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Multi-Objective Parameter Calibration and Multi-Attribute Decision-Making: An Application to Conceptual Hydrological Model Calibration

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Abstract For calibrating the conceptual hydrological models (CHM), the traditional calibration method with a single objective cannot properly measure all the behaviors of the hydrological system. To obtain a successful parameters calibration, in this paper, we propose a multi-objective cultural self-adaptive electromagnetism-like mechanism (MOCSEM) algorithm, which is first implemented in solving the parameters calibration problem of CHM. In this algorithm, a self-adaptive parameter is applied in local search operation for adjusting the values of parameters dynamically. Meanwhile, cultural algorithm (CA) is adopted to keep a good diversity and uniformity of Pareto-optimal solutions (POS). MOCSEM is tested, firstly, by several benchmark test problems. After achieving satisfactory performance on the test problems, a case study is implemented for parameter calibration of a CHM by comparing the properties of POS obtained by the MOCSEM and other methods. Finally, when the optimization problem quickly becomes a decision-making problem because of the multiple objectives in CHM, fuzzy technique for order preference by similarity to an ideal solution method has been used to rank the POS and select the optimal scheme. The results show that the MOCSEM algorithm can provide high-accuracy parameters of CHM on various decision-making scenarios.

Keywords Parameters calibration · Conceptual hydrological models · Multi-objective cultural self-adaptive electromagnetism-like mechanism · Decision-making problem

1 Introduction

There are a lot of parameters in the conceptual hydrological models (CHM), and parameters value will influence the forecast effect directly. Manual parameters calibration of CHM

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requires great experience and time of the operator. The development of computer skills during the last decades has contributed to implementation of the automatic calibration (Dakhlaoui et al. 2012). Most parameters of CHM have clear physics meaning, which, in principle, can directly determinate the values of parameters. However, due to the lack of measurement and experiment of all facts in the process of rainfall-runoff formation, in reality, we can merely deduce the parameter value by system identification methods. The definition of the value still faces great difficulties because of the huge amount of parameters while little information. In practice, we usually work it out by concept analysis, which means we can get the preliminary range of the values according to measurement and physics meaning, then input them and gain the output through model calculation, compare the output progress with the measurement and optimize it to decide the optimal value of the parameters under specific condition.

Traditional calibration of CHM mainly is done by considering single objective. The optimal methods include the Rosenbrock's method (Rosenbrock 1960), shuffled complex evolution (SCE-UA) algorithm (Wang et al. 2013), genetic algorithm (GA) (Sahay 2012), particle swarm optimization (PSO) (Afshar et al. 2013), and so on. However, the engineering practice shows that the traditional calibration scheme is hard to reflect the dynamic behavior characteristics of the hydrologic system because the parameter optimization calibration only considers one aspect of the features of hydrological process (Vrugt et al. 2003a). Therefore, it is necessary to develop the multi-objective model calibration of CHM so that different aspects of the system characteristics can take a full consideration. Multi-objective optimization method will produce a series of optimization schemes, which will not only allow an analysis of the trade-offs among the different objective functions but also enable hydrologists to better understand the limitations of current hydrologic model structure.

In recent years, researchers have begun to apply multi-objective evolutionary algorithms to multi-objective model calibration problems and achieved various degrees of success. Yapo et al. (1998) demonstrated that the multi-objective calibration approach is practical and relatively simple to implement and can help to better understand the different aspects of CHM, see also Cheng et al. (2002); Deckers et al. (2010); Dumedah et al. (2010); Dumedah (2012); Boyle et al. (2013). However, the optimization problem quickly becomes a decision-making problem because the multi-objective functions have been incorporated in CHM, and the users of hydrological model calibration have to face the task of selecting a set of suitable model parameters from the numerous Pareto-optimal sets (Khu and Madsen 2005).

In this paper, we propose a multi-objective cultural self-adaptive electromagnetism-like Mechanism (MOCSEM), which is an improvement over the Electromagnetism-like Mechanism (EM) algorithm. EM algorithm is proposed by Birbil and Fang (2003) and has been proven to be a flexible and effective global optimization algorithm for solving various optimization problems successfully (Chang et al. 2009; Wei et al. 2012). The method is first implemented in solving CHM problem. In MOCSEM, a self-adaptive parameter is applied in local search operation for adjusting the values of parameters dynamically to help the proposed method escape from local Pareto optimal front. In addition, cultural algorithm (CA) is adopted to keep a good diversity and uniformity of Pareto-optimal solutions (POS). MOCSEM is tested, firstly, by several benchmark test problems and compared with some previous research. After that, a case study is implemented for parameter calibration of a CHM by comparing the properties of POS obtained by MOCSEM method and those of the state-of-art algorithm SPEA2 (Zitzler et al. 2001), MOSCEM-UA (Vrugt et al. 2003a), and MOSCDE (Guo et al. 2013). Finally, the Fuzzy Technique for Order Preference by Similarity to an Ideal Solution (fuzzy TOPSIS) method has been used to rank the POS and select the optimal scheme.

This paper is organized as follows: Section 2 we briefly describe the CA scheme, detail the procedure of MOCSEM and fuzzy TOPSIS method. Section 3 introduces the multi-objective parameter calibration; in Section 4, the performance of the MOCSEM is first tested, and the computational results of a practical CHM problem are shown; while conclusions are made in Section 5.

2 Methodology

2.1 Framework of Cultural Algorithm (CA)

Cultural algorithm (CA), proposed by Reynolds in 1994, is a computational model inspired by the principle of cultural evolution that exceeds the rate of biological evolution based upon a process of dual inheritance (Reynolds 1994). The cultural evolution consists of three major processes: individuals' evolution in the population space, macro-evolutionary in the belief space and the communication between the population space and belief space. In CA, population space consists of a set of individuals, one of which corresponds to a solution of the problem, and it can be optimized by using any population-based algorithm, such as genetic algorithms (Reynolds 1994), differential evolution (Qin et al. 2010), particle swarm optimization (Zhang et al. 2012). As the evolutionary process unfolds, the belief space is a collectively information library where the knowledge acquired from individuals is stored. To communicate knowledge throughout the two spaces, a communication channel is established. In belief space, the function *update*() is the function to refine population experience by classifying the belief space. The belief space is established by using the function *accept*() to collect the population experience from the top individuals of the population space. In turn, the cultural knowledge controls the evolution of the population by a function *influence*() to improve the algorithm's searching efficiency. The function *objective*() is used to evaluate the fitness of every individual and the function *select*() selects the individuals for next generation evolved by function *generate*(). The framework of cultural algorithm is illustrated by (Qin et al. 2010).

2.2 Electromagnetism-Like Mechanism (EM) Algorithm

EM algorithm is a population-based stochastic algorithm which is quite simple, strong robust, significantly fast and effective. EM originates from the electromagnetism theory of physics by considering each sample point as a charged particle spread around the solution space (Tsou and Kao 2008). The fundamental procedures of EM include initialization of population, local search, calculation of total force, and movement of particles. The basic strategy of EM is detailed by Birbil and Fang (2003).

2.3 MOCSEM

As mentioned earlier, EM is primordially proposed for solving a single objective optimization problem. Thus, EM needs to make some modifications of its operations for dealing with multi-objective problems (MOP). Hence, in MOCSEM, we mainly focus on the preservation of POS, the modification of EM's operations and avoiding premature convergence effectively to try to achieve a successful application in dealing with MOP.

2.3.1 Knowledge Structures Defined in Belief Space

In Cultural Algorithm framework, There are at least five basic categories of cultural knowledge such as situational, normative, history or temporal, domain and topographic knowledge

(Saleem 2001). In MOCSEM, we redefine three knowledge structures (Situational Knowledge, Normative Knowledge and History Knowledge). These knowledge structures are introduced as follows:

(1) Situational Knowledge

Situational knowledge is composed of outstanding individuals along the population evolution process. These individuals are nearest to the true Pareto optimal front of multi-objective optimization. The structure of the redesigned situational knowledge is $[P_1,\ldots,P_i,\ldots,P_{NQ}]$. P_i is an outstanding Pareto solution; i is the index of the generation, and NQ is the size of situational knowledge.

Because of the limitation of computation source, the size N_Q of situational knowledge is usually a constant. Crowding distance, which was introduced by Deb et al. (2002) is often used to calculate the sharing fitness of POS. Moreover, the following updating strategy is implemented to maintain the size of situational knowledge while keeping POS spreading uniformly: (1) if situational knowledge is empty, P_i will be joined in situational knowledge directly; (2) if P_i is not dominated by any individual in situational knowledge, the individuals dominated by P_i are deleted, and P_i will be joined in situational knowledge; (3) if the number of POS is greater than N_Q, the cut operation is executed to reject redundant individuals according to the crowding distance.

(2) Normative Knowledge

Normative knowledge saves the feasible region of decision variables on each dimension where POS have been found. The structure of normative knowledge is $[L_1,\ldots,L_i,\ldots,L_{NQ}; U_1,\ldots,U_i,\ldots,U_{NQ}]$. L_j and U_j are the minimum and maximum values on the i-th dimension of the individuals. When the boundary is acquired from situational knowledge, normative knowledge needs to be reformed as the situational knowledge has changed. The upper and lower limits of individuals in new situational knowledge are introduced as new normative knowledge. When the original variables' boundaries are replaced by normative knowledge, the searching direction may be directed toward the optimization direction. The strategy can be shown as follow:

$$X_k^i = \begin{cases} L_k, & if\ X_k^i \prec L_k \\ U_k, & if\ X_k^i \succ U_k \end{cases} \quad (k = 1, 2, \cdots, D) \tag{1}$$

Where D is the dimensionality of decision variable X^i; $X^i{}_k$ denotes the k-th of the X^i; U_k and L_k are the upper and lower bound of $X^i{}_k$, respectively.

(3) History Knowledge

This knowledge source is used to monitor searching process of evolution algorithm and keep the distribution characteristics of decision variables. The structure of history knowledge is $[S_1,\ldots,S_i,\ldots,S_{NQ}]$. S_i is a convergence performance metric, which evaluates convergence of current POS to a reference set. We calculate S_k as follow:

$$S_k = \sqrt{\frac{1}{N_Q}\sum_{i=1}^{N_Q}\left(\frac{X_k^i - \overline{X_k}}{U_k - L_k}\right)^2} \tag{2}$$

Where $\overline{X_k}$ is the average value of the k-th dimension variables. For updating the history knowledge, S_k of k-th dimensions in situational knowledge is recalculated when situational knowledge and normative knowledge are evolved.

2.3.2 Modification of EM Operators

(1) Local search

The procedure of local search is going to move the points of solutions toward the local minimums that are near them. The method used in this procedure is very simple. In this paper, a novel self-adaptive mechanism is added to the local search operation, which improves the accuracy of solution and avoids the premature convergence. The modification of this step revises the evolution step of EM, as follow:

$$X_k^i = \begin{cases} X_k^i + \lambda \cdot decaypara(g) \cdot (\max(U_k - L_k)) & if \ \ rnd() > 0.5 \\ X_k^i - \lambda \cdot decaypara(g) \cdot (\max(U_k - L_k)) & otherwise \end{cases} \tag{3}$$

Where δ is local search parameter; λ is a uniform distributed random parameter; *rnd*() is a uniform random number between [0, 1]; *decaypara*(*g*) is the *g*-th self-adaptive function; *g* is the index of the generation:

$$decaypara(g) = \begin{cases} \delta \cdot \exp(-\alpha \cdot count \cdot g/G) & if \ \ count > r \\ \delta \cdot \exp(-g/G) & otherwise \end{cases} \tag{4}$$

Where *G* denotes the total evolution number; α is the self-adaptive parameter; *count* and *r* are the number and threshold value of stagnation, respectively.

(2) Calculation of total force

In general, MOP don't have a single solution that could optimize all objectives simultaneously. To solve this problem, the calculation for the charge q^i of *i*-th particle is revised by Tsou and Kao (2008). The technology determines q^i of X^i by its minimum distance to the non-dominated front:

$$q^i = \exp\left(-D \frac{Aprox(X^i) - Aprox(X^{best})}{\sum_{k=1}^{N} \left(Aprox(X^k) - Aprox(X^{best}) \right)} \right), \forall i \tag{5}$$

Where $Aprox(X^i)$ is the minimum distance of X^i from belief space; X^{best} is the nearest one to the belief space, as follows:

$$Aprox(X^i) = \min_{Y \in Qset} \left(\left\| f(X^i) - f(Y) \right\| \right) \tag{6}$$

$$X^{best} = \arg\min\left(Aprox(X^i)\right), \quad i = 1, 2, \ldots, N \tag{7}$$

Where $f(X^i)$ is the objective function value of X^i. The dominance relationship of two particles can be judging by the distance of X^i to belief space: (1) if $Aprox(X^j) < Aprox(X^i)$, X^i is attracted by X^j; (2) if $Aprox(X^j) > Aprox(X^i)$, the relationship change. The function for calculate total force F^i is modified to the following equation.

$$F^i = \sum_{j \neq i}^{N} \left\{ \begin{array}{ll} (X^j - X^i) \dfrac{q^i q^j}{\|X^j - X^i\|^2} & if \ Aprox(X^j) < Aprox(X^i) \\ (X^i - X^j) \dfrac{q^i q^j}{\|X^j - X^i\|^2} & if \ Aprox(X^j) > Aprox(X^i) \end{array} \right\}, \forall i \tag{8}$$

2.3.3 Procedures of MOCSEM

The flow chart of MOCSEM is shown as Fig. 1.

2.4 Fuzzy Technique for Order Preference by Similarity to an Ideal Solution (Fuzzy TOPSIS)

For selecting a set of suitable model parameters from Pareto-optimal sets, fuzzy TOPSIS (Baykasoglu et al. 2013) method has been implemented to sort the schemes of solution set. TOPSIS, proposed by (Hwang and Yoon 1981), is an ordering method according to proximity of evaluation objects and idealized goal. There are two types of ideal solutions, the positive ideal solution (or optimal solution) and negative ideal solution (or worst solution). The chosen one of the evaluation objects should be the nearest to positive ideal solution and is far away from negative ideal solution. To adapt the fuzzy set, Hausdorff distance (Rucklidge 1997) is used to measure the proximity of the evaluation objects to the idealized goal. The detailed steps of fuzzy TOPSIS are as follows:

Step1 Synthesize attribute weights $\widetilde{\omega}$ to decision matrix $\widetilde{R}$, and structure weight normalized matrix.

$$\widetilde{D} = \widetilde{\omega} \otimes \widetilde{R} = \begin{bmatrix} \widetilde{d}_{11} & \widetilde{d}_{12} & \cdots & \widetilde{d}_{1n} \\ \widetilde{d}_{21} & \widetilde{d}_{21} & \cdots & \widetilde{d}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{d}_{m1} & \widetilde{d}_{m2} & \cdots & \widetilde{d}_{mn} \end{bmatrix}, \quad \widetilde{d}_{ij} = \widetilde{\omega}_j \widetilde{r}_{ij} \tag{9}$$

Where $\widetilde{r}_{ij}$ is j-th attribute value of i-th solution; $\widetilde{d}_{ij}$ is the normalized value of i-th solution on j-th attribute; $\widetilde{\omega}_j$ is weight value of i-th attribute.

Step2 Determine the fuzzy ideal solution and fuzzy negative ideal solution by using fuzzy maximum set and fuzzy minimum set.

Fuzzy ideal solution $\widetilde{C}^+$ and fuzzy negative ideal solution $\widetilde{C}^-$ are defined as follows:

$$\widetilde{C}^+ = \left(\widetilde{C}_1^+, \widetilde{C}_2^+, \cdots, \widetilde{C}_n^+\right) \tag{10}$$

$$\widetilde{C}^- = \left(\widetilde{C}_1^-, \widetilde{C}_2^-, \cdots, \widetilde{C}_n^-\right) \tag{11}$$

Where $\widetilde{C}_j^+$ is the fuzzy maximum set on the j-th attribute.

$$\widetilde{C}^+ = \max_j \left\{\widetilde{d}_{ij}\right\} \tag{12}$$

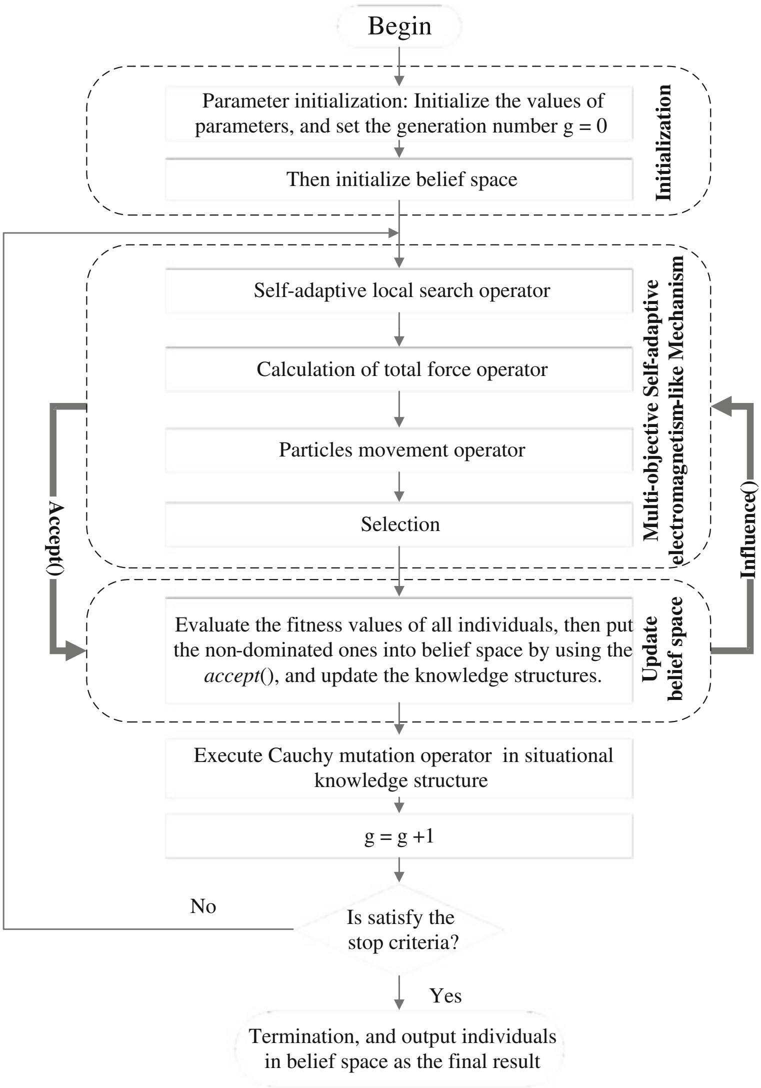

Fig. 1 The flow chart of MOCSEM

Its subordinating degree function is formulated as:

$$\mu^{+}_{\underset{\sim}{C}}(x) = \sup_{x=\max(x_1, x_2 \cdots x_m)} \min\left\{ \mu_{\underset{d_{1j}}{\sim}}(x_1), \mu_{\underset{d_{2j}}{\sim}}(x_2), \cdots, \mu_{\underset{d_{mj}}{\sim}}(x_m) \right\} \tag{13}$$

Where $\widetilde{C}_j^-$ is the fuzzy minim set on the *j*-th attribute.

$$\widetilde{C}^- = \min_j \left\{ \widetilde{d}_{ij} \right\} \tag{14}$$

Its subordinating degree function is formulated as:

$$\mu_{\underset{\sim}{C}}^{+}(x) = \sup_{x=\max(x_1, x_2 \cdots x_m)} \min\left\{ \mu_{\underset{d_{1j}}{\sim}}(x_1), \mu_{\underset{d_{2j}}{\sim}}(x_2), \cdots, \mu_{\underset{d_{mj}}{\sim}}(x_m) \right\} \tag{15}$$

Step3 Calculate fuzzy distance scale between $\widetilde{C}^{+}$ and $\widetilde{C}^{-}$.

A_i is the *i*-th alternative in solution set, its Hausdauff distances to $\widetilde{C}^{+}$ and $\widetilde{C}^{-}$, $\widetilde{d}_i^{+}$ and $\widetilde{d}_i^{-}$ respectively, are calculated as:

$$\widetilde{d}_i^{+} = \sum_{j=1}^{n} d_\lambda\left(\widetilde{d}_{ij}, \widetilde{C}_j^{+}\right) = \sum_{j=1}^{n} \vee_{\lambda\in[0,1]} \lambda \vee \left(\left| \widetilde{d}_{ij}^{L} - \widetilde{C}_j^{+L} \right|, \left| \widetilde{d}_{ij}^{R} - \widetilde{C}_j^{+R} \right| \right) \tag{16}$$

$$\widetilde{d}_i^{-} = \sum_{j=1}^{n} d_\lambda\left(\widetilde{d}_{ij}, \widetilde{C}_j^{-}\right) = \sum_{j=1}^{n} \vee_{\lambda\in[0,1]} \lambda \vee \left(\left| \widetilde{d}_{ij}^{-L} - \widetilde{C}_j^{L} \right|, \left| \widetilde{d}_{ij}^{R} - \widetilde{C}_j^{-R} \right| \right) \tag{17}$$

Step4 The close degree d_i of alternatives in solution set can be obtained by follow:

$$\frac{d_i = \widetilde{d}_i^{-}}{\widetilde{d}_i^{-} + \widetilde{d}_i^{+}} \tag{18}$$

Then the schemes of solution set can be descending sorted by the value of their d_i.

3 Study Area and Hydrologic Model

To implement a multi-objective parameters calibration of CHM, there are many selections of: a multi-objective algorithm to search the parameter space; a decision-making method to choose a scheme from solution set; a CHM; a period of historical data against which to calibrate CHM; multiple objective functions used to represent different characteristics of the runoff hydrograph and multiple evaluation indexes introduced to measure the merits of schemes. The multi-objective algorithm and decision-making optimization method are detailed in the previous section, and the other selections will appear in the following sections.

3.1 Selections of Study Area and the Data Used

Leaf River, a principal tributary of Pascagoula River, is located at 42°7′35″N and 89°24′11″W (42.126350, −89.402976). Pascagoula River is a river, about 80 miles long, in southeastern Mississippi in the United States, which flows into Mississippi Sound of Gulf of Mexico. Leaf River basin, draining an area of about 1,944 km^2, belongs to the typical wet basin. A long series of hydrological data about 11 years (30 September 1952 to 30 September 1962), which are investigated intensively in previous studies (Vrugt et al. 2003a, b; Blasone et al. 2008), are used for model parameter calibration. To eliminate the influence of initial condition, we set a 65-day warm-up period before the model calibration.

3.2 Selections of Conceptual Hydrological Model

The hydrological model (HYMOD), a five-parameter conceptual rainfall-runoff model, is used to illustrate the advantage of MOCSEM algorithm for multi-objective parameter calibration. In a catchment, the five parameters of HYMOD include maximum storage capacity C_{max}, degree of spatial variability of soil moisture capacity b_{exp}, factor distributing the flow between the two series of reservoirs Alpha, and residence times of the linear quick and slow response reservoirs R_q and R_s, respectively. This five parameters is the decision variable of multi-objective parameters calibration. HYMOD consists of a simple rainfall excess model and two series of linear reservoirs: three identical reservoirs for quick flow response and a single reservoir for slow flow response. This model, described in detail by Moore (1985), and recently used by Guo et al. (2013).

3.3 Selections of the Objective Functions

In general terms, to simulate the hydrological behavior of the catchment as closely as possible, different characteristics of the runoff hydrograph should be considered in a multi-objective framework. The following objective functions, mean squared logarithmic error (MSLE) (Hogue et al. 2000; Guo et al. 2013) and mean fourth-power error (M4E) (de Vos and Rientjes 2008), are selected during the optimization process for HYMOD model:

$$\mathrm{MSLE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\log Q_{t,sim} - \log Q_{t,obs}\right)^2} \tag{19}$$

$$\mathrm{M4E} = \frac{1}{T}\sum_{t=1}^{T}\left(Q_{t,sim} - Q_{t,obs}\right)^4 \tag{20}$$

$$Q = \mathrm{HYMOD}\left(\mathrm{C_{max}}, \mathrm{b_{exp}}, \mathrm{Alpha}, \mathrm{R_q}, \mathrm{R_s}\right) \tag{21}$$

Where T is the number of the samples; $Q_{t,sim}$ and $Q_{t,obs}$ are the simulated and observed runoff value of t-th sample, respectively; the range of C_{max}, b_{exp}, Alpha, R_q and R_s are [1.0,500.0], [0.1,2.0], [0.1,0.99], [0, 0.1], [0.1,0.99].

Because of the logarithmic transformation, MSLE function emphasizes fitting of low flows. M4E function pays more emphasizes on high flows events. It means trade-offs exist between MSLE and M4E. Therefore, MSLE and M4E are adopted as the objective functions for the multi-objective parameter optimization of a hydrological model.

3.4 Selections of the Multiple Evaluation Indexes

In this respect, selections of multiple evaluation indexes are intended to measure the solution quality. In order to obtain an optimal scheme, it is significative to formulate the multiple evaluation indexes that reflect different characteristics of Leaf River. The following four evaluation indexes (MSLE, M4E, coefficient of determination (R^2) and qualified rate (QR)) are used here for this study. MSLE and M4E are appeared in the previous section.

R^2, another index of how well parameters calibration using MOCSEM, is computed as:

$$R^2 = 1 - \frac{Dos}{Dom} = 1 - \frac{\sum_t^T \left(Q_{t,obs} - Q_{t,sim}\right)^2}{\sum_t^T \left(Q_{t,obs} - Q_{t,obs}^{ave}\right)^2} \tag{22}$$

Dos is the deviations of observations from forecasted runoff values; *Dom* means the deviations of observations from their mean values.

QR, the fourth index, is formulated as follow:

$$\mathrm{QR} = \frac{M}{T} \times 100\% \tag{23}$$

M said the number of the eligible sample, whose relative error of runoff is less than 20 %.

4 Result and Decision Analysis

4.1 Numerical Simulation

4.1.1 Test Functions and Performance Measures

In this paper, the optimization performance of MOCSEM is measured by adopting four well known test functions (denotes as ZDT2, ZDT3, ZDT4 and ZDT6) of the Zitzler–Deb–Thiele (ZDT) series. It is different to the main properties of their true optimal Pareto fronts, and the true fronts of these functions are known. These advantages for testing the optimization ability of MOCSEM are expatiated by Deb et al. (2002).

Generally, there are two types of measures in multi-objective optimization: convergence measure and diversity measure, for verifying the performance of multi-objective optimization algorithms. This paper applies two widely used measures: Convergence metric CM and Diversity metric DM (Deb et al. 2002), and their expressions are as follows:

$$\mathrm{CM} = \sum\nolimits_{i=1}^{n} D_i \Big/ n \tag{24}$$

$$\mathrm{DM} = \frac{d_f + d_l + \sum_{i=1}^{n-1} |d_i - d_{ave}|}{d_f + d_l + (n-1) d_{ave}} \tag{25}$$

where n is the number of POS, D_i is the minimum Euclidean distance between the i-th optimal solution and its corresponding point in true Pareto optimal front, d_i is the Euclidean distance between the i-th and the $i+1$-th solution, and d_{ave} is the average distance of all d_i, d_f and d_l are the Euclidean distance between upper and lower boundary solutions and its corresponding boundary points in true Pareto optimal front. From the description mentioned above, the desired value for CM is zero, which means obtained POS coincide with true Pareto optimal front perfectly. Meanwhile, the obtained POS will distribute equidistantly if the value of DM is zero.

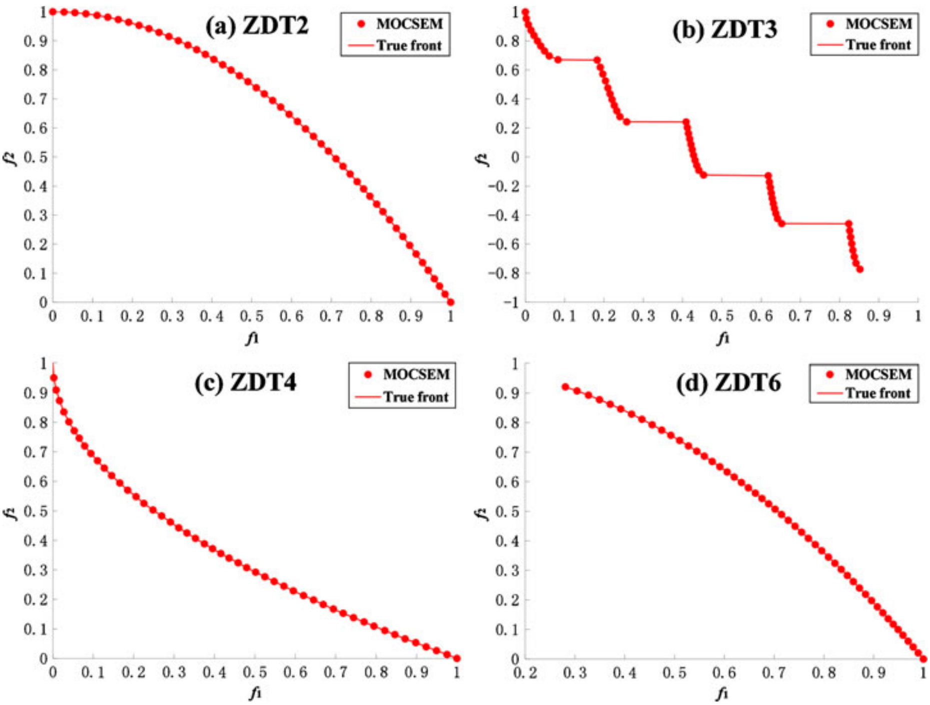

Fig. 2 Pareto frontiers for ZDT2, ZDT3, ZDT4 and ZDT6

Table 1 Statistics of results on CM and DM

Measures	Test functions	NSGA-II	SPEA2	ADEA	MOSCDE	MOCSEM
CM	ZDT 2	0.072391	0.167620	0.002203	0.000280	**0**
		0.031689	0.000815	0.000297	0.000045	0
	ZDT 3	0.114500	0.018409	0.002741	0.000468	**0.000154**
		0.007940	0	0.000120	0.000042	0
	ZDT 4	0.513053	4.9271	0.100100	0.000094	**0.000084**
		0.118460	2.703	0.446200	0.000007	0
	ZDT6	0.296564	0.232551	0.000624	0.000082	**0**
		0.013135	0.004945	0.000060	0.000006	0
DM	ZDT 2	0.430776	0.339450	0.329151	0.097607	**0.034171**
		0.004721	0.001755	0.032408	0.008595	0.000413
	ZDT 3	0.738540	0.469100	0.525770	0.125722	**0.330506**
		0.019706	0.005265	0.043030	0.010446	0.002243
	ZDT 4	0.702612	0.823900	0.436300	0.091203	**0.044314**
		0.064648	0.002883	0.110000	0.012373	0.000107
	ZDT 6	0.668025	1.04422	0.361100	0.073717	**0.054237**
		0.009923	0.158106	0.036100	0.007271	0.000115

4.1.2 Experimental Results and Comparison

To solve the four test functions, main parameters of MOCSEM are set as follows in this paper: the population size $N=100$, the size of belief space $N_Q=50$, the maximum number of generation G is selected as 2000, the maximum iteration number for local search operation L_{max} is set as 10, threshold value of stagnation r is set as 5, the local area parameter $\delta=0.1$ and the self-adaptive parameter α is selected as 0.8955. Moreover, trying to prove the optimizing performance of the proposed MOCSEM, the experimental results of these four test functions obtained by NSGA-II (Deb et al. 2002), SPEA2 (Zitzler et al. 2001), ADEA (Qian and Li 2008) and MOSCDE (Guo et al. 2013).

Figure 2 and Table 1 show the optimal solutions and its technical indexes, the CM and DM, obtained by MOCSEM. Meanwhile, the true Pareto optimal fronts of four ZDT test functions are also displayed in these figures to check the properties of the proposed method. Moreover, the mean and variance values of the technical indexes, which are acquired averaged over 10 runs by MOCSEM and other comparative algorithms, are displayed in Table 1. Among, the mean and variance values are shown in upper and lower rows respectively, and the value which is smaller than 10^{-6} will be denoted as 0.

From Fig. 2, we can see that the results acquired by MOCSEM are excellent for the four test problems. Because the results attain the true Pareto optimal front accurately. While the convergence and distribution performance of results converges well to the true Pareto frontiers and distributes uniformly.

From Table 1, MOCSEM can achieve better performance than other comparative methods for these ZDT test problems except ZDT3. About ZDT3, the result of MOCSEM is better than

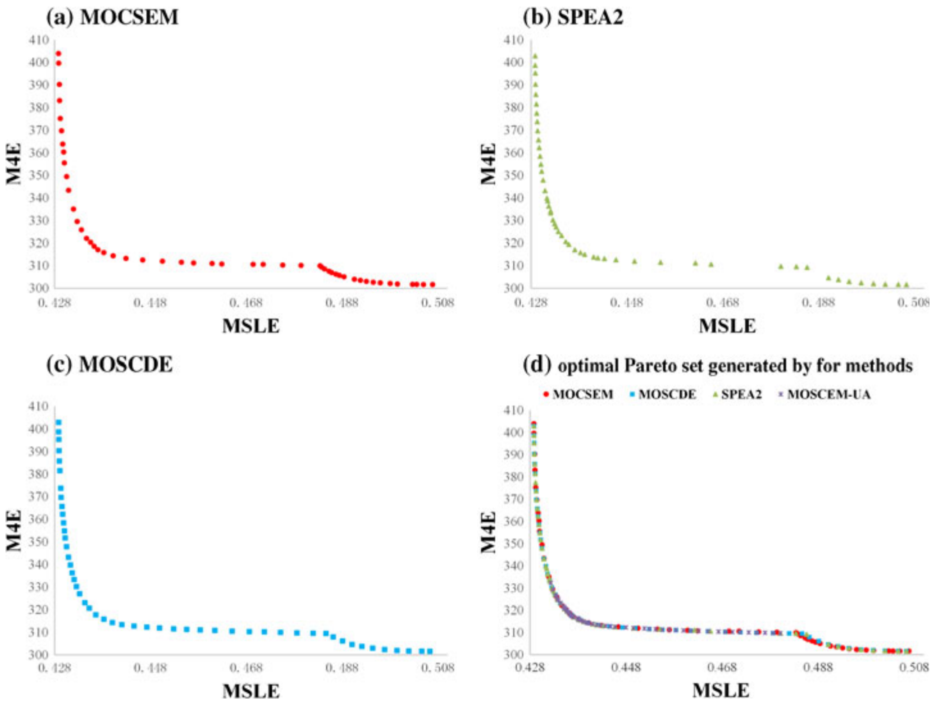

Fig. 3 The set of non-dominated solutions generated by MOCSEM, MOSCDE, SPEA2 and MOSCEM-UA methods

NSGA-II, SPEA2 and ADEA, but the result of MOSCDE is equally good on this problem. Besides, NSGA-II and SPEA2 may have some difficulties in dealing with ZDT2 and ZDT6 while MOCSEM has much better convergence performance for these two test problems.

All in all, MOCSEM can obtain excellent convergence performance and diversity property compared to other comparative methods.

4.2 Application to Multi-Objective Parameter Optimization of HYMOD

4.2.1 Parameter Settings of the Algorithms

MOCSEM is implemented to solve the multi-objective parameters calibration problems of CHM, and parameters settings of MOCSEM are as follows: D=5, N_y=2, N=100, the size of belief space N_Q=30, G is selected as 1000, *LocalNum* is set as 100, r is set as 5, δ=0.29 and α is selected as 0.8955. To verify the effectiveness of MOCSEM, the famous SPEA2, MOSCEM-UA and MOSCDE method is implemented with solving the same case. The parameter settings of the remnant algorithms are done according to Guo et al. (2013).

Table 2 Four evaluation indexes value of calibration solutions obtained by MOCSEM

Scheme	MLSE	M4E	R^2	QR	Scheme	MLSE	M4E	R^2	QR
1	0.4288	403.92	0.8182	42.21 %	26	0.4609	310.90	0.8296	40.01 %
2	0.4289	399.64	0.8187	42.29 %	27	0.4629	310.72	0.8297	40.01 %
3	0.4290	390.21	0.8207	42.53 %	28	0.4695	310.55	0.8292	37.83 %
4	0.4291	383.06	0.8213	42.24 %	29	0.4716	310.52	0.8295	36.99 %
5	0.4292	375.11	0.8229	42.13 %	30	0.4756	310.26	0.8293	37.21 %
6	0.4295	369.68	0.8233	42.21 %	31	0.4794	310.09	0.8299	35.92 %
7	0.4297	363.79	0.8239	42.05 %	32	0.4834	309.90	0.8325	36.83 %
8	0.4299	360.26	0.8243	41.81 %	33	0.4837	309.36	0.8314	36.91 %
9	0.4300	355.45	0.8252	41.94 %	34	0.4843	308.53	0.8316	36.86 %
10	0.4305	349.38	0.8267	41.65 %	35	0.4853	307.51	0.8322	36.80 %
11	**0.4309**	**343.32**	**0.8274**	**41.86 %**	36	0.4859	306.90	0.8322	36.72 %
12	0.4320	335.06	0.8277	41.73 %	37	0.4867	306.21	0.8325	36.29 %
13	0.4327	329.50	0.8291	41.35 %	38	0.4875	305.65	0.8320	36.35 %
14	0.4336	325.83	0.8288	41.03 %	39	0.4884	305.05	0.8330	36.24 %
15	0.4347	322.05	0.8302	41.40 %	40	0.4906	303.93	0.8320	36.19 %
16	0.4355	320.31	0.8298	40.92 %	41	0.4918	303.48	0.8319	36.05 %
17	0.4363	318.55	0.8294	40.54 %	42	0.4931	303.02	0.8323	35.49 %
18	0.4371	316.96	0.8297	40.68 %	43	0.4945	302.64	0.8335	35.49 %
19	0.4383	315.72	0.8296	40.44 %	44	0.4960	302.34	0.8328	35.19 %
20	0.4403	314.32	0.8287	41.08 %	**45**	**0.4980**	**302.10**	**0.8338**	**34.92 %**
21	0.4429	313.15	0.8294	41.11 %	46	0.4996	301.87	0.8334	34.73 %
22	0.4464	312.51	0.8290	40.49 %	47	0.5027	301.71	0.8331	34.09 %
23	0.4505	311.91	0.8295	41.78 %	48	0.5036	301.64	0.8330	34.03 %
24	0.4545	311.50	0.8303	40.33 %	49	0.5050	301.59	0.8332	33.79 %
25	0.4571	311.20	0.8303	40.01 %	50	0.5069	301.58	0.8333	33.47 %

4.2.2 Results and Discussion

In this section, MOCSEM is implemented to solve model parameters calibration problem of HYMOD. Figure 3 displays the optimal solutions obtained by the algorithms: MOCSEM, MOSCDE, SPEA2 and MOSCEM-UA.

From Fig. 3, the POS verifies that it is hardly to get an optimal solution for MSLE and M4E corresponding to the lower and higher runoff respectively. When comparing Fig. 3, it is obvious that MOCSEM can generate more uniform Pareto solutions than SPEA2, MOSCEM-UA and MOSCDE. The ranges of the MSLE and M4E obtained by MOCSEM are [0.4288, 0.5069] and [301.58, 403.92] which is a broader scale than MOSCDE and other methods. In addition, we can see that although a small part of the solutions are missed by MOSCEM-UA and SPEA2. The MSLE value of these solutions is in the range [0.465, 0.480]. This is because the belief space is a collectively information library where the knowledge acquired from the comprehensive top individuals of the population space is stored. Moreover, the self-adaptive local search is used for escaping the local minimums of the problem. In Fig. 3d, we can infer that M4E reduces faster when the value of MSLE is increasing in the range from 0.4288, 0.45. While, the value of MSLE mushroom within the range [0.45, 0.50] as the value of M4E is reducing from 310 to 301.

4.3 Optimal Selection of Calibrated Parameters Schemes

Moreover, to clearly measure the solution quality of runoff forecasting and to further verify the trade-off relationship between objective functions MSLE and M4E in parameter calibration of HYMOD, we use fuzzy TOPSIS method in two types of decision-making scenarios, emphasizing low flows (scenario 1) and emphasizing peak flows (scenario 2). The above mentioned four evaluation indexes (MSLE, M4E, R^2

Table 3 the predilection of experts and weights for the indexes on two decision-making scenarios

Experts number	Expert weight	Scenario 1				Scenario 2			
		MLSE	M4E	R^2	QR	MLSE	M4E	R^2	QR
1	0.174	extreme high	very low	common	Slightly lower	very low	extreme high	common	Slightly lower
2	0.159	very high	extreme low	low	Slightly lower	extreme low	very high	low	Slightly lower
3	0.145	extreme high	very low	low	low	very low	extreme high	low	low
4	0.174	very high	very low	common	Slightly lower	very low	very high	common	Slightly lower
5	0.203	extreme high	extreme low	lower	low	extreme low	extreme high	lower	low
6	0.145	extreme high	extreme low	Slightly lower	lower	extreme low	extreme high	Slightly lower	lower
Specialist weight		0.649	0.067	0.144	0.140	0.085	0.637	0.141	0.137
Objective weight		0.273	0.243	0.243	0.241	0.273	0.243	0.243	0.241
Comprehensive weight		0.461	0.155	0.1935	0.1905	0.179	0.44	0.192	0.189

Table 4 Sequencing optimization result of scheme set

Decision-making scenarios	Sequences
Scenario 1	111312151096714………. 31404642444150494748
Scenario 2	454643504923474844………. 10987654321

and QR) of calibration solutions obtained by MOCSEM are calculated here and displayed in the Table 2 for this study. According to two different decisions, various attributes predilections of the experts are determined to evaluate the importance of each attribute. Table 3 illuminates the predilection of five specialists for the four evaluation indexes and specifies the specialist weights, objective weights and comprehensive weights of this four indexes on the two decision-making scenarios. Then the schemes in Table 2 are graded and sorted using fuzzy TOPSIS method. The sequences are shown in Table 4, and the chosen two schemes are marked by bold in Table 2. The parameters values (C_{max}, b_{exp}, Alpha, R_q, R_s) of this two schemes, scheme 11 and scheme 45, are (215.4852, 0.2559, 0.7819, 0.0024, 0.4586) and (210.7721, 0.2235, 0.9900, 0.0000, 0.4700) respectively.

From Table 4, in scenario 1, the evaluation will be more focused on MSLE, the rank of schemes is generally following the change trend of MSLE. On another hand, the evaluation optimization is emphasizing M4E to consider peak flows in scenario 2. The pros and cons of sorting result for schemes are based on the value of M4E. Except the inclination, the choice of schemes is also influenced by three other properties. This two scenarios are the extreme situations, and the weights of four decision index will be changed to adapt to different forecast premises. Besides, to

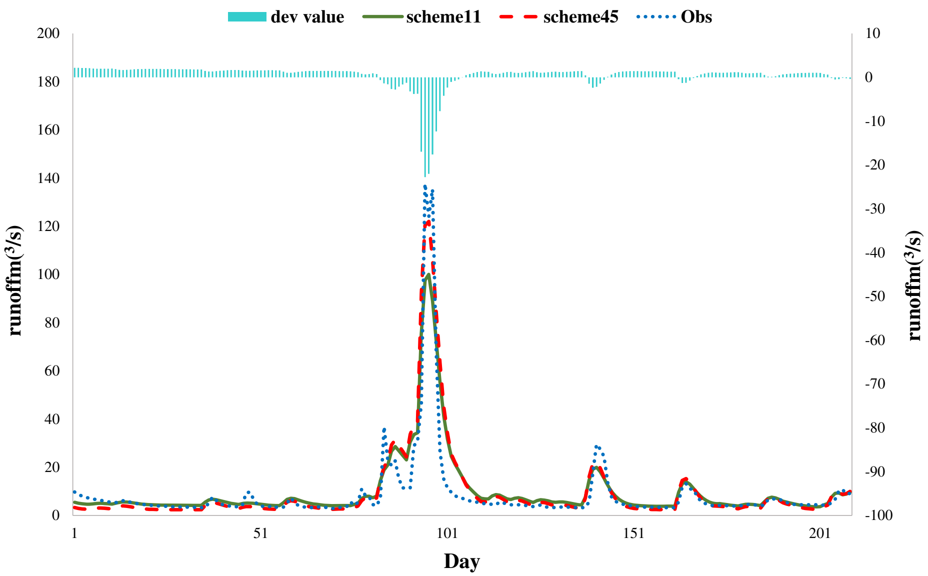

Fig. 4 Calibration results of two extreme Pareto solutions for a 300-day portion of the calibration period (21 May 1960 to 16 March 1961)

specifically display the performance of runoff forecasting of the chosen schemes on objective functions MSLE and M4E in parameter calibration of HYMOD model, the calibration results associated with the two solutions are plotted in Fig. 4 for a 300-day portion of the calibration period (21 May 1960 to 16 March 1961) which contain low flows period and peak flows period. The Dev value is the difference between runoffs of scheme 11 minus runoffs of scheme 45.

The above results indicate that expert's subjective preferences and objective decision-making information can be embodied in the parameters scheme as well. The parameters schemes calibrated by purposed method can provide a feasible set of model parameters to policymakers for different operating situations of CHM.

5 Conclusions

Parameters calibration of hydrological model is one of the most important works in the field of hydrology. To obtain a successful parameters calibration, in this paper, we propose a multi-objective cultural self-adaptive electromagnetism-like Mechanism (MOCSEM) algorithm, which is first implemented in solving the parameters calibration problem of the conceptual hydrological model (CHM). In this algorithm, a self-adaptive parameter is applied in the local search operation for adjusting the values of parameters dynamically. Meanwhile, considering the complicated constraints and objectives of CHM problem, cultural algorithm (CA) is adopted to keep a good diversity and uniformity of POS. MOCSEM is tested, firstly, by several benchmark test problems and the results show that MOCSEM algorithm outperforms other algorithms proposed in previous research. After that, a case study is implemented for parameters calibration of a CHM by comparing the convergence properties and diversification of POS obtained by MOCSEM, SPEA2, MOSCEM-UA and MOSCDE method. Finally, when the optimization problem quickly becomes a decision-making problem because of the multiple objectives in CHM, fuzzy technique for order preference by similarity to an ideal solution (fuzzy TOPSIS) method has been used to rank the POS and select the optimal scheme on decision-making scenarios. It is found that MOCSEM algorithm can provide high-accuracy parameters of CHM on various decision-making scenarios.

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