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A Re-Parameterized and Improved Nonlinear Muskingum Model for Flood Routing

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Abstract The nonlinear form of the Muskingum model has been widely applied to river flood routing. There are four variants of the nonlinear Muskingum model based on alternative formulations of the nonlinear storage equation. This paper proposes a new Muskingum model with an improved, seven-parameter, nonlinear storage equation. The proposed model provides more degrees of freedom in fitting observed hydraulic data than other nonlinear Muskingum models. The proper estimation of the proposed Muskingum nonlinear model's parameters is essential to achieve accurate flood-routing predictions. This paper introduces a hybrid method for the estimation of Muskingum parameters. The parameter-estimation method combines the shuffled frog leaping algorithm (SFLA) and the Nelder-Mead simplex (NMS). The proposed Muskingum model and parameter estimation method were applied to the routing of several hydrographs. Our results indicate improved performance of the methodology described in this work when compared with those of other Muskingum models.

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Abbreviations

AD	Absolute deviation between observed and computed outflows
BFGS	Broyden-Fletcher-Goldfarb-Shannon
DE	Differential evolution
DPO	Deviations of peak of routed and Observed outflows
GA	Genetic algorithm
GRG	Generalized reduced gradient
HS	Harmony search
LM	Lagrange multiplier
MAR	MARquardt
NL1	Non-linear 1
NL2	Non-linear 2
NL3	Non-linear 3
NL4	Non-linear 4
NL5	Non-linear 5
N-LSM	Nonlinear least-squares method
NMS	Nelder-Mead simplex
PS	Pattern search
ICSA	Immune clonal selection algorithm
PSO	Particle swarm optimization
PSF-HS	Parameter setting free harmony search
SA	Simulated annealing
SFLA	Shuffled frog leaping algorithm
S-LSM	Segmented least-squares method
SS	Standard Search
SAD	Sum of the absolute deviations between observed and computed outflows
SSQ	Sum of squared deviations between observed and computed outflows
SQ	Squared deviation between observed and computed outflow

1 Introduction

Many techniques of operations research have been developed and applied in various fields of water resources systems. The applications encompass diverse fields of inquiry, such as linear estimation (Loáiciga and Church 1990), reservoir operation (Afshar et al. 2011; Bozorg Haddad et al. 2008a, b, 2009, 2011a; Fallah-Mehdipour et al. 2011a), cropping patterns (Moradi-Jalal et al. 2007; Fallah-Mehdipour et al. 2012), pumping scheduling (Bozorg Haddad and Mariño 2007; Bozorg Haddad et al. 2011b; Rasoulzadeh-Gharibdousti et al. 2011), water distribution networks (Bozorg Haddad et al. 2008c; Soltanjalili et al. 2010; Fallah-Mehdipour et al. 2011b; Seifollahi-Aghmiuni et al. 2011; Ghajarnia et al. 2011; Sabbaghpour et al. 2012), water quality problems (Cozzolino et al. 2005a, b, 2006, 2011), urban and rural drainage networks (Cimorelli et al. 2013a, 2014a; Palumbo et al. 2014; Cozzolino et al. 2015), operation of aquifer systems (Bozorg Haddad and Mariño 2011), and site selection of infrastructures (Karimi-Hosseini et al. 2011). Several of these works dealt with the development flood routing methods.

Floods are one of the costliest natural phenomena (Garcia and Loáiciga 2013). The prevention of flood damages includes the accurate prediction of hydrograph propagation along river reaches. This is, in fact, the domain of flood routing (Tewolde and Smithers 2006). There are hydraulic and hydrologic approaches for flood routing through river channels (Chow et al. 1988). The hydraulic approach is based on the numerical solution of either the convective-diffusion equations or the one-dimensional Saint-Venant equations of gradually varied unsteady flow in open channels (Cunge et al. 1980). This approach performs well typically but it is data-intensive and computationally burdensome (Samani and Shamsipour 2004). Sometimes, in order to carry out faster computations, simplified hydraulic approaches are used, such as the Parabolic Model, linearized (Cimorelli et al. 2013b, 2014b, 2015) or not (Ponce 1990; Santillana and Dawson 2010) and Kinematic Model, in turn linearized or not (Singh 1996).

The hydrologic approach is based on the conservation of mass principle and uses a conceptual relation between storage and discharge in place of the dynamic flow equation (Chow et al. 1988). The Muskingum model is a hydrologic flood routing approach introduced by McCarthy (1938) while conducting flood control studies in the Muskingum river in the United States.

The standard procedure for applying the Muskingum method involves two steps (Das 2004): (1) calibration and (2) prediction. The calibration step determines the parameters of the Muskingum model by using historical inflow-outflow hydrograph data of the investigated river reach. The prediction step solves for the outflow hydrograph given an inflow hydrograph using Muskingum routing equations.

A routing scheme for the Muskingum model is proposed for situations where the storage and weighted flow relationship is nonlinear. In previous research, four nonlinear versions of the Muskingum model have been reported (Chow 1959; Gavilan and Houck 1985; Gill 1978; Easa 2013). In the first (NL1 model) and second (NL2 model) version of the nonlinear Muskingum model, the exponent parameters are associated with the inflow and outflow variables of the storage equation. The third (NL3 model) version of the nonlinear Muskingum model associates its exponent parameter with the weighted flow of the storage equation. The fourth (NL4 model) version of the nonlinear Muskingum model combines the storage equations of the NL1 and NL3 model.

Easa (2013) pointed out that the aim of modifying the structure of a flood routing model is to produce more degrees of freedom in model calibration. He also stated that the NL4 model has more degrees of freedom than other nonlinear Muskingum models and, hence, would generally yield a closer fit to the observed outflow data.

In practical applications the calibration step is of utmost importance for applying nonlinear Muskingum models (Chow et al. 1988). Several researchers have applied various methods to estimate the parameters of nonlinear Muskingum models.

The available methods for parameter estimation of nonlinear Muskingum models can be classified in three groups (Barati 2011a). The first group consists of mathematical techniques, such as segmented least-squares method (S-LSM), nonlinear least-squares method (N-LSM), the Broyden-Fletcher-Goldfarb-Shannon (BFGS) technique, the Lagrange multiplier (LM) method, the Nelder-Mead simplex (NMS) method, and the generalized reduced gradient (GRG) method. These techniques rely on local search algorithms, which may converge in a few iterations but lack global optimality, in general. In addition, they achieve global optimal solutions contingent on the specification of suitable initial parameter estimates, a nontrivial task (Geem 2011). The second group of nonlinear Muskingum parameter estimation methods comprises phenomenon-mimicking algorithms, such as the pattern search (PS), genetic algorithm (GA), harmony search (HS), particle swarm optimization (PSO), immune Clonal selection algorithm (ICSA), differential evolution (DE), parameter setting free harmony search (PSF-HS) algorithm, simulated annealing (SA) algorithm, and shuffled frog leaping algorithm (SFLA). These algorithms search randomly for the near-global optimal solution. However, they are poor in terms of convergence performance. The third group of parameter estimation methods consist of the hybrid methodology that combines phenomenon-mimicking algorithms and mathematical techniques, such as the hybridizing GA and NMS (GA-NMS), the hybridizing HS and BFGS (HS-BFGS), and the hybridizing GA and GRG (GA-GRG). These algorithmic procedures offer the advantages of both optimization methods (phenomenon-mimicking algorithms and mathematical techniques) while offsetting their disadvantages.

Table 1 show details (solution methods, model type, number of parameters, and reference) of research works carried out over recent decades on the estimation of parameters of various nonlinear Muskingum models. This table indicates that previous research work has focused mostly on estimating the parameters of the NL3 model using different methods. The methods used for estimating the parameters of the NL3 Muskingum model include the S-LSM (Gill 1978), the hybrid Hooke-Jeeves pattern search and the Davidson-Fletcher-Powell (HJ + DFP) algorithm (Tung 1985), the N-LSQ (Yoon and Padm Anabhan 1993), the GA (Mohan 1997), the HS (Kim et al. 2001), the LM method (Das 2004), the BFGS technique (Geem 2006), the PSO (Chu and Chang 2009), the ICSA (Luo and Xie 2010), the PSF-HS algorithm (Geem 2011), the DE algorithm (Xu et al. 2012), the GA-NMS (Barati 2011a), the NMS method (Barati 2011b), the SFLA and SA (Orouji et al. 2013), the hybrid HS-BFGS (Karahan et al. 2013), the hybridizing NMS and Big Bang–Big Crunch (BBBC) (NMS-BBBC) (Karahan 2013), and the GRG (Barati 2013a; and Hamedi et al. 2014). A few studies have applied various methods to estimate the parameters of the NL1, NL2, and NL4 models, and some of those studies have compared the performances of the latter models with that of the NL3 model (Table 1).

Several researches have focused recently on altering the structure of the storage equation of the nonlinear Muskingum model with the aim of introducing greater flexibility in fitting observed hydrograph data. This paper introduces a seven-parameter nonlinearMuskingum model that modifies the structure of the storage equationof the NL4 model. This model exhibits more degrees of freedom in model calibration than the standard NL4 model. In addition, this study proposes a novel hybrid method that combines parameter-estimation algorithms, namely, the SFLA (Orouji et al. 2013) and NMS (Barati 2011b), to calibrate the seven parameters of the proposed Muskingum model. Sections 2 and 3 present the formulation details of the nonlinear Muskingum models and the proposed nonlinear Muskingum model, respectively. Section 4 details the method to estimate the model's parameters. Section 5 presents applications and results of the proposed model using three case studies involving single-peak, non-smooth hydrographs, and multi-peak hydrographs.

2 Formulation of the Nonlinear Muskingum Model

The Muskingum model is based on the continuity equation for a river reach, which is given by:

$$\frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} = I - O \tag{1}$$

where I = inflow; O = outflow; S = channel storage volume; t = time. The Muskingum model provides a second equation relating S, I, O, and the model parameters. In the original (linear) Muskingum model, the following storage equation was used:

$$S = K[XI + (1-X)O] \tag{2}$$

where K = storage constant, is greater than 0; and X = dimensionless weighting factor that represents the inflow-outflow relative effects on the storage. X ranges between 0.0 and 0.5 for reservoir storage, and 0 and 0.3 for stream channels (Mohan 1997; Geem 2006).

It is common to observe a nonlinear storage-discharge relationship in natural stream that induces significant error in flood routing by the linear Muskingum model (Gill 1978; Tung 1985).n Four forms of the nonlinear Muskingum models have been suggested in previous research for taking into account the nonlinearity between storage and discharge (Chow 1959; Gavilan and Houck 1985; Gill 1978; Easa 2013). The NL1 model was introduced by Chow (1959). The latter author proposed

Table 1 Non-linear Muskingum models reported in previous studies

Author and year of publication	Model type	Number of parameters	Solution method
Gill (1978)	NL3	3	S-LSM
Gavilan and Houck (1985)	NL1 and NL2	3 and 4	SS
Tung (1985)	NL3	3	PS
Yoon and Padm Anabhan (1993)	NL1 and NL3	3 and 3	N-LSM
Papamichail and Georgiou (1994)	NL2, NL1, and NL3	3, 4, and 3	MAR
Mohan (1997)	NL1 andNL3	3 and 3	GA
Kim et al. (2001)	NL3	3	HS
Das (2004)	NL1 and NL3	3 and 3	LM
Geem (2006)	NL3	3	BFGS
Chu and Chang (2009)	NL3	3	PSO
Luo and Xie (2010)	NL3	3	ICSA
Barati (2011a)	NL3	3	GA-NMS
Barati (2011b)	NL3	3	NMS
Geem (2011)	NL3	3	PSF-HS
Xu et al. (2012)	NL3	3	DE
Orouji et al. (2013)	NL3	3	SA and SFLA
Karahan et al. (2013)	NL3	3	HS-BFGS
Barati (2013a)	NL1, NL2, and NL3	3, 4, and 3	GRG
Karahan (2013)	NL3	3	NMS-BBBC
Easa (2013)	NL4	4	GA-GRG

the following formulas for inflow (I) and outflow (O) in a river reach, and for the water storages at the upstream (S_{in}) and downstream (S_{out}) sections of the reach:

$$I = ay^n \tag{3}$$

$$O = ay^n \tag{4}$$

$$S_{in} = by^m \tag{5}$$

$$S_{out} = by^m \tag{6}$$

where y = flow depth; a and n = coefficients that express the discharge-depth characteristics of the upstream and downstream end sections of a river reach; b and m = coefficients that express the mean storage-depth characteristics of the reach; and S_{in} and S_{out} = the storages at the upstream and downstream end sections, respectively. Eliminating y from Eqs. (3) and (5), and Eqs. (4) and (6), S_{in} and S_{out} are expressed as follows:

$$S_{in} = b\left(\frac{I}{a}\right)^{m/n} \tag{7}$$

$$S_{out} = b\left(\frac{O}{a}\right)^{m/n} \tag{8}$$

Chow (1959) proposed the following equation for the storage in a channel (S) at any given time:

$$S = XS_{in} + (1-X)S_{out} \tag{9}$$

Substituting S_{in} and S_{out} from Eqs. (7) and (8) into Eq. (9) and simplifying, yields the storage equation of the NL1 model:

$$S = K[XI^{\alpha} + (1-X)O^{\alpha}] \quad (NL1) \tag{10}$$

where $\alpha = \frac{m}{n}$; and $K = \frac{b}{a^{\alpha}}$. Chow (1959) showed that α is larger than 0.6 in natural channels.

The NL2 model was introduced by Gavilan and Houck (1985). This is a generalization of the NL1 model, given that $\alpha_2 \neq \alpha_1$:

$$S = K[XI^{\alpha_1} + (1-X)\,O^{\alpha_2}] \quad (NL2) \tag{11}$$

The NL3 model was introduced by Gill (1978), who added an exponent parameter β to the Muskingum Eq. (2):

$$S = K[XI + (1-X)\ \ O]^{\beta} \quad NL3 \tag{12}$$

The NL4 model was presented by Easa (2013). He combined the NL1 and NL3 models to produce the NL4 version:

$$S = [XS_{in} + (1-X)S_{out}]^{\beta} = K[XI^{\alpha} + (1-X)\,O^{\alpha}]^{\beta} \tag{13}$$

where $K = \left(\frac{b}{a^{\alpha}}\right)^{\beta}$. The NL4 model has more degrees of freedom (i.e., parameters) than the other nonlinear Muskingum models. Hence, it would generally yield a better fit to the observed outflow data. Easa (2013) showed that the NL4 model produced better performance with lower value of the sum of squared deviations (*SSQ*) and the sum of the absolute deviations (*SAD*) between observed and computed outflows than the other nonlinear Muskingum models. Notice that the NL1, NL2, and NL3 models [Eqs. (9)–(11)] are obtained from the NL4 model [Eq. (12)] by setting $\beta=1$, $\beta=1$ and $\alpha 1=\alpha 2$, or $\alpha=1$, respectively.

3 The Proposed Nonlinear Muskingum Model

Section characteristics in natural channels (rivers) are related to the 'formative discharge' and the sediment transport capacity (Wolman and Miller 1960; Andrews 1980; Pianese 1992; Leopold 1994). Since the formative discharge and sediment transport capacity vary along the natural channels, the upstream and downstream sections of the reach are not necessarily equal in natural channels. In this work the difference in morphological changes between the upstream and downstream sections of a river reach are captured by varying the a and n coefficients in the following manner:

$$S_{in} = b\left(\frac{I}{a_1}\right)^{m/n_1} \tag{14}$$

$$S_{out} = b\left(\frac{O}{a_2}\right)^{m/n_2} \tag{15}$$

where $a1$ and $n1$=express the discharge-depth characteristics of the upstream section; and $a2$ and $n2$=express the discharge-depth characteristics of the downstream section. Substituting S_{in} and S_{out} from Eqs. (13) and (14) into the equation $S=[XS_{in}+(1-X)S_{out}]^{\beta}$, and simplifying the resulting expression, produces the nonlinear Muskingum model (referred to as the NL5 model in this work):

$$S = K[X(C_1 I^{\alpha_1}) + (1-X)(C_2 O^{\alpha_2})]^{\beta} \quad (NL5) \tag{16}$$

where

$$K = b^{\beta} \tag{17}$$

$$\alpha_1 = \frac{m}{n_1} \tag{18}$$

$$\alpha_2 = \frac{m}{n_2}; \tag{19}$$

$$C_1 = \left[\left(\frac{1}{a_1}\right)^{\alpha_1}\right]^{\beta} \tag{20}$$

and

$$C_2 = \left[\left(\frac{1}{a_2}\right)^{\alpha_2}\right]^{\beta} \tag{21}$$

where $\alpha 1$, $\alpha 2$, β, $C1$, and $C2$ are greater than 0. Model NL4 [Eq. (12)] is obtained from model NL5 by setting $C1=C2=1$ and $\alpha 1=\alpha 2=\alpha$. The proposed NL5 model [Eq. (15)] has seven parameters (K, X, $\alpha 1$, $\alpha 2$, β, $C1$, and $C2$), and it is, in this sense, more complex than the other known nonlinear Muskingum models. The flood-routing problem is formulated as a mathematical optimization model that minimizes the sum of the squared deviations between observed and estimated outflows.

4 Estimating the Parameters of the Proposed NL5 Model

A simulation–optimization procedure is used to estimate the parameters of the proposed NL5 model. Sections 4.1 and 4.2 presents the simulation and optimization procedures, respectively.

4.1 Simulation Procedure of the Proposed NL5 Model

This paper employs Tung's (1985) flood-routing method, also employed by Geem (2006), to simulate flood routing with the NL5 model. The observed inflow, calculated outflow, and calculated storage at i-th time interval are I_i, $\hat{O}_i$, and S_i, respectively, where i=0, 1, 2, …, N denotes the simulation time intervals. The steps of the proposed NL5 flood-simulation model are:

Step 1 Assume values for the seven hydrologic parameters (K, X, α_1, α_2, β, C_1, C_2).

$$S_0 = K\left[X\left(C_1 I_0^{\alpha_1}\right) + (1-X)\left(C_2 \widehat{O}_0^{\alpha_2}\right)\right]^{\beta} \quad i=0 \tag{22}$$

Step 2 Calculate the initial storage S_0, letting the initial calculated outflow be equal to initial observed inflow ($\hat{O}_0=I_0$):

$$\frac{\Delta S_i}{\Delta t} = I_i - \left\{\left[\frac{1}{C_2(1-X)}\right]\left(\frac{S_i}{K}\right)^{\frac{1}{\beta}} - \left[\frac{1}{C_2(1-X)}\right]\left[X\left(C_1 I_i^{\alpha_1}\right)\right]\right\}^{\frac{1}{\alpha_2}} \tag{23}$$

Step 3 Calculate the time rate of change of the storage volume at time interval i (starting with i=1):

$$S_i = S_{i-1} + \Delta t\left(\frac{\Delta S_{i-1}}{\Delta t}\right) \tag{24}$$

Step 4 Calculate the storage at time i:

Step 5 Calculate the outflow at time interval i:

$$\widehat{O}_i = \left\{ \left[\frac{1}{C_2(1-X)}\right] \left(\frac{S_i}{K}\right)^{\frac{1}{\beta}} - \left[\frac{1}{C_2(1-X)}\right] \left[X\left(C_1 I_{i-1}^{\alpha_1}\right)\right] \right\}^{\frac{1}{\alpha_2}} \tag{25}$$

Notice that I_{i-1} rather than I_i is used in Eq. (24), following the approach of Geem (2006).

Step 6 Increase the index i by 1 and repeat Steps (3)–(5) until the simulation has reached time N.

The objective function used to evaluate the optimal values for the parameters of the proposed NL5 model is given by:

$$Min\ SSQ = \sum_{i=1}^{N} \left(O_i - \widehat{O}_i\right)^2 \tag{26}$$

where SSQ=sum of the square deviations between the observed outflow and computed outflow at time interval i; and O_i= observed outflow at time interval i. The objective function can also be set to minimize the sum of the absolute deviations between the observed outflow and computed outflow at i-th time interval. This is given by:

$$SAD = \text{min}\mathit{i}\text{mini}\mathit{ze}\sum_{i=1}^{N} \left|O_i - \widehat{O}_i\right| \; i = 0, 1, 2, \ldots, N \tag{27}$$

Or, the objective function could minimize the difference between the peak observed and the peak routed streamflow:

$$DPO = \text{min}\mathit{imize}\left|O_P - \widehat{O_P}\right| \tag{28}$$

where O_P= the value of peak of observed outflow; and $\hat{O}_P$= the value of peak of routed outflow. It should be noted that the DPO is an important variable in flood routing predictions. Flood damage is decreased by the improved accuracy of DPO estimated in the downstream reach of rivers (Orouji et al. 2013). Thus, the SSQ is the main objective function in the calibration processes, and the SAD and DPO are alternative objective functions whose minima are also satisfied.

4.2 Optimization Procedure of the Proposed NL5 Model

This section describes a hybrid optimization method for flood-routing parameter calibration that combines the shuffled frog leaping algorithm (SFLA) with the Nelder-Mead (NMS) method. In addition, the SFLA and NMS method are briefly reviewed.

4.2.1 Hybrid of the SFLA and the NMS Method

Barati (2011b) reported that the NMS method yields the best results among leading parameter estimation techniques. Orouji et al. (2013) indicated that the SFLA is the most efficient among well-known algorithms such as the genetic algorithm (GA). This paper proposes a new hybrid optimization technique that merges the SFLA with the NMS method. The proposed algorithm overcomes the disadvantages of the NMS (requirement of one good initial vector of estimates, lack of global optimality, and numerical divergence) and the SFLA (poor in terms of convergence performance, difficulty in locating global optima).

The hybrid SFLA-NMS method has two phases: (1) obtaining a vector of parameters by SFLA that is used as the initial solution for the NMS method, and (2) estimation of final parameter values by NMS using the initial solution obtained in the previous step.

4.2.2 The Shuffled Frog Leaping Algorithm (SFLA)

The SFLA is a meta-heuristic algorithm for solving optimization problems that is inspired by research on the hunting behavior of frogs. The algorithm uses memetic evolution in the form of influencing of ideas from one individual to another in a local search. Conceptually, the local search is similar to particle swarm optimization (PSO). A shuffling strategy allows the exchange of information among local searchers, leading them toward a global optimum. Based on this abstract model of virtual frogs, the SFLA draws on the PSO as a local search tool and the idea of competitiveness and mixing information from parallel local searches to move toward a global solution.

In SFLA, the frog of order j in the D dimensional space coordinates can be described as the form $x_j = [x_{j1}, x_{j2}, \ldots, x_{jD}]$, where $j=1, 2, \ldots, P$. The steps of the SFLA can be summarized in a flowchart as shown in Fig. 1. The details of this flowchart can be found in Orouji et al. (2013).

The local search and the shuffling processes are repeated until defined convergence criteria are satisfied, for example, a specific number of iterations. Accordingly, the Primary parameters of theSFLA are the number of frogs, the number of memeplexes, the number of generation for each memeplex before shuffling, and the number of shuffling iterations.

A computational function of the SFLA written in MATLAB (Orouji et al. 2013) was interfaced with a NL5 model to obtain a vector of the hydrologic parameters.

4.2.3 The Nelder-Mead Simplex (NMS) Method

The Nelder-Mead simplex (NMS) method (Nelder and Mead 1965) was originally developed for nonlinear and unconstrained optimization. It does not need derivatives calculations but only a numerical evaluation of the objective function is required (Nelder and Mead 1965). It converges to minima (assuming that the objective is minimization) by forming a simplex and using this simplex to search for its promising directions. A simplex is defined as a geometrical figure which is formed by $(n+1)$ vertices (n: the number of variables of a function). The NMS technique uses an initial guess (initial point) by the user to produce the initial simplex, which starts the algorithm. The initial guess is used as one of the vertices of the simplex. The remaining vertices of the initial simplex are found by adding ΔL (%) to each component of the initial guess vector. The algorithm uses four possible operations: reflection, expansion, contraction, and

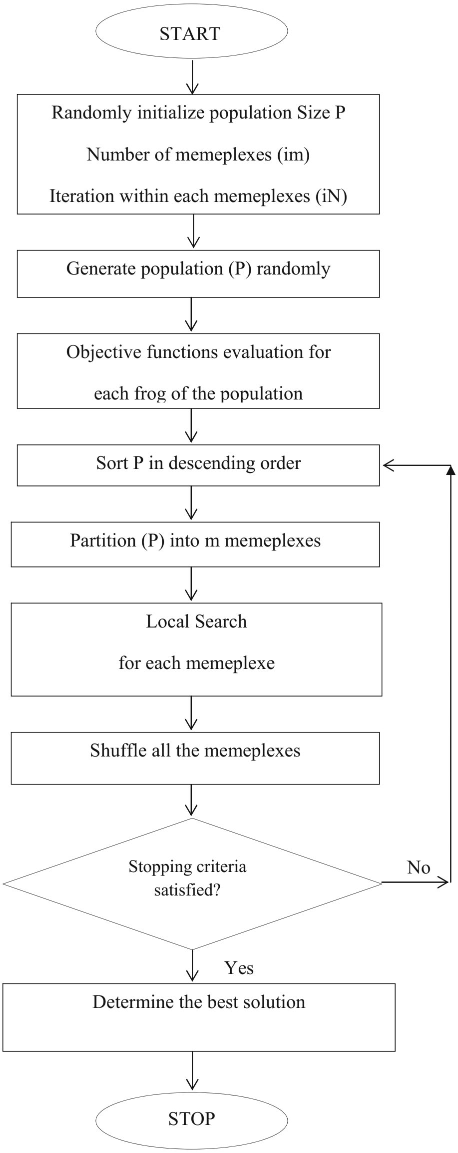

Fig. 1 Flow chart of the SFLA

shrinking. In each iteration of the NMS method the function values at each vertex are evaluated and the worst vertex (that with the largest value under minimization) is replaced by another (better) vertex which has just been found. Otherwise, a simplex is shrunk around the best vertex. This process is repeated iteratively until a desired convergence error value is satisfied. The convergence speed of the simplex method may be affected by four parameters: θ (reflection parameter), η (expansion parameter), γ (contraction parameter), and δ (shrink parameter). The values of these parameters

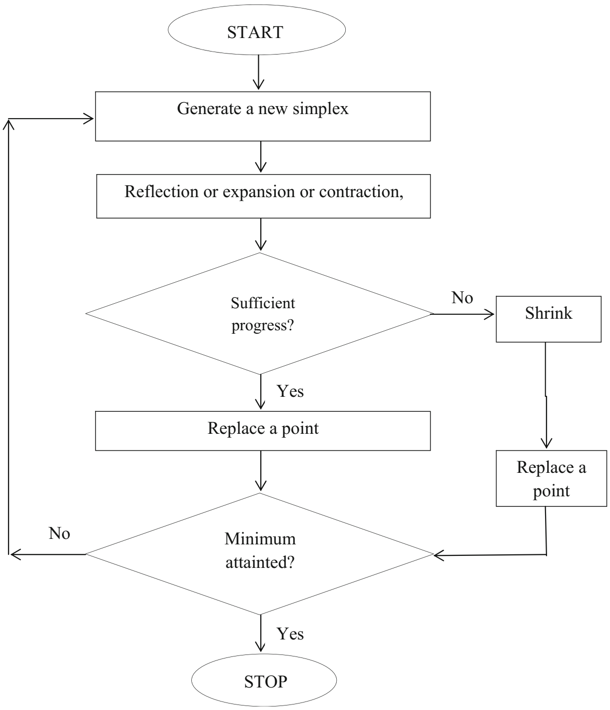

Fig. 2 Flowchart of the Nelder-Mead simplex algorithm

satisfy $\theta>0$, $\eta>1$, $0<\gamma<1$, and $0<\delta<1$. In the standard implementation of the Nelder-Mead method, the parameters are chosen to be $\theta=1$, $\eta=2$, $\gamma=0.5$, and $\delta=0.5$ (Nelder and Mead 1965; Lagarias et al. 1998), this set of parameters produces the shortest time of convergence. The steps of the NMS are shown in Fig. 2. Details ofthe NMS method can be found in Lagarias et al. (1998).

The NMS method was implemented in this paper by means of the *fminsearch* function of MATLAB (Yang et al. 2005), which was interfaced with the NL5 model to find the optimal values of the hydrologic parameters.

5 Applications and Results of the Proposed NL5 Model

Three different case studies (smooth single peak hydrograph, non-smooth single peak hydrograph and multiple peak hydrograph) were considered to test the performance of the proposed NL5 model compared to other nonlinear Muskingum models. In each of the three case studies the number of functional evaluations of the SFLA was 500, using 10 frogs and 50 iterations (Orouji et al. 2013). Moreover, although the SFLA is a random-based algorithm, the variation coefficient (standard deviation over the average) of the *SSQ* produced by the SFLA-NMS method in each of the case studies is very small. Thus, the results presented for each of three case studies were obtained with a single run of the SFLA-NMS method.

5.1 Case Study 1: Smooth Single Peak Hydrograph

The first case study uses the inflow and outflow hydrograph of Wilson (1974). The data reported by Wilson (1974) are known to present a nonlinear relationship between weighted discharge and storage (Yoon and Padm Anabhan 1993; Mohan 1997) and have been used by most of the previous studies for verification of a different procedure of parameter estimation of various nonlinear Muskingum models (especially the NL3 model) in the calibration step. The number of time steps and the duration of the time step in Wilson's (1974) data are Δt=6 h and N=21.

The optimal outflows and intermediate results of the proposed NL5 routing model include S_i, $\frac{\Delta S_i}{\Delta t}$, and O_i, squared deviation (SQ) between observed and computed outflows for time interval i, and absolute deviation (AD) between observed and calculated outflows for time interval i, which are listed in Table 2.

The comparison of the observed and calculated hydrographs of the proposed NL5 model is presented in Fig. 3. As is shown in Fig. 3, the computed hydrograph is well suited to the

Table 2 Optimal results obtained with the nonlinear Muskingum model NL5 for the first case study [data of Wilson (1974)]

i	Time (hour)	I_i (m^3/s)	O_i (m^3/s)	S_i (m^3)	$\frac{\Delta S_i}{\Delta t}$ (m^3/s)	$\hat{O}_i$ (m^3/s)	$(O_i-\hat{O}_i)^2$ (m^3/s)	$\lvert O_i-\hat{O}_i\rvert$ (m^3/s)
0	0	22	22	30.19	0.00	22.00	0.00	0.00
1	6	23	21	30.19	1.30	22.00	1.00	1.00
2	12	35	21	31.50	15.91	22.38	1.91	1.38
3	18	71	26	47.41	53.29	26.21	0.04	0.21
4	24	103	34	100.70	76.75	34.02	0.00	0.02
5	30	111	44	177.55	69.54	43.69	0.10	0.31
6	36	109	55	99.246	53.02	55.34	0.11	0.34
7	42	100	66	300.01	30.72	65.98	0.00	0.02
8	48	86	75	330.73	5.23	75.02	0.00	0.02
9	54	71	82	335.96	−17.61	81.78	0.05	0.22
10	60	59	85	318.34	−31.95	85.05	0.00	0.05
11	66	47	84	286.39	−43.31	84.07	0.00	0.07
12	72	39	80	243.08	−45.46	80.17	0.03	0.17
13	78	32	73	197.62	−44.58	72.81	0.04	0.19
14	84	28	64	15.04	−37.99	63.90	0.01	0.10
15	90	24	54	115.05	−31.93	53.95	0.00	0.05
16	96	22	44	83.12	23.42	44.50	0.25	0.50
17	102	21	36	59.70	−15.37	36.96	0.00	0.04
18	108	20	30	44.32	−9.81	29.43	0.32	0.57
19	114	19	25	34.52	−6.27	24.93	0.01	0.07
20	120	19	22	28.25	−2.88	21.88	0.01	0.12
21	126	18	19	25.37	–	20.24	1.55	1.24
Sum	–	–	–	–	–	–	5.44	6.69

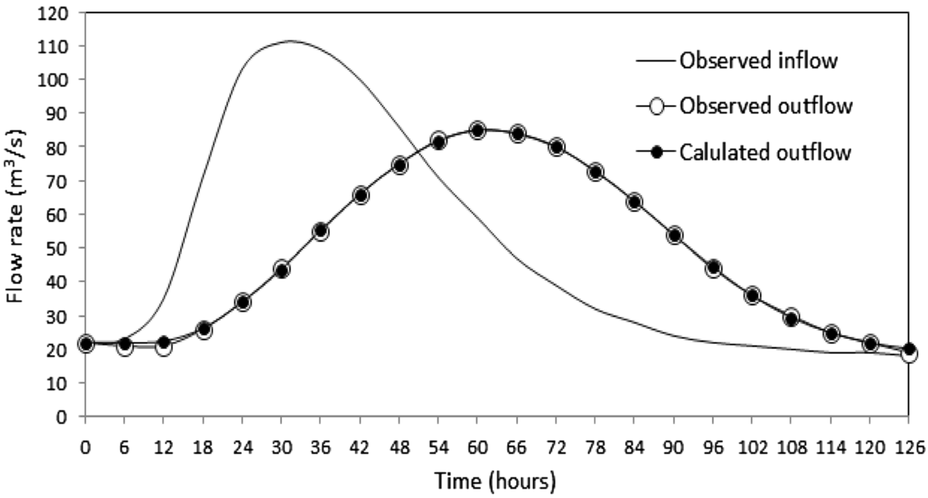

Fig. 3 Comparison of the observed hydrograph and calculated hydrograph obtained with the proposed NL5 model for the first case study [data from Wilson (1974)]

observed hydrograph. Moreover, this Figure shows that the NL5 model calculates accurately the outflow hydrograph peak, which is an important variable in hydrograph routing.

Table 2 lists the calculated outflows using the *SSQ*, *SAD*, and *DPO* objective functions and Wilson's (1974) obtained with the nonlinear Muskingum models NL1 (Barati 2013a), NL2 (Barati 2013a), NL3 (Karahan et al. 2013), NL4 (Easa 2013), and the proposed NL5 (this paper). It is clear from Table 2 that the use of the NL5 model improves the fit to observed outflows. The optimal values (minima) of the objective functions decrease with increasing model order, so that the proposed NL5 model features the best (smallest) values of the objective functions. The *SSQ*, *SAD*, and *DPO* values of the objective functions obtained with

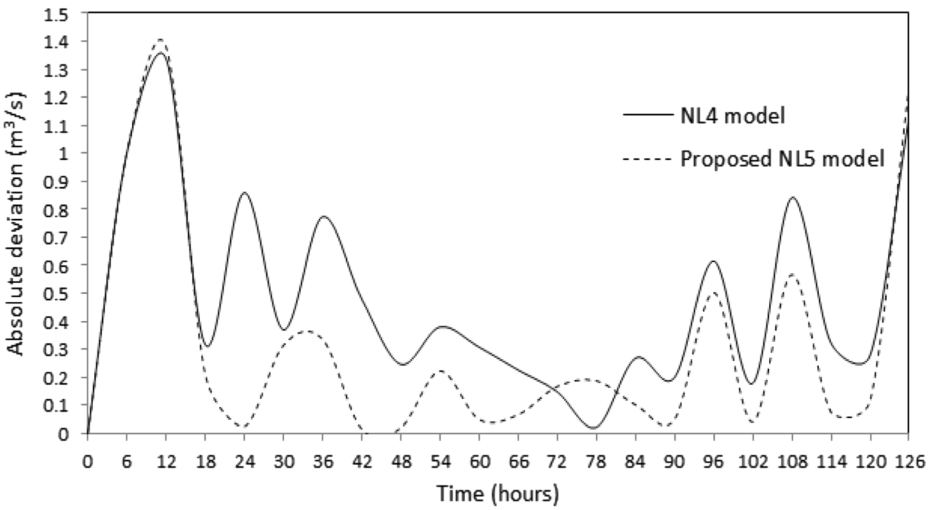

Fig. 4 Comparison of the *AD* values calculated with the NL4 model and the proposed NL5 model for the first case study [data from Wilson (1974)]

Table 3 Comparison of the optimal *SSQ*, *SAD*, and *DPO* parameter values obtained by the proposed and the existing nonlinear Muskingum models for the first case study, using the data by Wilson (1974)

Model type	Solution algorithm	Hydrologic parameters								Objective functions		
		K	X	A	α_1	α_2	β	C_1	C_2	*SSQ*	*SAD*	*DPO*
NL1	EV-GRG	0.461	0.229	1.501	–	–	–	–	–	258.45	58.25	1.85
NL2	EV-GRG	0.271	0.0003	–	3.042	1.568	–	–	–	184.32	32.18	0.55
NL3	HS-BFGS	0.086	0.278	–	–	–	1.868	–	–	36.77	23.47	0.90
NL4	GA-GRG	0.834	0.296	0.433	–	–	4.079	–	–	7.67	10.31	0.31
NL5	SFLA-NMS	0.478	0.088	–	0.696	0.425	3.817	0.619	0.735	5.44	6.69	0.05

the proposed NL5 model, for example, are 29, 35, and 84 %, respectively, lower than those with the NL4 model. These results demonstrate that the routed precision is satisfactory by using to NL5 model, which calculates the best objective function values when compared with the four other routing models reported in the literature.

A comparison of the *AD* values calculated with the of the NL4 model and the proposed NL5 model is shown in Fig. 4, which displays the superior fitting capacity of the proposed NL5 model compared to that of the NL4 model. Based on Fig. 4, the proposed NL5 model estimates the *AD* values more accurately than those obtained with the NL4 model.

The sensitivity of the objective function with respect to all model parameters of NL1, NL2, NL3, NL4, and NL5 was analyzed and the results are listed in Table 3. For a given parameter, the value shown is the percentage change in the objective function due to an increase of 1 % in the optimal value of that parameter (other parameters are kept at their optimal values). As noted, in all five models, K and X are relatively insensitive parameters, while the exponent parameters are sensitive parameters (exponent parameter α in the NL1 model, exponent parameters α_1 and α_2 in the NL2 model, exponent parameter β in the NL3 model, exponent parameters α and β in the NL4 model and exponent parameters α_2 and β in the NL5 model). Generally, the exponent parameters are the most sensitive when the storage vs. discharge is nonlinear.

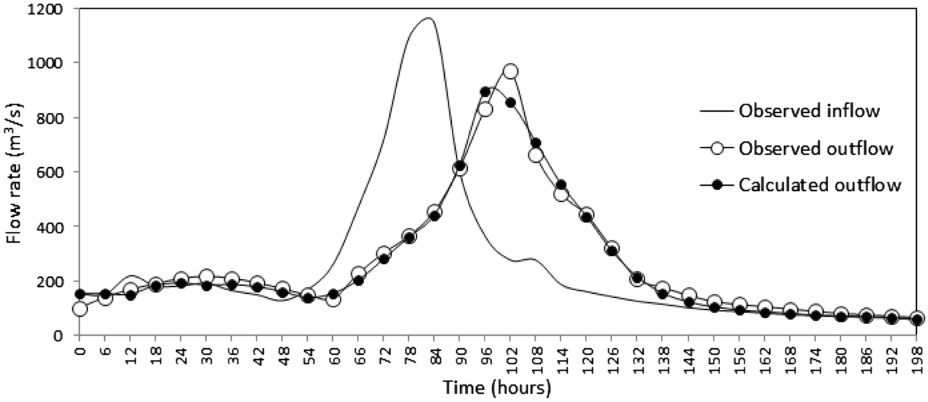

Fig. 5 Comparison of the observed hydrograph and the calculated hydrograph obtained with the proposed NL5 model for the second case study

Table 4 Sensitivity of the objective function to model parameters in the first case study

Parameter	Percentage change in the objective function due to 1 % increase in optimal parameter value				
	NL1	NL2	NL3	NL4	NL5
K	0.00	0.01	0.03	0.16	0.12
X	0.00	0.00	0.00	0.02	0.07
α	0.02	–	–	10.06	–
α_1	–	0.14	–	–	3.25
α_2	–	0.24	–	–	8.22
β	–	–	2.37	10.14	10.99
C_1	–	–	–	–	1.16
C_2	–	–	–	–	1.05

5.2 Case Study 2: Non-Smooth Single Peak Hydrograph

The second case study is a flood event that occurred in the Wye river in the United Kingdom (NERC 1975). The 69.75-km stretch of the River Wye from Erwood to Belmont has no tributaries and very small lateral inflow. Thus, this flood event is a good test case to test flood-routing methods (Bajracharya and Barry 1997). This flood was first studied by O'Donnell et al. (1988) with a linear Muskingum model. This flood event, similar to Wilson's (1974) data, presents a pronounced nonlinear relationship between flow and storage volume. This case study includes Δt=6 h and N=33. Karahan et al. (2013), Barati (2013b), and Hamedi et al. (2014) used this case study to estimate the parameters of the NL3 model with the HS-BFGS, NMS, and GRG methods. Easa (2013) used this case study to estimate the parameters of the NL4 model with the GA-GRG method.

A comparison of the observed hydrograph and calculated hydrograph using the proposed NL5 model of this case study is presented in Fig. 5. This clearly demonstrates that the calculated hydrograph obtained by proposed model is well fit to the observed hydrograph.

The *SSQ*, *SAD*, and *DPO* objective-function values calculated with the proposed NL5 model in this case study were 30,894.4, 731.7, and 72, respectively. Table 4 compares the *SSQ*, *SAD*, and *DPO* calculated considering different Muskingum models. Tables 4 shows a comparison of optimum parameters obtained from proposed NL5 model and various nonlinear Muskingum models, such as the NL3 model by

Table 5 Comparison of the optimal *SSQ*, *SAD*, and *DPO* parameter values obtained by the NL3 and NL4 models and the proposed NL5 model for the second case study

Model type	Solution algorithm	Hydrologic parameters								Objective functions		
		K	X	α	α_1	α_2	β	C_1	C_2	*SSQ*	*SAD*	*DPO*
NL3	GRG	0.076	0.415	–	–	–	1.59	–	–	34789.4	739.0	90
NL4	GA-GRG	0.437	0.404	1.197	–	–	1.332	–	–	32299.2	743.3	76
NL5	SFLA-NMS	0.600	0.609	–	1.056	1.163	1.398	0.96	1.02	30894.4	731.7	72

Table 6 Comparison of the computed outflows obtained with the NL3 and NL4 models and the proposed NL5 model for the second case study

i	Time	Observed data: m^3/s		Computed outflow: m^3/s		
	(hour)	I_i	O_i	NL3	NL4	NL5
0	0	154	102	154	154	154
1	6	150	140	154	154	154
2	12	219	169	152	152	152
3	18	182	190	183	184	184
4	24	182	209	192	192	193
5	30	192	218	185	186	186
6	36	165	210	187	187	187
7	42	150	194	178	179	176
8	48	128	172	161	162	162
9	54	168	149	139	141	140
10	60	260	136	154	155	155
11	66	471	228	201	203	204
12	72	717	303	267	281	281
13	78	1092	366	347	363	362
14	84	1145	456	419	443	443
15	90	600	615	602	624	626
16	96	365	830	879	893	896
17	102	277	969	839	849	855
18	108	277	665	689	709	711
19	114	187	519	531	560	554
20	120	161	444	414	424	439
21	126	143	321	290	307	311
22	132	126	208	203	219	216
23	138	115	176	150	160	156
24	144	102	148	123	127	124
25	150	93	125	102	107	105
26	156	88	114	94	94	93
27	162	82	106	88	88	88
28	168	76	97	81	82	81
29	174	73	89	75	75	75
30	180	70	81	72	73	72
31	186	67	76	69	69	69
32	192	63	71	66	66	66
33	198	59	66	62	62	61

GRG method (Hamedi et al. 2014) and the NL4 model by GA-GRG method (Easa 2013). The calculated optimal parameters of the proposed NL5 method in this study were obtained with the hybrid SFLA-NMS method. Based on Table 4 the least (best) value for the considered objective functions correspond to the NL5 method. Moreover, the *SSQ*, *SAD*, and *DPO* of the NL5 model were four, two, and five percent less

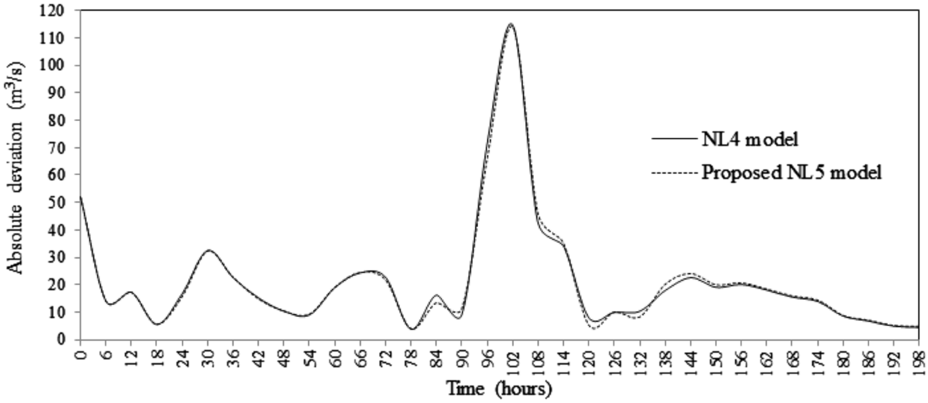

Fig. 6 Comparison of the *AD* values calculated with the NL4 model and the proposed NL5 model for the second case study

(better) than those obtained considering the NL4 nonlinear Muskingum model, respectively.

Table 5 presents a comparison between the outflows calculated with the NL3 Muskingum model (Hamedi et al. 2014), NL4 Muskingum model (Easa 2013), and the proposed NL5 Muskingum model introduced this study. It is evident from Table 5 that the outflows calculated with the NL5 model are more accurate than those obtained with other routing models (Table 6).

To further illustrate the ability of the proposed NL5 model to fit the data better than NL4 model, a comparison of the absolute outflow deviations of the two models is shown in Fig. 6. These values demonstrate that the NL5 model achieved better data fitting than the NL4 model.

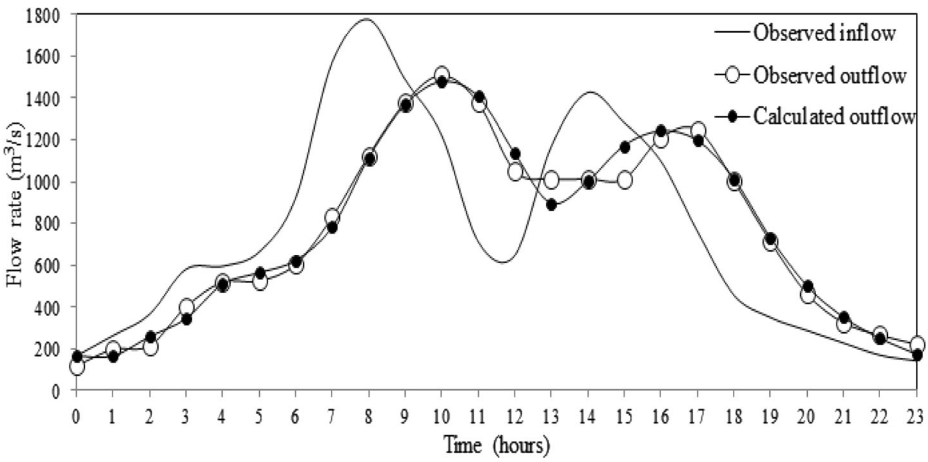

Fig. 7 Comparison of the observed hydrograph and the calculated hydrograph using the proposed NL5 model for the third case study

5.3 Case Study 3: Multiple-Peak Hydrograph

The third case study is a flood multiple peak hydrograph introduced by Viessman and Lewis (2003). This case study includes $\Delta t=1$ h and $N=23$. The observed inflow-outflow hydrographs are shown in Fig. 7. The outflow hydrograph estimated by the proposed model NL5 is also shown in Fig. 7. The *SSQ*, *SAD*, and *DPO* objective-function values were obtained with the proposed NL5 model for the parameter vector ($K=0.078$, $X=5\times10^{-7}$, $\alpha_1=3.121$, $\alpha_2=1.420$, $\beta=1.0861$, $C_1=0.99$, and $C_2=1.03$) were 69,860, 994, and 30, respectively. In addition, the best *SSQ*, *SAD*, and *DPO* objective-function values of the NL4 model for the parameter vector ($K=0.077$, $X=0.167$, $\alpha=0.921$, and $\beta=1.568$) were 73,399, 1034, and 50, respectively, obtained with the SFLA-NMS method in this paper. Thus, the calculated optimal *SSQ*, *SAD*, and *DPO* values with the proposed model NL5 for this example decreased (improved) 5, 4, and 40%, respectively, compared to those obtained with the NL4 model.

6 Concluding Remarks

The nonlinear Muskingum model is widely used for hydrologic flood routing. It relies on the continuity equation and an assumed nonlinear storage equation. Several researchers have improved the fitting performance of the nonlinear Muskingum model by modifying the structure of its nonlinear storage equation. Such modifications for the structure of the nonlinear storage equation have introduced more degrees of freedom in model calibration.

This paper proposed an improved nonlinear Muskingum model with a seven-parameter nonlinear storage equation. All existing forms of the Muskingum model are special cases of the proposed model. This provides the user with flexibility in evaluating all model forms easily. In addition, a new hybrid SFLA-NMS method was introduced in this paper to solve the hydrologic parameter calibration problem. The proposed algorithm finds the global or near-global minimum with fast convergence. The latter method found the best parameter values measured in terms of the sum of the square deviations, the sum of absolute deviations among observed and estimated outflows, and the absolute value of the difference between the observed peak the routed peak outflows.

The performance of the proposed nonlinear Muskingum model (NL5) was compared with those of other common nonlinear Muskingum models using three case studies. The proposed NL5 model produced better results than the existing non-linear models. The *SSQ* obtained with the proposed model for the first, second, and third case studies decreased (improved) 29, 4, and 5%, respectively, compared to those of the corresponding optimized values (NL4 model) reported in previous studies. Although the proposed model involves a more complex calibration procedure than other nonlinear Muskingum models, the additional complexity could results in a substantial improvement in data fitting. This paper's application of the proposed NL5 model shows that it could substantially (up to almost 29 %) improve the fit to observed outflows. The added model-calibration complexity was mitigated by a novel, hybrid, estimation method.

This paper has shown that the SFLA-NMS method can be successfully applied to estimate optimal parameter values of various nonlinear Muskingum models. The proposed nonlinear Muskingum model is recommended for future studies and applications in flood routing.

Conflict of interest No conflict of interest.

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