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Reservoir Inflow Forecasting Using Ensemble Models Based on Neural Networks, Wavelet Analysis and Bootstrap Method

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Abstract Accurate and reliable forecasting of reservoir inflow is necessary for efficient and effective water resources planning and management. The aim of this study is to develop an ensemble modeling approach based on wavelet analysis, bootstrap resampling and neural networks (BWANN) for reservoir inflow forecasting. In this study, performance of BWANN model is also compared with wavelet based ANN (WANN), wavelet based MLR (WMLR), bootstrap and wavelet analysis based multiple linear regression models (BWMLR), standard ANN, and standard multiple linear regression (MLR) models for inflow forecasting. Robust ANN and WANN models are ensured considering state of the art methodologies in the field. For development of WANN models, initially original time series data is decomposed using wavelet transformation, and wavelet sub-time series are considered to develop WANN models instead of standard data used for development of ANN model. To ensure a robust WANN model different types of wavelet functions are utilized. Further, a comparative analysis is carried out among different approaches of WANN model development using wavelet sub time series. Seven years of reservoir inflow data along with outflow data from two upstream reservoirs in the Damodar catchment along with rainfall data of 5 upstream rain gauge stations are considered in this study. Out of 7 years daily data, 5 years data are used for training the model, 1 year data are used for cross-validation and remaining 1 year data are used to evaluate the performance of the developed models. Different performance indices indicated better performance of WANN model in comparison with WMLR, ANN and MLR models for inflow forecasting. This study demonstrated the effectiveness of proper selection of wavelet functions and appropriate methodology for wavelet based model development. Moreover, performance

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of BWANN models is found better than BWMLR model for uncertainty assessment, and is found that instead of point predictions, range of forecast will be more reliable, accurate and can be very helpful for operational inflow forecasting.

Keywords Reservoir inflow · Neural networks · Wavelet analysis · Forecasting · Damodar catchment

1 Introduction

Accurate reservoir inflow forecasting is necessary for planning and management of available water resources. Inflow forecast can be applied for reservoir operation and management, flood control, drought management, water supply for irrigation, industrial and domestic uses and hydro-power generation. It emphasizes development of a model that is accurate and can be easily applied for the operational reservoir inflow forecasting. Several approaches have been applied to map the non-linear relationship between rainfall and runoff such as empirical, conceptual, physically and data driven (Verma et al. 2010; Paudel et al. 2011; Adamowski et al. 2013; Gad 2013). Data driven models have been applied in different fields of water resources with promising results (Jain et al. 2001; Mehta and Jain 2009; Mukerjee et al. 2009; Tiwari and Chatterjee 2010; Tiwari et al. 2013; Rath et al. 2013). In the previous years, complex nature of water resource variables has brought increased attention to the potential of soft computing technique methods including fuzzy logic and genetic programming (Kant et al. 2013), support vector regression (Herrera et al. 2010), and artificial neural networks (NN) (Adamowski 2008; Tiwari et al. 2013). Neural network information processing systems and capable of mimicking the functioning of human brain, and has been widely applied as an effective method for modeling highly non-linear phenomenon in hydrological processes (Abrahart et al. 2012).

One of the earliest uses of ANNs in reservoir inflow forecasting was used by Coulibaly et al. (2000). This research was subsequently followed by numerous studies on reservoir inflow forecasting (Jothiprakash and Magar 2012; Okkan 2012; Krishna 2014). Jothiprakash and Magar (2012) used artificial intelligent (AI) techniques such as artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS) and linear genetic programming (LGP) to predict daily and hourly multi-time-step ahead intermittent reservoir inflow of Koyna river watershed in Maharashtra, India. Similarly Valipour et al. (2013) developed ANN and ARIMA models for forecasting the inflow of Dez dam reservoir using monthly discharges from 1960 to 2007 and showed better performance of static and dynamic autoregressive artificial neural networks to forecast the inflow to the dam reservoirs. Among previous studies, feed forward back propagation neural networks (FFBP-NN), literally called as ANNs, is the most commonly used method.

Although ANN methods have been used extensively as useful tools for prediction of hydrological variables, it has some limitations in dealing with non-stationary data (Cannas et al. 2006; Partal 2009). A non-stationary time series data has a variable variance and mean that does not remain constant or same to their long-run mean over time, whereas the stationary time series data reverts around a constant long-term mean exhibits a constant variance independent of time. Daily flow time-series data are often non-linear and non-stationary (Rao et al. 2003; Wang et al. 2006). Non-stationarity such as seasonal variations and trends significantly affect modeling of time series and generally lead to poor predictability in practical applications (Francesco and Bernd 2000). Since the hydrological time series includes several frequency components and have non-linear relationships, hybrid model approaches which include different data-preprocessing and combine techniques, have been used to raise the prediction performance of neural networks. Wavelet analysis has emerged as an effective tool to simplify the non-stationarity in the dataset and has been widely applied by coupling with neural networks for water resource variables forecasting (Zhou et al. 2008; Makwana and Tiwari 2014; Kisi and Shiri 2012; Sahay and Sehgal 2013; Sahay and Srivastava 2014; Sehgal et al. 2014a, b). To develop WANN model, wavelet sub time series generated using discrete wavelet transformation (DWT) are used as inputs to the ANN models. The DWT decomposes original time series data into many components and each component has a distinct role in the original time series data. The low-frequency component or approximation generally reflects the identity (periodicity and trend) of the original data whereas the high-frequency components (i.e. details) uncover sharp fluctuations (Kucuk and Oglu 2006). There are several applications of WANN models in water resource variables forecasting and successful application inflow forecasting in the literature (Okkan 2012; Krishna 2014; Sehgal et al. 2014a, b). More recently, Krishna (2014) developed and demonstrated the potential of wavelet analysis and moving average (MA) methods in conjunction with two types of neural networks i.e. feed forward neural network and radial basis (RB) neural network and multiple linear regression (MLR) models in the prediction of the daily inflow values of Malaprabha reservoir, Belgaum, India. The results showed that WANN model performs better compared to an ANN and MLR models in forecasting the inflow hydrograph effectively. The author suggested undertaking further studies using data from upstream gauging stations to strengthen the findings.

ANN models including WANN models are prone to uncertainty depending on the input data arrangement in training and testing that leads to different optimal model structure and parametric values (Abrahart 2003; Arhami et al. 2013). Bootstrap resampling method that generates different realizations of dataset by resampling with replacement methodology is found to be efficient, simple and comparatively less complex than Bayesian method to assess uncertainty in the forecasts (Sharma and Tiwari 2009; Hinsbergen et al. 2009). Ensemble forecasting utilizing bootstrap method has improved the model performance compared to a single ANN model and provided assessment of uncertainty associated with the forecasts making it easier to implement in practice (Tiwari and Chatterjee 2011; Wang et al. 2013).

Even though wavelet analysis has improved the performance of ANN models significantly in different fields of water resources, the way wavelet sub time series are used to develop WANN models are different in different studies. Some of the previous work emphasized use of all the components for WANN model development (Wang and Ding 2003; Zhou et al. 2008; Nourani et al. 2009; Adamowski and Sun 2010; Maheswaran and Khosa 2012), other removed *d1* components, considering it as noise, as its correlation with the original data was very little and generated a new time series data by summing all the components except *d1* component of a particular time series data (Partal and Kisi 2007; Kişi 2009; Rajaee et al. 2010; Kisi and Cimen 2011). In some of the previous studies, new wavelet time series was developed by summing up effective wavelet components based on correlation analysis (Tiwari and Chatterjee 2010, 2011). In this study, in spite of considering a selective method to generate a new wavelet time series all the combination reported in the previous studies along with some new approaches are tested. Moreover, during WANN model development for forecasting water resources variables generally a particular wavelet transformation function and a level of decomposition are applied. However, each wavelet functions have their own strengths in capturing the different characteristics and physical structure of the hydrological processes and therefore, completely relying on a model based on a particular wavelet function often leads to predictions that capture some phenomena at the expense of others (Rathinasamy et al. 2013).

Therefore, in this study a combined bootstrap and wavelet based ANN model is proposed first time for inflow forecasting including different analysis such as (i) wavelet analysis for extracting non-stationarity from the dataset, (ii) uses combined strength of wavelet analysis and bootstrap resampling on a single platform to produce a model that is accurate and reliable, (iii) evaluate different combination approaches of discrete wavelet components, (iv) evaluate performance of different wavelet functions, and (v) produce the inflow forecast with confidence bands, assessing uncertainty associated with these forecasts.

2 Materials and Methods

2.1 Study Area

Damodar catchment, a part of the lower Ganges River, is located in the upper reaches of the Damodar river basin in Jharkhand state of India (Fig. 1). The area lies between 23° 34′ and 24° 09′ North latitude and 84° 42′ to 86° 46′ East longitudes with an elevation variation between

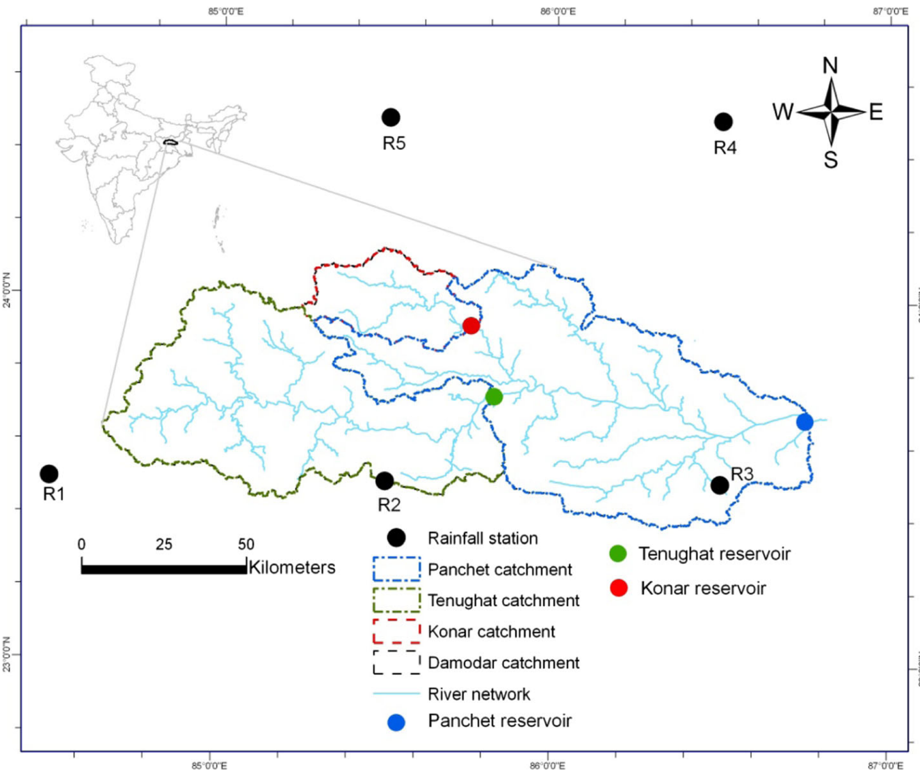


Fig. 1 Location map of study area along with reservoirs and rain gauge stations

122 and 1340 m above mean sea level. The catchment falls within sub-tropical climate and daily mean relative humidity varies from 40 to 95 % with alternating dry and wet periods. The daily mean temperature of the area ranges from 4 to 43 °C with average annual rainfall of 1390 mm most of which occurs in months from July to September. The study area comprises of mixed forest, mainly with deciduous and tropical moist forest along with many thorny bushes, and agricultural area. Damodar catchment consists of three reservoirs: Konar, Tenughat and Panchet, these reservoirs are constructed on Damodar river during 1955, 1972 and 1959, respectively with the intention of hydropower generation, water supply for irrigation, industrial and domestic uses, and flood control. Damodar catchment is further divided into three sub-catchments: Konar sub-catchment, Tenughat sub-catchment and Panchet sub-catchment with area of 997, 4480 and 5401 km^2, respectively. The total area of the catchment area is about 10878 km^2 with the length of the main stream being 350 km draining into the Panchet reservoir.

2.2 Dataset Used in the Study

Daily rainfall data were collected from Indian meteorological Department (IMD), Pune for five stations covering the study area (i.e. R_1, R_2, R_3, R_4 and R_5); daily outflow reservoir data for Konar (K_O) and Tenughat (T_O) reservoirs and daily inflow data for Panchet reservoir (P_I) were collected from for 7 years from 01 January, 2001 to 31 December, 2007 from the Reservoir Operation Department, Damodar Valley Corporation (DVC), Maithon, Jharkhand. From the available dataset, first 5 years of dataset (from 01 Jan. 2001 to 01 Jan 2005) were considered for training, whole data during 2006 were considered for cross validation and data during the year 2007 were for evaluation of the developed standard and combined ANN models (Table 1).

2.3 Artificial Neural Networks

Artificial Neural Networks (ANN) is a strong mathematical approach, based upon imitation of human brain functioning by forming a model structure with the capability to map complex non-linear relationships and processes that are inherent among several variables. In a simpler term it is networks with nodes in form of feed forward neural network, consisting of different layers with computational nodes such as an input layer, one or more hidden layers, and an output layer. As this approach is found very fast and efficient in highly complex and noisy environments to solve a wide range of problems, ANNs have been applied in numerous real-world applications including time series predictions (Abrahart et al. 2012). For detailed study on general properties of ANNs and its applications in water resource engineering, interested readers are directed to refer Bishop (1995), Haykin (1999), Maier and Dandy (2010) and Abrahart et al. (2012).

Table 1 Partitioning of data for ANN model development

Partition	Duration of the data	Number of data patterns
Training	01/01/2001 to 31/12/2005	1826
Cross-validation	01/01/2006 to 31/12/2006	365
Testing	01/01/2007 to 31/12/2007	365

2.4 Wavelet Analysis

Wavelet analysis utilizes a wavelet function known as mother wavelet defined as $\psi(t)=\int_{-\infty}^{+\infty}\psi(t)dt=0$ and successive wavelets can be derived as (Mallat 1989)

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right) \quad b\in R,\ a\in R,\quad a\neq 0 \tag{1}$$

where a and b are the scale and time factor, respectively, and R is the domain of real numbers.

For a time series with a finite energy signal $f(t)\in L^2(R)$, the continuous wavelet transform is defined as (Kisi 2010)

$$W_f(a,b) = |a|^{-\frac{1}{2}}\int_R f(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{2}$$

where $W_f(a, b)$ is the matrix of wavelet coefficient or a contour map known as scaleogram and ψ^* denotes a complex conjugate function.

To avoid generation of large number of coefficients discrete wavelet transform (DWT) is applied as it is a convenient way and very useful for solving practical problems. DWT is obtained by constraining the wavelet dilation (a) and translation (b) parameters and defining the DWT as (Mallat 1989)

$$\psi_{m,n}\left(\frac{t-b}{a}\right) = a_o^{-m/2}\psi^*\left(\frac{t-nb_o a_o^m}{a_o^m}\right) \tag{3}$$

where, integers m and n determine the magnitude of wavelet dilation and translation, respectively, a_0 represents a specified dilation step greater than 1 (most commonly $a_0=2$), and b_0 represents the location parameter which must be greater than zero (most commonly $b_0=1$).

2.5 Bootstrap Technique

There are three major sources of uncertainty such as parameter uncertainty, sub optimal training and insufficient input variables, which significantly affect the output of the ANN and WANN models. Bootstrapping a computational, data-driven simulation method has been used widely to assess the uncertainty by measuring the variance σ_s^2. The bootstrap samples are generated through intensive resampling of the data with replacement method, and these resample's or realizations of data provide a better understanding of the average and variability of the original, unknown distribution or process, that help to assess uncertainty associated with the estimate (Efron and Tibshirani 1993). Assuming a population T with unknown probability distribution F, where each sample is denote as $t_i = (x_i, y_i)$, where x_i is a input vector and y_i is the corresponding output vector, is a realization drawn as independently and identically distributed (i.i.d.) from T. T_n is a bootstrap resample denoted as $T_n = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$, where n is the size of original dataset obtained from empirical distribution function $\hat{F}$, by putting a mass of $1/n$ for each $t_1, t_2, \ldots, t_n$. Similarly, a set of bootstrap samples such as T^1, $T^2,\ldots, T^s,\ldots, T^S$ can be produced, where S is the total number of bootstrap samples, generally ranging from 50 to 200 samples (Efron and Tibshirani 1993).

In this study, several bootstrap resample's are generated and used to train several different ANN and WANN models, and an ensemble forecast is obtained and named as BANN and

BWANN, respectively (Tiwari and Chatterjee 2010, 2011). For each T^s, a ANN and WANN model is developed and trained using all n observations and the ANN and WANN outputs, $f_{ANN}(x_i, w_s/T^s)$ and $f_{WANN}(x_i, w_s/T^s)$, is then evaluated using a set A^s of observation pairs $t_i = (x_i, y_i)$ that are not included to generate bootstrap resample's. The performance of the ANNs and WANNs in these validation tests is subsequently averaged/ensemble. These ensembled models are also represents the generalization error for the ANN models relative to T_n that is denoted as E_0, and this can be estimated for ANN as (Twomey and Smith 1998)

$$\hat{E}_0 = \frac{\sum_{s=1}^{S} \sum_{i \in A_s} (y_i - f_{NN}(x_i, w_s/T^s))^2}{\sum_{s=1}^{B} (A_s)}. \tag{4}$$

where $f_{ANN}(x_i, w_b/T^s)$ is the output of the ANN developed from the bootstrap sample T^s, in which x_i is a particular input vector and w_s is the weight vector. Subsequently, the BANN estimate $\hat{y}(x)$ of all developed ANNs is given by the average of the S bootstrapped estimates (Jia and Culver 2006; Tiwari and Chatterjee 2010, 2011) as

$$\hat{y}(x) = \frac{1}{S} \sum_{s=1}^{S} f_{NN}(x, w_s). \tag{5}$$

and the variance is estimated as

$$\sigma^2(x) = \frac{\sum_{s=1}^{S} \sum_{i=A_s} [y_i - f_{NN}(x_i/w_s)]^2}{S-1} \tag{6}$$

Several forecasts obtained from the ANN and WANN models trained using several realizations of the training dataset are then used to generate the confidence band or confidence interval (CI) at the α% significance level. These CIs indicate the frequency containing the true value in the repeated simulation and is denoted as $100\times(1 - \alpha)$ % with value of α is generally taken as 0.05 that corresponds to 95 % confidence bands. The CIs covering the ensemble inflow $\hat{y}(x)$ are estimated as (Efron and Tibshirani 1993).

$$CI = (UB, LB) = \left[\hat{y}(x) + t_{n-p}^{\alpha/2}\sigma(x), \hat{y}(x) - t_{n-p}^{\alpha/2}\sigma(x)\right] \tag{7}$$

where UB and LB represent the upper and lower band, respectively, and $\sigma(x)$ represents the standard deviation of S number of forecasts, $t_{n-p}^{\alpha/2}$ is the $\alpha/2$ percentile for the t distribution with n - p degrees of freedom. n and p are the number of inflow data pattern and total number of parameters, respectively in the ANN and WANN models.

2.6 Development of ANN Models

To develop a robust ANN model, different structures of ANN model were developed by using different combinations of input variables along with 1–10 numbers of hidden neurons with learning coefficient equal to 0.2 and momentum equal to 0.9 for which the generalization error is found to be minimum. The dataset were standardized and normalized to scale in the range of 0–1. The Levenberg–Marquardt method, considered as most efficient and fast second order training method, is used to minimize the mean squared error between the forecast and observed reservoir inflows. To forecast inflows of Panchet reservoir at 1 day (t) lead time, initially inflow of Panchet reservoir at previous time step (i.e. t-1) is considered and then subsequently other input variables were considered starting from one lag time, until performance of model start deteriorating.

Selection of suitable wavelet function called as mother wavelet and a suitable level of wavelet decomposition is a crucial issue, as there is no such study showing the best performance of model for a particular wavelet function or decomposition level. Another important property of wavelet function is its vanishing moment that limits wavelets' ability to represent polynomial behavior of the signal. It can be considered that higher vanishing moments should capture the variations more effectively and efficiently and should improve the model forecasting ability. Therefore in this study different wavelet functions with different vanishing moments such as db2, db5, db10, db20, Bior1.1, Bior3.3, Bior6.8, Haar, Coif1, Coif3, Coif5 were considered to develop WANN models.

Optimum number of decomposition level for DWT of the time series are estimated using the following formula (Nourani et al. 2008):

$$L = \mathrm{int}[\log(N)] \tag{8}$$

where

L = Decomposition level
N = Number of time series data
This study uses N=1826, which produces L=3.

In this way for each wavelet function three level of decomposition is carried out such as approximation (*A3*) and three details *d1*, *d2* and *d3*. Inflow data at Panchet reservoir, outflow data from Konar and Tenughat reservoir and rainfall data at five gauging stations were initially decomposed using more frequently used mother wavelet *db5* into approximation (*A3*) and details (*d1*, *d2* and *d3*) for each component. As an illustration, only wavelet sub-time series of the rainfall at station DVC R2 and inflow to Panchet reservoir are shown from 01 Jan. 2001 to 01 Jan 2007 in Fig. 2. The time series data were also decomposed separately for training, cross-validation and testing dataset, but just for representation they are shown together. Table 2 shows the correlation between different wavelet components of a particular time series to the inflow of Panchet dam. Some of the components have very good correlation and some have no correlation. In spite of considering a selective method as discussed above to generate a new wavelet time series, all the combinations reported in previous studies along with some new approaches were tested in this study.

For several water resource variables forecasting, wavelet analysis has improved the ANN model performance significantly, but there is no specific method for selection of appropriate wavelet components for WANN model development and different studies have applied different approaches to develop WANN models. In this study, a

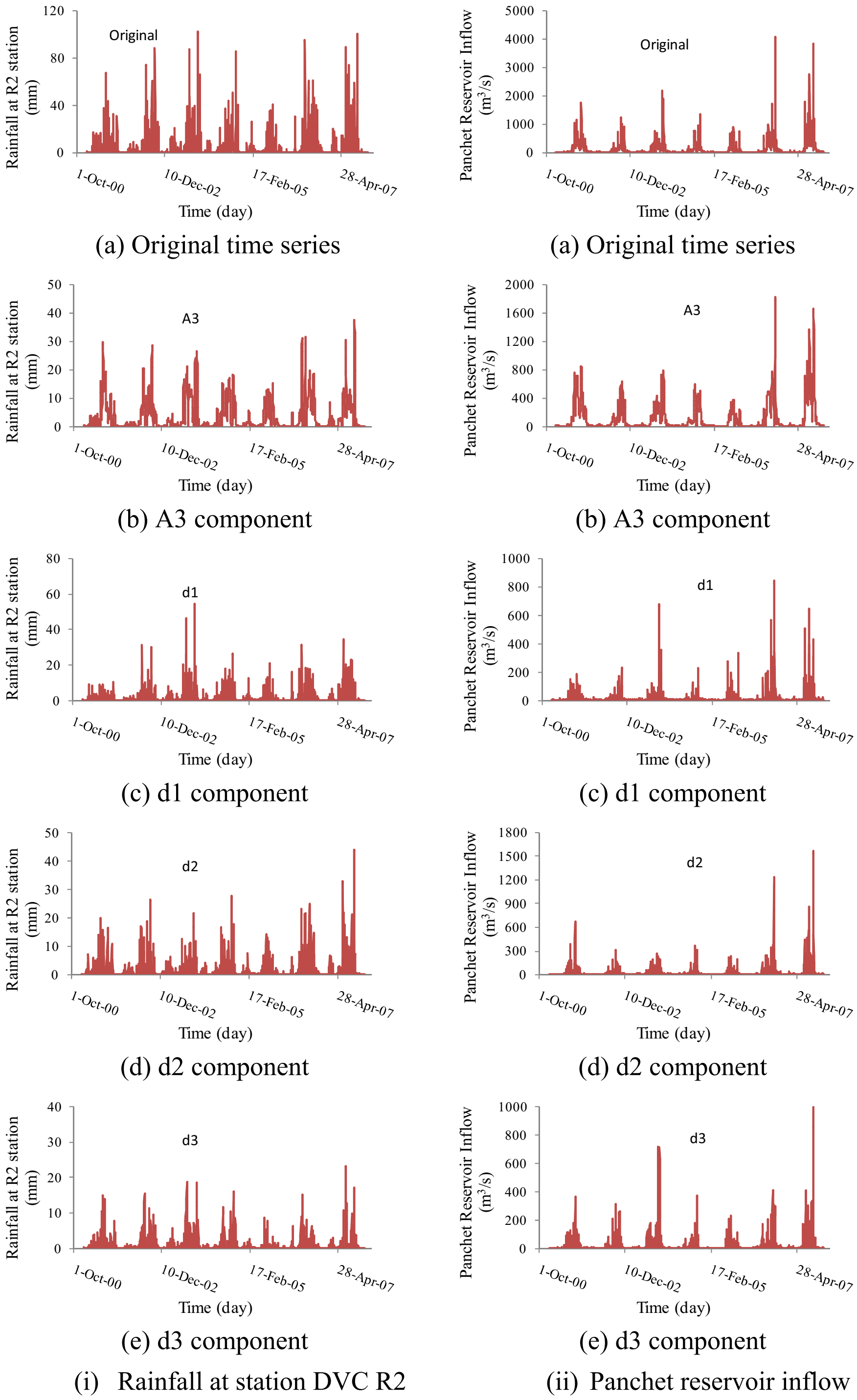


Fig. 2 Wavelet sub-time series of the reservoir release from (i) Rainfall at station DVC and (ii) inflow to Panchet dam from 01 Jan. 2001 to 31 Dec 2007

Table 2 Correlation between inflow of Panchet reservoir and different wavelet components of input variables

Variables	Correlation of variables with inflow of Panchet reservoir				
	Original	*A3*	*d1*	*d2*	*d3*
R1	0.27	0.48	−0.02	−0.02	0.01
R2	0.26	0.47	−0.02	−0.04	−0.02
R3	0.29	0.50	−0.02	−0.05	0.03
R4	0.27	0.42	−0.02	0.03	0.06
R5	0.24	0.42	−0.02	0.00	0.04
Konar outflow	0.68	0.56	0.12	0.30	0.26
Tenughat outflow	0.71	0.77	−0.03	0.13	0.20
Panchet inflow	1.00	0.82	0.22	0.40	0.35

comprehensive examination is carried out to compare and select the best approach for inflow forecasting. Five approaches tested in this study are: (i) all the 4 components (i.e. A_3, d_1, d_2 and d_3) of 8 input variables (i.e., R_1, R_2, R_3, R_4,R_5, K_O, T_O, P_I) with lag 1 (Approach 1) (ii) all the four components of input variables and lags found best for ANN model (i.e., A_3, d_1, d_2 and d_3 *of* P_I with lag 1; A_3, d_1, d_2 and d_3 of T_O with lag 1; A_3, d_1, d_2 and d_3 of R_1 with lag 1, 2 and 4; A_3, d_1, d_2 and d_3 of R_2 with lag 1 and 2) (Approach 2) (iii) all the significant wavelet components separately having correlation >0.10 with lag 1 (A_3 of R_1, R_2, R_3, R_4,R_5, K_O, T_O, P_I; d_1 of K_o, P_I, d_2 of K_O, T_O, P_I; d_3 of K_O, T_O, P_I) (Approach 3) (iv) newly constructed time series adding significant components of each parameter excluding d1 component (lag input as best ANN) (A_3, d_2 and d_3 *of* P_I with lag 1; A_3, d_2 and d_3 of T_O with lag 1; A_3, d_2 and d_3 of R_1 with lag 1, 2 and 4; A_3, d_2 and d_3 of R_2 with lag 1 and 2) (Approach 4), and (v) newly constructed time series adding significant components having correlation >0.10 of each variable (A_3 of R_1, R_2, R_3, R_4,R_5, K_O, T_O, $P_{I;}$ A_3+ d_1+, d_2+d_3 of K_O; A_3+ d_2+d_3 of T_O; A_3+ d_1+, d_2+d_3 of P_I) (Approach 5).

In this study, MLR models are also developed to compare the performance of different ANN models. To further evaluate the effectiveness of wavelet analysis similar to WANN models, BWMLR models are also developed and evaluated to forecast 1 day lead time reservoir inflow of Panchet dam. Next, BWANN and BWMLR models are developed by combining several WANN and WMLR models trained using different realizations of the dataset generated using bootstrap resample's. In this way BWANN and BWMLR hybrid models contain capabilities of both wavelet analysis and powerful bootstrap resampling techniques. Different realizations of data patterns were generated using Bootstrap.xla an Excel-add-in (Barreto and Howland 2006). The WANN models were developed using all the approaches discussed above. The BWANN and BWMLR models are the ensemble of WANN and MLR models, respectively, trained using 100 realizations of training dataset generated using bootstrap resampling. A flow chart showing the development of different models is shown in Fig. 3.

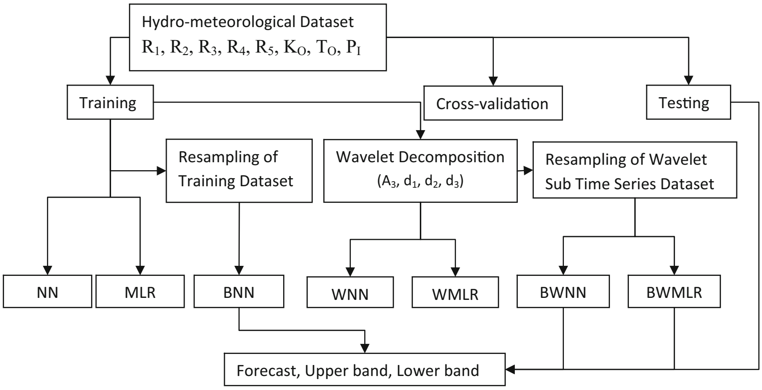


Fig. 3 Flow chart showing the development of different models

2.7 Performance Indices

The performance of the developed ANN, WANN, MLR, WMLR, BWANN and BWMLR models were evaluated using five performance indices defined below:

(i) The coefficient of determination (R^2):

$$R^2 = \left(\frac{\sum_{i=1}^{n} \left(O_i - \overline{O}\right)\left(P_i - \overline{P}\right)}{\sqrt{\sum_{i=1}^{n} \left(O_i - \overline{O}\right)^2 \sum_{i=1}^{n} \left(P_i - \overline{P}\right)^2}} \right)^2 \tag{9}$$

where O_i and P_i are the observed and forecasted inflow, respectively, $\bar{O}$ and $\overline{P}$ are the means of the observed and forecasted inflow, respectively, and n is the number of data patterns. Range of R^2 varies from 0 to 1, with 1 presents a perfect forecasting model.

(ii) The Nash-Sutcliffe coefficient (E) is defined as:

$$E = 1 - \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} \left(O_i - \overline{O}_i\right)^2} \tag{10}$$

The Nash–Sutcliffe efficiency varies from -∞ to 1. The value of 1 shows the perfect model.

(iii) Root mean square error (RMSE):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (O_i - P_i)^2} \tag{11}$$

Table 3 Best ANN model structure for different input variables for testing dataset

Model	Input variables	Number of optimal hidden neurons	ANN architecture	Performance indices				
				R^2	E (%)	RMSE (m^3/s)	P_{dv} (%)	MAE (m^3/s)
1	P_I(t-1)	2	1-2-1	0.80	64.49	263.51	19.36	90.20
2	P_I(t-1;t-2)	3	2-3-1	0.81	64.21	264.84	62.34	113.11
3	P_I(t-1); T_O(t-1)	2	2-2-1	0.86	69.55	244.03	47.46	74.05
4	P_I(t-1); T_O(t-1;t-2)	9	3-9-1	0.86	71.68	235.59	33.01	74.94
5	P_I(t-1); T_O(t-1); K_O(t-1)	7	3-7-1	0.79	62.14	272.09	33.37	86.11
6	P_I(t-1); T_O(t-1); K_O(t-2)	7	3-7-1	0.84	70.49	240.49	0.09	75.63
7	P_I(t-1); T_O(t-1); R_1(t-1)	3	3-7-1	0.86	70.03	242.12	49.74	74.30
8	P_I(t-1); T_O(t-1); R_1(t-1;t-2)	5	4-5-1	0.87	73.20	229.17	29.17	73.77
9	P_I(t-1); T_O(t-1); R_1(t-1;t-2;t-3)	6	5-6-1	0.86	70.57	240.42	42.51	73.39
10	P_I(t-1); T_O(t-1); R_1(t-1;t-2;t-4)	9	5-9-1	0.89	75.01	221.80	15.38	71.80
11	P_I(t-1); T_O(t-1); R_1 (t-1;t-2;t-4;t-5)	4	6-4-1	0.88	72.75	231.86	28.83	75.24
12	P_I(t-1); T_O(t-1); R_1 (t-1;t-2;t-4); R_2(t-1)	2	6-2-1	0.90	77.94	208.39	4.09	69.87
13	**P_I(t-1); T_O(t-1); R_1 (t-1;t-2;t-4); R_2(t-1;t-2)**	**7**	**7-7-1**	**0.90**	**80.74**	**194.74**	**2.85**	**68.10**
14	P_I(t-1); T_O(t-1); R_1(t-1;t-2; t-4); R_2(t-1); R_3(t-1)	7	7-7-1	0.89	79.06	203.04	3.62	72.28
15	P_I(t-1); T_O(t-1); R_1 (t-1;t-2;t-4); R_2(t-1); R_4(t-1)	4	7-4-1	0.90	78.32	206.60	15.91	68.12
16	P_I(t-1); T_O(t-1); R_1 (t-1;t-2;t-4); R_2(t-1); R_5(t-1)	6	7-6-1	0.89	77.95	208.36	28.15	85.37

P_I, T_O, K_O, R_1, R_2, R_3, R_4 and R_5 represent inflow of Panchet, Outflow of Tenughat, outflow of Konar, Rainfall at stations 1, 2,3,4 and 5, respectively. Whereas t represents at time t and t-1 represents at time t-1, etc. The entries in bold show the best fit models found during the study

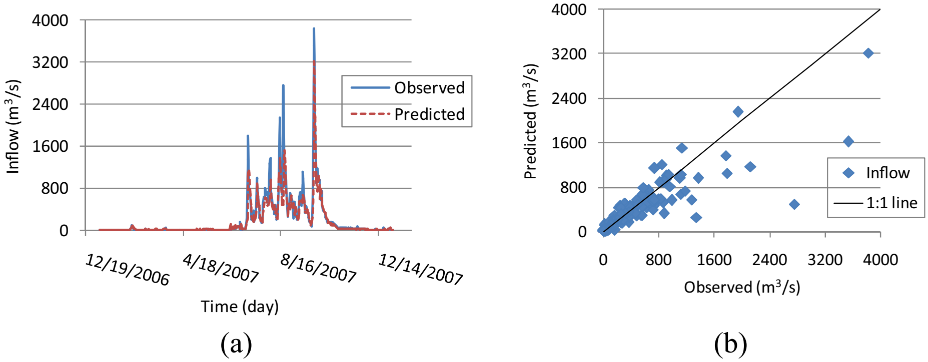


Fig. 4 Performance of NN model (**a**) Hydrograph (**b**) Scatter plot

Table 4 Performance of WANN models with different mother wavelets using all the wavelet components of best found ANN model #13

Mother wavelet function	Vanishing moment	Hidden neurons	Performance indices				
			R^2	E (%)	RMSE (m^3/s)	P_{dv} (%)	MAE (m^3/s)
db2	2	8	0.74	53.08	303.90	16.91	145.99
db5	**5**	**7**	**0.93**	**85.47**	**169.11**	**22.27**	**67.48**
db10	10	8	0.74	44.81	329.62	40.23	158.88
db20	20	5	0.47	16.15	406.27	76.47	270.08
Bior1.1	1	10	0.61	35.92	355.15	45.82	155.59
Bior3.3	3	6	0.67	44.70	329.93	21.70	179.08
Bior6.8	6	1	0.71	49.65	314.82	59.44	157.35
Haar	1	10	0.61	35.92	355.15	45.82	155.59
Coif1	2	6	0.80	52.60	305.45	15.61	140.87
Coif3	6	10	0.87	71.83	235.50	1.49	94.53
Coif5	10	7	0.80	62.75	270.79	14.69	130.53

The entries in bold show the best fit models found during the study

(iv) RMSE is always greater than 0, with value 0 the model fits the data perfectly. Percentage deviation in peak (P_{dv}):

$$P_{dv} = \frac{P_p - O_p}{O_p} 100 \tag{12}$$

where O_p and P_p represents peak values of observed and forecasted inflow, respectively.

Table 5 Performance of WANN models developed using different approaches for 1 day lead inflow forecasting

Input variables	Input variables	Hidden neurons	Performance indices				
			R^2	E (%)	RMSE (m^3/s)	P_{dv} (%)	MAE (m^3/s)
Approach 1	All the 4 components of 8 variables with lag 1	9	0.91	82.84	183.21	37.79	67.72
Approach 2	All the four components of input parameters and lags found best for ANN model	7	0.93	85.47	169.11	22.27	67.48
Approach 3	All the significant wavelet components separately having correlation >0.10 with lag 1	6	0.88	76.63	213.79	42.33	73.70
Approach 4	Newly constructed time series adding significant components of each parameter excluding d1 component (same lag input as found for best ANN)	7	0.93	87.15	159.05	11.31	65.83
Approach 5	Newly constructed time series adding significant components of each parameter	8	0.84	69.39	245.46	19.32	82.22

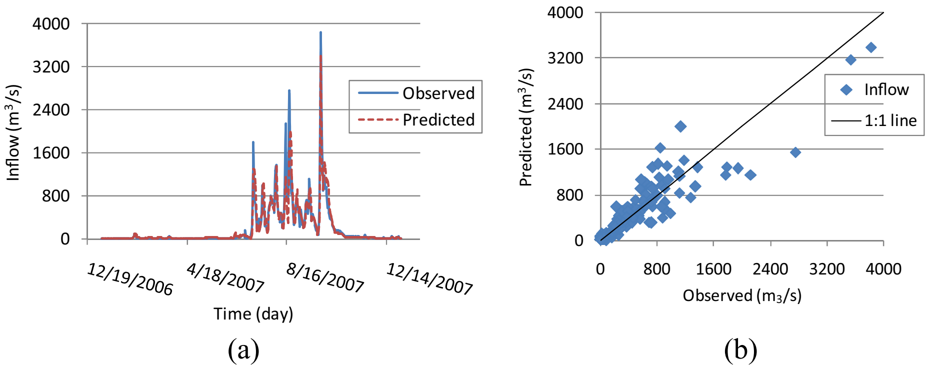


Fig. 5 Performance of WANN model (**a**) Hydrograph (**b**) Scatter plot using Model 4

(v) Mean absolute error (*MAE*):

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |O_i - P_i| \tag{13}$$

MAE is always a positive number, with its minimum value 0 representing a perfect model.

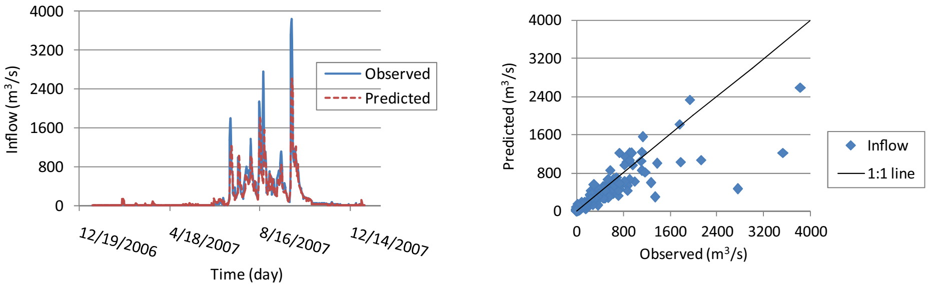
3 Results and Discussion

3.1 Performance of ANN Models

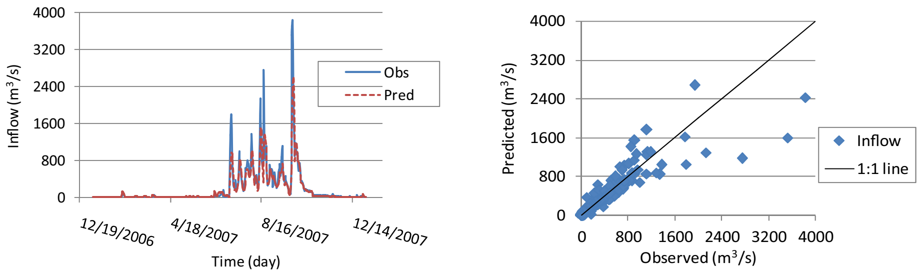
The performance of ANN model for different input variables and for optimum number of hidden neuron in terms of different performance indices is shown in Table 3. Considering all the performance indices, performance of model #13 was found to be the best. Hydrograph and scatter plot between observed and forecasted values are shown in Fig. 4. It is observed that simulated values show the general behavior of the observed values, even though performance of model is not very good for medium and high inflow values forecast. It can be considered that for best ANN model, outflow of Tenughat, inflow of Panchet reservoirs and rainfall values at stations R_1 and R_2 are found effective, whereas some of the input variables are not found effective in

Table 6 Performance of MLR and WMLR models for 1 day lead inflow forecasting

Model	Input variables	Performance indices				
		R^2	E (%)	RMSE (m^3/s)	P_{dv} (%)	MAE (m^3/s)
MLR	Same input variables as used for best ANN model	0.76	73.72	227.43	32.43	71.69
WMLR	Same input variables as used for best WANN model	0.80	77.41	210.88	32.36	66.28



(a) MLR



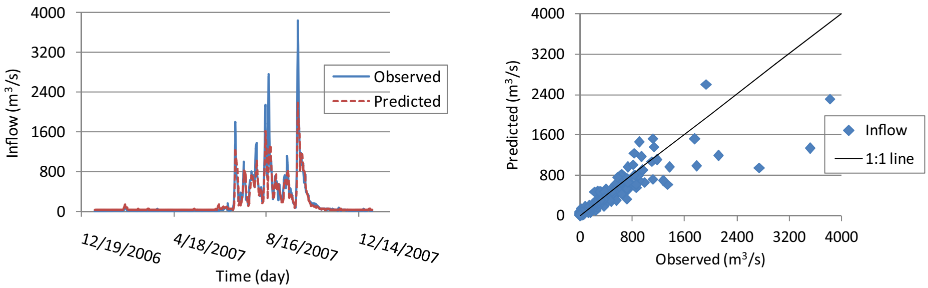
(b) WMLR

Fig. 6 Hydrographs and scatter plots for performance comparison of (**a**) MLR and (**b**) WMLR models

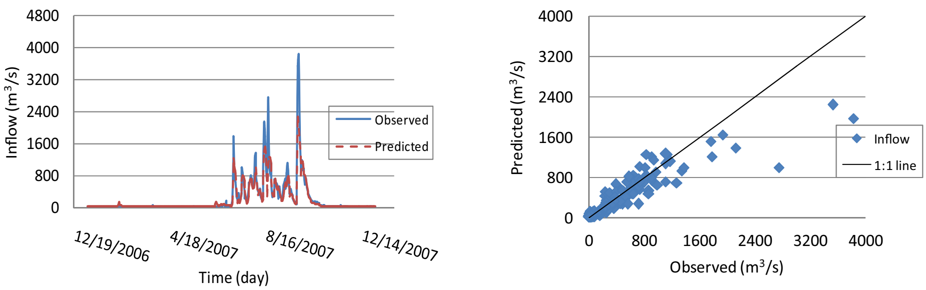
simulating inflow forecast at Panchet reservoirs such as outflow of Konar, rainfall at stations R_3, R_4 and R_5. This may be due to the reason that these data have high co-linearity with other hydro-climatic data and add negligible information for the improvement of daily inflow forecasts at Panchet reservoir. With R^2=0.90, E=78.32, RMSE=206.60 m^3/s and P_{dv}=15.91 and MAE=68.12 m^3/s, performance of best ANN model can be considered satisfactory, but higher value of RMSE compared to MAE shows that model is not able to simulate high inflow values accurately. It may be due to the reason that ANN models are not able to extract non-stationarity from the training dataset.

Table 7 Performance of BWMLR and WBANN models for 1 day dead inflow forecasting

Model	Input variables	Performance indices				
		R^2	E (%)	RMSE (m^3/s)	P_{dv} (%)	MAE (m^3/s)
BWMLR	Ensembled forecast of 100 MLR models calibrated using realization of training data, input same as for best ANN model	0.80	77.44	210.74	32.09	66.23
BWANN	Ensembled forecast of 100 ANN models trained using realization of training data, input same as for best WANN model	0.86	82.91	183.40	41.30	70.97



(a) BWMLR



(b) BWANN

Fig. 7 Hydrographs and scatter plots for performance comparison of (**a**) BWMLR and (**b**) WBANN models

3.2 Performance of WANN Models

WANN models were initially developed using all the wavelet sub time series components of input variables and lagged information for all the components were the same as found best for ANN model (Model #13) (i.e. Approach 2). Subsequently, performance of several wavelet functions with different vanishing moments was considered to select an appropriate wavelet function. The performance of WANN models using different wavelet functions and vanishing moment is shown in Table 4. It can be observed that wavelet function db5 performed best among all the wavelet functions considered. It can also be observed that performance of WANN model developed using db5 wavelet function with 5 vanishing moment performed better than standard ANN model in terms of different performance indices.

Once performance of WANN model was found better than ANN model, another issue addressed was to found the appropriate selection of wavelet components or wavelet sub time series to develop a robust WANN model. The performance of WANN models considering different approaches are presented in Table 5. It can be observed that performance of WANN models developed using newly constructed time series (i.e. Model 4) by adding significant components of each variables excluding *d1* component and by considering the same lagged time information as found best for ANN, resulted in improved performance.

The performance of best found WANN modeling approach is also shown in form of hydrograph and scatter plot as shown in Fig. 5. It can be observed that the observed values

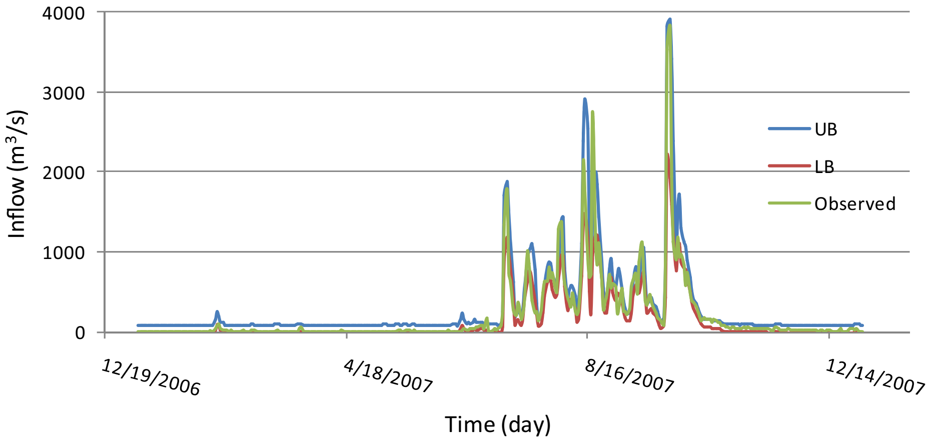


Fig. 8 Predicted 95 % confidence band and observed values using BWANN for testing period

are simulated very well and performance is considerably improved particularly for peak values as these are very close to 1:1 regression line. It can be observed that wavelet analysis with proper selection of wavelet function and vanishing moment along with suitable wavelet selection method can significantly improve the model performance.

3.3 Performance of MLR and WMLR Models

Similar to several previous studies, developed ANN and WANN models are compared with MLR model to benchmark the performance. In this study in addition to MLR models, wavelet based MLR models (WMLR) are developed and the performance is compared with ANN and WANN models. MLR and WMLR models are developed by using the same input variables as used for best ANN and WANN models, respectively, and the performance of both the models is shown in Table 6. It can be observed that the performance of WMLR models is better than MLR models, but WANN model performed better than both the MLR and WMLR models as these two models are not able to simulate medium and high inflow values satisfactorily (Fig. 6). This can also be observed from the higher RMSE values compared to MAE values for both the models.

3.4 Performance of BWANN and BWMLR Models

Performance of BWANN and BWMLR models is shown in Table 7 in terms of different performance indices and hydrograph and scatter plots using models (a) BWMLR and (b) BWANN are shown in Fig. 7. It can be observed that ensemble forecasts obtained using BWANN models is better than BWMLR models. It can also be seen that performance of BWANN models is not as good as obtained using WANN models, but BWANN models can be considered as more robust and accurate as these models are ensemble of several WANN models trained using different realizations of the training dataset. All these individual WANN models are developed using 100 different realizations of the training dataset, these realizations are applied to train the ANN models, and the ensemble of all these models are used to develop a new forecast for a new dataset. Further, the ensemble models initially generate 100 forecasts for a new testing/validation dataset using all these 100 trained models. Though the performance of ensemble BWANN models could have been better by generating ensemble of some of the better performing models, but these forecasts are the ensemble using all the 100 forecasts instead of omitting any of the non-performing models. Therefore it is advocated that the performance of BWANN model is better as these are the ensemble of 100 forecasts and are more reliable and accurate even if the new dataset for the forecasts has different complexity and variability compared to those applied for training of these ensemble models.

Instead of better performance of WANN models their reliability in generating same forecast for the newer even or dataset is questioned. Moreover, it cannot be guaranteed that if the training and testing dataset are changed or interchanged then their performance during forecast will remain the same. Therefore, assessment of uncertainty associated with the forecast is very important to know the reliability of the models and to implement the model in operational forecasting. The another advantage of BWANN model is that using different forecasts, uncertainty band or confidence bands can be constructed to see the uncertainty associated with the forecasts. Figure 8 depicts 95 % forecasted confidence bands and the corresponding observed values. Confidence bands not only show general behavior of the observed time series but that the values are simulated very well. It can be observed that higher inflow values contain higher uncertainty whereas lower inflow values have low uncertainty. Moreover, it can be observed that ensemble forecasts underestimate higher values (Fig. 7b), but BWANN models are able to assess the uncertainty associated with these forecasts. It shows that bootstrap technique is capable of uncertainty assessment and increases the reliability of WANN model forecasts.

4 Conclusions

Bootstrap wavelet based ANN model (BWANN) is developed in this study for inflow forecasting of Panchet reservoir in India and performance is compared with standard ANN, wavelet based ANN (WANN), bootstrap wavelet based MLR (BWMLR) and wavelet based MLR (WMLR) models. A robust ANN model was developed by considering several combinations of parameters such as input variables, optimization parameters, training algorithms and hidden neurons. WANN models are developed by considering different wavelet functions and approaches to ensure an efficient WANN model. Based on this study following conclusions are drawn:

- Wavelet analysis with proper selection of wavelet function and vanishing moment along with suitable wavelet selection method can significantly improve the WANN model performance.
- WANN model perform better than standard ANN, MLR and WMLR models for inflow forecasting.
- Optimum number of input selected for ANN model development are also best with different number of wavelet sub time series excluding d1 components for best WANN model development.
- Out of several wavelet functions and vanishing moments, db5 wavelet function with 5 vanishing moment provide best WANN model for inflow forecasting.

- WANN model not only have capabilities to simulate all the observed values very well, it simulates peak inflow values far better compared to remaining models.
- Performance of ANN model is improved significantly by including reservoir outflow and rainfall information in the upstream and nearby areas.
- Selection of significant input is very crucial as inclusion of randomly selected input variables may significantly reduce the model performance.
- Best WANN model can be developed by taking different wavelet components excluding d3 components of wavelet sub time series of input variables similar to those found best for ANN model. Second best approach found is by considering all the wavelet sub time series of raw input variables found best for ANN model.
- WANN model can deal with non-stationary dataset effectively and can be used as suitable tool for inflow forecasting. It has the potential to perform better for different non-stationary water resource variable forecasting.
- Forecasts obtained using BWANN models are not as good as WANN models, but they are more stable and consistent in case of change in training data pattern.
- BWANN models are very effective to assess the uncertainty associated with the inflow forecasts and have high applicability in operational inflow forecasting.
- Inflow forecasts can be improved by considering discharge releases from upstream reservoirs, and rainfall values in upstream and nearby locations in the upstream boundary.
- Ensemble forecasts not only provide quantitative point estimate but also provide probabilistic forecasts by generating confidence bands, which would be helpful for flood control and reservoir management authority in decision making.

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