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Comparing Low and High-Level Hybrid Algorithms on the Two-Objective Optimal Design of Water Distribution Systems

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Abstract This paper presents the comparison of two hybrid methodologies for the two-objective (cost and resilience) design of water distribution systems. The first method is a low-level hybrid algorithm (LLHA), in which a main controller (the non-dominated sorting genetic algorithm II, NSGA-II) coordinates various subordinate algorithms. The second method is a high-level hybrid algorithm (HLHA), in which various sub-algorithms collaborate in parallel. Applications to four case studies of increasing complexity enable the performances of the hybrid algorithms to be compared with each other and with the performance of the NSGA-II. In the case study featuring low/intermediate complexity, the hybrid algorithms (especially the HLHA) successfully capture a more diversified Pareto front, although the NSGA-II shows the best convergence. When network complexity increases, instead, the hybrid algorithms (especially the LLHA) turn out to be superior in terms of both convergence and diversity. With respect to both the HLHA and the NSGA-II, the LLHA is capable of detecting the final front in a single run with a lower computation burden. In contrast, the HLHA and the NSGA-II, which are more affected by the initial random seed, require numerous runs with an attempt to reach the definitive Pareto front. On the other hand, a drawback of the LLHA lies in its reduced ability to deal with general problem formulations, i.e., those not relating to water distribution optimal design.

Keywords low-level hybrid algorithm · High-level hybrid algorithm · Multi-objective design · Water distribution system

1 Introduction

Water is one of most important resources and water distribution infrastructures are essential in maintaining an adequate high quality, continuous drinking water supply to our homes. Rapid

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urbanisation generates considerable pressure on water engineers tasked with extending existing and designing new water distribution infrastructures. Research into optimal design of water distribution systems (WDSs) has a long history (Cisty 2010). Majority of optimisation methods use the least-cost approach aimed at minimising one objective function (network cost) while satisfying constraints concerning the range of variables (e.g., available pipe sizes), the physical conditions (e.g., water mass and energy conservation) and the operation-related criteria (e.g., minimum nodal pressure and maximum flow velocity).

However, such a problem formulation may lead to network solutions featuring poor hydraulic performance since it is only based on economic concerns (Walski 2001). Consequently, the multi-objective formulation has been gaining more attention and various multi-objective evolutionary algorithms (MOEAs) have been applied to solve small-to-medium sized benchmark problems and some large problems based on the real-world networks (Cheung et al. 2003; Farmani et al. 2005; Fu et al. 2012; Raad et al. 2009). These algorithms are capable of approximating the Pareto-optimal front (PF) in a single run. The non-dominated sorting genetic algorithm II (NSGA-II) (Deb et al. 2002) is one of the most popular MOEAs, which is frequently used to solve optimisation problems of different kinds and involving complex WDS configurations. For instance, it was adopted by most teams in The Battle of the Water Networks II (Marchi et al. 2013).

Despite the features of flexibility and robustness, the MOEAs are often criticised (Kollat and Reed 2006; Creaco and Franchini 2013) due to the issue of parameterisation and extensive function evaluations to reach a near-optimal PF (Fu et al. 2012). In order to overcome their limitations and achieve a better numerical performance, hybrid algorithms that combine different components and strategies have been proposed in the scientific literature. According to Talbi's (2002) classification, these algorithms can be subdivided into two categories: the low-level hybrid algorithms (LLHA), in which the component metaheuristics are embedded in other metaheuristics as functional parts, and the high-level hybrid algorithms (HLHA), in which the component metaheuristics work on their own without mutual dependence.

Raad et al. (2009) addressed three benchmark problems as well as a real case in South Africa using a HLHA for the first time. This HLHA was based on the framework of a multi-algorithm, genetically adaptive multi-objective method (AMALGAM) (Vrugt and Robinson 2007) and introduced two new sub-algorithms which differed from those within the original AMALGAM. They also conducted a comparative study extensively by testing up to 23 alternative algorithms for the multi-objective design of 9 small-to-large sized WDS benchmarks (Raad et al. 2011). Three novel variants based on the structure of AMALGAM and NSGA-II turned out to be the four top-performing algorithms according to various metrics. Creaco and Franchini (2012) proposed a LLHA as a fast tool dedicated for the multi-objective design of large WDSs. This method embedded a Linear Programming in the NSGA-II. Unlike the traditional definition of decision variables (the diameter option for each single pipe), only three genes were considered for individuals of a population (independent from the number of pipes), thus yielding significant computational efficiency especially on larger networks. When compared with the traditional approach (i.e., NSGA-II), the hybrid approach demonstrated convincing benefits in terms of quality of solutions and CPU time. In a more recent work, Creaco and Franchini (2013) presented an upgraded version of LLHA (with number of individual genes extended to five), being able to consider more complex objective functions (network resilience) and constraints (maximum flow velocity) within the WDS design. Wang et al. (2014) compared two HLHAs (including the original AMALGAM) with NSGA-II on a wide range of benchmark problems and found that AMALGAM outperformed its competitors for small-to-medium sized cases. However, both HLHAs deteriorated for larger problems due to the loss of their adaptive capabilities.

Most of the aforementioned studies only compared the performance of hybrid algorithms with other popular MOEAs (like NSGA-II). Therefore, there is a lack of sound comparative studies between LLHAs and HLHAs. This deficiency in comparative work makes it difficult to assess the state of the art, particularly those aspects of hybrid development and application still requiring research. Dissemination of best practice to practitioner and research communities is also hampered. Thus, the major objectives of the current study are: (1) to develop a sound framework (problem definition, performance evaluation and data sets) for the comparison of multi-objective design algorithms for real-world WDSs; (2) to perform a direct comparison between LLHA, HLHA and NSGA-II on real-world WDSs; (3) to investigate differences between the LLHA and the HLHA considered from multiple perspectives, including conceptual and algorithmic performance in both objective and solution space; and (4) to provide recommendations on the most efficient way in dealing with optimisation of complex WDS design problems in real life. To achieve these objectives, the LLHA developed by Creaco and Franchini (2013) and the original AMALGAM (Vrugt and Robinson 2007) were selected, tested and compared together with the NSGA-II on four medium-to-large sized design problems based on the real-world networks in Italy.

The remainder of this paper is arranged as follows. Section 2 provides the two-objective formulation of a WDS and the concise introduction to the LLHA and the HLHA considered. Section 3 briefly describes the cases used for the comparative study. The results and discussion is given in Section 4. Section 5 concludes the whole paper.

2 Methodology

2.1 Two-Objective Design of a WDS

The optimal WDS design is aimed at determining the size and location of different components (e.g., pipes, pumps and tanks) in order to convey the treated water in a safe and efficient manner, with respect to a number of constraints, such as conservation of mass and energy as well as other service standards (e.g., quantity and quality). More often, only the size of pipes is considered under a single demand loading condition given the configuration of the network system. This is also known as a pipe sizing problem. The task is to choose the best combination of pipe diameters from within a number of discrete options, which are available in the market or from the manufacturers. It is difficult to solve such a problem due to a discontinuous, highly nonlinear, constrained and multi-modal search space (di Pierro et al. 2009), featuring non-deterministic polynomial-time hard (NP-hard) characteristics (Papadimitriou and Steiglitz 1998).

Minimising the cost (mainly the capital cost) is one of the main concerns during the process as the design and construction of a WDS usually require a great amount of expenditure. The capital cost is, then, the first objective function (to minimise) in the WDS design. In the present work, it takes on the following form:

$$\min C = \sum_{i=1}^{np} c_i(D_i) \times L_i \tag{1}$$

Where C=total cost (monetary units problem dependant); c_i=unit cost of pipe i depending on the specific diameter; np=number of pipes; D_i=diameter of pipe i; L_i=length of pipe i.

Besides the economic considerations, hydraulic performance should also be well addressed to ensure the reliability and service standard of a WDS. Compact reliability indicators, based on nodal pressure (Cheung et al. 2003; Prasad and Park 2004; Farmani et al. 2005) can be used to characterise network performance and formulate the second objective function. In this work, the network resilience indicator I_n proposed by Prasad and Park (2004), which represents an upgrade of the Todini (2000) resilience, is considered. This indicator expresses the ratio of the power excess delivered to the users, corrected in order to consider the uniformity of the pipes connected to each network demanding node, to the power excess leaving the source node(s). I_n has been advised as a better indirect reliability index for both simple and complex networks (Creaco et al. 2013).

$$\max I_n = \frac{\sum_{j=1}^{nn} C_j Q_j \left(H_j - H_j^{req}\right)}{\sum_{k=1}^{nr} Q_k H_k - \sum_{j=1}^{nn} Q_j H_j^{req}} \tag{2}$$

$$C_j = \frac{\sum_{i=1}^{np_j} D_i}{np_j \times \max\{D_i\}} \tag{3}$$

Where I_n=network resilience; nn=number of demand nodes; C_j, Q_j, H_j and H_j^{req}=uniformity coefficient, demand, actual head (evaluate by means of a hydraulic simulator, e.g. EPANET software, Rossman 2000) and minimum head of node j; nr=number of reservoirs; Q_k and H_k=discharge and actual head of reservoir k; np_j=number of pipes connected to node j; D_i=diameter of pipe i connected to demand node j.

For the HLHA and the NSGA-II, EPANET2 software (Rossman 2000), which is based on the Global Gradient Algorithm (Todini and Pilati 1988), is taken to run the hydraulic simulation from where the variables required for the evaluation of network resilience I_n are obtained. The LLHA, instead, uses the Global Gradient Algorithm specifically implemented in the Matlab2011b® environment.

2.2 Hybrid Optimisation Algorithms

Although the two algorithms, LLHA (Creaco and Franchini 2013) and HLHA (Vrugt and Robinson 2007) have very different structures, both combine various sub-algorithms.

The LLHA selected is based on a cascade of sub-algorithms coordinated by a main controller, which is the NSGA-II. Each solution of the NSGA-II contains the instructions (5 decision variables in all, as shown below) for the sub-algorithms, which are executed in series by performing various hydraulic simulations. Being tailored to the pipe design problem, the LLHA is not a general optimisation method. The fixed decision space for the LLHA (i.e., 5 decision variables) does not depend on the network topological complexity. This leads to a highly efficient and robust numerical performance of the LLHA.

The HLHA selected is based on the collaboration of various multi-objective algorithms arranged in parallel. The main controller of HLHA decides how the work in generating offspring individuals has to be divided among the various sub-algorithms. A single hydraulic simulation is performed for each HLHA solution to test its hydraulic performance. The decision space is made up of the entire set of variables which have to be designed (i.e., the pipe sizes) and thus depends on the topological complexity of the network. This makes the HLHA more flexible than the LLHA since other aspects than those considered in this work can be easily incorporated in the optimisation. However, it renders the computational burden of

HLHA higher than that of the LLHA, particularly in the case of complex networks. Furthermore, for each HLHA optimisation, numerous runs have to be performed in order to eliminate the influence of the initial random seed, since optimisation results may change significantly from a run to another.

A summary of key features of the LLHA and HLHA selected is provided in Table 1. The NSGA-II method is not compared in Table 1 because the main difference between the HLHA and the NSGA-II lies in the fact that the former employs four different search operators, rather than only one as the NSGA-II for reproduction.

2.2.1 Low-Level Hybrid Algorithm

The LLHA (Creaco and Franchini 2013) is made up of two blocks (see Fig. 1) and based on the combination of various algorithms. The first preliminary block makes it possible to detect one or more decompositions of the looped network each one generating a set of single source branched networks. The second main block encompasses a cascade of four different algorithms for the network multi-objective design. The first and main algorithm (A1) is the NSGA-II. The individuals of the population of this algorithm are made up of only five genes: the first makes it possible to detect time by time which of the decompositions detected in the preliminary block has to be applied to the looped network; the second and third genes are parameters that have to be supplied to the second algorithm, i.e. to the linear programming (A2) for the branched network design, and relate to the minimum pressure head and resilience constraints respectively; the fourth and fifth genes are parameters that have to be supplied to the third algorithm (heuristic algorithm A3), which re-closes network loops with the smallest diameter considered in the design phase and then improves the uniformity of the diameters of the pipes connected to each network node; the fourth algorithm (heuristic algorithm A4) modifies some pipe diameters in order that maximum flow velocity constraints are respected all over the network. The final network configuration is assessed in terms of *Cost* and I_n, which are the objective functions of A1.

In this context, it is worth highlighting that, naturally, the rationale behind the procedure herein presented (based on the design of the branched networks concealed inside the looped network, loop re-closure and diameter modification) comes from a significant simplification of the design problem, which entails that the design of a looped network comes from the design of a system of branched networks concealed inside the network itself and from the correction of the generic network solution by the application of two heuristic algorithms. This significant simplification may then result in a reduction in the research space. However, this weakness is balanced by its simplicity, which leads to the procedure easily converging and finding good

Table 1 Key features of the LLHA and HLHA selected

Features	LLHA	HLHA
type of hybridisation	low-level relay hybrid	high-level teamwork hybrid
number of decision variables	fixed (5)	problem-dependent
hydraulic simulation	multiple run per solution	single run per solution
flexibility	low	high
computational burden	low	high
robustness to random seed	high	low

The type of hybridisation is termed according to the taxonomy of hybrid algorithm in (Talbi 2002). Flexibility in the above table refers to how much effort has to be made to adapt the algorithm to other cases or problem formulations

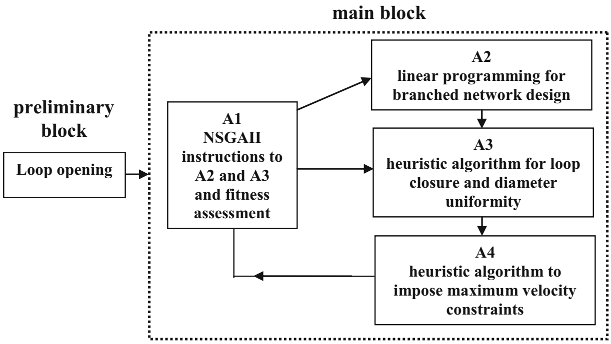


Fig. 1 Flowchart of the LLHA

solutions, as will be shown in the next sections. More details about this low-level hybrid algorithm can be found in Creaco and Franchini (2013).

2.2.2 High-Level Hybrid Algorithm: AMALGAM

AMALGAM (Vrugt and Robinson 2007) is a high-level hybrid optimisation framework which employs simultaneously four sub-algorithms within its structure, including NSGA-II, adaptive metropolis search (Haario et al. 2001), particle swarm optimisation (Kennedy and Eberhart 2001) and differential evolution (Storn and Price 1997). It is designed to overcome the drawbacks of using an individual algorithm and to be suitable for a wide range of problems. The strategies of global information sharing and genetically adaptive offspring creation are implemented in the process of population evolution. Each sub-algorithm is allowed to produce a specific number of offspring individuals based on the survival history of its solutions in the previous generation. The pool of current best solutions is shared among sub-algorithms for reproduction. The operation of AMALGAM can be summarised as follows (see also Figure 2). Firstly, an initial population P_0 of individuals, with a number N of genes equal to the number of pipes to be designed, is generated using Latin hypercube sampling (LHS). Then, P_0 is ranked via the fast non-dominated sorting (FNS) procedure (Deb et al. 2002). The offspring Q_0 of size N is yielded from P_0 using four sub-algorithms simultaneously, with each algorithm contributing the same number of individuals (i.e., $N/4$). Next, a combination of the parents (P_0) and the offspring (Q_0), namely R_0 (size $2N$), is produced and ranked via the FNS. A number N of members from R_0 are selected based on their rank and crowding distance (CR), forming the population in the next generation. The latest population is then taken to create the offspring using the adaptive multi-method search technique, which is detailed in the subsequent paragraph. The aforementioned procedure is repeated until the stopping criteria are met (e.g., number of function evaluations and/or prescribed precision).

The basic idea of adaptive multi-method search is to take full advantage of the most efficient sub-algorithm and to keep a balance in using diverse methods. That is, each algorithm is allowed to produce a number of children according to the reproductive rate (ratio of the children alive to the children created) in the previous generation. However, if one fails to contribute even a single individual in the latest population, a minimum number of individuals (5 here as the bottom line) are consistently maintained for it to generate the offspring.

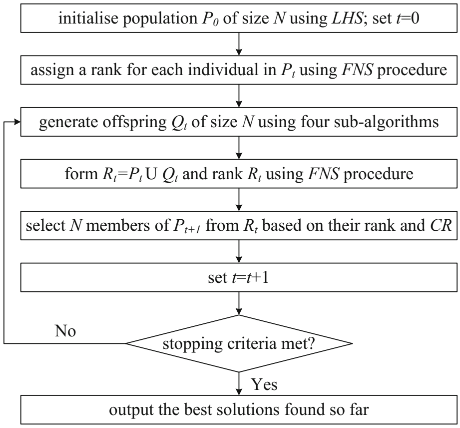


Fig. 2 Flowchart of AMALGAM

Therefore, the most successful algorithm (with highest reproductive rate) is favored by giving more slots in the process of reproduction, but no one is completely discarded even though it exhibits the worst performance. In addition, AMALGAM provides a general template which is flexible and extensible, and can easily accommodate any other population-based algorithm (Raad et al. 2009, 2011).

3 Applications

3.1 Case Studies

Four WDS design problems were used to assess the performance of the aforementioned hybrid optimisation algorithms and the NSGA-II. These problems are based on different real-world WDSs in Italy with varied complexity in terms of search space size. The first three cases were originally introduced by Bragalli et al. (2008), while the last case is a WDS of a city in Northern Italy. For confidentiality reasons, it is named 'Town X' in this paper. A brief summary of four design problems is provided in Table 2.

Table 2 Characteristics of benchmark design problems considered for the comparative study

Problem (Network)	No. of				C-Value	Size of Search Space	Design Criteria		
	Reservoir	Node	Pipe	Size			P_{min} (m)	P_{max} (m)	V_{max} (m/s)
Fossolo	1	36	58	22	150	7.25×10^{77}	40	varied	1
Pescara	3	68	99	13	130	1.91×10^{110}	20	varied	2
Modena	4	268	317	13	130	1.32×10^{353}	20	varied	2
Town X	2	536	825	13	130	1.01×10^{919}	25	30	2

C-Value: Hazen-Williams roughness coefficient (unitless). The size of search space is estimated by computing the number of diameter sizes to the power of the number of pipes. For example, the search space size of the Fossolo problem is $22^{58} \approx 7.25 \times 10^{77}$

3.2 Benchmarking Setup

The LLHA, HLHA and the NSGA-II were run on a 2.70 GHz CPU. In the experiments, no parallel computing was used and thus each optimisation run was executed on a single core.

In order to investigate the performance of the hybrid algorithms and compare them with that of the NSGA-II under low and high computational burdens, short and long runs on each benchmark problem were applied concurrently. Since the algorithmic frameworks of the LLHA and the HLHA are different, the computational budgets are set to keep the execution time of a single run as close as possible for the LLHA and the HLHA. The NSGA-II adopts the same budget as the HLHA because they take very similar CPU time to finish a single run. The details of the computational budgets in terms of CPU time for each design problem in a single run are given in Table 3. Table 4 and Table 5 show, instead, the general parameter settings, i.e., population size and number of function evaluations (NFE), of the LLHA and the HLHA, respectively for the low and high computational burdens.

The analysis of Table 4 shows that in the LLHA the population size is always the same (equal to 50 individuals) and number of function evaluations (NFE) does not vary significantly as the network complexity increases (from case study 1 to case study 4). This is a direct consequence of the fact that the number of individual genes used in the LLHA (equal to 5 – see Section 2.2) does not depend on the network size. Furthermore, the simple genetic structure results in the best PF in each optimisation run of the LLHA.

In the HLHA, instead, the influence of the initial random seed is much stronger. In order to obtain a 'best PF', each problem was solved independently 30 times using three varied population sizes (see Table 5) (10 times for each population size). The idea of such a plan for the HLHA is to capture a PF as widespread as possible in the objective space of *Cost* against I_n. In this context, it is worth stressing that the results indicated in Table 3 for the HLHA and the NSGA-II refers to an average run time.

A comparison between Tables 4 and 5 proves that the population size and NFE required by the LLHA are smaller than those featured by the HLHA for pre-fixed computation time (of a single run). This is due to the fact that in the LLHA each objective function evaluation requires linear programming and various hydraulic simulations to be performed (see algorithms A2, A3 and A4 in Section 2.2); in the HLHA, instead, each objective function evaluation simply requires a single hydraulic simulation to be performed.

Table 3 Computational times Used in Analyses

Case Study	Computational Budget (minutes)					
	Short Run			Long Run		
	LLHA	HLHA	NSGA-II	LLHA	HLHA	NSGA-II
Fossolo	0.7	0.8	0.8	3	3	3
Pescara	0.7	0.7	0.7	5	7	7
Modena	9	9	9	55	58	58
Town X	17	18	18	100	90	90

Table 4 Parameter settings of LLHA

Case study	Population Size	Computational Budget in Terms of NFE	
		Low Burden	High Burden
Fossolo	50	500	2000
Pescara	50	500	3000
Modena	50	800	3000
Town X	50	500	3000

4 Results & Discussion

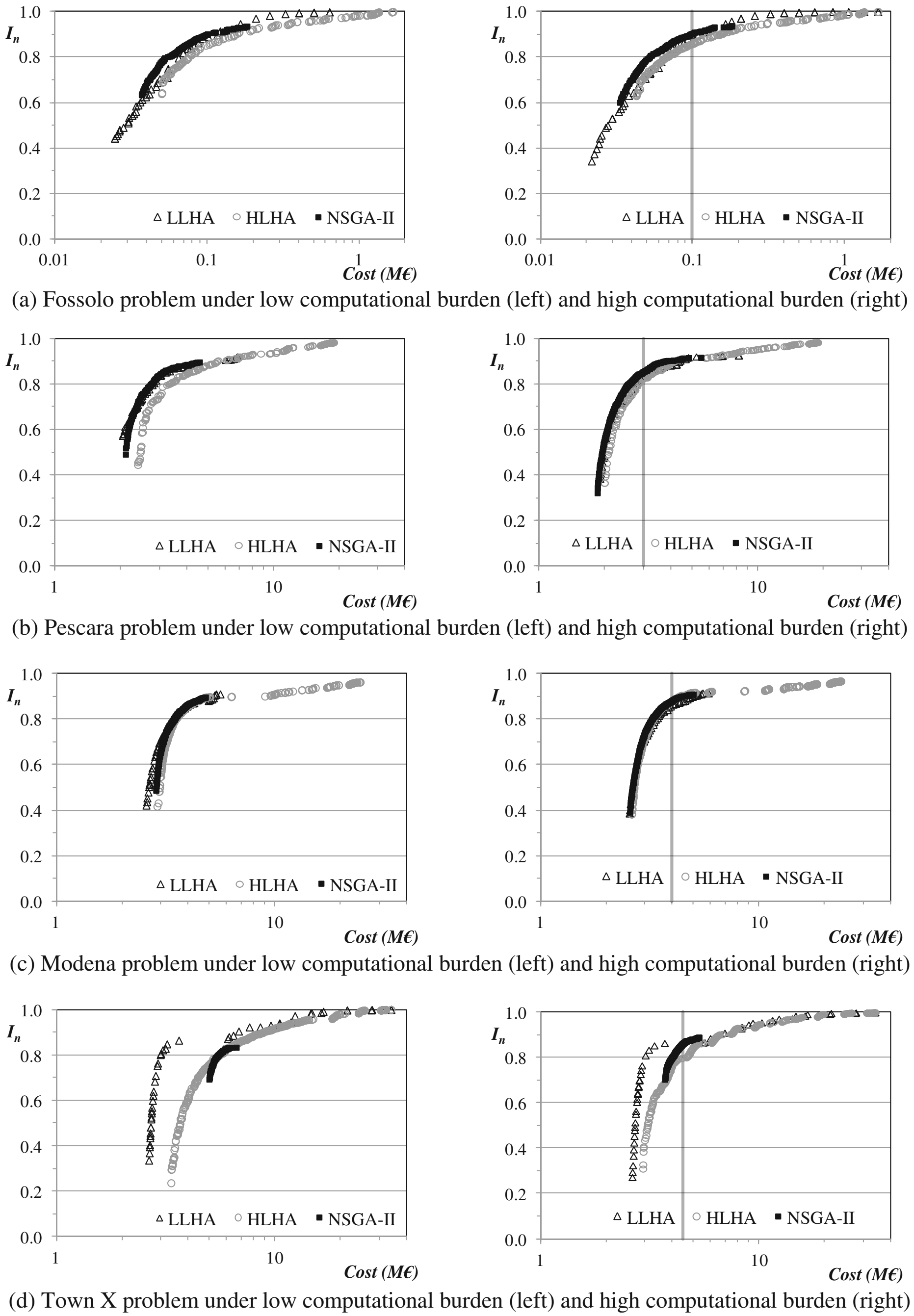
The results of the optimisations carried out by means of the hybrid algorithms and the NSGA-II are reported in Figs. 3 and 4. The first analysis was made for pre-fixed computational burden. In Fig. 3, graphs on the left and right correspond to the small and large computational burdens respectively.

For the Fossolo problem (lowest complexity case study) the positions of the PFs obtained by the LLHA and the HLHA, considering both the small and large computational burden, are close. The PFs are slightly dominated by those obtained by the NSGA-II, which shows a higher convergence performance on a reduced front length. The only remarkable difference between the hybrids lies in the fact that the LLHA lends itself better to detecting the solutions featuring both low cost and resilience (left side of the front). The fact that the LLHA procedure performs better for low cost solutions and worse for high cost solutions than the HLHA can be ascribed to its basic assumptions: the design based on the looped network decomposition (basic assumption of the LLHA) is more effective to yield solutions featuring low cost and resilience. In the case of high cost and resilience solutions, the simplifications contained in the LLHA structure can, instead, endanger its performance. Results in graph (a) on the left of Fig. 3, obtained considering a small computational burden, indicate a slight predominance of the HLHA in detecting solutions featuring high cost and resilience.

The applications to the Pescara and Modena problems of intermediate complexity yield similar results regarding both the comparison of the hybrid procedures and against the NSGA-II. Under both computational burdens, the superiority of the HLHA in detecting the high cost and high resilience solutions on the PF is highlighted. The comparison with the PF of the NSGA-II shows that the latter yields very close results to the LLHA, with a slightly better convergence performance for the high computation burden. The inability of the LLHA to detect high cost and high resilience solutions in these two cases is due to the fact that, in the high cost solutions generated by the LLHA for such networks with multiple sources, the

Table 5 Parameter settings of HLHA and NSGA-II

Case Study	Population Size		Computational Budget in Terms of NFE		
	Size 1	Size 2	Size 3	Low Burden	High Burden
Fossolo	100	200	400	50,000	80,000
Pescara	100	200	400	40,000	150,000
Modena	200	400	800	200,000	800,000
Town X	400	800	1600	113,600	454,400



(a) Fossolo problem under low computational burden (left) and high computational burden (right)

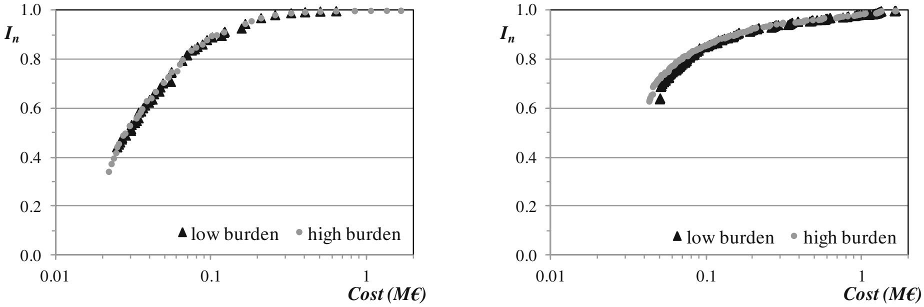
(b) Pescara problem under low computational burden (left) and high computational burden (right)

(c) Modena problem under low computational burden (left) and high computational burden (right)

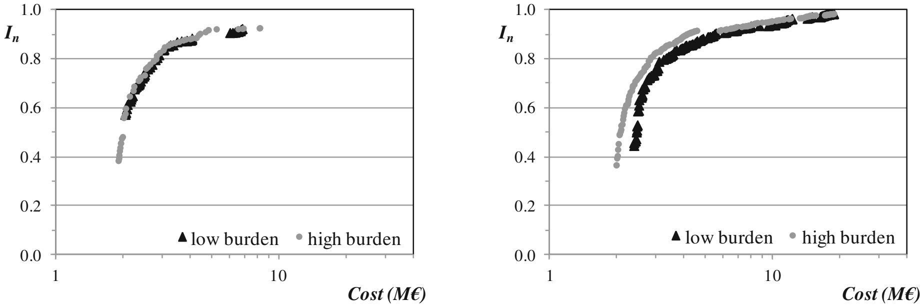
(d) Town X problem under low computational burden (left) and high computational burden (right)

Fig. 3 Comparison of LLHA, HLHA and NSGA-II using low and high computational burdens (Cost axis in logarithmic scale)

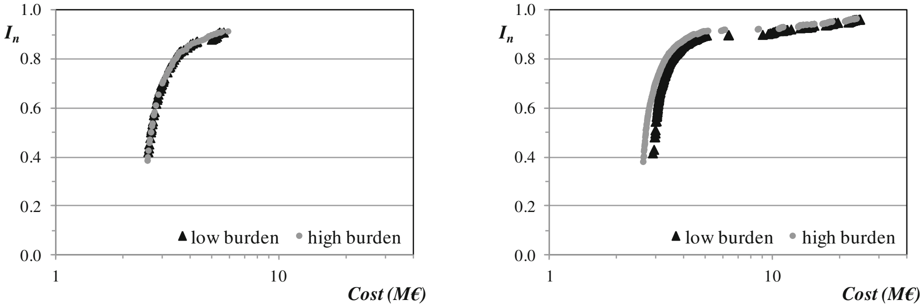
installation of large and uniform diameter pipes encourages the formation of water exchanges between the reservoirs. This results in the increase in network head losses and then the



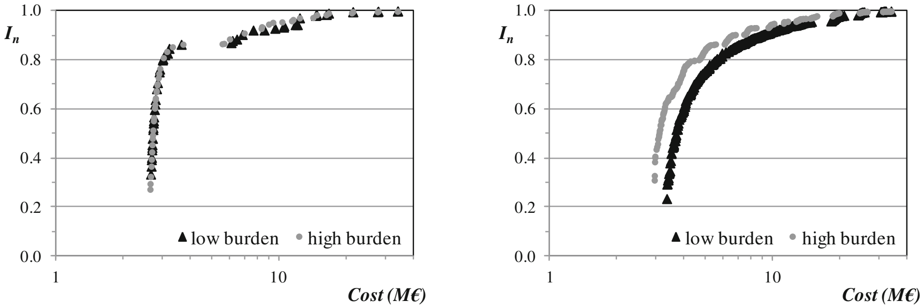
(a) Fossolo problem by LLHA (left) and HLHA (right) using low and high computational burden



(b) Pescara problem by LLHA (left) and HLHA (right) using low and high computational burden



(c) Modena problem by LLHA (left) and HLHA (right) using low and high computational burden



(d) Town X problem by LLHA (left) and HLHA (right) using low and high computational burden

Fig. 4 Comparison of low burden with high burden for LLHA and HLHA (Cost axis in logarithmic scale decrease in nodal pressure heads. As a consequence, a decrease in the network resilience (Eq. 2) takes place. High cost solutions, i.e., large size pipes in the network selected by the LLHA, are then discarded as being dominated in terms of resilience by the low cost solutions on the PF. On the other hand, the HLHA avoids this situation by allowing small size pipes to be selected in suitable sites. The incapacity of the NSGA-II to detect high cost and resilience solutions, instead, has to be ascribed to its ability to yield high convergence performance in a reduced front length (see also case study 1).

For the problem of highest complexity, Town X, the LLHA yields better results than HLHA in the case of both low and high computational burdens and for either side of the PF (low cost and low resilience solutions on the left and high cost and high resilience solutions on the right). This better performance is achieved due to the reduced search space in LLHA. Unlike case studies 2 and 3, in case study 4 high cost and high resilience solutions are also present in the PF yielded by the LLHA; this happens because the elevation of the two sources and their mutual distance spontaneously hinder the formation of inter-source water transfer and thus the single-source branched-networks do not produce negative effects as in the case of Pescara and Modena networks. The comparison between the hybrid algorithms and the NSGA-II in case study 4 highlights that, for high network complexity, the hybrid procedures turn out to have a much better performance in terms of both convergence and front diversification.

In Fig. 4, another viewpoint of the optimisation results is reported. In particular, graphs on the left report the PFs obtained by the LLHA considering small and large computational burdens; those on the right report, instead, the results obtained by the HLHA considering small and large computational burdens. The comparison of the results obtained by the LLHA in each case study showed that the fronts obtained with the small computational burden are almost coincident with those obtained with the longer runs. This means that only a small computational effort is needed to obtain the best results achievable. In the case of the HLHA, instead, the increase in computational burden improves the effectiveness of the results significantly since the fronts obtained by running the procedure long enough dominate those obtained in short runs. The latter effect becomes more and more evident when network complexity increases, i.e., moving from graph (a) to graph (d) in Fig. 4.

Whereas the previous analyses are concerned with the comparison of the algorithms in the objective space, the following remarks are made for the solution space. Several methods have recently been proposed on how to choose solutions from the PF to facilitate a posteriori analysis, such as weighted stress function method (Ferreira et al. 2007) and cluster analysis (Dumedah et al. 2010). In this study a solution from the 'knee' point on the PF was chosen as the most interesting region for decision makers. After fixing a cost value around the knee point in each case study (see vertical lines in Fig. 3) the solution featuring the closest cost value is taken for each algorithm considered in this work. For each case study, the comparison of the three solutions featuring similar cost values made it possible to analyse to which extent diameter distribution in the network changes when the algorithm used for the optimisation changes. To this end, 4 diameter classes were constructed for each case study, and the network pipe length associated with its class was calculated. The graphs in Fig. 5 report the length of pipes as a function of the diameter class for the three solutions selected in each of the four case studies (obtained by LLHA, HLHA and NSGA-II, respectively).

Overall the graphs show that the LLHA, HLHA and NSGA-II led to similar diameter distributions. In the Fossolo network, no one of the algorithms yielded pipes for the class featuring diameters lower than or equal to 51.0 mm. In the three remaining classes, HLHA yielded larger network pipe length for 61.4–90 mm and 163.6–229.2 mm classes, whereas the NSGA-II in 102.2–147.2 mm class. The LLHA yielded intermediate pipe length values in 61.4–90 mm and 102.2–147.2 mm classes and a length equal to 0 in 163.6–229.2 mm class. In

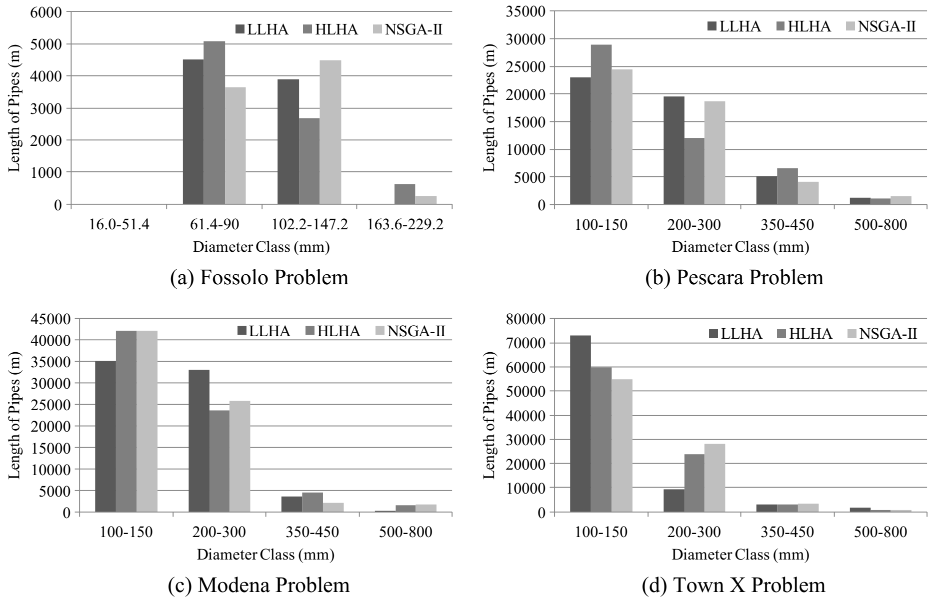


Fig. 5 Comparison of solutions obtained by three algorithms in solution space

the Pescara network, unlike the LLHA and the NSGA-II, the HLHA tends to prefer 100–150 mm class with respect to 200–300 mm class. In the Modena network, the LLHA tends to prefer 200–300 mm class with respect to 100–150 mm class and yields a pipe length close to 0 in the last class with pipe size ranging from 500 to 800 mm. Finally, in the Town X network, the hybrid algorithms, especially the LLHA, tend to prefer 100–150 mm class to 200–300 mm class.

It is worth stressing that we did not use any quantitative indicator to compare the performance of the hybrid algorithm and that of the NSGA-II. Such a decision is related to the following observations:

(1) Most of quantitative indicators existing in the literature require a reference set (i.e., often a known optima) to measure certain aspect (convergence or diversity) of an approximation set, such as the general distance (convergence), the gamma (convergence) and delta indicators (diversity) (Deb et al. 2002). However, due to the NP-hard nature of the optimal design of Water Distribution Systems, it is very difficult to obtain such a reference set beforehand, especially for large and complex networks.
(2) Using some quantitative indicators in the context of multi-objective design of Water Distribution Systems can be misleading, because these indicators are mainly developed for continuous problems. However, as shown above, the PF is discrete and unevenly distributed in the objective space. In such a situation, a wrong interpretation may be easily derived from a numerical indicator. In addition, there is no single indicator which can measure both aspects (convergence and diversity) of multi-objective optimisation in a clear and definitive way. In other words, in the context of multi-objective optimisation, the comparison itself should be multi-objective.
(3) It is relatively easy and computationally cheap to compare the non-dominated solutions produced by the different algorithms visually by plotting their approximation sets in the

objective space. This approach has enabled a direct comparison and facilitated drawing of reasonable conclusions.

5 Conclusions

This paper developed a sound comparative framework for multi-objective WDS design algorithms. The two objectives used are the minimisation of cost and maximisation of network resilience. The performance comparison of two different types of hybrid search procedures and the baseline NSGA-II was presented in detail. The first type of hybrid procedure considered was a low-level hybrid algorithm, where various inner algorithms are embedded within the NSGA-II. The second type of hybrid procedure was a high-level hybrid algorithm, where various search operators co-operate in parallel.

Applications to case studies of increasing complexity showed that performances of the LLHA and HLHA are complementary. Due to the fact that optimisation with LLHA is not significantly affected by the initial random seed and that the size of the search space of LLHA does not increase with the growth in network complexity, selection of the LLHA is recommended for networks from low to high complexity, particularly in the latter case where the pursuit of solutions with high accuracy using the other algorithms, would inevitably lead to exceedingly high computational overhead in terms of both the number of function evaluations and the running time. On the other hand, when computation efficiency is not a concern (i.e., it is possible to consider a large number of individuals as well as to repeat optimisation several times in order to eliminate the influence of the initial random seed), selection of the HLHA is expected to improve the accuracy of the results as much as required under the various circumstances. Unfortunately, this approach may fail when the network size is really too large. Alternatively, a combination usage of the LLHA and the HLHA may overcome this limitation and yield better results in shorter time. That is, the LLHA could be employed first to quickly approximate the Pareto-optimal front, and then this approximation could be further improved by the HLHA. This approach will be investigated thoroughly in a future work.

Overall, the comparison between the hybrid algorithms and the NSGA-II demonstrates the advantage of using the hybrid algorithms in order to obtain a more diversified PF. Their superiority in terms of convergence also emerges when network complexity increases. In the future, more objectives should be taken into account for the optimal design of a WDS, for example minimising leakage and water age (an indicator of water quality), or minimising carbon emissions in the pipe manufacturing process and carbon emissions during pipe installation and operation, transforming the task from two-objective to many-objective (four or more) optimisation. As indicated by Fu et al. (2013), the optimal solutions obtained in a lower dimensional formulation often tend to have a worse performance in other objectives considered in a higher dimensional formulation. Although it supports more informed and transparent decision making in the design stage, the many-objective formulation will greatly challenge the capabilities of the current algorithms, including both LLHAs and HLHAs, in approximating the PF in higher (thus more complex) dimensional space. Furthermore, more complex benchmark problems, not only based on large networks with/without multiple loading conditions, but also the ones associated with operational cost (typically requiring extended period simulation), should also be considered for the comparison.

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