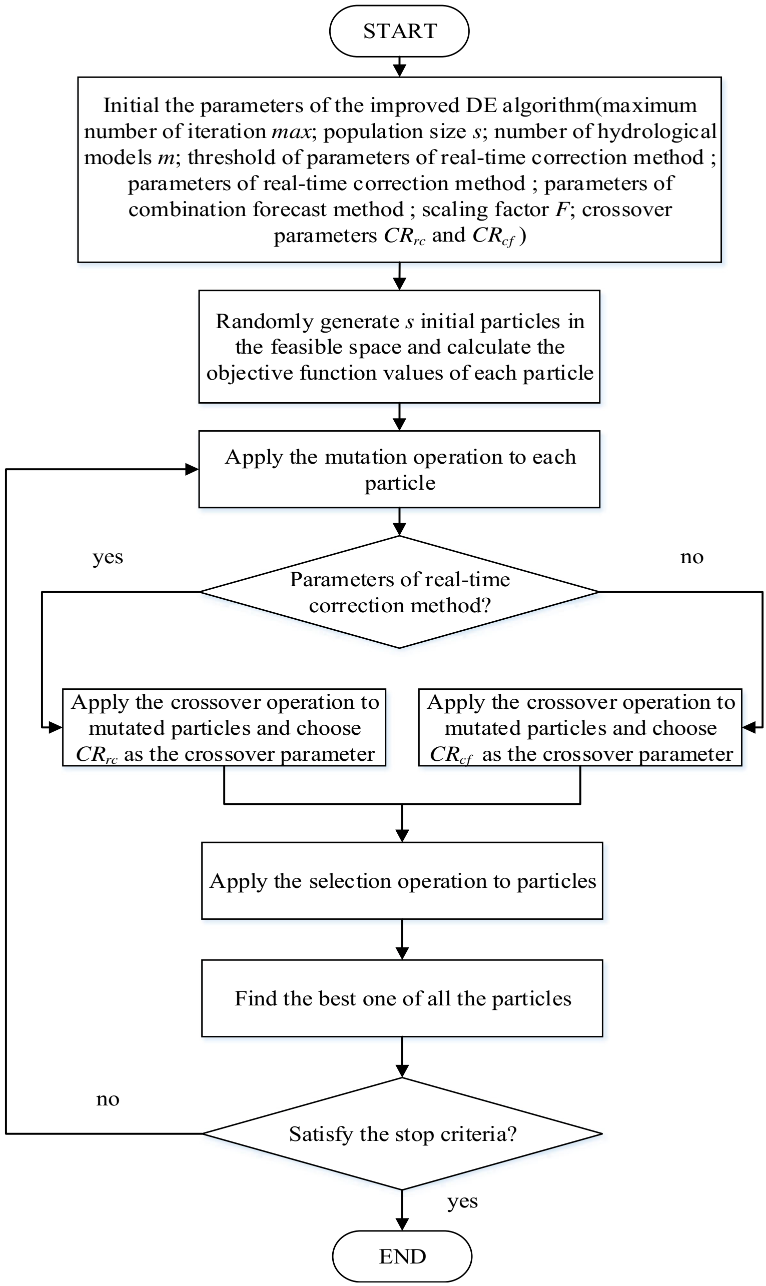

Fig. 1 Flowchart of the solution to IPO model

into many hydrological unit watersheds, and the outflow of each hydrological unit watershed is obtained by runoff calculation. Then the process of river flood routing is enforced to get the outflow of the export of hydrological unit watershed. Finally, the outflow of whole basin is calculated by summing the outflow of all the hydrological unit watersheds.

API model is a rainfall and runoff related hydrological forecast model (Fedora and Beschta 1989; Heggen 2001; Descroix et al. 2002), which has been adopted as a main tool for flood forecasting in Yangtze River over 10 years. The model consists of three parts: rainfall and runoff related model based on flood for runoff calculation, synthetic unit hydrograph model for watershed confluence calculation, and Muskingum model for river confluence calculation.

Tank model is a conceptual runoff model. For its character of simply formulating runoff formation process, it has been widely adopted (Tingsanchali and Gautam 2000; Lee and Singh 2005; Li and Gowing 2005). The complicated process of converting rainfall to runoff is summed up as the relationship between storage capacity of river basin and outflow to simulate. Units of flood process that consist of runoff, slope surface confluence, and river confluence is formulated by several water tanks contact with each other. Based on the water depths of all the water tanks, the processes of runoff, confluence, and infiltration are formulated.

2.3.3 Indices of Accuracy Assessment

Mean absolute error (*MAE*), mean relative error (*MRE*), certainty coefficient (R^2) and root-mean-square error (*RMSE*) are taken as the indices for accuracy assessment in this paper. The definitions of all the indices are expressed as:

(1) Mean absolute error (*MAE*)

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|Q_i - \hat{Q}_{\mathrm{i}}\right| \tag{14}$$

Where Q_i denotes the observed series, $\hat{Q}_i$ denotes the forecast series, n denotes the length of the series Q_i and $\hat{Q}_i$.

(2) Mean relative error (*MRE*)

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left[\left(Q_i - \hat{Q}_i\right)/Q_i\right]^2 \tag{15}$$

(3) Certainty coefficient (R^2)

$$R^2 = 1 - \left[\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2\right] / \sum_{i=1}^{n}\left(Q_i - \overline{Q}_i\right)^2\Bigg] \tag{16}$$

Where $\overline{Q}_i$ denotes the average value of observed series.

(4) Root-mean-square error (*RMSE*)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2} \tag{17}$$

3 Results and Discussion

3.1 Forecast Results of Hydrological Models

After calibrating parameters of all the hydrological models including XAJ model, API model, and Tank model, three different results of outflow are calculated by each hydrological model. Here, we take the forecast outflow of Xiangjiaba station from 1 June 2013 to 30 September 2013 as the basic data for the later research on coupling forecast methods. The observed series and three forecast series of Xiangjiaba station from 1 June 2013 to 30 September 2013 are showed in Fig. 2. Figure 2a~c are the forecast results of XAJ, API and Tank models respectively. The basic data are divided into two parts, the first part is taken as calibration period from 1 June 2013 to 31 October 2013, and the second part is taken as validation period from 1 September 2013 to 30 September 2013. The time interval is one hour. The results of accuracy assessment in calibration and validation periods are listed in Table 1. From the indices of *MAE*, *MRE*, R^2, and *RMSE* in Table 1, we can see that three hydrological models have similar forecast accuracy, and they behave a little better in calibration period than in validation period.

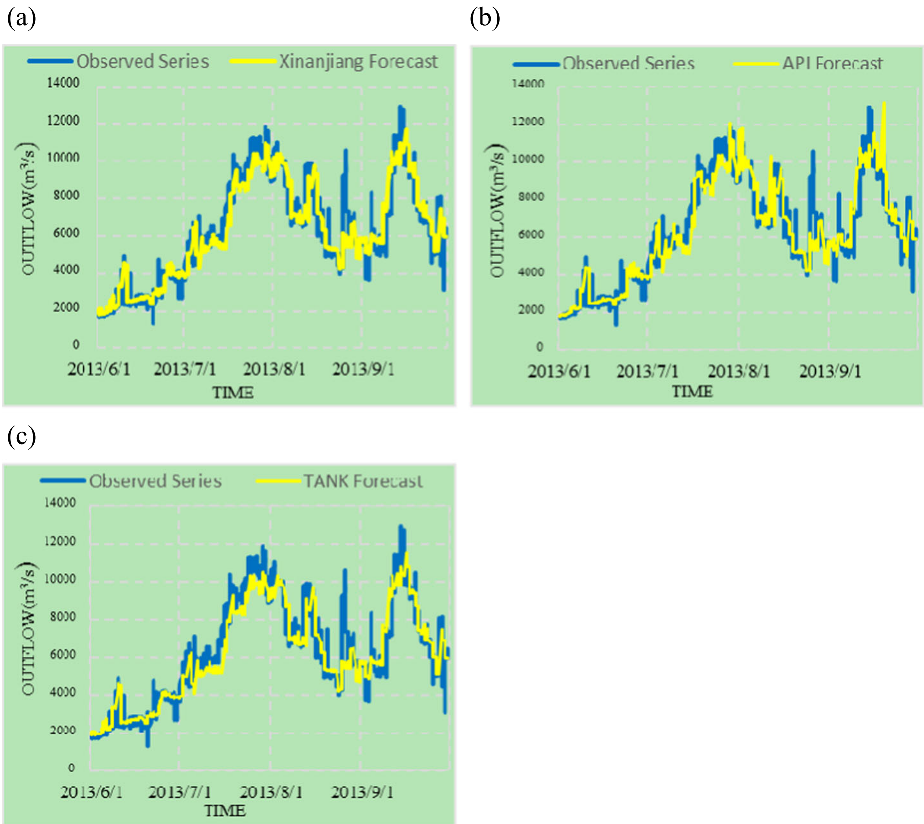

Fig. 2 Accuracy assessment results of the original forecast series calculated by XAJ, API, and Tank models

Table 1 Accuracy assessment results of hydrological models

Model	Period	*MAE* (m^3/s)	*MRE*	R^2	*RMSE* (m^3/s)
XAJ	Calibration	549	0.098	0.914	815
	Validation	790	0.122	0.752	1053
API	Calibration	564	0.095	0.906	850
	Validation	792	0.119	0.736	1088
TANK	Calibration	620	0.106	0.894	903
	Validation	740	0.112	0.768	1020

3.2 Results of Real-Time Correction and Combination Forecast Methods

The real-time correction and combination forecast results are calculated based on the data in calibration period, and are verified based on the data in validation period. The accuracy assessment results of real-time correction and combination forecast methods in calibration and validation periods are listed in Table 2. The indices R^2 in calibration of real-time correction results obtained by AR models are 0.992, 0.99 and 0.992 respectively for XAJ, API, and Tank models, and there are remarkable improvement of forecast accuracy in contrast to the original forecast results which obtain 0.914, 0.906 and 0.894 of indices R^2 in calibration period. In the validation period, the indices R^2 of real-time correction results are 0.98, 0.973 and 0.982 respectively for XAJ, API, and Tank models, and the indices R^2 of original forecast results are 0.752, 0.736 and 0.768. Due to the strong correlation of the data that we adopt, the performance of AR model is very well. The AR model works not so well in the article (Zhao and Chen 2015), based on the ensemble empirical mode decomposition (EEMD) and Auto-Regressive (AR), the author proposes the EEMD-AR hybrid model and compare with EMD-AR and single AR models to prove that EEMD-AR hybrid model gives better accuracy in predicting annual runoff. Since AR model works well here, so we don't try any other ways as described in the article above to improve the AR model alone. In contrast to the perfect performance of real-time correction method, the combination forecast method may not behave as good as the real-time correction method, but it also shows promotion of forecast accuracy. Under the

Table 2 Accuracy assessment results of real-time correction and combination forecast methods

Method	Model	Period	*MAE* (m^3/s)	*MRE*	R^2	*RMSE* (m^3/s)
Real-time correction	XAJ	Calibration	161	0.030	0.992	242
		Validation	204	0.030	0.980	298
	API	Calibration	172	0.031	0.990	275
		Validation	218	0.032	0.973	349
	TANK	Calibration	171	0.031	0.992	254
		Validation	189	0.028	0.982	288
Combination forecast	/	Calibration	521	0.099	0.923	768
		Validation	903	0.140	0.706	1148

extreme case, when individual models are very similar to each other in their flood forecasting ability, the combination system will produce forecasts only marginally better than those of the individual models (Xiong et al. 2001). Also, it is likely that the combination of the different forecasts might lead to bigger errors than the individual ones at some particular time steps. The performance of combination forecast method would not be so well under some situations.

3.3 Results of Coupling Forecast Methods

The accuracy assessment results of RC-CF, CF-RC and IPO methods in calibration and validation periods are listed in Table 3. From the accuracy assessment results of three coupling forecast methods, it's obvious that these three methods show superiority when they compare with both real-time correction method and combination forecast method. When comparing the indices *RMSE* of these three methods, it's not hard to find that the IPO method performs better than other two methods which behave undifferentiated. For the RC-CF and CF-RC methods, the indices *RMSE* of RC-CF and CF-RC methods are 240, 244 m^3/s in the calibration period and 311, 343 m^3/s in the validation period respectively. Both of the two methods take the process of real-time correction and combination forecast as isolated parts, and the performance of them are close to each other. For the IPO method, the index *RMSE* is 207 m^3/s in the calibration period and 204 m^3/s in the validation period. Compared with the index *RMSE* of original forecast results and the results of real-time correction method, combination method, RC-CF method and CF-RC method, the IPO method performs best. Since the optimization algorithms like DE algorithm have excellent search ability, the IPO method is capable of searching more suitable parameters of AR model and combination model under a same target to obtain more accurate forecast results. In the article (Jiang et al. 2014), the author adopts three widely used real-time satellite precipitation products for ensemble stream flow simulation with the gridded xinanjiang (XAJ) model and shuffled complex evolution metropolis (SCEM-UA) algorithm. Different input data are taken used for one rainfall-runoff model. However, streamflow simulation performed poorly when the raw satellite precipitation data were taken as input and the model parameters were calibrated with gauged data. By implementing the precipitation error multiplier and the precipitation error model and then recalibrating the model, the behavior of the simulated stream flow and calculated uncertainty boundary were significantly improved. It indicates that error correction combined with ensemble forecast method would be an available way to improve the accuracy of hydrological forecasting. Moreover, by introducing the optimization algorithm, the IPO method are more capable of searching better solution to improve the accuracy of hydrological forecasting.

Table 3 Accuracy assessment results of RC-CF, CF-RC and IPO methods

Method	Period	*MAE* (m^3/s)	*MRE*	R^2	*RMSE* (m^3/s)
RC-CF	Calibration	159	0.030	0.993	240
	Validation	218	0.033	0.978	311
CF-RC	Calibration	163	0.031	0.992	244
	Validation	237	0.035	0.974	343
IPO	Calibration	114	0.022	0.994	207
	Validation	129	0.018	0.987	240

From the expression of auto-regressive model, we can figure out that the forecast period of these forecast methods is just one time step (one hour in this paper). Forecast period of just one hour may hardly satisfy the need for actual flood forecasting. For the purpose of prolonging forecast period, two methods are proposed to deal with this problem.

(a) **Method A**: Suppose the forecast period is n, and the time interval is one hour. We take the RC-CF method for example, and other two coupling forecast methods are applied the same. In the calibration period, parameters which contain (α_1^1,α_1^2), (α_2^1,α_2^2),…, (α_m^1,α_m^2) and $\omega_1,\omega_2,\ldots,\omega_m$ are calculated for the AR(2) model and combination forecast method. While in the validation period, for the first n points, they are calculated as:

$$\hat{Q}_i=\sum_{j=1}^{m}\omega_j\left[Q_j^i-\left(\alpha_j^1e_j^{i+1}+\alpha_j^2e_j^i\right)\right]\quad i=1,2,\ldots,n \tag{18}$$

Where $\hat{Q}_i$ denotes the i-th point of final forecast series, Q_j^i denotes the i-th point of j-th hydrological model' original forecast series, e_j^i is defined as:

$$e_j^i=\begin{cases} e_j^{1*}=\hat{Q}_j^{i-2}-Q_j^{i-2} & i=1\\ e_j^{2*}=\hat{Q}_j^{i-1}-Q_j^{i-1} & i=2\\ \alpha_j^1e_j^{i-1}+\alpha_j^2e_j^{i-2} & i=3,4,\ldots,n\end{cases} \tag{19}$$

Where e_j^{1*} denotes the penult point of error series of j-th hydrological model in calibration period, e_j^{2*} denotes the last point of error series of j-th hydrological model in calibration period, $\hat{Q}_j^{i-2}$ denotes the original forecast series of j-th hydrological model at point i-2 which also denotes the penult point of original forecast series of j-th hydrological model in calibration period, $\hat{Q}_j^{i-1}$ denotes the original forecast series of j-th hydrological model at point i-1 which also denotes the last point of original forecast series of j-th hydrological model in calibration period, Q_j^{i-2} denotes the penult point of observed series in calibration period, Q_j^{i-1} denotes the last point of observed series in calibration period.

Then, for the left points in validation period, the final forecast results for every continuous n points are calculated as described above.

(b) **Method B**: Suppose the forecast period is n, and the time interval is still one hour. We take the RC-CF method for example, and other two coupling forecast methods are applied the same. In the calibration period, n groups of parameters are calculated for the AR(2) model and combination forecast method, and they are expressed as:

$$\begin{gathered}\left\{\left\{\left(\alpha_1^{1,1},\alpha_1^{2,1},\omega_1^1\right),\left(\alpha_2^{1,1},\alpha_2^{2,1},\omega_2^1\right),\ldots,\left(\alpha_m^{1,1},\alpha_m^{2,1},\omega_m^1\right)\right\},\right.\\ \left\{\left(\alpha_1^{1,2},\alpha_1^{2,2},\omega_1^2\right),\left(\alpha_2^{1,2},\alpha_2^{2,2},\omega_2^2\right),\ldots,\left(\alpha_m^{1,1},\alpha_m^{2,1},\omega_m^2\right)\right\},\ldots,\\ \left.\left\{\left(\alpha_1^{1,n},\alpha_1^{2,n},\omega_1^n\right),\left(\alpha_2^{1,n},\alpha_2^{2,n},\omega_2^n\right),\ldots,\left(\alpha_m^{1,n},\alpha_m^{2,n},\omega_m^n\right)\right\}\right\}\end{gathered} \tag{20}$$

Where $\{(\alpha_1^{1,i},\alpha_1^{2,i},\omega_1^i),(\alpha_2^{1,i},\alpha_2^{2,i},\omega_2^i),\ldots,(\alpha_m^{1,i},\alpha_m^{2,i},\omega_m^i)\}$ are the parameters when the AR(2) model which is expressed as $x_{t+i-1}=\alpha_1 x_{t-1}+\alpha_2 x_{t-2}$ and combination method are applied to calculate final forecast result.

While in the validation period, for the first n points, they are calculated as:

$$\hat{Q}_i=\sum_{j=1}^{m}\omega_j^i\left[Q_j^i-\left(\alpha_j^{1,i}e_j^{2*}+\alpha_j^{2,i}e_j^{1*}\right)\right]\quad i=1,2,\ldots,n \tag{21}$$

Where $\hat{Q}_i$ denotes the final forecast series of point i, Q_j^i denotes the original forecast series of j-th hydrological model at point i, e_j^{1*} denotes the penult point of error series of j-th

Table 4 Accuracy assessment results of RC-CF, CF-RC and IPO methods under different forecast periods by method A

Method	Period	Forecast period (h)	MAE (m^3/s)	MRE	R^2	$RMSE$ (m^3/s)
RC-CF	Calibration	/	159	0.030	0.993	240
	Validation	1	218	0.033	0.978	311
		2	334	0.050	0.946	490
		3	432	0.065	0.912	627
		4	562	0.086	0.860	793
		5	609	0.093	0.839	850
		6	654	0.100	0.814	912
		12	779	0.120	0.756	1046
		18	804	0.123	0.745	1069
		24	816	0.126	0.740	1078
CF-RC	Calibration	/	163	0.031	0.992	244
	Validation	1	237	0.035	0.974	343
		2	354	0.053	0.943	504
		3	456	0.069	0.908	642
		4	580	0.089	0.857	800
		5	631	0.096	0.834	862
		6	677	0.104	0.810	922
		12	811	0.125	0.745	1068
		18	842	0.130	0.732	1096
		24	857	0.132	0.725	1109
IPO	Calibration	/	114	0.022	0.994	207
	Validation	1	129	0.018	0.987	240
		2	208	0.029	0.968	378
		3	258	0.037	0.951	469
		4	321	0.045	0.933	546
		5	336	0.048	0.930	560
		6	402	0.059	0.904	656
		12	544	0.079	0.843	838
		18	748	0.109	0.737	1085
		24	658	0.097	0.751	1056

hydrological model in calibration period, e_j^{2*} denotes the last point of error series of j-th hydrological model in calibration period.

Then, for the left points in validation period, the final forecast results for every continuous n points are calculated as described above.

After introducing the concept of two methods (**method A & method B**) for prolonging the forecast period, and results of different forecast periods that consist of 1~6, 12, 18 and 24h are calculated. The results of accuracy assessment for **method A** and **method B** are showed in Tables 4 and 5, respectively. When calculating the forecasting results of method A, only one type of AR(2) model is taken used to fit the error series, so there is only one accuracy assessment result in the calibration period. When calculating the forecasting results of method B, n (the length of forecast period) types of AR(2) model is taken used to fit the error series, so

Table 5 Accuracy assessment results of RC-CF, CF-RC and IPO methods under different forecast periods by method B

Method	Period	Forecast period (h)	MAE (m^3/s)	MRE	R^2	$RMSE$ (m^3/s)
RC-CF	Calibration	/	/	/	/	/
	Validation	1	218	0.033	0.978	311
		2	220	0.031	0.970	367
		3	279	0.040	0.954	453
		4	339	0.048	0.938	526
		5	369	0.053	0.930	561
		6	415	0.061	0.916	614
		12	574	0.086	0.855	806
		18	662	0.100	0.796	955
		24	695	0.106	0.784	985
CF-RC	Calibration	/	/	/	/	/
	Validation	1	237	0.035	0.974	343
		2	220	0.032	0.971	358
		3	289	0.042	0.956	444
		4	346	0.051	0.943	504
		5	397	0.058	0.928	568
		6	440	0.066	0.916	613
		12	647	0.100	0.836	856
		18	770	0.119	0.767	1021
		24	847	0.132	0.732	1095
IPO	Calibration	/	/	/	/	/
	Validation	1	129	0.018	0.987	240
		2	200	0.028	0.974	342
		3	252	0.036	0.960	423
		4	302	0.043	0.947	485
		5	329	0.048	0.939	523
		6	371	0.055	0.926	577
		12	505	0.075	0.879	737
		18	643	0.096	0.816	908
		24	605	0.093	0.819	901

the accuracy assessment results in calibration period are not listed for the reason of similarity of each AR model and tedious data. Then, we choose the indices *RMSE* in Tables 4 and 5, and Fig. 3 is drawn for following comparison and analysis. Figure 3a~c show *RMSE* results of a same coupling forecast method that include RC-CF, CF-RC, and IPO methods calculated by method A and method B. While Fig. 3d~e show *RMSE* results of three coupling forecast methods calculated by method A or method B. From Fig. 3a~c, two points are clearly discovered: (a) with the forecast period increasing, the *RMSE* increase the same which means decrease of forecast accuracy and (b) method B is superior to method A. There is three reasons:

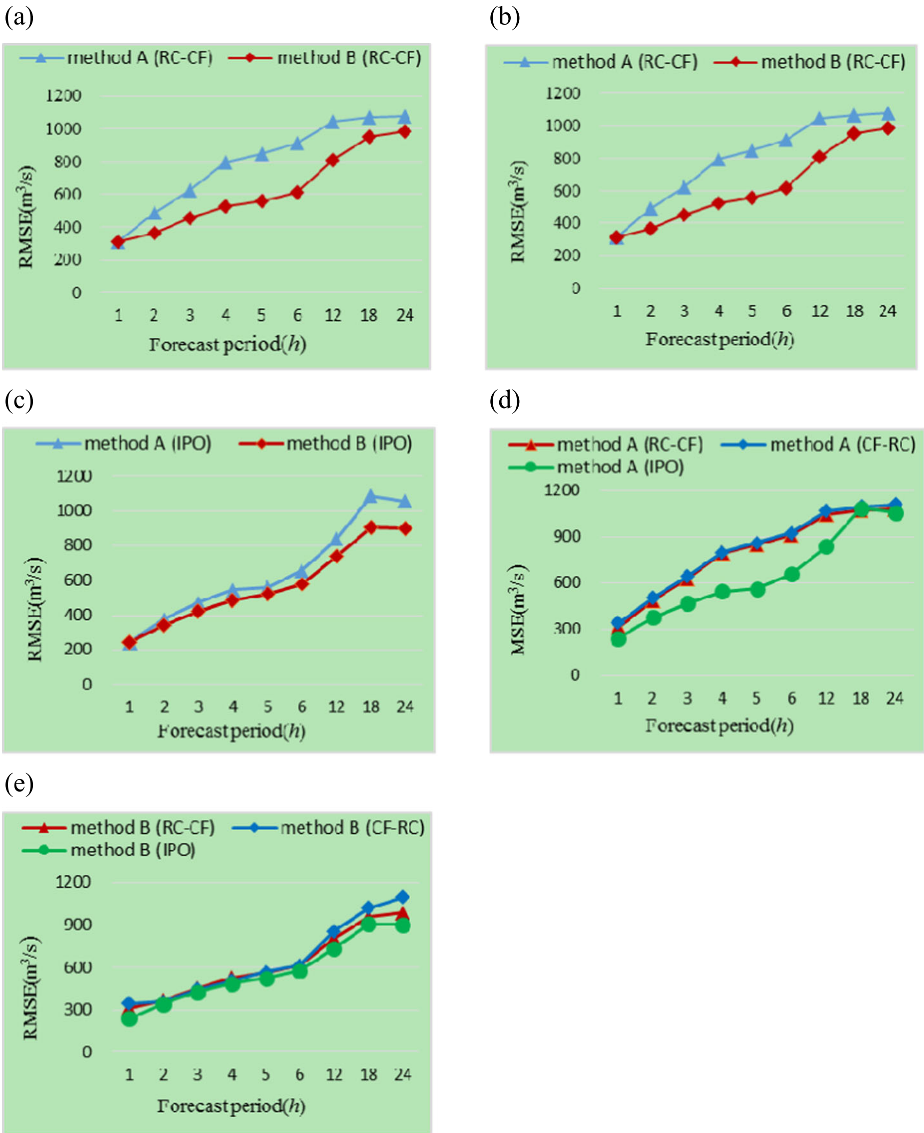

Fig. 3 Indices *RMSE* in validation period of RC-CF, CF-RC and IPO methods under different forecast periods by method A & method B

(a) for method A, all the points except the first one of one forecast period are calculated based on corrected error but not the real error and (b) for method A, all the points in one forecast period are calculated based on real error by different AR(2) models and combination method and (c) with the increasing of forecast period, much more error are accumulated for method A and less precision for method B. From Fig. 3d~e, we can figure out that IPO method has an advantage over RC-CF and CF-RC methods which have fairly effect. The reason of this phenomenon is that the IPO method is capable of searching better solution for the problem involving in this paper. In conclusion, IPO method combined with the proposed method B for prolonging forecast period should be a good choice for improving the forecast accuracy.

4 Conclusions

On the basis of real-time correction and multi-model combination forecast methods, three coupling forecast methods including RC-CF, CF-RC, and IPO methods are proposed in this paper. By the utilization of three classical models which consist of XAJ, API, and Tank model, three groups of forecast series are obtained for the research on coupling forecast methods. Through applying coupling forecast methods based on the information collected, forecast results that considering both real-time error correction and combination forecast are calculated. The results show that the RC-CF and CF-RC methods have fairly effects and the IPO method is superior to previous two methods. Besides, two methods are proposed for the purpose of prolonging the forecast period when the problem of one time step forecast period is realized. Thereafter, three coupling forecast methods are applied under different forecast periods based on two methods proposed for prolonging the forecast period respectively. From the analysis on the comparison of each forecast result, the IPO method combined with the second method for prolonging the forecast period is testified to be the most effective method for improving the forecast accuracy. It is found that the IPO method can be a viable alternative for flood forecasting of multiple hydrological models. It is hoped that more efforts focus on improving forecast accuracy and prolonging length of forecast period to develop better coupling forecast method of multiple rain-runoff models.

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