Development of a Fuzzy-Boundary Interval Programming Method for Water Quality Management Under Uncertainty

J. Liu · Y. P. Li · G. H. Huang · S. Nie

Abstract This study develops a fuzzy-boundary interval programming (FBIP) method for tackling dual uncertainties expressed as crisp intervals and fuzzy-boundary intervals. An interactive algorithm and a vertex analysis approach are proposed for solving the FBIP model and solutions with α-cut levels have been generated. FBIP is applied to planning water quality management of Xiangxi River in the Three Gorges Reservoir Region, China. Biological oxygen demand (BOD), total nitrogen (TN), and total phosphorus (TP) are selected as water quality indicators to determine the pollution control strategies. Results reveal that the highest discharge of BOD is observed at the Baishahe chemical plant, among all point and nonpoint sources; crop farming is the main nonpoint source with the excessive nitrogen loading due to too much uses of livestock manures and chemical fertilizers; phosphorus discharge derives mainly from point sources (i.e. chemical plants and phosphorus mining companies). Abatement of pollutant discharges from industrial and agricultural activities is critical for the river pollution control; however, the implementation of management practices for pollution control can have potentials to affect the local economic income. These findings can help generate desired decisions for identifying various industrial and agricultural activities in association with both maximizing economic income and mitigating river-water pollution.

Keywords Fuzzy-boundary · Interval · Management · Optimization · Water quality · Dual uncertainty

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Nomenclatures of FBIP-WQM

i	Chemical plant, 1 = Gufu (GF), 2 = Baishahe (BSH), 3 = Pingyikou (PYK), 4 = Liucaopo (LCP), 5 = Xiangjinlianying (XJLY)
j	Agricultural zone, and j =1, 2, 3, 4
k	Main crop, 1= citrus, 2 = tea, 3 = wheat, 4 = potato, 5 = rapeseed, 6 = alpine rice, 7 = second rice, 8 = maize, 9 = vegetables
p	Phosphorus mining company; 1 = Xinglong (XL) 2 = Xinghe (XH), 3 = Xingchang (XC), 4 = Geping (GP), 5 = Jiangjiawan (JJW), 6 = Shenjiashan (SJS)
r	Livestock, 1 = pig, 2 = ox, 3 = sheep, 4 = domestic fowls
s	Town, 1 = Gufu, 2 = Nanyang, 3 = Gaoyang, 4 = Xiakou
t	Planning time period, 1 = dry season, 2 = wet season
L_t	Length of period (day)
$BC_{it}^{\pm}$	Net benefit from chemical plant i in period t (RMB¥/t)
$PLC_{it}^{\pm}$	Production level of chemical plant i in period t (t/day)
$BP_{pt}^{\pm}$	Average benefit for per unit phosphate ore (RMB¥/t)
$PLM_{pt}^{\pm}$	Production level of phosphorus mining company p during period t (t/day)
$BW_{st}^{\pm}$	Net benefit from water supply to municipal uses (RMB¥/m^3)
$QW_{st}^{\pm}$	Quantity of water supply to town s in period t (m^3/day)
$CY_{jkt}^{\pm}$	Yield of crop k planted in agricultural zone j during period t (t/ha)
$BA_{jkt}^{\pm}$	Average benefit for agricultural product (RMB¥/t)
$PA_{jkt}^{\pm}$	Planning area of crop k in agricultural zone j during period t (ha)
$BL_r^{\pm}$	Average benefit from livestock r (RMB¥/unit)
$NL_r^{\pm}$	Number of livestock r in the study area (unit)
$WC_{it}^{\pm}$	Wastewater generation rate of chemical plant i during period t (m^3/t)
$CC_{it}^{\pm}$	Wastewater treatment cost of chemical plant i during period t (RMB¥/m^3)
$FW_{it}^{\pm}$	Water consumption of per unit production of chemical plant i during period Rt (m^3/t)
$WSP_t^{\pm}$	Price for industrial water supply (RMB¥/m^3)
$GT_{st}^{\pm}$	Wastewater generation rate at town s during period t (m^3/m^3)
$CT_{st}^{\pm}$	Cost of municipal wastewater treatment (RMB¥/m^3)
$CM_{jt}^{\pm}$	Cost of manure collection/disposal in agricultural zone j during period t (RMB¥/t)
$CF_{jt}^{\pm}$	Cost of purchasing fertilizer in agricultural zone j during period t (RMB¥/t)
$AM_{jkt}^{\pm}$	Amount of manure applied to agricultural zone j with crop k during period t (t)
$AF_{jkt}^{\pm}$	Amount of fertilizer applied to agricultural zone j with crop k during period t (t)
$WSC_t^{\pm}$	Price for agricultural water supply (RMB¥/ha)
$TPC_{st}^{\pm}$	Capacity of wastewater treatment capacity (WTPs) (m^3/day)
$TPD_{it}^{\pm}$	Capacity of wastewater treatment capacity (chemical plants) (m^3/day)
$IC_{it}^{\pm}$	BOD concentration of raw wastewater from chemical plant i in period t (kg/m^3)
$\eta_{BOD,it}^{\pm}$	BOD treatment efficiency in chemical plant i during period t (%)
$ABC_{it}^{\pm}$	Allowable BOD discharge for chemical plant i in period t (kg/day)
$BM_{st}^{\pm}$	BOD concentration of municipal wastewater at town s during period t (kg/m^3)
$\eta'^{\pm}_{BOD,st}$	BOD treatment efficiency of WTPs at town s during period t (%)

$\widetilde{ABW}_{st}^{\pm}$ Allowable BOD discharge for WTPs at town s during period t (kg/day)

$AML_{rt}^{\pm}$ Amount of manure generated by livestock r during period t [t/ (unit·day-1)]

$AMH_{t}^{\pm}$ Amount of manure generated by humans [t/ (unit·day-1)]

$RP_{t}^{\pm}$ Total rural population in the study area during period t (unit)

$MS_{t}^{\pm}$ Manure loss rate in period t (%)

$\varepsilon_{NM}^{\pm}$ Nitrogen content of manure (%)

$ACW_{t}^{\pm}$ Wastewater generation of per capita water consumption during period t [m^3/ (unit·day-1)]

$DNR_{t}^{\pm}$ Dissolved nitrogen concentration of rural wastewater during period t (t/m^3)

$ANL_{t}^{\pm}$ Maximum allowable nitrogen loss from rural life section in period t (t)

$NS_{jk}^{\pm}$ Nitrogen content of soil in agricultural zone j planted with crop k (%)

$SL_{jkt}^{\pm}$ Average soil loss from agricultural zone j planted with crop k in period t (t/ha)

$RF_{jkt}^{\pm}$ Runoff from agricultural zone j with crop k in period t (mm)

$DN_{jkt}^{\pm}$ Dissolved nitrogen concentration in runoff from agricultural zone j planted with crop k in period t (mg/L)

$MNL_{jt}^{\pm}$ Maximum allowable nitrogen loss in agricultural zone j during period t (t/ha)

$TA_{jt}^{\pm}$ Tillable area of agricultural zone j during period t (ha)

$PCR_{it}^{\pm}$ Phosphorus concentration of raw wastewater from chemical plant i in period t (kg/m^3)

$\eta_{TP,it}^{\pm}$ Phosphorus treatment efficiency in chemical plant i in period t (%)

$ASC_{it}^{\pm}$ Amount of slag discharged by chemical plant i in period t (kg/t)

$SLR_{it}^{\pm}$ Slag loss rate due to rain wash in chemical plant i during period t (%)

$PSC_{it}^{\pm}$ Phosphorus content in slag generated by chemical plant i in period t (%)

$APC_{it}^{\pm}$ Allowable phosphorus discharge for chemical plant i in period t (kg/day)

$\varepsilon_{PM}^{\pm}$ Phosphorus content of manure (%)

$DPR_{t}^{\pm}$ Dissolved phosphorus concentration of rural wastewater during period t (t/m^3)

$APL_{t}^{\pm}$ Maximum allowable phosphorus loss from rural life during period t (t)

$PCM_{st}^{\pm}$ Phosphorus concentration of municipal wastewater at town s in period t (kg/m^3)

$\eta_{TP,st}^{\pm}$ Phosphorus treatment efficiency of WTP at town s in period t (%)

$APW_{st}^{\pm}$ Allowable phosphorus discharge for WTP at town s in period t (kg/day)

$WPM_{pt}^{\pm}$ Wastewater generation from phosphorus mining company p in period t (m^3/t)

$MWC_{pt}^{\pm}$ Phosphorus concentration of wastewater from mining company p in period t (kg/ m^3)

$\eta_{TP,pt}^{\pm}$ Phosphorus treatment efficiency in mining company p (%)

$ASM_{pt}^{\pm}$ Amount of slag discharged by mining company p during period t (kg/t)

$PCS_{pt}^{\pm}$ Phosphorus content in generated slag (%)

$SLW_{pt}^{\pm}$ Slag loss rate due to rain wash (%)

$APM_{pt}^{\pm}$ Allowable phosphorus discharge for mining company p during period t (kg/day)

$PS_{jk}^{\pm}$ Phosphorus content of soil in agricultural zone j planted with crop k (%)

$SL_{jkt}^{\pm}$ Average soil loss from agricultural zone j planted with crop k in period t (t/ha)

$DP_{jkt}^{\pm}$ Dissolved phosphorus concentration in runoff from agricultural zone j with crop k (mg/L)

$MPL_{jt}^{\pm}$	Maximum allowable phosphorus loss in agricultural zone j during period t (t/ha)
$MSL_{jt}^{\pm}$	Maximum allowable soil loss agricultural zone j in period t (t/ha)
$NVF_{t}^{\pm}$	Nitrogen volatilization/denitrification rate of fertilizer in period t (%)
$NVM_{t}^{\pm}$	Nitrogen volatilization/denitrification rate of manure in period t (%)
$\varepsilon_{NF}^{\pm}$	Nitrogen content of fertilizer (%)
$\varepsilon_{PF}^{\pm}$	Phosphorus content of fertilizer (%)
$\varepsilon_{NM}^{\pm}$	Nitrogen content of manure (%)
$\varepsilon_{PM}^{\pm}$	Phosphorus content of manure (%)
$NR_{jkt}^{\pm}$	Nitrogen requirement of agricultural zone j with crop k during period t (t/ha)
$PR_{jkt}^{\pm}$	Phosphorus requirement of crop k in agricultural zone j during period t (t/ha)
$TAH_{jt}^{\pm}$	Dry farmland of agricultural zone j during period t (ha)
$TAS_{jt}^{\pm}$	Paddy farmland of agricultural zone j during period t (ha)
$MFP_{t}^{\pm}$	The government requirement for minimum area of farmland during period t (ha)
$PLC_{it,\ min}^{\pm}$	Minimum production level of chemical plant i in period t (t/day)
$PLC_{it,\ max}^{\pm}$	Maximum production level of chemical plant i in period t (t/day)
$NL_{r,\ min}^{\pm}$	Minimum number of livestock r in the study area (unit)
$NL_{r,\ max}^{\pm}$	Maximum number of livestock r in the study area (unit)
$QW_{st,\ min}^{\pm}$	Minimum quantity of water supply to town s in period t (m³/day)
$QW_{st,\ max}^{\pm}$	Maximum quantity of water supply to town s in period t (m³/day)
$PLM_{pt,\ min}^{\pm}$	Minimum production level of phosphorus mining company p during period t (t/day)
$PLM_{pt,\ max}^{\pm}$	Maximum production level of phosphorus mining company p during period t (t/day)

1 Introduction

Effective planning of water quality management is important for facilitating socio-economic development and eco-environmental sustainability in watershed systems (Carroll et al. 2013). A wide range of mathematical techniques have been developed to examine the temporal and spatial economic, environmental and ecological impacts of alternative pollution-control actions, and thus aid the planners or decision-makers in formulating and adopting cost-effective water-quality management plans and policies (Lung et al. 1999; Islam et al. 2013; Wang et al. 2014). Water-quality management often requires not only the reinforcement of established principles and technologies but also their extension to much wider, higher and freer scope for the realization of sustainability. Moreover, in water quality management problems, various uncertainties exist in a number of system components as well as their interrelationships; these uncertainties can be further amplified by not only interactions among various uncertain and dynamic impact factors, but also their associations with economic implications of satisfied or violated environmental requirements (Zhang et al. 2014).

A number of optimization methods were developed for water quality management under uncertainty (Chaves and Kojiri 2007; Kataria et al. 2010; Li et al. 2011; Prakash et al. 2012; Mohammad and Najmeh 2013; Xu and Qin 2013). Fuzzy programming (FP), derived from the fuzzy set theory, is effective in reflecting ambiguity and vagueness in decision-making problems; there are two major FP approaches: possibilistic programming and flexibility programming (abbreviated as FPP and FFP). Fuzzy parameters can be introduced into the FPP modeling framework, which represent fuzzy regions where the parameters are regarded as possibility distributions; however, when many uncertain parameters are expressed as fuzzy

sets, interactions among these uncertainties may lead to serious complexities, particularly for large-scale practical problems. In practical water-quality management problems, some parameters (e.g., discharge allowance) can be expressed as possibility distributions (i.e. objectively determined relying on some available historical data that are analogous to the probability concepts), which represent the possible degree of event occurrence for imprecise data (Torabi and Hassini 2008; Zhang et al. 2009). Furthermore, some uncertainties can be estimated as interval values and, at the same time, the lower and upper bounds of these intervals are also fuzzy in nature, leading to dual uncertainties (Liu et al. 2014). These complexities have placed many water quality management problems beyond the conventional fuzzy programming methods.

Therefore, this study aims at developing a fuzzy-boundary interval programming (FBIP) method for tackling dual uncertainties expressed as crisp intervals (i.e. with deterministic lower and upper bounds) and fuzzy-boundary intervals (i.e. the lower and upper bounds of some intervals may rarely be acquired as deterministic values, and they may be fuzzy in nature). FBIP will incorporate interval-parameter programming (IPP) and fuzzy possibilistic programming (FPP) within a general framework. The FBIP method will be applied to planning municipal, industrial and agricultural activities of Xiangxi River watershed, and results will help decision makers to identify desired pollution-mitigation strategies with a maximized system benefit and minimized environmental impact.

2 Model Development

When coefficients in the objective and constraints are ambiguous and can be expressed as possibility distributions, the problem can be formulated as a fuzzy programming (FP) model as follows:

$$\text{Max } \tilde{f} = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{1a}$$

subject to:

$$\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \leq \widetilde{b_i}, \quad i = 1, 2, \ldots, m \tag{1b}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{1c}$$

where x_j ($j = 1, 2, \ldots, n$) are decision variables; $\tilde{c}_j$, $\widetilde{a_{ij}}$ and $\widetilde{b_i}$ are fuzzy coefficients of the objective and constraints. The possibility distributions of fuzzy parameters can be characterized as fuzzy sets; application of the extension principle to fuzzy sets can be viewed as its extension to α-cuts when the membership functions are continuous (Zadeh 1975; Zimmermann 1995). Vertex analysis based on α-cut concept is useful for dealing with fuzzy sets, and the detailed definitions related to vertex analysis can be found in a number of literatures (Dong and Shah 1987; Kaufmann and Gupta 1991; Chen et al. 1998; Li et al. 2009).

Interval-parameter programming (IPP) is an alternative for handling uncertainties that cannot be quantified as membership or distribution functions, since interval numbers are acceptable as its uncertain inputs (Huang 1996). However, in many real-world problems, the lower and upper bounds of some interval parameters can rarely be acquired as deterministic values. Instead, they may often be given as subjective information that can only be expressed as fuzzy sets; this leads to dual uncertainties due to the fact that decision makers express different subjective judgments upon a same problem. For example, in water quality management problems, decision makers may estimate that there is no possibility for total phosphorus (TP) discharge from one point source

lower than [30.49, 50.89] kg/day or higher than [70.71, 90.23] kg/day. Such dual uncertainties cannot be addressed through the conventional IPP and FP methods. As a result, techniques of IPP and FP will be coupled in a general framework to handle such complexities; this leads to a fuzzy-boundary interval programming (FBIP) model as follows:

$$\text{Max } \widetilde{f}^{\pm} = \sum_{j=1}^{n} \left(\widetilde{c}_{j}^{\pm}, +, e_{j}^{\pm} \right) x_{j}^{\pm} \tag{2a}$$

subject to:

$$\sum_{j=1}^{n} a_{rj}^{\pm} x_{j}^{\pm} \le b_{r}^{\pm}, \quad r = 1, 2, \ldots, m_1 \tag{2b}$$

$$\sum_{j=1}^{n} \widetilde{a}_{ij}^{\pm} x_{j}^{\pm} \le \widetilde{b}_{i}^{\pm}, \quad i = 1, 2, \ldots, m_2 \tag{2c}$$

$$x_{j}^{\pm} \ge 0, \quad \forall j \tag{2d}$$

where '−' and '+' superscripts represent the lower and upper bounds of interval parameters/variable; $e_{j}^{\pm}$, $a_{rj}^{\pm}$ and $b_{r}^{\pm}$ form single intervals with deterministic lower and upper bounds; $\widetilde{c}_{j}^{\pm}$, $\widetilde{b}_{i}^{\pm}$ and $\widetilde{a}_{ij}^{\pm}$ form dual intervals with fuzzy lower and upper bounds; r is the number of single-interval constraints; i is the number of dual-interval constraints.

In the FBIP model, assume that there is no intersection between the fuzzy sets at the two bounds (e.g. let $\widetilde{b}_{r}^{\pm} = \left[\widetilde{b}_{r}^{-}, \widetilde{b}_{r}^{+} \right] = \left[[\underline{b}^{-}_{r}, \overline{b}_{r}], \left[\underline{b}^{+}_{r}, \overline{b}_{r}^{+} \right] \right]$, where $\widetilde{b}_{r}^{-}$ and $\widetilde{b}_{r}^{+}$ are fuzzy lower and upper bounds of $\widetilde{b}_{r}^{\pm}$; $\underline{b}^{-}_{r}$ and $\overline{b}_{r}$ are the lower- and upper-boundary of $\widetilde{b}_{r}^{-}$; $\underline{b}^{+}_{r}$ and $\overline{b}_{r}^{+}$ are the lower- and upper-boundary of $\widetilde{b}_{r}^{+}$). This is due to satisfy the definition of an interval value that its lower bound should not be larger than its upper bound. Secondly, interval numbers are used to express uncertainties without distribution information. If the fuzzy sets of an interval's lower and upper bounds intersect, then the so-called "interval" is actually described by fuzzy membership functions, such that the interval representation becomes unnecessary (Nie et al. 2007; Suo et al. 2013). Thirdly, if the fuzzy sets of lower and upper bounds intersect, the interactive algorithm for solving the interval-parameter programming problem cannot be used for solving such a FBIP model.

A two-step solution method is proposed for solving model (2) based on the interactive algorithm and vertex analysis approach. In the first step, a set of submodels corresponding to $\widetilde{f}^{+}$ can be first formulated based on the interactive algorithm; for each $\widetilde{f}^{+}$ submodel, take one end point from each of the fuzzy intervals (i.e., $\left[\underline{c}^{+}_{j}, \overline{c}_{j}^{+} \right]$, $\left[\underline{a}_{ij}^{-}, \overline{a}_{ij}^{-} \right]$, and $\left[\underline{b}^{+}_{i}, \overline{b}^{+}_{i} \right]$); then, the obtained end points can be combined into an n-array, leading to 2^{n} combinations for n fuzzy sets (Dong and Shah 1987; Chen et al. 1998). In detail, for each α-cut level, a set of $\widetilde{f}^{+}$ submodels can be formulated as follows (assume that the right-hand sides and objective are both greater than zero):

$$\text{Max } \widetilde{f}^{+} = \sum_{j=1}^{j_1} \left(\left[\underline{c}^{+}_{j}, \overline{c}_{j}^{+} \right] + e_{j}^{+} \right) x_{j}^{+} + \sum_{j=j_1+1}^{n} \left(\left[\underline{c}^{-}_{j}, \overline{c}_{j}^{+} \right] + e_{j}^{+} \right) x_{j}^{-} \tag{3a}$$

subject to:

$$\sum_{j=1}^{j_1} |a_{rj}|^- \text{Sign}\left(a_{rj}^-\right) x_j^+ + \sum_{j=j_1+1}^{n} |a_{rj}|^+ \text{Sign}\left(a_{rj}^+\right) x_j^- \leq b_r^+, \ \forall r \tag{3b}$$

$$\sum_{j=1}^{j_1} \left[|\underline{a}_{ij}|^- \text{Sign}\left(\underline{a}_{ij}^-\right), \ |\overline{a}_{ij}|^- \text{Sign}\left(\overline{a}_{ij}^-\right) \right] x_j^+$$

$$+ \sum_{j=j_1+1}^{n} \left[|\underline{a}_{ij}|^+ \text{Sign}\left(\underline{a}_{ij}^+\right), \ |\overline{a}_{ij}|^+ \text{Sign}\left(\overline{a}_{ij}^+\right) \right] x_j^- \leq \left[\underline{b}_i^+, \ \overline{b}_i^+ \right], \ \forall i \tag{3c}$$

$$x_j^+ \geq 0, \ j = 1, 2, \dots, j_1 \tag{3d}$$

$$x_j^- \geq 0, \ j = j_1 + 1, \ j_1 + 2, \dots, n \tag{3e}$$

where x_j^+ ($j=1, 2, \dots, j_1$) are upper bounds of the decision variables ($x_j^{\pm}$) with positive coefficients in the objective function, and x_j^- ($j = j_1 + 1, \ j_1 + 2, \dots, n$) are lower bounds with negative coefficients. Through solving 2^n submodels, a set of values ($f_1^+, \ f_2^+, \ \cdots, \ f_{2^n}^+$) can be obtained. The upper-bound interval for the objective-function value (under an α-cut level) can be identified as follows:

$$\left[\underline{f}_{\text{opt}}^+, \ \overline{f}_{\text{opt}}^+ \right]^{\alpha} = \left[\min\left(f_1^+, \ f_2^+, \ \cdots, \ f_{2n}^+ \right), \ \max\left(f_1^+, \ f_2^+, \ \cdots, \ f_{2n}^+ \right) \right]^{\alpha} \tag{4}$$

Submodels corresponding to $\widetilde{f}^-$ can be formulated as:

$$\text{Max } \widetilde{f}^- = \sum_{j=1}^{j_1} \left(\left[\underline{c}_j^-, \ \overline{c}_j^- \right] + e_j^- \right) x_j^- + \sum_{j=j_1+1}^{n} \left(\left[\underline{c}_j^-, \ \overline{c}_j^- \right] + e_j^- \right) x_j^+ \tag{5a}$$

subject to:

$$\sum_{j=1}^{j_1} |a_{rj}|^+ \text{Sign}\left(a_{rj}^+\right) x_j^- + \sum_{j=j_1+1}^{n} |a_{rj}|^- \text{Sign}\left(a_{rj}^-\right) x_j^+ \leq b_r^-, \ \forall r \tag{5b}$$

$$\sum_{j=1}^{j_1} \left[|\underline{a}_{ij}|^+ \text{Sign}\left(\underline{a}_{ij}^+\right), \ |\overline{a}_{ij}|^+ \text{Sign}\left(\overline{a}_{ij}^+\right) \right] x_j^-$$

$$+ \sum_{j=j_1+1}^{n} \left[|\underline{a}_{ij}|^- \text{Sign}\left(\underline{a}_{ij}^-\right), \ |\overline{a}_{ij}|^- \text{Sign}\left(\overline{a}_{ij}^-\right) \right] x_j^+ \leq \left[\underline{b}_i^-, \ \overline{b}_i^- \right], \ \forall i \tag{5c}$$

$$0 \leq x_j^- \leq x_{j\ opt}^+, \quad j = 1, 2, \ldots, j_1 \tag{5d}$$

$$x_j^+ \geq x_{j\ opt}^-, \quad j = j_1 + 1, j_2 + 1, \ldots, n \tag{5e}$$

where $x_{j\ opt}^+$ ($j=1, 2, \ldots, j_1$) and $x_{j\ opt}^-$ ($j=j_1 + 1, j_1 + 2, \ldots, n_1$) are solutions corresponding to $\underline{f}_{opt}^+$. Through solving 2^n deterministic problems, a set of values ($f_1^-, f_2^-, \cdots, f_{2n}^-$) can be obtained. The lower-bound interval for the objective-function value (under an α-cut level) can be identified as follows:

$$\left[\underline{f}_{opt}^-, \overline{f}_{opt}^-\right]^\alpha = \left[\min\left(f_1^-, f_2^-, \cdots, f_{2n}^-\right), \max\left(f_1^-, f_2^-, \cdots, f_{2n}^-\right)\right]^\alpha \tag{6}$$

Then, through integrating the computational results of the two sets of submodels, the solution for the objective function value (under an α-cut level) can be obtained. Iteratively, the computational process can be repeated with the other α-cut levels.

3 Application

3.1 Overview of the Study Area

The Xiangxi River, located at around 40 km upstream of the Three Gorges Reservoir, is one of the largest tributaries of Yangtze River. It stretches Xingshan County and Zigui County of Hubei Province, with a length of 94 kilometer (km) and an area of 3,099 square kilometer (km^2). In this study, a length of about 51 km river stretch (from Gufu town to estuary) with two tributaries (Baisha River and Gaolan River) is examined, which receives the majority of point and nonpoint source pollutants generated in the entire catchment. In this area, water quality management is becoming more and more important for its sustainable development, due to serious water pollution from point sources (i.e. those sources directly discharging into a receiving water at a fixed and geographically identifiable location) and nonpoint sources (i.e. those sources discharging into a receiving water in a diffuse manner where the point of discharge cannot be defined geographically or easily measured). Based on field investigations and related literatures, biological oxygen demand (BOD), total nitrogen (TN) and total phosphorus (TP) are selected as water quality indicators to determine the pollution control plans.

As shown in Fig. 1, fifteen main point sources scatter along the river stretch, including five chemical plants (i.e. GF, BSH, PYK, LCP, and XJLY), six phosphorus mining companies (i.e. XL, XH, XC,GP, JJW, and SJS), and four wastewater treatment plants (abbreviated as WTPs, i.e. Gufu, Nanyang, Gaoyang, and Xiakou); four main agricultural zones (AZs) are also taken into consideration marked 1 to 4, which can result in non-point source pollution due to manure/fertilizer applications. A one-year planning horizon is selected in this study, and further sorted into two periods: non-flood season (i.e. November to May of the next year) and flood season (i.e. June to October). Generally, different crops have specific growth periods; wheat, potato, rapeseed and alpine rice are determined as main crops in non-flood season; second
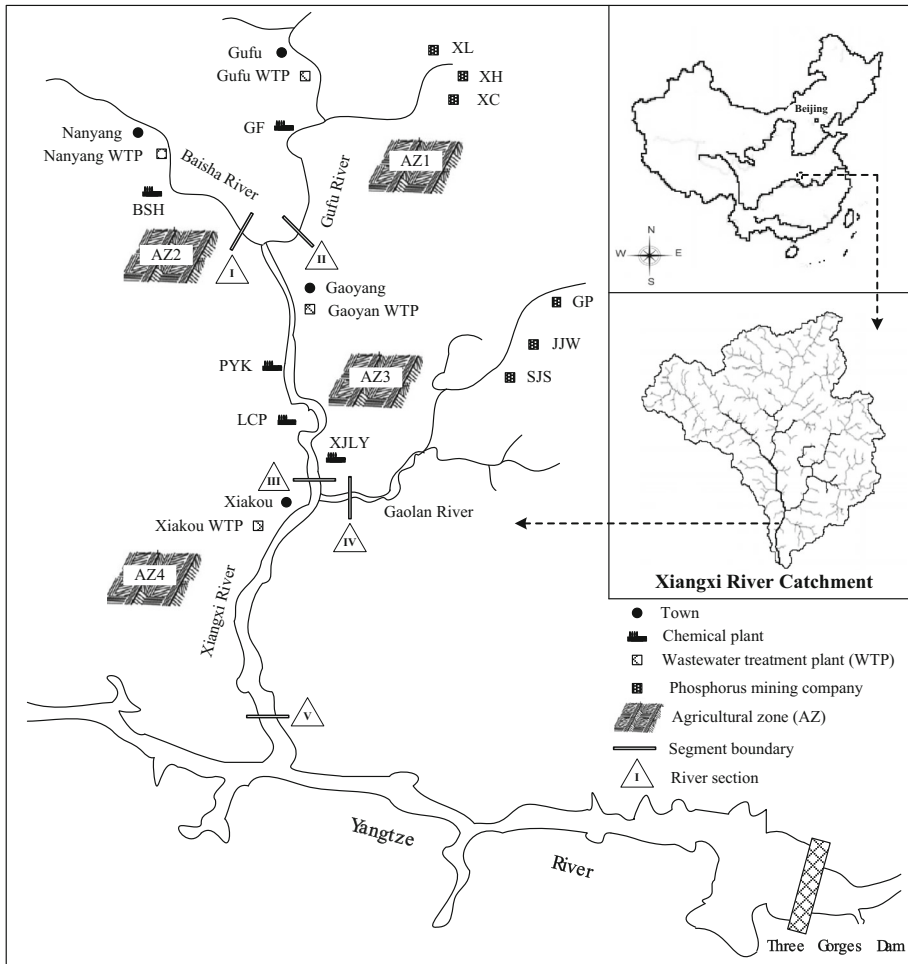
Fig. 1 Study area

rice, maize and vegetables are identified as main crops during flood season; citrus and tea grow up over the entire planning horizon.

3.2 Objective Function

Based on the FBIP method, an inexact water quality-management model (abbreviated as FBIP-WQM) is formulated. The objective is to maximize the net system benefit (i.e. industrial and agricultural incomes) subject to a set of constraints for relationships between decision variables and water-related restrictions. Decision variables represent industrial production, water supply, cropping area, manure/fertilizer applied, and livestock husbandry size. The objective function can be presented as:

$$\text{Max } \widetilde{f^{\pm}} = ICP^{\pm} + \widetilde{ILC^{\pm}} - CIA^{\pm} - CFM^{\pm} \tag{7}$$

(1) Income from industrial activities ($ICP^{\pm}$):

$$L_t\left(\sum_{i=1}^{5}\sum_{t=1}^{2} BC_{it}^{\pm}\cdot PLC_{it}^{\pm} + \sum_{p=1}^{6}\sum_{t=1}^{2} BP_{pt}^{\pm}\cdot PLM_{pt}^{\pm} + \sum_{s=1}^{4}\sum_{t=1}^{2} BW_{st}^{\pm}\cdot QW_{st}^{\pm}\right) \tag{7a}$$

(2) Income from agricultural activities ($\widetilde{ILC^{\pm}}$):

$$\sum_{j=1}^{4}\sum_{k=1}^{9}\sum_{t=1}^{2} \widetilde{CY_{jkt}^{\pm}}\cdot BA_{jkt}^{\pm}\cdot PA_{jkt}^{\pm} + \sum_{r=1}^{4} BL_r^{\pm}\cdot NL_r^{\pm} \tag{7b}$$

(3) Cost for industrial activities ($CIA^{\pm}$):

$$L_t\left(\sum_{i=1}^{5}\sum_{t=1}^{2} PLC_{it}^{\pm}\cdot WC_{it}^{\pm}\cdot CC_{it}^{\pm} + \sum_{i=1}^{5}\sum_{t=1}^{2} PLC_{it}^{\pm}\cdot FW_{it}^{\pm}\cdot WSP_t^{\pm} + \sum_{s=1}^{4}\sum_{t=1}^{2} QW_{st}^{\pm}\cdot GT_{st}^{\pm}\cdot CT_{st}^{\pm}\right) \tag{7c}$$

(4) Cost for agricultural activities ($CFM^{\pm}$):

$$\sum_{j=1}^{4}\sum_{k=1}^{9}\sum_{t=1}^{2} CM_{jt}^{\pm}\cdot AM_{jkt}^{\pm} + \sum_{j=1}^{4}\sum_{k=1}^{9}\sum_{t=1}^{2} CF_{jt}^{\pm}\cdot AF_{jkt}^{\pm} + \sum_{j=1}^{4}\sum_{k=1}^{9}\sum_{t=1}^{2} PA_{jkt}^{\pm}\cdot WSC_t^{\pm} \tag{7d}$$

where $\widetilde{f^{\pm}}$ is the net system benefit, which equals to the total returns (from chemical plant production, phosphate mining, water supply, livestock husbandry, and crops cultivation) minus the total costs (for purchasing water for productions, wastewater treatment, manure collection/disposal, and fertilizer application).

3.3 Constraints

(1) Wastewater treatment capacity constraints:

$$QW_{st}^{\pm}\cdot GT_{st}^{\pm} \leq TPC_{st}^{\pm}, \quad \forall s, \; t \tag{8a}$$

$$PLC_{it}^{\pm}\cdot WC_{it}^{\pm} \leq TPD_{it}^{\pm}, \quad \forall i, \; t \tag{8b}$$

The above constraints guarantee that the capacity of wastewater treatment at each source should be greater than amount of wastewater generated from human-daily life and industrial-production process. Raw wastewater from each point source (i.e., WTPs, chemical plants, and phosphorus mining companies) must be treated before they are discharged into the river body.

(2) BOD discharge constraints:

$$PLC_{it}^{\pm}\cdot WC_{it}^{\pm}\cdot IC_{it}^{\pm}\cdot\left(1\text{-}\eta_{BOD,it}^{\pm}\right)\leq ABC_{it}^{\pm},\quad\forall i,\ t \tag{9a}$$

$$QW_{st}^{\pm}\cdot GT_{st}^{\pm}\cdot BM_{st}^{\pm}\cdot\left(1\text{-}\eta_{BOD,st}^{\pm}\right)\leq\widetilde{ABW}_{st}^{\pm},\quad\forall s,\ t \tag{9b}$$

Among all point sources, chemical plants and WTPs are major contributors for BOD discharges. The raw wastewater at each chemical plant and each WTP is required to be treated before entering into the river, and the ultimate BOD discharge should not exceed the allowable requirement level.

(3) Nitrogen discharge constraints:

$$\left(L_t\cdot\sum_{r=1}^{4}AML_{rt}^{\pm}\cdot NL_r^{\pm}\ +\ L_t\cdot AMH_t^{\pm}\cdot RP_t^{\pm}\text{-}\sum_{j=1}^{4}\sum_{k=1}^{9}AM_{jkt}^{\pm}\right)\cdot MS_t^{\pm}\cdot\varepsilon_{NM}^{\pm}$$

$$+L_t\cdot RP_t^{\pm}\cdot ACW_t^{\pm}\cdot DNR_t^{\pm}\leq ANL_t^{\pm},\quad\forall t \tag{10a}$$

$$\sum_{k=1}^{9}\left(NS_{jk}^{\pm}\cdot SL_{jkt}^{\pm}\ +\ RF_{jkt}^{\pm}\cdot DN_{jkt}^{\pm}\cdot 10^{-5}\right)\cdot PA_{jkt}^{\pm}\leq MNL_{jt}^{\pm}\cdot TA_{jt}^{\pm},\quad\forall j,\ t \tag{10b}$$

Due to the excessive application of animal manure and commercial fertilizer in watershed system, unused nutrients are transported to the canal water via soil erosion and surface runoff, resulting in eutrophication of canal water (Saadatpour and Afshar 2013). In the study area, nitrogen losses mainly from crop farming and agricultural life (i.e. nonpoint sources) contribute significantly to the river water pollution.

(4) Phosphorus discharge constraints:

$$PLC_{it}^{\pm}\cdot\left[WC_{it}^{\pm}\cdot PCR_{it}^{\pm}\left(1\text{-}\eta_{TP,it}^{\pm}\right)\ +\ ASC_{it}^{\pm}\cdot SLR_{it}^{\pm}\cdot PSC_{it}^{\pm}\right]\leq\widetilde{APC}_{it}^{\pm},\quad\forall i,\ t \tag{11a}$$

$$\left(L_t\sum_{r=1}^{4}AML_{rt}^{\pm}\cdot NL_r^{\pm}\ +\ L_t\cdot AMH_t^{\pm}\cdot RP_t^{\pm}\ -\sum_{j=1}^{4}\sum_{k=1}^{9}AM_{jkt}^{\pm}\right)MS_t^{\pm}\cdot\varepsilon_{PM}^{\pm}$$

$$+\ L_t\cdot RP_t^{\pm}\cdot ACW_t^{\pm}\cdot DPR_t^{\pm}\leq APL_t^{\pm},\quad\forall t \tag{11b}$$

$$QW_{st}^{\pm}\cdot GT_{st}^{\pm}\cdot PCM_{st}^{\pm}\left(1\text{-}\eta_{TP,st}^{\pm}\right)\leq APW_{st}^{\pm},\quad\forall s,\ t \tag{11c}$$

$$PLM_{pt}^{\pm}\cdot\left[WPM_{pt}^{\pm}\cdot MWC_{pt}^{\pm}\left(1\text{-}\eta_{TP,pt}^{\pm}\right)\ +\ ASM_{pt}^{\pm}\cdot PCS_{pt}^{\pm}\cdot SLW_{pt}^{\pm}\right]\leq\widetilde{APM}_{pt}^{\pm},\quad\forall p,\ t \tag{11d}$$

$$\sum_{k=1}^{9}\left(PS_{jk}^{\pm}\cdot SL_{jkt}^{\pm} + DP_{jkt}^{\pm}\cdot RF_{jkt}^{\pm}\cdot 10^{-5}\right)PA_{jkt}^{\pm}\leq MPL_{jt}^{\pm}\cdot TA_{jt}^{\pm}, \quad \forall j, t \qquad (11e)$$

As one of the three richest phosphate-ore regions of China, the amount of its reserve phosphorus is round 357 million tonne; high concentration phosphorus-containing wastewater and industrial-solid waste (e.g. chemical wastes, slags, and tailings) can be generated, which pose serious threat to the river-water quality (Li et al. 2012). By setting relevant discharge thresholds, the severe situation can be ameliorated. These constraints regulate that phosphorus discharges from point and nonpoint sources should not to be greater than the thresholds.

(5)   Soil loss constraints:

$$\sum_{k=1}^{9} SL_{jkt}^{\pm}\cdot PA_{jkt}^{\pm}\leq MSL_{jt}^{\pm}\cdot TA_{jt}^{\pm}, \quad \forall j, t \qquad (12)$$

In the study area, due to the special geography and heavy rainfall, the catchment possesses high potential for generating soil erosion and surface runoff. Particularly, most agricultural croplands centralize in the hillside along with the river; it is easy to lead to effluent discharge directly into the water body of the river through the soil losses. Thus, each agricultural zone is imposed a discharge permit to control its sediment loads.

(6)   Fertilizer and manure constraints:

$$\left(1- NVF_{t}^{\pm}\right)\cdot\varepsilon_{NF}^{\pm}\cdot AF_{jkt}^{\pm} + \left(1- NVM_{t}^{\pm}\right)\cdot\varepsilon_{NM}^{\pm}\cdot AM_{jkt}^{\pm}\geq NR_{jkt}^{\pm}\cdot PA_{jkt}^{\pm}, \quad \forall j, k, t \ (13a)$$

$$\varepsilon_{PF}^{\pm}\cdot AF_{jkt}^{\pm} + \varepsilon_{PM}^{\pm}\cdot AM_{jkt}^{\pm}\geq PR_{jkt}^{\pm}\cdot PA_{jkt}^{\pm}, \quad \forall j, k, t \qquad (13b)$$

$$\sum_{k=1}^{9}\left(\varepsilon_{NF}^{\pm}\cdot AF_{jkt}^{\pm} + \varepsilon_{NM}^{\pm}\cdot AM_{jkt}^{\pm}- NR_{jkt}^{\pm}\cdot PA_{jkt}^{\pm}\right)\leq MNL_{jt}^{\pm}\cdot TA_{jt}^{\pm} \quad \forall j, t \qquad (13c)$$

$$\sum_{k=1}^{9}\left(\varepsilon_{PF}^{\pm}\cdot AF_{jkt}^{\pm} + \varepsilon_{PM}^{\pm}\cdot AM_{jkt}^{\pm}- PR_{jkt}^{\pm}\cdot PA_{jkt}^{\pm}\right)\leq MPL_{jt}^{\pm}\cdot TA_{jt}^{\pm}, \quad \forall j, t \qquad (13d)$$

$$L_{t}\left(\sum_{r=1}^{4}AML_{rt}^{\pm}\cdot NL_{r}^{\pm} + AMH_{t}^{\pm}\cdot RP_{t}^{\pm}\right)\geq\sum_{j=1}^{4}\sum_{k=1}^{9}AM_{jkt}^{\pm}, \quad \forall t \qquad (13e)$$

These constraints guarantee the nutrients are kept within acceptable levels through controlling the amount of manure and fertilizer applications. For agricultural production, livestock manures and chemical fertilizers are essential to satisfy nutrient (i.e., nitrogen

and phosphorus) demands of crops since the soil fertility in the study area is low; however, their excessive use sometimes causes environmental problems such as water and air pollution. The study area is one of the largest producers and consumers of chemical fertilizers in the Three Gorges Region, and the excessive nutrient loading from agricultural activities is considered to be the principal pollution source. The mitigation of nonpoint source pollution needs to consider the type and amount of fertilizer application.

(7)   Cropland resources constraints:

$$\sum_{j=1}^{4}\sum_{k=1}^{6} PA_{jkt}^{\pm} \geq MFP_t^{\pm}, \quad t = 1 \tag{14a}$$

$$\sum_{j=1}^{4}\sum_{k=1}^{2} PA_{jkt}^{\pm} + \sum_{j=1}^{4}\sum_{k=8}^{9} PA_{jkt}^{\pm} \geq MFP_t^{\pm}, \quad t = 2 \tag{14b}$$

$$PA_{jkt}^{\pm} \leq TAS_{jt}^{\pm}, \quad k = 6, \ t = 1 \tag{14c}$$

$$\sum_{k=1}^{5} PA_{jkt}^{\pm} \leq TAH_{jt}^{\pm}, \quad t = 1 \tag{14d}$$

$$PA_{jkt}^{\pm} \leq TAS_{jt}^{\pm}, \quad k = 7, \ t = 2 \tag{14e}$$

$$\sum_{k=1}^{2} PA_{jkt}^{\pm} + \sum_{k=8}^{9} PA_{jkt}^{\pm} \leq TAH_{jt}^{\pm}, \quad t = 2 \tag{14f}$$

$$TAS_{jt}^{\pm} + TAH_{jt}^{\pm} = TA_{jt}^{\pm}, \quad \forall j, t \tag{14g}$$

Crop farming is an important source of income for rural households, which accounts for 73 % of the total population. There are diversiform land use patterns, and multiple crops are cultivated, such as rice, maize, wheat, citrus, tea, potato and vegetables. These constraints are established to ensure that the planning crop area is less than or equal to the limited tillable land resources (including paddy field and dry farmland). Moreover, the total planning croplands in each period are more than or equal to the minimum farmland areas required by the local government.

(8)   Industrial production scale constraints:

$$PLC_{it,\ min} \leq PLC_{it}^{\pm} \leq PLC_{it,\ max}, \quad \forall i, t \tag{15a}$$

$$NL_{r,\ min} \leq NL_r^{\pm} \leq NL_{r,\ max}, \quad \forall r \tag{15b}$$

$$QW_{st,\,min} \leq QW_{st}^{\pm} \leq QW_{st,\,max}, \quad \forall s,\, t \tag{15c}$$

$$PLM_{pt,\,min} \leq PLM_{pt}^{\pm} \leq PLM_{pt,\,max}, \quad \forall p,\, t \tag{15d}$$

These constraints require that the scale of each industrial production unit should be restricted within a suitable range.

(9)   Non-negative constraints:

$$PLC_{it}^{\pm},\ PA_{jkt}^{\pm},\ NL_{r}^{\pm},\ QW_{st}^{\pm},\ PLM_{pt}^{\pm},\ AM_{jkt}^{\pm},\ AF_{jkt}^{\pm} \geq 0 \tag{16}$$

These constraints assure that only positive activities are considered in the solution, eliminating infeasibilities while calculating the solution.

In this study, input parameters expressed as crisp intervals and fuzzy-boundary intervals are investigated according to field surveys, statistical data, government reports, and related literatures. Table 1 lists the data related to agricultural activities, including crop yield and net benefit. The allowable pollutant discharge is an indispensable factor in making regional environmental planning and water pollution control, which reflects the capacity of the water body provided to receive pollutants during a certain time without destroying its function. Table 2 provides the allowable BOD and TP discharge as regulated by the local authority.

4 Results and Discussion

Figure 2 presents the net system benefit under different α-cut levels. Different combinative combinations on the uncertain inputs would lead to varied solutions for objective function values and decision variables. For example, when $\alpha=0$, the system benefit would be RMB ¥[[309.32, 446.46], [1464.01, 1790.11]] $\times 10^6$ (i.e. dual interval). The solution for the objective-function value under each α-cut level provides four options of maximized system benefit corresponding to different conditions (i.e. different reliability levels). A medium value of system benefit [i.e. $\underline{f}^- + \overline{f}^- + \underline{f}^+ + \overline{f}^+)/4$] could be calculated under each α-cut level. The marginal variation of mid-value with α-cut level (i.e. $\Delta f_{mid}/\Delta \alpha$) would be RMB¥$24.7 \times 10^6$ ($\alpha=0.2$), RMB¥20.3×10^6 ($\alpha=0.4$), RMB¥19.8×10^6 ($\alpha=0.6$), RMB¥17.3×10^6 ($\alpha=0.8$), and RMB¥ 16.2×10^6 ($\alpha=1$). Results indicate that marginal benefit would decrease as α-cut level is raised.

Table 3 presents the solutions for industrial production and water supply, which would vary with α-cut level due to the uncertainties that exist in the system components. In period 1, the production of LCP would be [[70.4, 112.4], [345.3, 399.6]] t/day (i.e. tonne/day) under $\alpha=0.2$, and [[80.9, 101.9], [387.8, 399.6]] t/day under $\alpha=0.6$. Significant variations in water supply would exist for different towns and different periods. More water would be delivered to Gufu (i.e. [[4527, 5533], [19277, 21739]]m^3/day in period 1 and [[4705, 5682], [19512, 21739]] m^3/day in period 2 under $\alpha=0.2$). Generally, different α-cut levels correspond to different river water-quality requirements and pollutant allowances, and thus result in varied industrial production scales and water

Table 1 Crop yield and net benefit

	AZ1	AZ2	AZ3	AZ4
Crop yield (t/ha):				
Citrus	[[11.1, 15.0], [16.7, 21.6]]	[[10.2, 11.6], [16.4, 22.1]]	[[8.4, 10.3], [11.4, 18.2]]	[[9.2, 11.6], [14.8, 19.2]]
Tea	[[0.1, 0.3], [0.5, 0.7]]	[[0.1, 0.3], [0.4, 0.7]]	[[0.3, 0.9], [1.2, 1.6]]	[[0.2, 0.7], [1.0, 1.4]]
Wheat	[[1.4, 2.8], [4.5, 7.1]]	[[1.3, 2.7], [4.2, 7.8]]	[[1.2, 3.6], [5.4, 7.5]]	[[2.3, 4.7], [6.4, 8.5]]
Potato	[[2.4, 4.7], [6.3, 9.2]]	[[2.0, 4.1], [6.5, 8.2]]	[[1.9, 3.8], [5.9, 7.9]]	[[2.8, 4.1], [5.4, 7.9]]
Rapeseed	[[1.6, 3.8], [5.1, 6.7]]	[[1.3, 2.1], [4.1, 5.8]]	[[1.3, 2.7], [3.6, 5.8]]	[[1.7, 2.5], [3.9, 5.4]]
Alpine Rice	[[6.9, 10.4], [14.2, 17.6]]	[[6.7, 9.4], [13.4, 16.4]]	[[4.4, 8.4], [11.6, 15.2]]	[[5.7, 9.6], [12.3, 15.4]]
Second rice	[[7.4, 10.2], [15.6, 19.6]]	[[7.3, 10.4], [14.6, 18.9]]	[[4.7, 8.5], [10.9, 16.6]]	[[6.2, 9.1], [14.5, 17.0]]
Maize	[[2.9, 5.4], [7.6, 10.2]]	[[2.7, 4.1], [6.8, 9.6]]	[[2.5, 4.7], [6.9, 10.4]]	[[4.0, 7.0], [9.5, 12.4]]
Vegetable	[[22.6, 27.8], [31.4, 37.9]]	[[10.4, 17.8], [19.2, 22.9]]	[[21.8, 25.4], [29.4, 32.6]]	[[26.2, 30.5], [34.6, 38.4]]
Benefit from crop (RMB¥/t):				
Citrus	[1,082, 1,350]	[1,010, 1,260]	[1,126, 1,405]	[1,203, 1,502]
Tea	[18,000, 18,900]	[19,125, 20,081]	[20,170, 21,178]	[22,500, 23,625]
Wheat	[1,031, 1,242]	[972, 1,169]	[945, 1,134]	[1,010, 1,218]
Potato	[915, 1,001]	[827, 897]	[893, 969]	[846, 917]
Rapeseed	[2,435, 2,934]	[2,191, 2,640]	[2,305, 2,778]	[2,490, 3,000]
Alpine Rice	[1,105, 1,214]	[1,047, 1,153]	[1,084, 1,178]	[1,149, 1,265]
Second rice	[1,159, 1,274]	[1,098, 1,209]	[1,137, 1,236]	[1,205, 1,327]
Maize	[1,295, 1,451]	[1,267, 1,422]	[1,230, 1,378]	[1,308, 1,467]
Vegetable	[2,360, 2,691]	[1,613, 1,865]	[1,927, 2,197]	[1,845, 1,950]

Symbol "AZ" means "agricultural zone"

Table 2 Allowable pollutant discharge

	Period 1 (non-flood season)	Period 2 (flood season)
Allowable phosphorus discharge for each chemical plant (kg/day)		
Gufu (GF)	[[0.60, 0.80], [1.05, 1.25]]	[[0.70, 0.90], [1.15, 1.28]]
Baishahe (BSH)	[[195.63, 328.36], [432.56, 759.91]]	[[162.64, 308.34], [407.87, 704.18]]
Pingyikou (PYK)	[[30.49, 50.89], [70.71, 92.03]]	[[50.03, 70.74], [90.30, 132.17]]
Liucaopo (LCP)	[[124.7, 224.88], [294.7, 488.17]]	[[213.40, 318.90], [409.5, 637.01]]
Xiangjinlianying (XJLY)	[[126.22, 218.21], [321.76, 423.76]]	[[174.4, 274.45], [364.8, 475.28]]
Allowable phosphorus discharge for each mine company (kg/m^3)		
Xinglong (XL)	[[47.41, 85.68], [123.49, 153.49]]	[[78.31, 112.94], [191.59, 211.59]]
Xinghe (XH)	[[23.68, 40.92], [86.87, 108.87]]	[[39.19, 67.92], [86.87, 108.87]]
Xingchang (XC)	[[30.54, 42.58], [62.75, 92.48]]	[[52.74, 72.64], [115.76, 137.86]]
Geping (GP)	[[48.39, 69.22], [109.36, 139.79]]	[[70.03, 103.68], [173.95, 193.78]]
Jiangjiawan (JJW)	[[25.91, 40.47], [40.13, 70.38]]	[[38.49, 54.59], [87.51, 112.52]]
Shenjiashan (SJS)	[[20.23, 30.67], [42.31, 66.47]]	[[28.19, 60.57], [82.61, 102.81]]
Allowable BOD discharge for each WTP (kg/day)		
Gufu	[[70, 90], [110, 130]]	[[75, 95], [115, 132]]
Nanyang	[[3, 5], [8, 10]]	[[4, 6], [9, 12]]
Gaoyang	[[10, 18], [25, 35]]	[[12, 20], [27, 37]]
Xiakou	[[15, 23], [28, 38]]	[[17, 25], [30, 40]]

supply amounts. Solutions for crop area and fertilizer application under different α-cut levels are summarized in Table 4. The areas of citrus and tea would maintain low levels over the planning horizon due to their high pollutant losses. In period 1, wheat, potato, rapeseed, and alpine rice would be cultivated; in period 2, wheat, potato, rapeseed, and alpine rice would be harvested and second rice, maize, and vegetables would be sown. Moreover, most of the tillable lands would be planted with vegetables (i.e. [359.0, 3284.9] ha) due to its high crop yield, good market price, and low pollutant losses. In addition, results show that fertilizer application would vary with crop areas. The largest fertilizer user is rapeseed in period 1, while vegetable is the largest fertilizer user in period 2.

Figure 3 presents incomes from industries (including water supply, chemical plant, and phosphorus mining company). A higher α-cut would result in a higher lower-bound but a lower upper-bound income from industrial activities (i.e. leading to a narrow interval). When $\alpha=1$, the income from water supply is RMB¥[113.6, 450.8] × 10^6 ([30.5, 32.1]% of the total income); the income from chemical production is RMB¥[180.6, 699.3] × 10^6 ([48.5, 49.8]% of the total income); the income from producing phosphorus ore is RMB¥[78.1, 253.6] × 10^6 ([18.1, 21.2]% of the total income). Results reveal that chemical production brings the highest income for the study area. Income from industry activities would be RMB¥[[327.3, 381.1], [1275.7, 1340.8]] × 10^6, occupying [[78.3, 80.3], [93.4, 93.7]]% of the net system benefit, implying that industry is the major contributor to the local economy. Such an industry-oriented pattern is linked to the advantages of mineral resources (phosphate ore) and high economic return.

Different α-cut levels correspond to varied production levels, and thus result in changed pollutant loadings. BOD discharges from chemical plants are presented in Fig. 4, which are
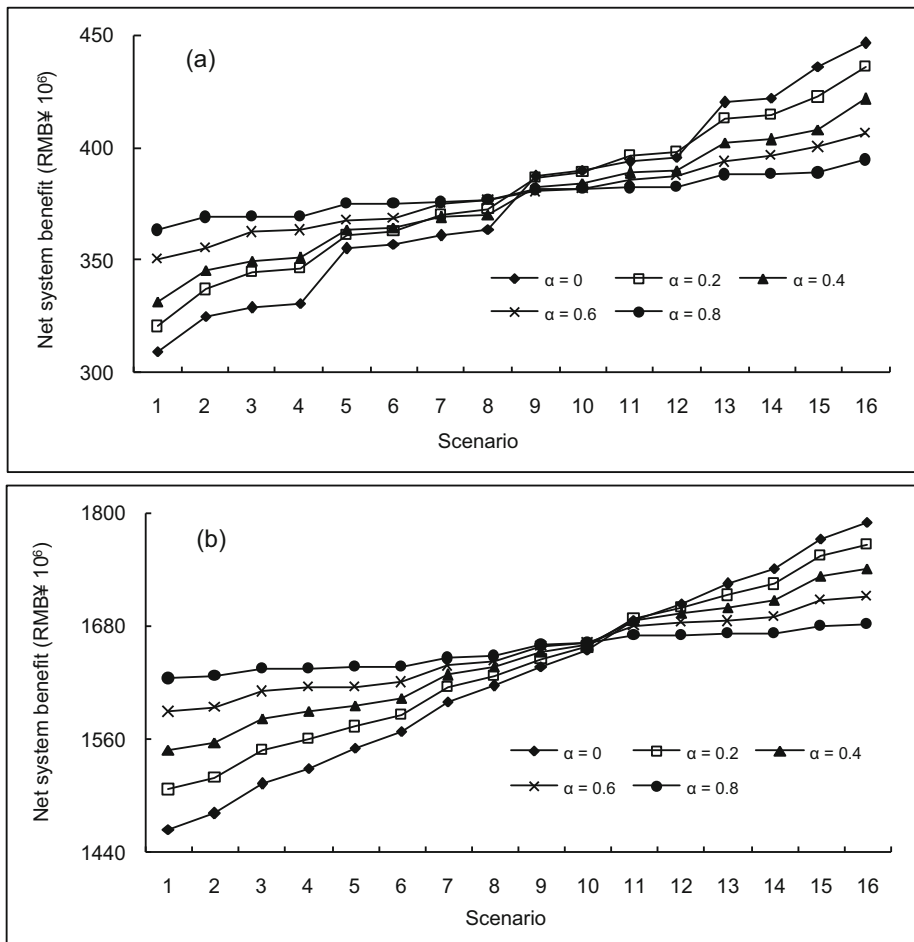
Fig. 2 Net system benefits under different scenarios [(**a**) lower-bound, (**b**) upper-bound]

associated with various factors such as generated product, product amount, and wastewater treatment efficiency. The amount of BOD discharge of each chemical plant during every period under each α-cut level is BSH > LCP > XJLY > PYK > GF, and the BOD discharged from BSH occupies 74.9 % of the total BOD emission. This is due to its relatively lower treatment efficiency, higher wastewater-generation rate, and higher concentration of raw wastewater.

The study area is one of the largest producers and consumers of chemical fertilizers in the Three Gorges Region, and the excessive nutrient loading from agricultural activities is the principal pollutant source. Figure 5 presents the solutions for TN discharges from crop farming and agricultural life. The average TN losses from agricultural activity would be [[117.00, 145.80], [364.17, 366.40]] tonne over the planning horizon (i.e. [[38.3, 42.7], [62.3, 62.4]]% of the total TN discharge). Specifically, TN losses from crop farming is dominated by nitrogen in soil erosion (i.e. [[94.1, 95.4], [96.9, 97.3]]% of TN losses from crop farming). This is associated with high runoff, high soil loss, and

Table 3 Solutions for industrial production and water supply

Point sources	α=0.2		α=0.6		α=1	
	$t=1$	$t=2$	$t=1$	$t=2$	$t=1$	$t=2$
Chemical plant (t/day)						
GF	[17.7, 25.0]	[15.1, 27.4]	[17.7, 25.0]	[15.1, 27.40]	[17.7, 24.9]	[15.1, 27.4]
BSH	[[176.2, 265.7], 534.1]	[[70.9, 117.6], [283.0, 337.0]]	[[198.5, 243.3], 534.1]	[[82.6, 105.9], [321.3, 337.0]]	[220.9, 536.1]	[94.3, 337.0]
PYK	[20.3, 30.5], 76.0]	[[19.8, 26.0], [68.9, 93.3]]	[22.9, 28.0], 76.0]	[21.4, 24.4], [75.0, 87.2]]	[25.4, 76.0]	[22.9, 81.1]
LCP	[70.4, 112.4], [345.3, 399.6]]	[81.7, 112.5], [276.5, 392.9]]	[80.9, 101.9], [387.8, 399.6]]	[89.4, 104.8], [305.6, 363.8]]	[91.4, 399.6]	[97.1, 334.7]
XJLY	[77.9, 120.2], 313.8]	[68.6, 98.3], [277.4, 342.7]]	[88.5, 109.7], 313.8]	[76.0, 90.9], [293.8, 326.4]]	[99.1, 313.8]	[83.1, 310.1]
Water supply (m³/day)						
Gufu	[4,527, 5,533], [19,277, 21,739]]	[4,705, 5,682], [19,512, 21,739]]	[4,778, 5,281], [19,966, 21,343]]	[4,949, 5,438], [20,081, 21,218]]	[5,030, 20,654]	[5,194, 20,649]
Nanyang	[237, 355], [1,309, 1,538]]	[306, 422], [1,461, 1,538]]	[266, 325], [1,373, 1,500]]	[335, 393], 1,538]	[296, 1,436]	[364, 1,538]
Gaoyang	[593, 944], 3,225]	[692, 1,038], 3,225]	[680, 856], 3,225]	[778, 951], 3,225]	[768, 3,225]	[2,962, 3,225]
Xukou	[808, 1,135], [2,426, 2,703]]	[910, 1,237], [2,629, 2,703]]	[890, 1,053], [2,593, 2,703]]	[991, 1,155], 2,703]	[971, 2,703]	[1,074, 2,703]
Phosphorus mining company (t/day)						
XL	[397.3, 634.8], 1,261.0]	[304.1, 407.2], 1,261.0]	[456.7, 575.5], 1,261.0]	[329.9, 381.4], 1,261.0]	[516.1, 1,261.0]	[355.7, 1,261.0]
XH	[117.7, 181.6], [364.0, 400.0]]	[93.5, 144.5], [320.8, 384.2]]	[133.7, 165.7], 400.0]	[106.2, 131.8], [336.7, 368.3]]	[149.7, 400.0]	[119.0, 360.4]
XC	[130.8, 170.5], 420.0]	[108.2, 139.6], [361.5, 415.4]]	[140.8, 160.6], 420.0]	[116.0, 131.8], [374.9, 401.9]]	[150.7, 420.0]	[123.9, 395.2]
GP	[270.4, 359.6], 778.0]	[188.5, 257.7], [743.0, 778]]	[292.7, 337.3], 778.0]	[205.8, 240.4], [759.9, 778.0]]	[315, 778.0]	[223.1, 778.0]
JJW	[167.4, 238.7], [488.1, 547.0]]	[117.7, 155.5], [429.4, 524.8]]	[185.2, 220.9], 547.0]	[127.1, 146.0], [453.3, 500.9]]	[203.1, 547.0]	[136.6, 489.0]
SJS	[167.8, 233.7], 685.0]	[118.9, 216.9], [565.7, 672.6]]	[184.2, 217.2], 685.0]	[143.4, 192.4], [592.4, 645.9]]	[200.7, 685.0]	[167.9, 632.5]

Table 4 Solutions for crop area and fertilizer application

	Growth period	α=0.2	α=0.6	α=1
Crop area (ha)				
Citrus	whole year	[0, 80.8]	[0, 80.8]	[0, 80.8]
Tea	whole year	[0, 30.0]	[0, 30.0]	[0, 30.0]
Wheat	non-flood season	[[404.6, 450.52], 800.5]	[[404.6, 507.8], 800.5]	[408.7, 800.5]
Potato	non-flood season	[411.3, 524.4], 900.0]	[[411.3, 497.3], 900.0]	[576.91, 900.0]
Rapeseed	non-flood season	[[844.1, 949.3], [1,388.5, 1,389.2]]	[[844.1, 914.6], [1,388.5, 1,389.2]]	[812.8, 1,388.7]
Alpine Rice	non-flood season	[[209.3, 253.8], 424.0]	[[185.7, 253.8], 424.0]	[230.9, 424.0]
Second rice	flood season	[[222.2, 341.5], 545.0]	[[222.2, 341.5], 545.0]	[341.5, 545.0]
Maize	flood season	[0, 81.2]	[0, 81.2]	[0, 81.2]
Vegetable	flood season	[359.0, 3,284.9]	[359.0, 3,284.9]	[359.0, 3,284.9]
Fertilizer application (t)				
Citrus	whole year	27.0	27.0	27.0
Tea	whole year	1.02	1.02	2.5
Wheat	non-flood season	[461.4, 518.8], [562.4, 566.2]]	[461.4, 562.4]	[466.1, 562.4]
Potato	non-flood season	[403.4, 539.5]	[403.4, 599.5]	603.5
Rapeseed	non-flood season	[1,086.6, 1,215.1]	[1,072.4, [1,086.6, 1,215.1]]	1,116.6
Alpine Rice	non-flood season	[[550.6, 580.3], [652.0, 811.6]]	[[580.4, 610.6], [755.2, 811.6]]	[511.3, 515.1]
Second rice	flood season	[671.5, 967.9]	[671.5, 976.9]	967.9
Maize	flood season	[176.3, 285.9]	[176.3, 285.9]	[176.3, 285.9]
Vegetable	flood season	[906.5, 1,055.6], 2,470.3]	[[906.5, 1,055.6], [2,470.3, 2,480.5]]	[904.9, 2,470.3]

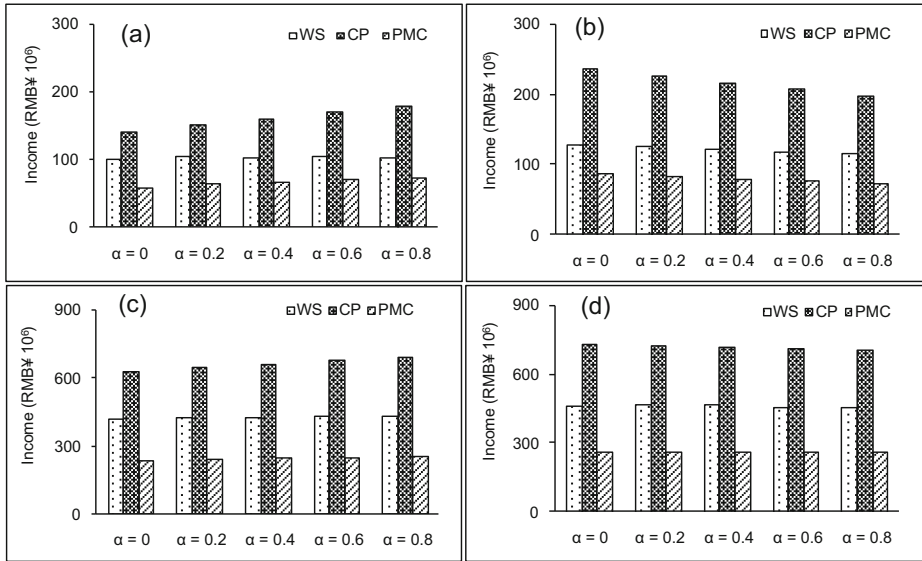**Fig. 3** Incomes from industries [(**a**) lower-min, (**b**) lower-max, (**c**) upper-min, and (**d**) upper-max]

high nitrogen concentration in the study area. Most agricultural croplands centralize in the hillside along with the river; it is easy to lead to effluent discharge directly into the water body of the river. For agricultural life, the amount of TN discharged from livestock wastes would be higher than that from untreated rural domestic sewage. The average TN losses from livestock wastes would be [[125.25,129.49], [175.8, 186.59]] t (i.e. [[65.6,
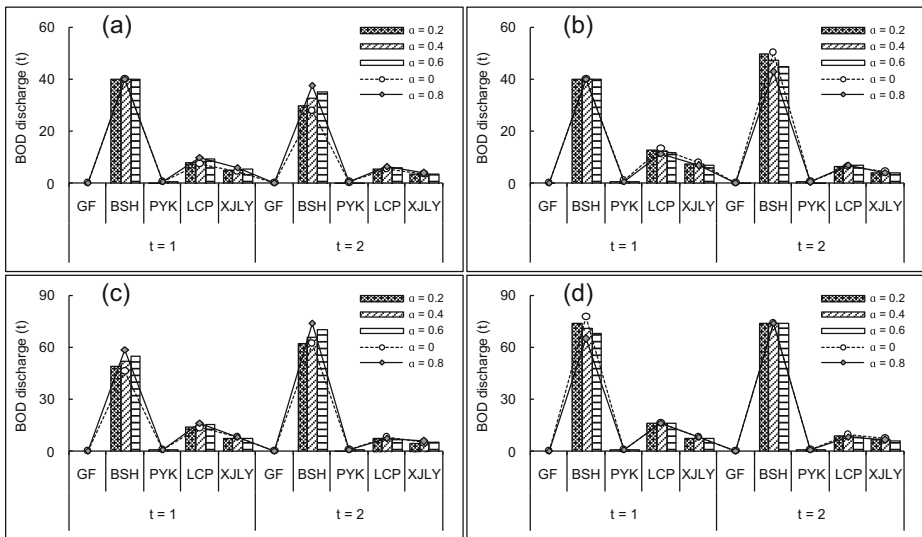
Fig. 4 BOD discharges under different α-cut levels [(**a**) lower-min, (**b**) lower-max, (**c**) upper-min, and (**d**) upper-max]
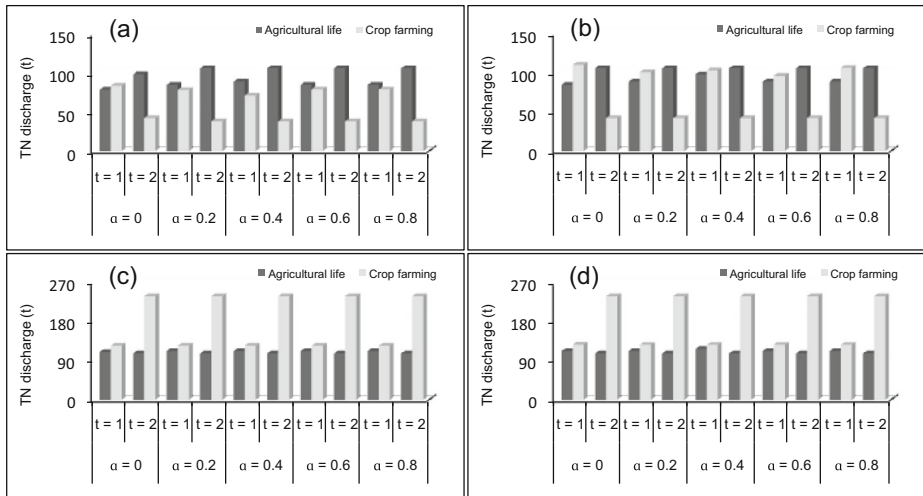
Fig. 5 TN discharges under different α-cut levels [(**a**) lower-min, (**b**) lower-max, (**c**) upper-min, and (**d**) upper-max]

66.4], [79.9, 84.7]]% of TN losses from agricultural life). This is attributed to large scales of livestock and population, and relatively higher nitrogen concentration.

The study watershed is abundant in phosphorus resources and phosphorus related industry is of great importance to the local economy. Phosphorus pollutants were generated from point and nonpoint sources (as shown Fig. 6). Generally, chemical plants are the largest ones among all sources, occupying 51.9 % of the total TP discharges. For chemical plants, phosphorus can be discharged from wastewater and solid wastes (i.e. chemical wastes, slags and tailings), and the latter is in a large proportion. This is because the wastewater should be sluiced strictly according to the integrated discharge standards to maintain TP discharge stay at a low level, while phosphorus-containing
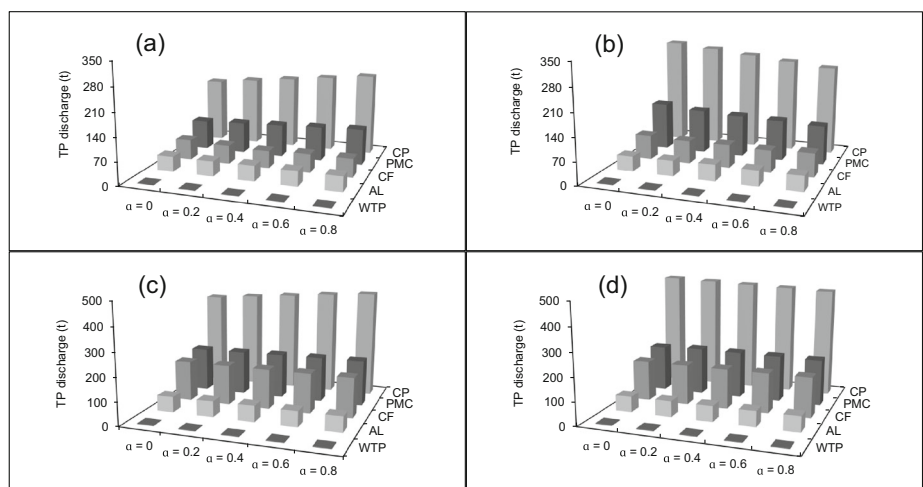

Fig. 6 TP discharges under different α-cut levels [(**a**) lower-min, (**b**) lower-max, (**c**) upper-min, and (**d**) upper-max]

wastes would be discharged directly and washed by rainfall. In addition, results also reveal that TP discharge derives mainly from point sources, particularly from chemical plants and phosphorus mining companies (occupying 77.4 % of the total amount).

5 Conclusions

A fuzzy-boundary interval programming (FBIP) method has been developed to aid planning river-water pollution control under uncertainty. FBIP incorporates interval-parameter programming (IPP) and fuzzy possibilistic programming (FPP) within a general framework. FBIP can deal with uncertainties expressed as crisp intervals and fuzzy-boundary intervals in the objective function and the left- and right-hand sides of constraints. The FBIP model can be solved through an interactive algorithm and vertex analysis technique, solutions under a number of α-cut levels can be generated. Then, a FBIP-based water quality-management model (FBIP-WQM) has been formulated for planning water quality management of the Xiangxi River in the Three Gorges Reservoir Region, China. A variety of pollutants (BOD, TN and TP) discharged from various point and nonpoint sources have been considered. Useful solutions for planning river water-quality management have been generated, reflecting tradeoffs among municipal water supply, industry production, agricultural activity, environmental requirement, and economic performance.

Results indicate that chemical production brings the highest income for the study area among all industrial and agricultural activities, while the average amount of BOD discharged from chemical plants could reach approximately 141tonne over the planning horizon. In addition, abundant phosphorus resources, extensive soil disturbance, and heavy fertilizer application cause point and nonpoint source losses of nitrogen and phosphorus nutrients, resulting in the river's water quality being gradually eutrophicated. The excessive nitrogen loading from agricultural activities is the principal pollution source due to unreasonable uses of livestock manures and chemical fertilizers; most agricultural croplands centralize in the hillside along with the river, such that it is easy to lead to effluent discharge directly into the water body of the river. Results also reveal that phosphorus discharge derives mainly from point sources, particularly from the chemical plants and phosphorus mining companies (occupying 77.4 % of the total TP).

Abatement of pollutant contaminations from industrial and agricultural activities is important for the river pollution control. However, the implementation of management practices for river pollution control can have potentials to affect the local economic income. Limitations of fertilizer application can reduce crop yields, thereby causing reductions of livestock herds fed by the crop and decreasing the agricultural benefit. Owing to advantageous phosphorus resources conditions, chemical plants and phosphate mining companies are the main sources to local financial income. Abatement of their production scales would lead to a reduced economic income. There is a tradeoff between the economic income and river-water pollution control. Summarily, several suggestions that could help local authority generate desired decision schemes for planning industrial and agricultural activities in association with both maximizing economic income and reducing river pollution are: (i) wastewater treatment technologies (e.g. tertiary treatment and depth processing technologies) should be enhanced to improve the pollutant removal efficiency; (ii) for chemical plants and phosphorus mining companies, phosphorus-containing wastes should be controlled strictly in the production processes, and effective treatments as well as disposal measures should be taken simultaneously to reach the goal of achieving TP abatement; (iii) soil erosion control is desired to reduce the transports of the nitrogen and phosphorus pollutants into the river.

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