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Optimized Sustainable Groundwater Extraction Management: General Approach and Application to the City of Lucknow, India

Ashutosh Singh · Claudius M. Bürger · Olaf A. Cirpka

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Abstract In rapidly developing urban areas of emerging countries, increased water demand has led to enormous groundwater withdrawal, calling out for sustainable groundwater management. We suggest implementing a sustainable pumping rate concept based on numerical modeling of the managed aquifer. Sustainability is achieved by constraints regarding (1) a minimum groundwater discharge rate to gaining rivers (ecological constraint) and (2) a maximum drawdown along the city boundaries (social constraints) to prevent excessive groundwater depletion in the neighboring peri-urban and rural areas. The total groundwater extraction is maximized subject to these constraints, leading to specific extraction patterns throughout the city, depending upon the values set for the constraints. The optimization is performed by linear programming. For a given extraction rate, the two constraints can be traded off by the groundwater manager, causing different wells to be activated or deactivated. We demonstrate the applicability of the methodology by the example of the city of Lucknow, India, but it can be transferred to other cities facing conflicts of managing groundwater resources.

Keywords Urban hydrogeology · Groundwater management · MODFLOW · Numerical modeling · Response matrix · Optimization

1 Introduction

Major urban settlements worldwide face depletion of water resources due to increasing water demands of fast growing population and rising industrial water needs (e.g., Munoz et al. 2003; Lorenzen et al. 2010; Liu et al. 2008; Romano and Preziosi 2010). The city of Lucknow, India is a representative example of such a case, imposing

A. Singh · C. M. Bürger · O. A. Cirpka (✉)
Center for Applied Geoscience (ZAG), University of Tübingen, Hölderlinstr. 12, 72074 Tübingen, Germany
e-mail: olaf.cirpka@uni-tuebingen.de

obligations of integrated water resources management (Livingston 2009; Foster and Choudhary 2009; CGWB 2009). The inefficient water distribution network and growing urban population further accentuates the water demand in the city. This has resulted in water pilferation and the installation of a number of licensed and unlicensed groundwater extraction wells, resulting in reduced groundwater discharge to River Gomti and drawdown in the areas outside the city limits, thus raising environmental and social concerns, which were termed important by Pearson et al. (2010) for planning sustainable groundwater extraction in an urban setting. While sustainable groundwater extraction within the city is necessary to meet the demands of the urban population and industry, the extraction should be managed in such a way that the resulting drawdown at the city limits remain acceptable for the groundwater users outside of the city (social constraint) and the groundwater discharge into River Gomti is held at a level guaranteeing an acceptable ecological status of the river (environmental constraint). Land subsidence due to excessive groundwater extraction has posed an additional major problem in areas of some overexploited aquifers (e.g. Bayer et al. 2009; Xu et al. 2013), but this is not the case in the city of Lucknow (Foster and Choudhary 2009) and thus not considered in the present study.

Managing sustainability of groundwater resources must be based on the requirements of the specific situation. For instance, a study based on groundwater sustainability to calculate groundwater extraction rates in North China (Liu et al. 2008) considered two scenarios: The first one aimed at maximizing total groundwater extraction under the constraints of minimum and maximum groundwater extraction at individual wells and of hydraulic heads at the groundwater wells, respectively, whereas the second scenario focused on the cost optimization of extraction under the same constraints. In order to achieve these aims, a transient groundwater model was developed and coupled to an evolutionary algorithm for optimization. The author concluded that the constraint should be based on heads considering river discharge (referred as ecological water requirement) in the model domain.

Baú (2012) focused on groundwater extraction design using stochastic modeling and evolutionary algorithms. This author aimed at minimizing the total costs of the extraction system while meeting several technical, economical, and hydrological constraints. The extraction schemes were based upon trade-offs between the costs and the violation of the constraints.

While the cited studies used constraints to achieve sustainability and planning extraction systems, they remained focused on cost optimization and the issue of sustainability itself stayed unaddressed. Sustainable management of groundwater resources is commonly attempted by the concept of sustainable pumping rates (SPR) (Bredehoeft 1997; Stanghellini and Collentine 2008; Devlin and Sophocleous 2005; Ouessar et al. 2009; Armstrong and Rose 1999; Romano and Preziosi 2010), though other terms such as safe yield and sustainable yield have also been used (e.g., Bredehoeft 1997; Maimone 2004), which are often called to be abandoned due to ambiguity (van der Gun and Lipponen 2010; Kalf and Woolley 2005; Alley and Leake 2004).

Most of the studies cited above aimed at achieving sustainable pumping rates by considering the water balance of an aquifer, thus defining sustainability as the ability of the aquifer to balance the extraction by induced groundwater recharge and reduced groundwater discharge into surface water bodies. In transient flows, a water-balanced related definition of sustainability would imply that these fluxes balance in the average over the time scales of hydrological fluctuations, typically a year

(Devlin and Sophocleous 2005; Romano and Preziosi 2010). However, the pumping rates determined from meeting the water balance alone may lead to unacceptable conditions at the time period of the year when hydraulic heads are the lowest, even though recovery is possible. In this study, we thus use the term "sustainable" in an extended sense: Sustainable pumping rates must not lead to unacceptable ecological or social side effects. Within this framework, meeting the long-term water balance without continuous abstraction of water from the storage is only a prerequisite. Therefore, the key of the following analysis lies in setting ecological and social constraints to achieve sustainable (and acceptable) pumping rates.

In the following, we will consider a vector of sustainable pumping rates $\mathbf{q}_{\mathbf{spr}}$, in which each vector element refers to an individual well. The total pumping rate P equals $\mathbf{1} \cdot \mathbf{q}_{\mathbf{spr}}$, with $\mathbf{1}$ being a vector of ones. We also define a critical loss of groundwater discharge q_{crit} to the river, whereas the induced lateral recharge is not considered explicitly. Instead, we define a critical drawdown value d_{crit} along the city limit.

We use a transient groundwater model to evaluate the maximally possible pumping rate $P(\mathbf{q}_{\mathbf{spr}}) = \mathbf{1} \cdot \mathbf{q}_{\mathbf{spr}}$ meeting the constraints regarding the change of groundwater discharge q_{crit} and the drawdown at the city limits d_{crit} at a target time point. It should be clear that meeting these constraints is most difficult at the end of the dry season when the groundwater levels are the lowest. Thus, sustainable pumping rates, that are also environmentally and socially acceptable, cannot be identified from an average water balance. Considering the transient nature of groundwater flow most likely leads to smaller pumping rates that guarantee sustainability at all times.

This approach is similar to that of Liu et al. (2008) in the sense that the sum of groundwater extraction rates is maximized for a fixed duration of extraction. Our approach mainly helps a water manager to estimate $P(\mathbf{q}_{\mathbf{spr}})$ based on the concept of sustainable pumping rates for varying constraint values. The analysis of different $\mathbf{q}_{\mathbf{spr}}$ provides general suggestions about the wells to be preferred or avoided for groundwater extraction requiring lower values of either drawdown at the city limit or reduction in groundwater discharge to the river.

The optimization problem is stated here as the maximization of total extraction for given constraints. In practice, however, the problem often occurs reversely, i.e., a certain extraction rate must be achieved, and the water manager needs to decide which sustainability constraint must be sacrificed to which extent in order to meet the demand. We have set up the optimization problem in such a way that the two constraints are free parameters and the manager can decide how to trade off between environmental and social constraints regarding groundwater drawdown for an intended water demand.

2 Groundwater Management Problem of Lucknow

2.1 Problem Statement and Current Management Scheme

The northern alluvial plains of India have shown declining groundwater levels over the last few decades due to increasing groundwater over-exploitation (Livingston 2009; Foster and Choudhary 2009; CGWB 2009). Lucknow, the capital city of the state of Uttar Pradesh in the north of India may be assessed as a representative case.

Situated in the Sai-Gomti basin, the Lucknow district faces a threat to its natural groundwater resources: Half of the city's total water demand of $4.9 \times 10^5 m^3/day$ is met by directly treating the water of River Gomti, while the other half is supplied by government tube wells from the aquifers underlying the city (CGWB 2009). Leaking water-supply pipes and urban population growth have led to installation of up to 10,000 wells in the unconfined aquifer, resulting in annual decrease of 0.7 m since 2001 (Livingston 2009; Foster and Choudhary 2009).

The central groundwater board (CGWB) in India manages the groundwater resources of the city by estimating the groundwater balance in the first step followed by a groundwater management strategy. The water-table-fluctuation method has been applied in water management studies by Chatterjee et al. (2009) and CGWB (2009), as described in Healy and Cook (2002) and suffers procedural drawbacks, related to (1) the consideration of a block as a hydrogeological unit, (2) neglecting spatial variation of groundwater levels within a block, (3) uncertain specific yield within a block, and (4) unaccounted groundwater inflow/outflow across the lateral boundaries of the block.

A sustainable groundwater extraction plan should consider the groundwater rights of the neighboring communities and decreasing groundwater drainage to the river as constraints in optimization. Hence, using the sustainable-pumping-rate concept in conjunction with groundwater modeling is a suitable approach to overcome the aforementioned drawbacks of the existing groundwater management scheme.

2.2 Study Area

Lucknow is situated in the Sai-Gomti basin, which is a part of the Central Ganga basin of India as shown in the upper left inset of Fig. 1, and stretches across both banks of River Gomti. Except during monsoon, the river is fed by groundwater. The climate in the region is subtropical with three distinct seasons: summer, monsoon, and winter. The average rainfall is 1,110 mm/yr, out of which 1000 mm/yr occur during the 45 monsoon days between June and August (Livingston 2009; Foster and Choudhary 2009).

2.3 Data Inventory

The hydrogeological setting of the city and the Sai-Gomti sub-basin is described by Bhatnagar (1966) and CGWB (2009). In addition, a soil map from the state soil testing department, published and unpublished borehole data from the Geological Survey of India (Bhatnagar 1966) and various private drilling companies of the city, respectively, were acquired to formulate a conceptual hydrogeological model.

Bhatnagar (1966), CGWB (2009), and acquired borehole data provide hydrogeological details for the five topmost hydrogeological layers. They show a top unit of sand and silt with intercalations of clay. A map of hydraulic conductivity constructed from values found in the literature (Srivastava et al. 2003; Foster and Choudhary 2009; Bhatnagar 1966) and the above mentioned geological data helped identifying eight hydraulic conductivity zones in the basin as shown in the right map of Fig. 1. The geological cross-sections of Bhatnagar (1966) provide additional information regarding the plausible ranges of riverbed conductances of River Gomti in various parts of the city.

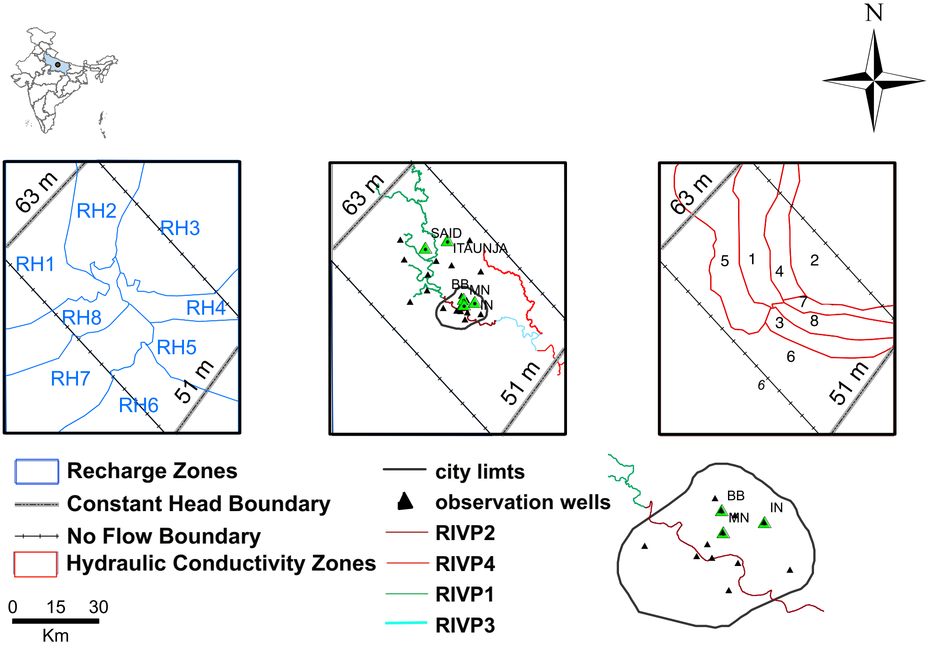

Fig. 1 Conceptual model of the Lucknow groundwater model. *Left map* definition of recharge zones (RH1–RH8); *center map* definition of river-conductance zones (indicated by colors of the river reaches), city limits (*bold, black, closed line*) and observation wells (*triangles*); *right map* definition of hydraulic conductivity zones. All three maps show the constant head boundaries (at 51 and 63 m a.s.l., respectively) and the no-flow boundaries of the domain

Since most of licensed/unlicensed groundwater extraction occurs within the topmost unconfined aquifer, only a single aquifer layer is considered in the study. The general direction of the groundwater flow is from the north-west to the south-east part of the basin (Bhatnagar 1966; CGWB 2009; Foster and Choudhary 2009). The current groundwater extraction ($\mathbf{q}_{\mathbf{ext}}$) in all parts of the city is estimated on the basis of population and groundwater extraction data provided by Foster and Choudhary (2009), Livingston (2009) and Bhatnagar (1966). Based on this information, the spatial extraction pattern was implemented through 29 wells in areas of high extraction (see Fig. 2) considering a total extraction of $1.9 \times 10^5 \text{m}^3/\text{day}$. These 29 wells represent the public and the ≈10,000 private wells.

The natural and anthropogenic factors affecting the groundwater levels in the city are recharge through precipitation, water exchange with the river, and the urban extraction in the various parts of the city, respectively. Since short-duration peak flow occurs rarely in the river, CGWB (2009) and Bhatnagar (1966) characterize River Gomti as a gaining river. This interpretation is also followed in the current study.

966 groundwater observations, $d(\mathbf{x}, t)$ (depth to the water table from the surface) from 23 locations were acquired from the Central Groundwater Department, Lucknow, where $\mathbf{x}$ refers to the location and t to the time of the observation. The groundwater observations were recorded monthly for each location, spanning a period from September 2003 to February 2007. The groundwater elevation $y(\mathbf{x}, t)$,

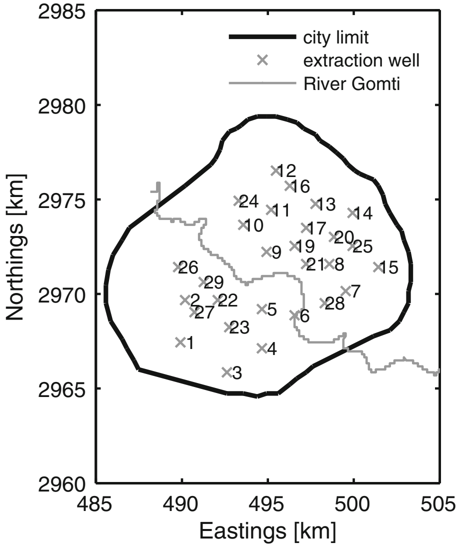

Fig. 2 Location of the groundwater extraction locations, the city limit (*bold black line*) and River Gomti (*gray line*). Easting and northing according to the UTM coordinate system, zone 44N

was calculated using $y(\mathbf{x}, t) = h_{\text{elev}}(\mathbf{x}) - d(\mathbf{x}, t)$, where $h_{\text{elev}}(\mathbf{x})$ is the elevation of the location $\mathbf{x}$ acquired from the Shuttle Radar Topographic Mission (SRTM) data, which was provided by the German aerospace center, DLR (Taubenboeck et al. 2009). Figure 2 provides the locations of the groundwater extraction wells, the city limit, where the social constraint acts, and River Gomti, where the ecological constraint is defined.

The meteorological data used for estimating groundwater recharge is measured at the meteorological station within the city and was acquired from the World Data Center for Meteorology. The mean potential evapotranspiration is estimated to be 500 mm/yr based on the Penman equation (Monteith 1965). Figure 3 shows the difference between measured daily precipitation and calculated potential evapotranspiration as function of time. Due to the limited spatial resolution of the meteorological data, a recharge potential map (obtained from CGWB (2009)) based on pre- and post-monsoon groundwater levels was used.

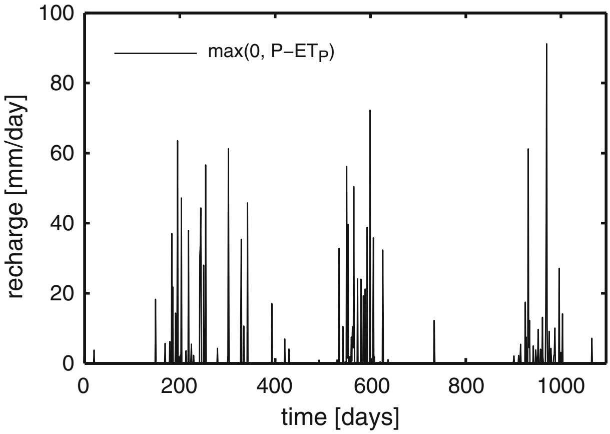

Fig. 3 Time series of the spatially averaged groundwater recharge applied within the model

3 Numerical Groundwater Model

3.1 Model Set-Up

The conceptual model of the basin shown in Fig. 1 reflects the known hydrogeology of the city, the key meteorological forcing, the gaining stream and the continued groundwater extraction within the city. The boundary of the city outlines mainly the groundwater extraction region of the city. The representation of streams in the conceptual model complies with the position of streams in topographic maps. The boundary conditions at the model boundary could not be well defined based on the literature and/or available data, so that an ambient groundwater flow in the direction from the north-east to the south-west was insured by using constant-head boundary conditions of 51 m and 63 m, respectively, at the north-west and south-east boundaries and no-flux boundary conditions at the remaining boundaries. All domain boundaries were placed far from the city (distance > 13 km), ensuring regional groundwater flow in the right direction but hardly affecting hydraulic heads and groundwater discharge within the city and in its direct vicinity.

Transient groundwater flow is simulated with a two-dimensional unconfined groundwater flow model subject to boundary conditions as shown in Fig. 1 using MODFLOW (Harbaugh 2005):

$$S_y \frac{\partial h}{\partial t} - \nabla \cdot (K(h - z_{\text{bot}})\nabla h) = q_{\text{in/out}} \tag{1}$$

$$q_{\text{in/out}} = RCH \cdot \max(0, P - E_{PT}) - RIVP \cdot (h - h_{\text{drain}}(\mathbf{x}_{\text{dr}})) - \delta(\mathbf{x} - \mathbf{x}_w) q_{\text{ext}} \tag{2}$$

in which S_y is the specific yield [-], K is the hydraulic conductivity [LT^{-1}], assumed isotropic, h is the hydraulic head [L], z_{bot} is the geodetic height of the aquifer bottom [L], and $q_{\text{in/out}}$ is the source/sink term in the model [LT^{-1}]. $RCH[-]$ is a recharge flux factor, P is the amount of precipitation [LT^{-1}], E_{PT} is the calculated potential evapotranspiration [LT^{-1}], $RIVP$ is the drainage bed conductance [T^{-1}], and $h_{\text{drain}}(\mathbf{x}_{\text{dr}})$ is the elevation of the riverbed cell [L] at location $\mathbf{x}_{\text{dr}}$. q_{ext} [L^3T^{-1}] represents groundwater extraction and $\mathbf{x}_w$ represents its location. The Dirac delta function $\delta(\mathbf{x} - \mathbf{x}_w)$ is approximated within the model as distribution over an entire grid cell.

Equation 2 represents the river by a drainage boundary condition. Defining a more appropriate leakage boundary condition, allowing for either river infiltration or groundwater exfiltration depending on the river stage and groundwater head, would have required classified river-stage data of River Gomti, which were not available. Thus, the drainage boundary condition chosen here is a substitute that could be implemented with purely bathymetric data, which were accessible.

The initial conditions for the transient groundwater model on September 1, 2003 were generated using a steady-state model. The steady-state model had the same model setup as shown in Fig. 1 with recharge of 5×10^{-4} m/day. By this, the initial groundwater heads were consistent with the numerical groundwater model in terms of model structure and boundary conditions (Anderson and Woessner 1992). The steady-state groundwater model matched the annual averaged groundwater head for the year 2003 and the average baseflow of the river of 15 m^3/s.

The groundwater model was setup with the help of ModelMuse (Winston 2009) which is a graphical user interface for MODFLOW. The model domain is variably discretized in 354 rows and 362 columns encompassing a total area of $\approx$ 7755 km^2, with the size of each cell varying between $\approx$ 100 m and $\approx$ 300 m. The parameters S_y, K, RCH and $RIVP$ are spatially distributed (shown in Fig. 1) and constitute the parameter vector $\mathbf{p}$.

3.2 Sensitivity Analysis

In order to match the observed $y(\mathbf{x}, t)$ and simulated groundwater heads $h(\mathbf{x}, t, \mathbf{p}, \mathbf{q_{ext}})$ in the period of September 2003 to August 2006, the parameter vector $\mathbf{p}$ constituting of values of S_y, K, RCH and $RIVP$, is estimated. We computed the composite scaled sensitivities (CSS) of each parameter j in order to identify those parameters that are important to fitting the observations (Hill and Tiedeman 2007; Foglia et al. 2009):

$$Css_j = \sqrt{\frac{1}{n_{\text{obs}}} \sum_{i=1}^{n_{\text{obs}}} \left(\frac{\partial h_i}{\partial p_j}\bigg|_{\mathbf{p}} \frac{p_j}{\sigma_{h_i}} \right)^2} \tag{3}$$

in which $\partial h_i / \partial p_j$ is the sensitivity of the i-th simulated groundwater head with respect to the j-th parameter, $n_{\text{obs}} = 828$ is the total number of observations, σ_{h_i} is the measurement error of the i-th observation, and the total number of parameters is 28.

The correlation coefficient r_{jk}, between two parameters, j and k is calculated to identify whether variations in two parameters result in identical model outcome:

$$r_{jk} = \frac{C_{p_j, p_k}}{\sqrt{C_{p_j, p_j} C_{p_k, p_k}}} \tag{4}$$

in which the covariance matrix $\mathbf{C_{pp}}$ of the parameters is calculated by:

$$\mathbf{C_{pp}} = \left(\mathbf{J}^T \mathbf{C_{hh}^{-1}} \mathbf{J}\right)^{-1} \tag{5}$$

with the sensitivity matrix $J_{i,j} = \partial h_i / \partial p_j$ and the covariance matrix of measurement errors $\mathbf{C_{hh}}$, which is a diagonal matrix with elements $\sigma_{h_i}^2$.

The sensitivity analysis was performed using UCODE (Hill and Tiedeman 2007). The results of the correlation analysis are shown in Table 1. The parameters RH1, RIVP3, RH5 and RH6 were removed during model calibration due to high

Table 1 Parameter correlations greater than 0.85 of the parameters shown in the Fig. 1

Parameter names	Correlation (r_{jk})
RH1,RH4	−0.96
HK-Par6,SY-Par6	0.92
RH1,RH3	−0.94
RH2,RH8	0.90
RIVP3,RH5	0.86
RH1,RH2	−0.86
RH1,RH7	−0.89
RH2,RH3	0.86

correlation and low values of the composite scaled sensitivity (less than 5×10^{-3} of the highest composite scaled sensitivity) (Fig. 4).

3.3 Calibration and Validation

The transient model was calibrated by minimizing the negative log-likelihood function:

$$\chi^2 = \boldsymbol{\varepsilon}^T \mathbf{C}_{\mathbf{hh}}^{-1} \boldsymbol{\varepsilon} \tag{6}$$

in which $\boldsymbol{\varepsilon}$ is the vector of model errors consisting of n_{obs} entries $\boldsymbol{\varepsilon}_i = y_i - h_i$.

The four statistical criteria employed to quantitatively judge the goodness of fit between monthly observed and simulated groundwater heads were: the maximum error (ME), the root mean square error ($RMSE$), the coefficient of residual mass (CR), and the Nash-Sutcliffe model efficiency coefficient (NS) (Nash and Sutcliffe 1970):

$$ME = \max|\mathbf{y} - \mathbf{h}| \tag{7}$$

$$CR = \frac{\sum (y_i - h_i)^2}{\sum y_i} \tag{8}$$

$$RMSE = \sqrt{\frac{1}{n_{\text{obs}}} \sum (y_i - h_i)^2} \tag{9}$$

$$NS = 1 - \frac{\sum (y_i - h_i)^2}{\sum \left(y_i - \frac{1}{n_{\text{obs}}} \sum y_i \right)^2} \tag{10}$$

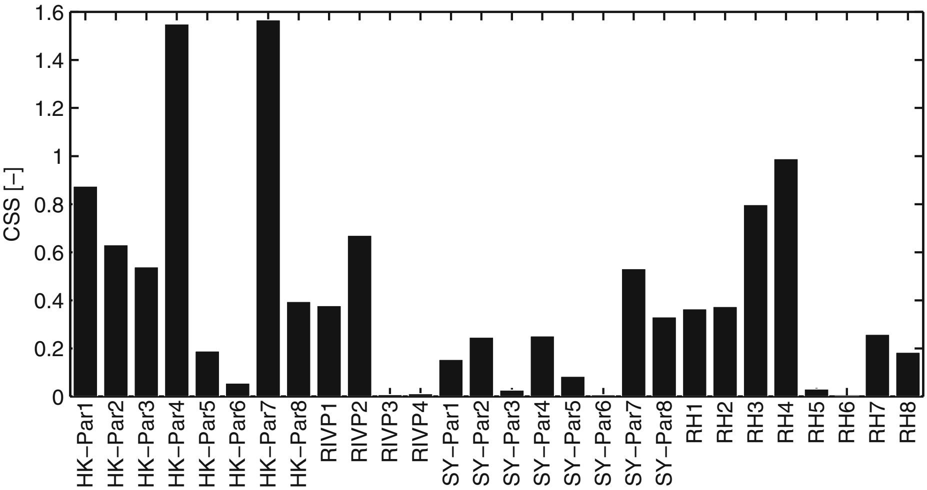

Fig. 4 Composite scaled sensitivities of the model parameters. HK-Par$x$, SY-Par$x$, are the hydraulic conductivities and specific yields respectively, of the x-th zones are shown in Fig. 1. All other parameters are also explained in Fig. 1

These metrics have been used previously in quantifying residuals and systematic characterization of underestimation and overestimation (e.g., Loague and Kyriakidis 1997; Jones et al. 2008; Perez et al. 2011). $RMSE$ represents an aggregated error of model precision; negative and positive values of CR are aggregated measures of the over- or underestimation of the simulated values, respectively; the Nash-Sutcliffe index compares the explained variation in the observations in comparison to a model made of the mean observation. The ideal values for ME, $RMSE$, CR and NS are 0, 0, 0 and 1, respectively, implying that the observed and simulated groundwater heads are identical.

The groundwater model was calibrated using monthly observed groundwater head measurements at 23 locations, between a period of September 2003 to August 2006. Automated calibration of the model was performed to minimize the value of χ^2 using the automated calibration code UCODE (Hill and Tiedeman 2007), using the same measurement error of 0.5 m for all observations in the covariance matrix $\mathbf{C_{hh}}$. The estimated parameter values were the hydraulic-conductivity values, ranging between 6 m/day and 170 m/day, the specific yield, ranging between 0.05 to 0.3, and the riverbed-conductance values, ranging between 285 and 864 day^{-1}, respectively. The recharge flux factor was estimated to range between 0.06 and 0.3. All hydrogeological parameters are reasonable for an alluvial aquifer with laterally varying clay content.

The observed and simulated groundwater heads for selected piezometers over the calibration period (first 36 months) are shown in Fig. 5. The simulated and observed groundwater heads match considerably well. It can be seen that occasionally the simulated heads are lower than the observed ones. This occurred during the small monsoon period when River Gomti infiltrates. Due to the unavailability of river-flow data, a river boundary condition accounting for river stages could not be used in the

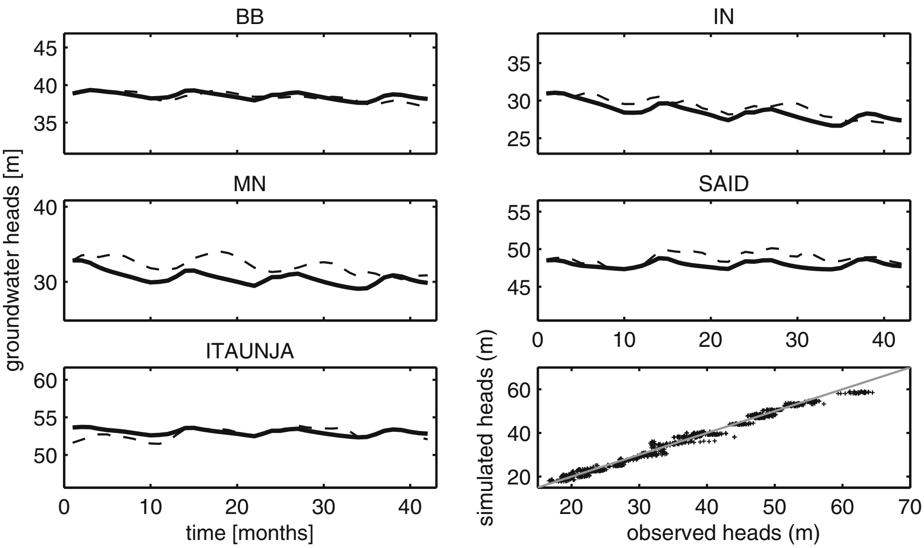

Fig. 5 Calibration and validation results at five selected locations. *Dashed lines* are observations while *continuous lines* represent simulated values. The first 36 months are for calibration and the remaining 6 months are for validation

Table 2 Model performance metrics for the calibration period September 2003 to August 2006 and the validation period (September 2006 to February 2007)

Run	ME [m]	RMSE [m]	CR [-]	NS [-]
Calibration	5.9	1.5	0.01	0.94
Validation	4.2	1.6	−0.01	0.93

ME: maximum error, RMSE: root mean square error; CR: coefficient of residual mass; and NS: the Nash-Sutcliffe coefficient

model. The groundwater model can well represent the seasonality of hydraulic heads and the generally declining groundwater levels in the city. In order to validate the groundwater model, the simulations were extended using the meteorological forcing of the next six months, keeping the model structure and parameter values obtained by calibration unchanged. The observed and simulated groundwater heads for these six months are also plotted in Fig. 5 and show good agreement.

The model performance is judged on the basis of calculated statistical metrics listed in Table 2. The maximum error ME of 5.9 m occurred at an observation well close to the model boundary, which can be ignored, considering that the area of interest lies inside the city limits. The ME at other locations inside the city remained below 1.6 m during calibration and validation. The value of $RMSE$ was observed to be 1.5 m and 1.63 m for calibration and validation, respectively. However, inside the city the $RMSE$ was estimated to be 1.2 m. The values of CR during calibration and validation were estimated to be 0.012 and −0.012, which is a measure of slight underestimation and overestimation of the groundwater heads. The underestimation of heads occurred due to unrepresented infiltrating river during the monsoon season. The slight overestimation of heads during validation might have occurred due to increased extraction in the city of Lucknow which was also unrepresented in the groundwater model. A further detailed analysis of overestimation can be performed but is unimportant for the scope of this study and its low value of −0.012. The values of NS for calibration and validation are 0.94 and 0.93, respectively, demonstrating the suitability of the model to reproduce groundwater head fluctuations. The slight mismatch between the simulated and observed groundwater heads might have been overcome if detailed river discharge data and hydrostratigraphic information had been available.

4 Calculation of Sustainable Pumping Rates

The groundwater model developed in the previous sections can be used to simulate the stress induced on the groundwater heads and the groundwater discharge to the river inside the city limits. This section focuses on estimating the total pumping rate P based in the sustainable pumping rate concept, which aims towards maximizing the total groundwater extraction subject to a set of constraints.

The model was simulated between a period of September 2006 and August 2012 using the same model structure, parameter vector $\mathbf{p}$ as obtained by calibration, groundwater extraction $\mathbf{q}_{\text{ext}}$ and repeating the meteorological forcing of the time period between September 2003 and August 2006. The simulated groundwater heads at the end of August serve as initial conditions for the groundwater model simulated from September 1, 2012 to the target date, May 1, 2024.

Keeping other factors unchanged, except the groundwater extraction rates, $\mathbf{q}$, the time period from September 2012 to May 2024 (≈ 12 years) is long enough to achieve dynamic steady-state conditions of groundwater heads and groundwater discharge to the river in the last year of simulation.

4.1 Formulation of the Optimization Problem

Similar to Shen et al. (2004), Liu et al. (2008), and Ahlfeld and Baro-Montes (2008), we evaluate sustainable pumping rates as a maximization problem $P = \mathbf{1} \cdot \mathbf{q_{spr}} = f(d_{\text{crit}}, q_{\text{crit}})$, in which P is the total pumping rate, d_{crit} is the maximum drawdown at the city limit (social constraint) and q_{crit} is the maximum reduction of groundwater discharge to the River Gomti within the city as shown in Fig. 2 (environmental constraint) at the target date, May 1, 2024. The bounds 0 m $< d_{\text{crit}} \leq 3$ m and 0 m^3/s$< q_{\text{crit}} \leq 1.35$ m^{3}s values were used in order to estimate P with a step size of 0.01:

$$\max_{\mathbf{q_{spr}}=\mathbf{q}} \quad P(\mathbf{q}) = \sum_{j=1}^{29} q_j \tag{11}$$

subject to the following constraints:

$$h(\mathbf{x}_1, t_1, \mathbf{p}, \mathbf{q}) \geq h(\mathbf{x}_1, t_1, \mathbf{p}, \mathbf{0}) - d_{\text{crit}} \tag{12}$$

$$\sum Q_{\text{sim}}(\mathbf{x}_{dr1}, t_1, \mathbf{p}, \mathbf{q}) \geq \sum Q_{\text{sim}}(\mathbf{x}_{dr1}, t_1, \mathbf{p}, \mathbf{0}) - q_{\text{crit}} \tag{13}$$

$$\mathbf{0} \leq \mathbf{q} \leq \mathbf{q}_{\text{ext}} \tag{14}$$

in which $\mathbf{x}_1$ and $\mathbf{x}_{dr1}$ denote the vector of locations at the city limit and along the river, respectively (as shown in Fig. 2), Q_{sim} is the simulated groundwater discharge into River Gomti within the city limits, and t_1 is the target date. The first two constraints are the social and environmental constraints, respectively, while the third constraint sets an upper limit $\mathbf{q_{ext}}$ to the individual groundwater extraction rates $\mathbf{q}$, since values above $\mathbf{q_{ext}}$ resulted in dry cells in the groundwater model.

Similar to the methodology of Ahlfeld et al. (2005) and Ahlfeld and Baro-Montes (2008), the transient groundwater model can be replaced by a matrix-vector product after verifying that the constraints depend linearly on the extraction rates. The social constraints are linear with respect to groundwater extraction if:

$$||h(\mathbf{x_1}, t_1, \mathbf{p}, \mathbf{q}) - h(\mathbf{x}_1, t_1, \mathbf{p}, \mathbf{0}) - \mathbf{Jq}|| < \varepsilon \tag{15}$$

with the Jacobian matrix:

$$J_{ij} = \frac{h(x_{1,i}, t_1, \mathbf{p}, \mathbf{q} + \delta_j \cdot \mathbf{e}_j \cdot q_j) - h(x_{1,i}, t_1, \mathbf{p}, \mathbf{q})}{\delta_j \cdot q_j} \tag{16}$$

in which δ_j represents the fraction by which the groundwater extraction rate of well j is perturbed, and $\mathbf{e}_j$ is a vector of zeros except for the j-th element, which is unity.

A similar approach was used for the environmental constraint. The tolerance criteria ε achieved by comparing the full model runs and $\mathbf{Jq}$ had maximum values of 0.3 m and 0.02 m^3/s for the social and environmental constraint, respectively, which is two orders of magnitude smaller than the minimum groundwater depth and minimum groundwater drainage to the river in the model (20 m and 1.7 m^3/s).

4.2 Solution of the Sustainable Pumping Rate Problem

$P(\mathbf{q}_{\mathbf{spr}})$ is obtained by using the `linprog` function implemented in MATLAB (Zhang 1998), in order to solve Eq. 11 subject to the constraints, Eqs. 12–14. The constraints used in Eq. 11 were slightly modified to estimate the constraint values by combining the social and environmental constraint in a single equation as following:

$$-\begin{pmatrix} \mathbf{J}_{\mathbf{social}} \\ \mathbf{J}_{\mathbf{env}} \end{pmatrix} \mathbf{q} = \begin{pmatrix} \mathbf{1} \cdot d_{\mathrm{crit}} \\ q_{\mathrm{crit}} \end{pmatrix} \qquad (17)$$

The social and environmental response matrices, $\mathbf{J}_{\mathbf{social}}$ and $\mathbf{J}_{\mathbf{env}}$, obtained from Eq. 16 are substituted into Eqs. 12 and 13 to evaluate the values of the first two constraints, whereas the third constraint (Eq. 14) and the function to be maximized (Eq. 11), stayed the same. The resulting problem is linear in both the objective function and all constraints.

Figure 6 shows a contour plot of the total pumping P as a function of the constraint d_{crit} and q_{crit}. Three wells, 18, 19, 20 were taken out of the analysis because well 18 is located far from the location of the constraints resulting in full extraction whereas wells 19 and 20 dried out during the MODFLOW simulations between September 2006 and August 2009. The first feature revealed by the contour map of Fig. 6 is that the same total amount of pumping, $P = (\mathbf{1} \cdot \mathbf{q}_{\mathbf{spr}})$, can be achieved by higher values of either d_{crit} or q_{crit}. We have bounded the solution space to $\mathbf{q}_{\mathbf{spr}}$-combinations corresponding to 3 $m^2/s > \partial q_{\mathrm{crit}}/\partial d_{\mathrm{crit}} > 0.1$ m^2/s along the contour lines $P(d_{\mathrm{crit}}, q_{\mathrm{crit}})$ shown in Fig. 6. The value of 3 m^2/s was chosen to select the points along the environmental curve (E1–E4) constituting of lower q_{crit}-values and resulting in higher discharge to the river. Similarly the value of 0.1 m^2/s was chosen to select the points along the social curve (S1–S4) constituting of lower d_{crit}-values

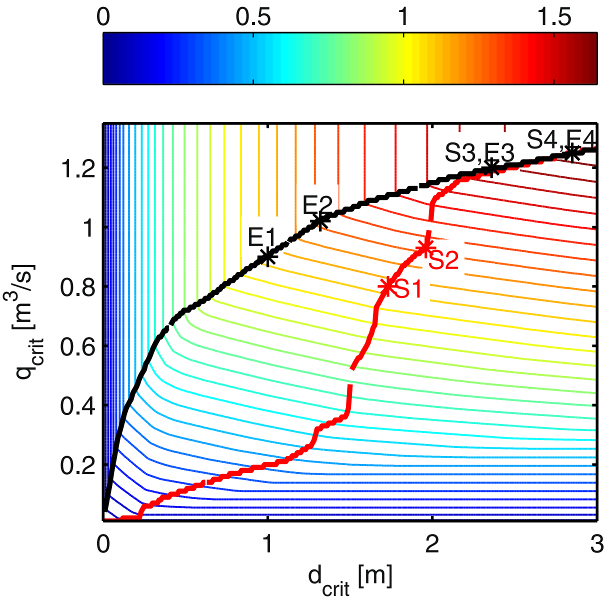

Fig. 6 Contour plot of total groundwater extraction [m^3/s] based on sustainable pumping rates as function of the critical drawdown d_{crit} along the city limit and the critical reduction of groundwater discharge q_{crit} to River Gomti. *Black curve*: environmental regime; *red curve* social regimes; S1, S2, S3, S4, E1, E2, E3, E4: arbitrarily chosen points on the social and environmental curves, respectively

Table 3 List of points used for performance analysis (shown in the Fig. 6)

	$d_{crit}[m]$	$q_{crit}[m^3/s]$	$P[m^3/s]$
E1	1.73	0.80	1.12
S1	1.00	0.90	1.12
E2	1.96	0.93	1.27
S2	1.32	1.02	1.27
S3 = E3	2.36	1.19	1.57
S4 = E4	2.85	1.25	1.67

leading to higher heads at the city limits. The water manager can choose points on one of the two curves if the other constraint has to be completely sacrificed. The second feature revealed is that on increasing the total pumping rate P the two curves favoring either the social or environmental constraint collapse to a single curve (S3 = E3, S4 = E4), which can be seen in Table 3.

Table 3 lists the values of the constraint d_{crit} and q_{crit}, and the total extraction rate P for points S1–S4 and E1–E4, respectively. Note that $P(S1) = P(E1)$ and $P(S2) = P(E2)$, respectively, giving the opportunity to compare constraint values for identical total extraction rates. The analysis of both the social and environmental curves shows that for a higher total extraction rate P, the wells favored for groundwater extraction became independent of giving more weight to either constraint. For further illustration, Fig. 7 shows the spatial maps of drawdown for the points E1-E4 (being more restrictive with respect to groundwater discharge to the river) and S1–S4 (giving more weights to social concerns). In the environmental scenarios, E1 and E2, a substantial drawdown occurs outside of the city limits, whereas, this is avoided in

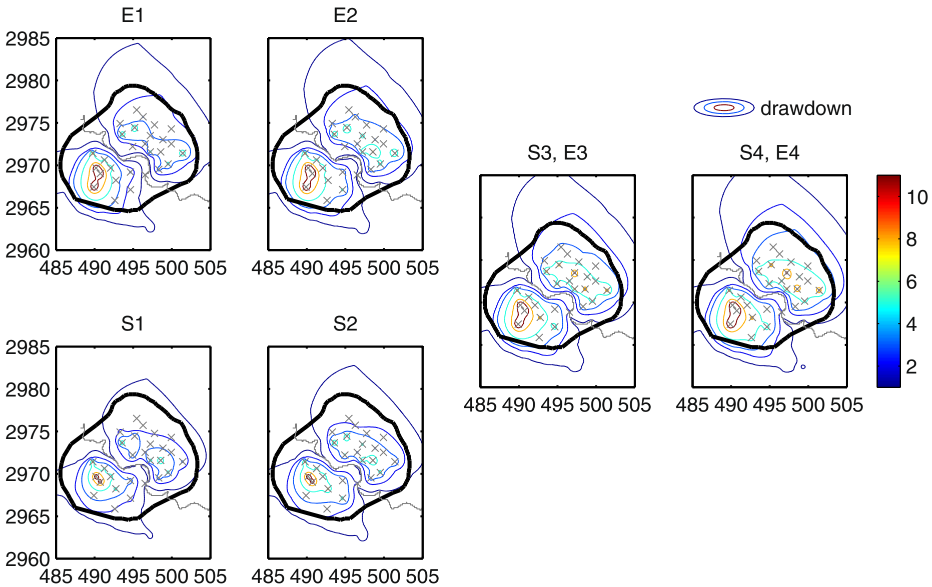

Fig. 7 Groundwater drawdown at time t_1 [m] for various groundwater extraction rates (given in Table 3), as compared to no groundwater extraction

the social scenarios S1 and S2. Conversely, it is not so clearly visible that the social scenarios S1 and S2 show stronger drawdown values close to the river.

5 Multivariate Analysis

Upon varying the constraints d_{crit} and q_{crit}, we obtain multiple vectors $\mathbf{q}_{\mathbf{spr}}$ of 29 individual pumping rates. In order to identify which wells are important in the schemes favoring either the social or environmental constraints, we perform a multivariate analysis. This analysis assists the water manager in preferring and avoiding particular wells depending upon the weights given to the two constraints. Towards this end, we analyze the individual pumping rates $\mathbf{q}_{\mathbf{spr}}$ along the S- and E-lines depicted in Fig. 6 by cluster analysis.

In order to perform the cluster analysis, we evaluate a characteristic total extraction rate P at which a particular well i starts pumping in either the social or environmental regime:

$$\mu_i = P_{\max} - \frac{1}{q_i^{\max}} \int_0^{P_{\max}} q_i(P)\, dP \tag{18}$$

in which μ_i is the characteristic total extraction rate at which the i-th well starts pumping, $P_{\max}$ is the maximum total extraction rate considered, $q_i(P)$ is the individual pumping rate of the i-th well for a given total extraction rate P, and $q_i^{\max} = q_i(P_{\max})$ denotes the maximum pumping rate of the i-th well. The functional relationship $q_i(P)$ differs for the two bounding lines depicted in Fig. 6. Thus, we obtain two values of μ_i, one for the upper curve of Fig. 6, representing the regime giving more weight on social concerns (μ_i^{soc}), and the other for the lower curve curve of Fig. 6, where the environmental constraint is stricter (μ_i^{env}). The data pairs of μ_i^{env} and μ_i^{soc} are plotted against each other for each well in Fig. 8. It may be noted that the wells 14, 19 and 20 did not show any extraction along any of the two curves, and are therefore excluded from this analysis.

The cluster analysis is performed by combining the characteristic total extraction rates, μ_i^{soc} and μ_i^{env}, of all wells in the two regimes, and separating the wells into three clusters using the k-means algorithm implemented in MATLAB (Seber 1984; Spath 1985). This algorithm minimizes the sum of squared distances from all points to the centroid. Wells belonging to the same cluster thus show similar behavior regarding at which total extraction rate P they start pumping in the two regimes.

The determined clusters are marked in Fig. 8 by different colors and dashed enclosing polygons. Cluster 1 includes wells that start pumping at low total extraction rates in the environmental regime, and at high total extraction rates in the social regime. Pumping these wells leads to a comparably strong drawdown along the city limit, while it does not cause strong reduction in groundwater discharge to River Gomti. Comparing the well numbers of cluster 1 with the map of Fig. 2 reveals that these wells are located close to the city limits. The wells of cluster 3 start pumping at high total extraction rates in the environmental regime, and at low total extraction rates in the social regime. These wells strongly affect discharge into River Gomti, but do not influence so much the drawdown at the city limit. Not surprisingly, these wells are located close to the river and far from the city limit. Finally, cluster 2 includes wells that are sensitive in meeting either of the two constraints.

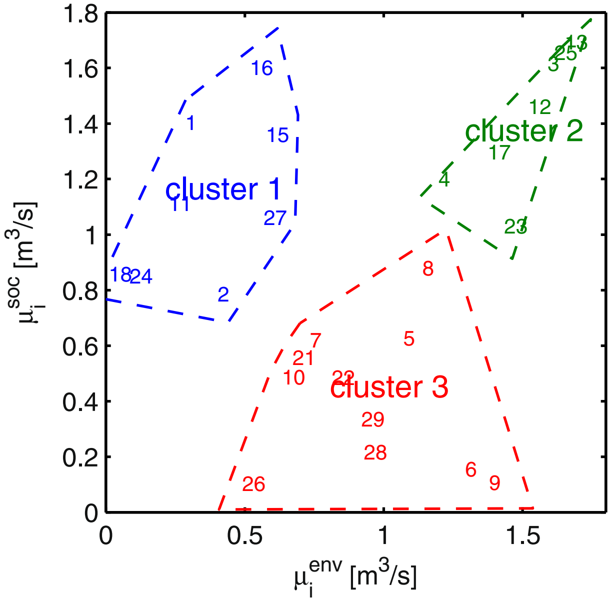

Fig. 8 Characteristic total extraction rates μ_i at which each well starts pumping, as stated in equation 18. Values for the social regime, μ_i^{soc}, are plotted versus those for the environmental regime, μ_i^{env}. *Numbers*: well numbers according as shown in Fig. 2; *dashed lines* outline of three well clusters

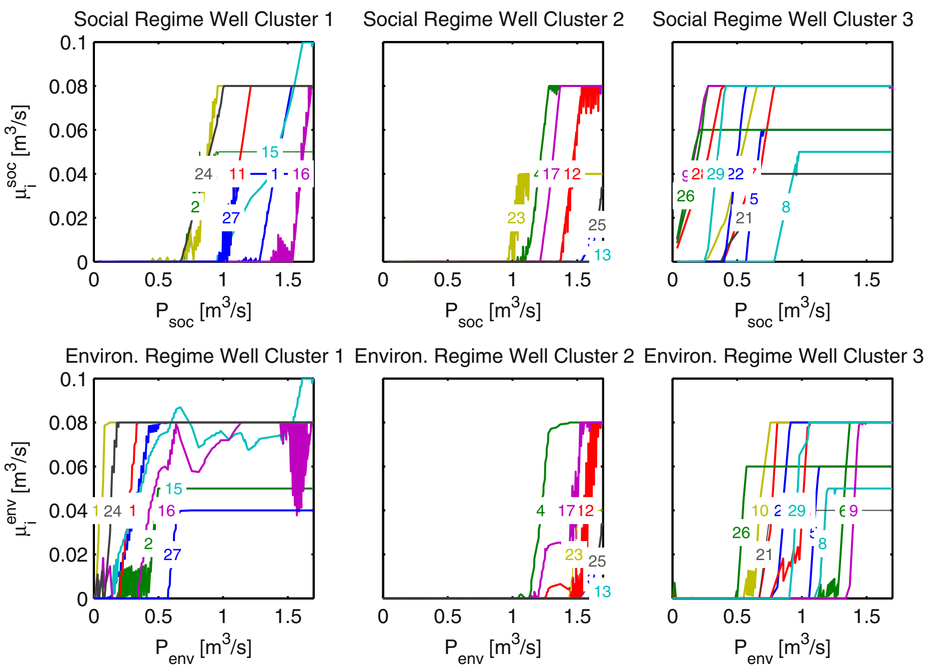

Fig. 9 Pumping rates along the respective regime of each well in three clusters. The number of the well is placed at the corresponding μ-value along the x-axis

Figure 9 shows the individual pumping rates $q_i(P)$ of the wells as function of the total extraction rates P in the social and environmental regimes, sorted by the well clusters depicted in Fig. 8. The well numbers in Fig. 9 are plotted at the μ-value of the corresponding well, indicating that Eq. 18 indeed yields a characteristic total extraction rate at which an individual well starts pumping. Also, it can be seen that the wells within each cluster share similar functional dependencies $q_i^{\mathrm{soc}}(P_{\mathrm{soc}})$ and $q_i^{\mathrm{env}}(P_{\mathrm{env}})$, as discussed above.

6 Linear Trade Off

Up to now, we have constructed two functional relationships, $\mathbf{q_{spr}}(P_{\mathrm{soc}})$ and $\mathbf{q_{spr}}(P_{\mathrm{env}})$, indicating which set of individual pumping rates for each well have to be selected for a given total water demand P, giving either more weight to social or environmental concerns, respectively. In practice, the water manager would have to choose which of the regimes to follow. Because of the linearity between pumping rates and constraints, any linear combination of $\mathbf{q_{spr}}(P_{\mathrm{soc}})$ and $\mathbf{q_{spr}}(P_{\mathrm{env}})$ for $P_{\mathrm{soc}} = P_{\mathrm{env}}$ would require the linear combination of the constraint values d_{crit} and q_{crit} of the two end members. We suggest to apply a linear trade off based on the ratio of demands at locations close to where the constraint d_{crit} and q_{crit} act. In case that the ratio of demands is $w_1 : w_2$, where w_1 is the total demand at well locations close to the city limit as shown in Fig. 2 and w_2 is the total demand at the other wells, the trade-off vector of pumping rates $\mathbf{q_{spr}^{tr}}$ given by linear combination:

$$\mathbf{q_{spr}^{tr}} = \frac{\mathbf{q_{spr}}(P_{\mathrm{env}}) \cdot w_1 + \mathbf{q_{spr}}(P_{\mathrm{soc}}) \cdot w_2}{w_1 + w_2} \tag{19}$$

resulting in identical trade offs in d_{crit}:

$$d_{\mathrm{crit}}^{tr} = \frac{d_{\mathrm{crit}}^{L}(P) \cdot w_1 + d_{\mathrm{crit}}^{U} \cdot w_2}{w_1 + w_2} \tag{20}$$

and similarly for q_{crit}.

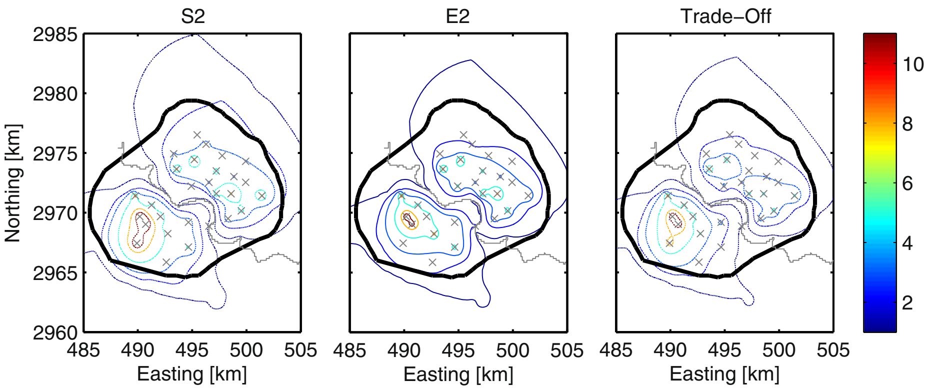

Fig. 10 Groundwater drawdown [m] after implementing 1:1 trade off in E2 and S2

As an example we consider the total extraction rate P of 1.27 m^3/s shared by the points S2 and E2 shown in Fig. 6. In case the demands are identical, $w_1 = w_2$, the constraints are also traded-off in the 1:1 ratio. Figure 10 shows the groundwater drawdown due to the three groundwater extractions $\mathbf{q}_{\mathbf{spr}}^{\mathbf{S2}}$, $\mathbf{q}_{\mathbf{spr}}^{\mathbf{E2}}$ and $\mathbf{q}_{\mathbf{spr}}^{\mathbf{tr}}$.

The trade off implementation resulted in more groundwater extraction of 100 % at well locations 4, 6, 9, 17, 23 and less groundwater extraction of 43 % at 1, 12, 15, 16 as compared to S2.

7 Discussion and Conclusions

In this study, we have presented a framework of estimating sustainable pumping rates by means of a numerical groundwater model and constrained optimization. We applied the method to the groundwater management of the city of Lucknow, India. The constraints set limits to the environmental and social drawbacks associated with excessive groundwater drawdown. Within the framework, the constraints were defined as acceptable loss of groundwater discharge to the draining river and maximum drawdown at the city limit, respectively. We are convinced that this approach of defining sustainable pumping rates is superior to classical steady-state analysis of induced recharge and discharge. However, to achieve sustainability, dynamic steady state must be reached; that is, under time-periodic forcing, the hydraulic heads at the end of the period are identical to those at the beginning.

In the given application, the dynamic groundwater model could be replaced by a response matrix, and optimization could be performed by linear programming. Most likely, these simplifications minimizing the computational effort would not be valid for excessive pumping in shallow phreatic aquifers. However, we have outlined how to check the validity of the linear approximation.

While the optimization problem has been formulated as maximizing the total extraction rates for given constraints, practical applications will often be reversed: for a given demand, the least harmful pumping scheme is sought, which is non-unique when several concerns are raised. We had constructed two functional relationships $P(\mathbf{q}_{\mathbf{spr}})$ emphasizing either environmental or social aspects. Due to the validity of the linear approximation, also the trade-off between the two solutions is linear and we suggest taking local water demands as trade-off criteria. However, other criteria may be used as well, including absolute limits of either constraint. It is quite possible that no acceptable solution can be found when water demand is too high. In such a case alternative water supply measures, such as artificial groundwater recharge, needs to be considered. The design of such schemes may be based on the same principles as the optimization of pumping rates.

The presented scheme may be extended to consider additional constraints reflecting other concerns. As long as the constraints react approximately linearly upon extraction, the extension is straightforward. Otherwise computationally more expensive schemes based on multiple transient model runs and nonlinear optimization become necessary.

We highly recommend clustering the wells according to the characteristic total extraction rates, at which they practically start pumping, in order to characterize which wells affect the environmental and social concerns the most. A simplified look-up figure of the cluster analysis may be used for practical groundwater management so that the full optimization scheme needs not to be operated by the water manager.

This study is a first step towards the groundwater management plan for the city of Lucknow. Taking a step forward in the groundwater management plan requires a cost-benefit analysis of any selected $\mathbf{q}_{spr}$ and the consideration of additional factors such as artificial groundwater recharge, land use and climate change. The continuing urban sprawl leads to increasing water demands and hence more groundwater extraction wells outside the current city limits may need to be considered. Another option could be importing water from nearby perennial streams. While this study shows the application to Lucknow city, the methodology put forward can be transferred to other urban areas facing similar groundwater management problems.

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References

Ahlfeld DP, Baro-Montes G (2008) Solving unconfined groundwater flow management problems with successive linear programming. J Water Resour Plan Manag 134:404

Ahlfeld DP, Barlow PM, Mulligan AE (2005) GWM–a Ground-water Management Process for the US Geological Survey Modular Ground-Water Model (MODFLOW-2000). US Dept. of the Interior, US Geological Survey

Alley WM, Leake SA (2004) The journey from safe yield to sustainability. Ground Water 42(1): 12–16

Anderson MP, Woessner WW (1992) Applied groundwater modeling. Simulation of flow and advective transport. Academic Press Inc., London

Armstrong A, Rose S (1999) Ditch water levels managed for environmental aims: effects on field soil water regimes. Hydrol Earth Syst Sci 3(3):385–394. doi:10.5194/hess-3-385-1999

Baú DA (2012) Planning of groundwater supply systems subject to uncertainty using stochastic flow reduced models and multi-objective evolutionary optimization. Water Resour Manage 26(9):2513–2536

Bayer P, Duran E, Baumann R, Finkel M (2009) Optimized groundwater drawdown in a subsiding urban mining area. J Hydrol 365(1–2):95–104. doi:10.1016/j.jhydrol.2008.11.028

Bhatnagar NC (1966) Progress report on the systematic geohydrological studies in lucknow district and its adjoining areas, uttar pradesh. Tech. rep., Geological Survey of India

Bredehoeft J (1997) Safe yield and the water budget myth. Ground Water 35(6):929

CGWB (2009) Annual report 2008-09. Tech. rep., Central Groundwater Board India, Ministry of Water Resources, Government of India

Chatterjee R, Gupta BK, Mohiddin SK, Singh PN, Shekhar S, Purohit R (2009) Dynamic groundwater resources of national capital territory, Delhi: assessment, development and management options. Environ Earth Sci 59(3):669–686. doi:10.1007/s12665-009-0064-y

Devlin JF, Sophocleous M (2005) The persistence of the water budget myth and its relationship to sustainability. Hydrogeol J 13(4):549–554. doi:10.1007/s10040-004-0354-0

Foglia L, Hill MC, Mehl SW, Burlando P (2009) Sensitivity analysis, calibration, and testing of a distributed hydrological model using error-based weighting and one objective function. Water Resour Res 45:W06427. doi:10.1029/2008WR007255

Foster S, Choudhary NK (2009) Lucknow city—India: groundwater resource use and strategic planning needs. Tech. rep., UP-SWaRA

Harbaugh AW (2005) MODFLOW-2005. The U.S. Geological Survey modular ground-water model—the Ground-Water Flow Process: U.S. Geological Survey Techniques and Methods

Healy RW, Cook PG (2002) Using groundwater levels to estimate recharge. Hydrogeol J 10(1): 91–109. doi:10.1007/s10040-001-0178-0

Hill MC, Tiedeman CR (2007) Effective groundwater model calibration: with analysis of data, sensitivities, predictions, and uncertainty. Wiley, Hoboken

Jones JP, Sudicky EA, McLaren RB (2008) Application of a fully-integrated surface-subsurface flow model at the watershed-scale: a case study. Water Resour Res 44(3):W03407

Kalf FRP, Woolley DR (2005) Applicability and methodology of determining sustainable yield in groundwater systems. Hydrogeol J 13(1):295–312

Liu J, Zheng C, Zheng L, Lei Y (2008) Ground water sustainability: methodology and application to the north china plain. Ground Water 46(6):897–909

Livingston M (2009) Deep wells and prudence: towards pragmatic action for addressing groundwater overexploitation in India. Report, World Bank

Loague K, Kyriakidis PC (1997) Spatial and temporal variability in the r-5 infiltration data set: Déjà vu and rainfall-runoff simulations. Water Resour Res 33(12):2883–2895

Lorenzen G, Sprenger C, Taute T, Pekdeger A, Mittal A, Massmann G (2010) Assessment of the potential for bank filtration in a water-stressed megacity (Delhi, India). Environ Earth Sci 61(7):1419–1434. doi:10.1007/s12665-010-0458-x

Maimone M (2004) Defining and managing sustainable yield. Ground Water 42(6):809–814

Monteith JL (1965) Evaporation and environment. Symp Soc Exp Biol 19:205–234

Munoz JF, Fernandez B, Escauriaza C (2003) Evaluation of groundwater availability and sustainable extraction rate for the Upper Santiago Valley Aquifer, Chile. Hydrogeol J 11(6):687–700. doi:10.1007/s10040-003-0292-2

Nash JE, Sutcliffe JV (1970) River flow forecasting through conceptual models part ia discussion of principles. J Hydrol 10(3):282–290

Ouessar M, Bruggeman A, Abdelli F, Mohtar RH, Gabriels D, Cornelis WM (2009) Modelling water-harvesting systems in the arid south of tunisia using SWAT. Hydrol Earth Syst Sci 13(10):2003–2021

Pearson LJ, Coggan A, Proctor W, Smith TF (2010) A sustainable decision support framework for urban water management. Water Resour Manage 24(2):363–376

Perez AJ, Abrahao R, Causape J, Cirpka OA, Buerger CM (2011) Simulating the transition of a semi-arid rainfed catchment towards irrigation agriculture. J Hydrol 409(3–4):663–681. doi:10.1016/j.jhydrol.2011.08.061

Romano E, Preziosi E (2010) The sustainable pumping rate concept: lessons from a case study in central Italy. Ground Water 48(2):217–226. doi:10.1111/j.1745-6584.2009.00628.x

Seber GAF (1984) Multivariate observations, vol 14 of Wiley Series in probability and statistics—applied probability and statistics section series, illustrated edn. Wiley

Shen W, Dong D, Yu M (2004) Optimal groundwater management model in a two-aquifer system. Pract Period Hazard Toxic Radioact Waste Manage 8:119–127. doi:10.1061/(ASCE)1090-025X(2004)8:2(119)

Spath H (1985) Cluster dissection and analysis: theory, FORTRAN programs, examples. Horwood, Halsted Press [distributor], Chichester, New York

Srivastava P, Singh IB, Sharma S, Shukla UK, Singhvi AK (2003) Late pleistocene-holocene hydrologic changes in the interfluve areas of the central Ganga plain, India. Geomorphology 54(3–4):279–292. doi:10.1016/S0169-555X(02)00361-6

Stanghellini PSL, Collentine D (2008) Stakeholder discourse and water management—implementation of the participatory model CATCH in a northern italian alpine sub-catchment. Hydrol Earth Syst Sci 12(1):317–331

Taubenboeck H, Wegmann M, Roth A, Mehl H, Dech S (2009) Urbanization in India—spatiotemporal analysis using remote sensing data. Comput Environ Urban Syst 33(3):179–188. doi:10.1016/j.compenvurbsys.2008.09.003

van der Gun J, Lipponen A (2010) Reconciling groundwater storage depletion due to pumping with sustainability. Sustainability 2(11):3418–3435. doi:10.3390/su2113418

Winston RB (2009) ModelMuse—A graphical user interface for MODFLOW-2005 and PHAST: U.S. Geological Survey Techniques and Methods 6–A29, 52 p, available only online at http://pubs.usgs.gov/tm/tm6A29

Xu YS, Huang RQ, Han J, Shen SL (2013) Evaluation of allowable withdrawn volume of groundwater based on observed data. Nat Hazards 67(2):513–522. doi:10.1007/s11069-013-0576-3

Zhang Y (1998) Solving large-scale linear programs by interior-point methods under the matlab environment. Optim Meth Software 10(1):1–31