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Optimization of Industrial Structure Considering the Uncertainty of Water Resources

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Abstract This paper developed a stochastic linear fractional programming model for industry optimization allocation base on the uncertainty of water resources incorporating chance constrained programming and fractional programming. In this paper, the stochastic linear fractional programming is used in the real word. The development SLFP has the following advantages: (1) The model can compare the two aspects of the targets; (2) The model can reflect the system efficiency intuitively; (3) The model can deal with uncertain issues with probability distribution; (4) The model can give different optimal plans under different risk conditions. The model has a significant value for the industry optimization allocation under uncertainty in local and areas to achieve the maximum economic benefits and the full use of the water resources.

Keywords Chance constrained programming · Water resources optimal allocation · Stochastic linear fractional programming · Industrial optimization · Uncertainty

1 Introduction

Water is not only the material basis of human survival, but also the important material of national economic and social development. China is a serious water shortage country with available fresh water resources of 2,043 m^3 per capita, accounting for a quarter of the world average level only. Among the 669 cities in China, there are more than 400 cities in the condition of water shortage, and 114 cities with severe water shortages condition. Water shortages and water pollution have become the very serious problems with the rapid development of the economy in China. The

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urban sewage water is in the rapid growth, because of china's rapid urbanization, city scale's rapid expansion and the industry's rapid development.

Therefore, many cities in China face the problem of water shortages, because urban water demands are increasing rapidly. Groundwater extraction increases dramatically because of the continued water shortage in cities, which may cause a series of geological environment problems, such as land subsidence. There are more than 40 cities and towns have land subsidence caused by the unreasonable exploitation of groundwater, causing serious economic losses and social issues, which distributed in the Northeast Plain, the North China Plain etc.

Lack of water resources is not only limits the development of the city, but also becomes the "bottleneck" to local economic and social development. Because of the shortage of water resources, industry and agriculture are competing for water, cities and towns are competing for water. So they over extract ground water and take up some ecological water. There are some policies of water restrictions in some places in order to relieve the water shortage problem, which have caused great negatively impact on people's lives. Moreover, water body has been polluted resulted by the intensification of the urbanization process, the sharp increasing of urban waste and industrial waste water emission. The increased water pollution has not only weakened the available water body, it deteriorates the problem of shortage of water resources to a higher degree, which has grave threats to the drinking water safety and people health.

According to monitoring, the groundwater has been polluted by point source and non-point source pollution at a certain degree, and the pollution trend is increasing year by year, which has a serious impact on the sustainable development strategy of China. The fundamental way to solve the water problem is to coordinate the relationship between water resources and socio-economic development. In other words, the industrial structure and productivity distribution in the study area should be analysed to balance the relationship between short-term interests and long-term interests and the relationship between partially development and overall development, in the circumstances of social stability and economic growth.

In the past 10 years, a series of mathematical methods were used to study water resources carrying capacity. Percia and Mehrez (1997) put forward a linear programming model to achieve the maximum economic benefit of the whole system and developed a water resources management model considering the requirements from different users. Khare et al. (2007) developed a linear programming model for economic project to explore the potential links between surface water and groundwater in the constraints of hydrology and management. The model can obtain the optimal planting pattern with the largest economic benefits. Kondili et al. (2010) developed a water supply system optimization model considering different water resources, different users and environmental problems. Liu et al. (2011) put forward a mixed integer linear programming for multi water resources planning problem, including sea water, wastewater, reuse water etc. Min et al. (2011) developed a comprehensive evaluation index and model system to calculate and analyze water resources carrying capacity in Jining, China.

Among the previous model study for water resources carrying capacity, they just focus on the maximum output or minimum output of the system. However, we have to resolve the fraction or ratio problems in the real life. Therefore, in order to solve this kind of problems, we put forward the fraction programming (FP). Compared with the above methods, the FP has two aspects of the advantages. On the one hand, we can able to compare objectives of multiple aspects directly through the original magnitudes. On the other hand, it is not only focus on system inputs or outputs, but also could measure the efficiency of water management that was related to output/input ratios (Zhu and Huang 2011). Generally speaking, the FP is mainly used in the management problems (Stancu-Minasian 1997; Gomez et al. 2006) that required comparison of two magnitudes (e.g. cost/time, or output/input). Moreover, FP was used when the efficiency of a system is to be measured (Charnes et al. 1978; Lara and Stancu-Minasian 1999). The optimization of ratio between environmental and economics (Mehra et al. 2007), quantities could well reflect the real-world complexities. In conclusion, FP can be used in the management of water resources.

However, the aforementioned studies are incapable of reflecting the features of fractional objective function under uncertainty (Chang 2009; Hladik 2010) in management of water resources. In addition, many real world problems are associated with random, fuzzy and/or other uncertain characters in the actual issues (Guo et al. 2010). Therefore, a series of optimal allocation models under uncertainty were developed, including Fuzzy Mathematical Programming (FMP), Stochastic Mathematical Programming (SMP) (Zhu et al. 2009; Lv et al. 2010; Yan et al. 2010), Interval Stochastic Programming (ISP) and so on. In the water resources optimal allocation system, many parameters, such as water storage, have random characters and can be expressed as the form of probability distribution (Guo et al. 2008; Li et al. 2007; Huang et al. 2001). Chance Constrained Programming (CCP) is very effective to solve optimal problem with the random parameters (Charnes and Cooper 1959; Miller and Wagner 1965) in the right hand of constraints. However, there was hardly any research handled both multiple uncertainties and fractional objective function.

Therefore, this paper aim to develop a stochastic linear fractional programming (SLFP) model for water resources optimal allocation by introducing CCP into the fraction programming framework to deal with uncertainties existing in the water resources system. Then the SLFP method will be applied to a real-world case study of industrial water resources management. The developed model can balance the relationship between water resources carrying capacity and economic development planning. Moreover, it can analyze system efficiency (It means the values can be produced by the unit of water. It shows in the objective function, the numerator and denominator denote the value of all industries and the investment of the water resources) and deal with stochastic uncertain existing in the planning system.

2 Methodology

2.1 Fraction Programming

The general form of fraction programming can be written as:

$$Max\ \ f(x) = \frac{CX + \alpha}{DX + \beta} \qquad (1-1)$$

$$AX \leq BX \geq 0 \qquad (1-2)$$

where, A is m by n matrix; X and B are column vectors with n and m components respectively; C and D are row vectors with n components; α and β are constants. Assumed that if $DX+\beta$ is constant for all X in the whole feasible region, the optimal solution of fraction programming above can be obtained by solving the linear programming model.

According to Chadha and Chadha (2007), if the model above fulfil: (1) $DX+\beta>0$ for all x; (2) Objective function is continuously differentiable; (3) the feasible region is non-empty and bounded, the following variable can be brought into the formula 1 based on Charnes-Cooper method.

$$Z = \frac{1}{DX + \beta} \qquad (2-1)$$

$$Y = ZX \qquad (2-2)$$

So the original linear fraction programming model can be translated into linear programming model:

$$Max\ \ f(x) = CY + \alpha Z \tag{3 – 1}$$

Subject to

$$AY \leq BZ \tag{3 – 2}$$

$$DY + \beta Z = 1 \tag{3 – 3}$$

$$Y \geq 0\ \ Z > 0 \tag{3 – 4}$$

So, LFP (1) can be translated into LP (3) by introducing the corresponding variables (2), and then the optimal solution would be got. LFP model can solve optimal proportion issues, but the model has difficult in solving the issues when the input parameters are uncertain (Chadha and Chadha 2007).

2.2 Chance Constraint Programming (CCP)

The characteristic of the parameters in the right hand side of constraint is "unpredictability" when solving the optimal model. The CCP model is very effective in solving the problems when the parameters mentioned above are random and can be expressed as the form of probability distribution. The typical CCP model can be described as:

$$Min\ \ f(x) \tag{4 – 1}$$

$$\Pr[A_i(t) \leq b_i(t)] \geq 1 - p_i, i = 1, 2, \cdots m \tag{4 – 2}$$

$$X \geq 0 \tag{4 – 3}$$

where, $A_i(t) \in A(t)$, $b_i(t) \in B(t)$, $t \in T$; $A(t)$, $B(t)$ are random elements in the probability space T; $p_i(p_i \in [0,1])$ are probability level of constraint event; m is the number of constraint event.

If the parameters are uncertain in both left hand and right hand in model (4), and constraint (4-2) are non-linear, the practical constraints would become very complex (Zare and Daneshmand 1995). However, not all of the parameters of *A(t)* and *B(t)* in the CCP model are random (Charnes et al. 1972), when the parameters in the left hand are determinate and the parameters in the right hand are variational, constraint (4-2) can be converted to linear programming and the feasible set of constraints can be described as:

$$A_i \leq b_i(t)^{(p_i)}, i = 1, 2, \cdots, m \tag{4 – 4}$$

where, $b_i(t)^{(p_i)} = F_i^{-1}(p_i)$ denotes the cumulative distribution function of b_i(*i.e.*,$F_i(b_i)$) and the probability of violating constraint $i(p_i)$; So CCP model can tackle the stochastic issues in the right hand of constraints through the following transition: (1) Give the probability value $p_i(p_i \in [0,1])$ of uncertainty event; (2) Each constraint should be satisfy the probability value $1-p_i$

2.3 Stochastic Linear Fraction Programming (LFP)

LFP model has difficult in dealing with the problems when the parameters in the model are uncertain, while CCP model can handle the problems when the parameters in the right hand have probability distribution effectively. So a potential method is to combine LFP and CCP model, so we have Stochastic Linear Fraction Programming (SLFP). The model can be written as:

$$Max\ f(x) = \frac{CX + \alpha}{DX + \beta} \tag{5 − 1}$$

$$A_i \le b_i(t)^{(p_i)}, i = 1, 2, \cdots, m \tag{5 − 2}$$

$$X \ge 0 \tag{5 − 3}$$

where, X is column vectors with n components; C and D are row vectors with n components; α and β are constants. A_i is vector constrained of constraint i, and it is a row vector with n components, b_i are parameters in the right hand of constraint i, p_i is probability level of uncertain event i.

The integrated SLFP model can settle optimization matching and probability distribution problems under uncertain. The solving steps of SLFP are:

(I): Build original SLFP model (5);
(II): Transform the stochastic constraint into deterministic constraint through CCP model (5-2); $A_i \le b_i(t)^{(p_i)}, i = 1, 2, \cdots, m$
(III): Build deterministic LFP model. Model (1) and the corresponding transfers model (2);
(IV): Solve the deformation model and get (Z, Y);
(V): Solve LFP model and get the corresponding optimal solution x through f (x) = z;
(VI): Repeat (II) ~ (V) under different p_i.

3 Application

3.1 Study Area

The study area is Jinchang city (101°04′35″~102°13′40″E,37°47′10″~39°00′30″N), Gansu Province, China, located in the mid-east of Hexi Corridor in Gansu Province, in the west of Qilian Mountain, and in the south of Desert Badanjilin, with the land area of 9,593 km^2. In order to put the proposed approach into a real-word case, we select Jinchang for study area. The investigation was made and the hydrological data of many years, the reports of government work plans, and the Statistic Yearbooks were collected (including the number of population, the number of working population, the added value of the industry ect). The numbers in the tables were gotten by the analysis of collected data. This study take year 2011 as current year, years 2011–2015 and 2016–2020 as the two planning periods. The economic society overall objective of Jinchang city 2010–2015 planning are: Keep the double-digit growth of economic development; Realize the increase rates of primary industry, secondary industry and tertiary industry are 5.5 %, 17.8 %, and 15 %, respectively; The structure rate of the three industry are expected to achieve 3.3:82:14.7, and the urbanization process is expected to achieve more than 65 %. Jinchang is a developing city based on heavy water using and water resources shortage has become a limiting factor to the development of Jinchang's economy. Industrial water and domestic water are increasing rapidly as the increasing population and the development of secondary industry and tertiary industry. Because of severe water shortages, the water demand and supply appeared inevitable unbalance. One of the sound solutions is to optimize the industrial structure and improve the carrying capacity of water resources. We get the minimum water demand of every industry by studying the plan of industrial restructure and the investigation of the industry in Jinchang. Then on the conditions of satisfying the demand of ecological water and the minimum of industry water, the remaining water resources were used to allocate to meet the maximum benefit of economic. Therefore, the Industry optimization allocation model under the condition of water shortage has not only the practical guiding significance, but also has great influence to the city development. On the whole, this study can solve the following problems: (1) Whether the water carrying capacity can meet the requirements of economic development of Jinchang city; (2) How to optimize the industrial structure to obtain the biggest economic benefits under the condition of limited water resources.

3.2 Model Building

The SLFP model for the industry optimization allocation can be written as:

Objective function:

$$Max\ f = \frac{A - B - C - Inv_t}{\sum_{t=1}^{2}\left(\mathrm{Pr}WS_{\mathrm{t}} + \sum_{j=1}^{m} TeWS_{\mathrm{jt}} + TeWS_{\mathrm{t}}\right) + EWS_t} \qquad (6-1)$$

$$A = \sum_{t=1}^{2}\left(\mathrm{Pr}V_t \cdot \mathrm{Pr}WS_t + \sum_{j=1}^{m} SeV_{jt} \cdot SeWS_{jt} + TeV_t \cdot TeWS_t\right) \qquad (6-2)$$

$$B = \sum_{t=1}^{2} STF_t \left(\begin{array}{l}\mathrm{Pr}RWT_t \cdot \mathrm{Pr}WD_t \cdot \mathrm{Pr}V_t \cdot \mathrm{Pr}WS_t + \\ \sum_{j=1}^{m} SeRWT_t \cdot SeWD_t \cdot SeV_{jt} \cdot SeWS_{jt} + TeRWT_t \cdot TeWD_t \cdot TeV_t \cdot TeWS_t\end{array}\right) \qquad (6-3)$$

$$C = \sum_{t=1}^{2} \frac{1}{RWP_t} \cdot IMWD_t \cdot STF_t \left(\begin{array}{l}\mathrm{Pr}WP_t \cdot \mathrm{Pr}V_t \cdot \mathrm{Pr}WS_t + \\ \sum_{j=1}^{m} SeWP_{jt} \cdot SeV_{jt} \cdot SeWS_{jt} + TeWP_t \cdot TeV_t \cdot TeWS_t\end{array}\right) \qquad (6-4)$$

Subject to

COD constraints:

$$\Pr V_t \cdot \Pr WS_t \cdot \Pr COD_t \cdot (1{-}\Pr RWT_t \cdot RSTCOD_t) + \frac{1}{RWP_t} \cdot DCOD_t \cdot \Pr WP_t \cdot \Pr V_t \cdot \Pr WS_t$$
$$\times \ (1{-}RDST_t \cdot RSTCOD_t) \ + \ \sum_{j=1}^{m} SeCOD_t \cdot SeV_{jt} \ \cdot SeWS_{jt} \left(1{-}SeRWT_{jt} \cdot RSTCOD_{jt}\right)$$
$$+\sum_{j=1}^{m} \frac{1}{RWP_t} \cdot DCOD_t \cdot SeWP_{jt} \cdot SeV_{jt} \cdot SeWS_{jt}(1{-}\Pr RWT_t \cdot RSTCOD_t) + \mathrm{Te}V_t \cdot TeWS_t \cdot TeCOD_t$$
$$\times \ \frac{1}{RWP_t} \cdot DCOD_t \ \cdot TeWP_t \ \cdot TeV_t \ \cdot TeWS_t \ (1{-}\Pr RWT_t \cdot RSTCOD_t) \ \le \ ECCOD^{P_i}$$

$$(6-5)$$

Water resources constraints:

$$\left(1 + \frac{1}{RWP_t} \cdot DWD_t \cdot \Pr WP_t \cdot \Pr V\right) \cdot \Pr WS_t + \sum_{j=1}^{m} \left(1 + \frac{1}{RWP_t} \cdot DWD_t \cdot SeWP_{jt} \cdot SeV_{jt}\right) \cdot SeWS_{jt}$$
$$\left(1 + \frac{1}{RWP_t} \cdot DWD_t \cdot TeWP_t \cdot TeV\right) \cdot TeWS_t \ + \ EWS_t \le W_t^{P_i}$$

$$(6-6)$$

$$Min\Pr WS_t \le \Pr WS_t MinSeWS_t \le SeWS_t MinTeWS_t \le TeWS_t \qquad (6-7)$$

Nonnegativity constraints:

$$\Pr WS_t \ge 0; \ \ SeWS_{jt} \ge 0; \ \ TeWS_t \ge 0; \qquad (6-8)$$

A, B, C represent water total output value, sewage treatment cost of the three industry, domestic waste water treatment cost, respectively. The above symbols' meanings are as follows:

$SeRWT_{jt}$	The rate of waste water treatment of j trade of the secondary industry in period t
$SeWD_{jt}$	The waste water discharge amount of j trade of the secondary industry per unit GDP in period t
STF_t	the waste water discharge amount of j trade of the secondary industry per unit GDP in period t
$PrRWT_t$	The rate of waste water treatment of the primary industry in period t
$PrWD_t$	The waste water discharge amount of the primary industry per unit GDP in period t
$TeRWT$	The rate of waste water treatment of the tertiary industry in period t
$TeWD_t$	The rate of waste water treatment of the tertiary industry in period t
$IMWD_t$	The individual annual municipal waste water discharge in period t
RWP_t	The ratio of the working population to the population of the study area in period t
$RDST_t$	The domestic sewage treatment rate in period t
$SeWP_{jt}$	Working population of j trade of the secondary industry per add-value in period t
$PrWP_t$	Working population of the primary industry per add-value in period t
$TeWP_t$	Working population of the tertiary industry per add-value in period t
$SeCOD_{jt}$	The COD in j trade of secondary industry waste water per unit GDP in period t
$RSTCOD_t$	The rate of COD disposal in sewage treatment works in period t
$DCOD_t$	The COD in individual domestic sewage per unit GDP in period t
$PrCOD_t$	The COD in primary industry waste water per unit GDP in period t
$TeCOD_t$	The COD in tertiary industry waste water per unit GDP in period t
$ECCOD_t$	Environment capacity of COD in certain environment aim in period t

DWD_t	The per capita domestic water demand in period t
$IDWD_t$	The individual domestic waste discharge in period t
$PrWS_t$	The water supply to the primary industry in period t
$SeWS_{jt}$	The water supply to the j trade of secondary industry in period t
$TeWS_t$	The water supply of the tertiary industry in period t
EWS_t	The water supply for ecological propose in period t
SeV_{jt}	The valve-added of j trade of the secondary industry per unit water supply in period t
PrV_t	The valve-added of the primary industry per unit water supply in period t
TeV_t	The valve-added of the tertiary industry per unit water supply in period t
$MinPrWS_t$	The minimum water supply to the primary industry in period t
$MinSeWS_{jt}$	The minimum water supply to the j trade of secondary industry in period t
$MinTeWS_t$	The minimum water supply of the tertiary industry in period t

The objective of the above model was to obtain ratio maximization between production value and the input water resources. The model reflected overall value

Table 1 Minimum water supply and the added value per unit of water resources

	Minimum water supply (m^3)		Added value (10^4yuan/t)	
	T=1	*T*=2	*T*=1	*T*=2
S1	480	528	0.03472	0.04586
S2	938	1032	0.03472	0.04586
S3	10362	11398	0.01681	0.02220
S4	33600	36960	0.01681	0.02220
S5	2800	3080	0.01832	0.02185
S6	18022	19824	0.00292	0.00349
S7	102	112	0.03322	0.03980
S8	666317	732949	0.00947	0.01250
S9	291	320	0.04000	0.04779
S10	473	521	0.00364	0.00435
S11	74335	81768	0.02222	0.02625
S12	133370	146707	0.02917	0.03209
S13	6387598	7026358	0.03156	0.03307
S14	13	14	0.02976	0.03562
S15	194	214	0.03448	0.04120
S16	2525968	2778565	0.02500	0.02698
S17	4438533	4882386	0.02500	0.02698
S18	1132192	1245412	0.03448	0.04120
S19	141117	155228	0.05000	0.05992
T1	5042367	5546604	0.03333	0.03939
P1	152798012	168077814	0.00061	0.00072

S1: Coal Mining and Dressing; *S2*: Coal Mining and Processing; *S3*: Farm and Sideline Processing; *S4*: beverage manufacturing; *S5*: textile industry; *S6*: Paper & Paper Products; *S7*: Printing and recording media copy; *S8*: Chemical raw materials and chemical products manufacturing; *S9*: Medical manufacturing; *S10*: Plastic Products; *S11*: Non-metallic mineral industry; *S12*: Black metal smelting and rolling processing industry; *S13*: Non-ferrous metal smelting and rolling processing industry; *S14*: Metal manufacturing; *S15*: Electric Equipment and Machinery; *S16*: Electric heat production and supply industry; *S17*: Production and Supply of Water; *S18*: Equipment for Special Purpose; *S19*: construction industry; *T1*: tertiary industry; *P1*: primary industry

Table 2 Working population per unit GDP, waste water discharge per unit GDP and COD discharge per unit GDP

	Working population (p/10^4yuan)		Waste water discharge (t/10^4yuan)		COD discharge (t/10^4yuan)	
	T=1	*T*=2	*T*=1	*T*=2	*T*=1	*T*=2
S1	0.3705	0.3555	46.1512	45.1512	0.0047	0.0046
S2	0.3100	0.2990	52.8549	51.8549	0.0092	0.0090
S3	0.0700	0.0500	65.7267	64.7267	0.0411	0.0401
S4	0.0500	0.0400	41.9187	40.9187	0.0250	0.0240
S5	0.2200	0.1800	74.0882	73.0882	0.0159	0.0155
S6	0.1600	0.1200	467.1656	466.1656	0.2184	0.2130
S7	0.7000	0.6000	4.6911	3.6911	0.0006	0.0006
S8	0.6000	0.5500	126.9129	125.9129	0.0205	0.0202
S9	0.3200	0.3000	35.0079	34.0079	0.0140	0.0134
S10	0.4500	0.4100	6.3360	5.3360	0.0018	0.0018
S11	0.6200	0.5900	26.9748	25.9748	0.0031	0.0030
S12	0.6000	0.5800	62.8383	61.8383	0.0057	0.0057
S13	0.2800	0.2600	35.2067	34.2067	0.0033	0.0033
S14	1.0000	0.9600	16.3852	15.3852	0.0015	0.0014
S15	0.5900	0.5700	5.0670	4.0670	0.0003	0.0003
S16	0.0370	0.0350	68.9861	67.9861	0.0032	0.0031
S17	0.0110	0.1020	82.7461	81.7461	0.0078	0.0077
S18	0.1800	0.1600	13.9240	12.9240	0.0013	0.0012
S19	0.3000	0.2800	9.7929	8.7929	0.0000	0.0000
T1	0.3100	0.2700	9.4671	8.4671	0.0048	0.0047
P1	0.8500	0.7000	20.4650	19.4650	0.0103	0.0102

added of the three industry and the water resources allocation of different industries objectively. The constraints reflected the relationship between decision variables and water resources allocation clearly.

Table 1 shows the minimum water requirement of different industry and added value per unit water resources in the two planning periods. For the convenience of calculation, production value is introduced into the paper. We define production value as different industries generated value by one tone of water. It can be got by that the GDP of the industry divide the water used to produce the GDP. Table 2 represents the working population per unit GDP, discharge of waste water and COD per unit GDP and ecological water consumption. Table 3 represents water storage capacity and Environmental Capacity of Chemical Oxygen Demand (ECCOD) under different

Table 3 Water storage capacity and ECCOD capacity

p_i	0	0.01	0.05	0.1	0.25	0.5	0.75	0.9	0.95	0.99	1
Water storage capacity ($10^8 m^3$)	3.521	3.561	3.736	3.974	4.670	5.374	6.618	7.760	8.677	9.597	9.856
ECCOD capacity (t)	14084	14244	14944	15896	18680	21486	26472	31040	34708	38388	39424

Table 4 The rest input parameters

	STF (10^4yuan)	IMWD (t/p)	DCOD (t/10^4yuan)	RDST	RSTCOD	DWD (t/p)	PrRWT	SeRWT	TeRWT	EWS (t)
T=1	0.000075	198.4688	0.10676	0.80	0.9875	205.315	0.345	0.951	0.853	8633481
T=2	0.000072	193.4500	0.11495	0.85	0.9900	196.188	0.563	0.992	0.901	14323911

illegal probability levels. This study suggested that water storage capacity and ECCOD are changeless in the two planning periods. The Chemical Oxygen Demand (COD) values accorded with the requirement of national water quality classification standard (Wei 2009; Gu et al. 2012). Table 4 shows the other parameters that related to the model.

4 Results Analysis

Table 5 gives the results of SLFP model. The water resources allocation situation varies as p_i changes in this study. Through the results, we can clearly analyze the ratio between the value of water resources and the investment of water resources, the growing trend of the value of water resources. As we all known, the reasonable industrial structure is capable of promoting total industrial output value and environmental carrying capacity. So the principle of regional water resources allocation is to give priority to the industry with higher output and less pollution. According to the production value that can be obtained in the planning periods from Tables 2 and 5, the production value under each p_i is lower than the economic plan growth value in Jinchang. It can be concluded that the economic planning has already beyond the water carrying capacity in Jinchang. The main factor restrict the development of economy in Jinchang is the water resources shortage. To accomplish the economic growth plan as much as possible in the planning periods of Jinchang, the best way is to optimize the water allocation plans between the three industries as well as optimize the industrial structure following the priority rule of regional water resources allocation. According to the results from Table 5, water resources are in favor of the production of electric power, heating power and supply industry after satisfying the water demand of people's livelihood, ecological environment and the three industries. As the matter of fact, electric power, heating power and supply industry are industries with higher output and less pollution. It also indicated that the results from the SLFP model could match with the actual life. Therefore, this study is able to reflect real problems objectively and provide solutions to actual issues.

Figures 1 and 2 show the ratio of the model and the total added value of water resources under different probability level. From the figures, we can conclude that the higher p_i level, the larger the objective ratio and the added value. For example, the objective ratio (i.e., system efficiency) is rising from 0.267 to 0.31 in Fig. 1. The relationship between the ratio of model and p_i level reveals the relationship between the system benefits and violating probability levels. The system risk will increase as p_i increase, so the decision space will be relatively broad. The higher the p_i level, the larger of the system benefits and the larger of the added value. That is, we can get a

Table 5 The results of SLFP model

	P_i=0		P_i=0.25		P_i=0.5		P_i=0.75	
	T=1 (m^3)	T=2 (m^3)	*T*=1 (m^3)	*T*=2 (m^3)	*T*=1 (m^3)	*T*=2 (m^3)	*T*=1 (m^3)	*T*=2 (m^3)
S1	480	528	480	528	480	528	480	528
S2	938	1032	938	1032	938	1032	938	1032
S3	10362	11398	10362	11398	10362	11398	10362	11398
S4	33600	36960	33600	36960	33600	36960	33600	36960
S5	2800	3080	2800	3080	2800	3080	2800	3080
S6	18022	19824	18022	19824	18022	19824	18022	19824
S7	102	112	102	112	102	112	102	112
S8	666317	732949	666317	732949	666317	732949	666317	732949
S9	291	320	291	320	291	320	291	320
S10	473	521	473	521	473	521	473	521
S11	74335	81768	74335	81768	74335	81768	74335	81768
S12	133370	146707	133370	146707	133370	146707	133370	146707
S13	6387599	7026359	6387598	7026358	6387598	7026358	6387598	7026358
S14	13	14	13	14	13	14	13	14
S15	194	214	194	214	194	214	194	214
S16	2525968	2213200931	2525968	3091539575	2525968	3629703673	2525968	4580664416
S17	4438534	4882387	4438533	4882386	4438533	4882386	4438533	4882386
S18	1132193	1245412	1132192	1245412	1132192	1245412	1132192	1245412
S19	141117	155228	141117	155228	141117	155228	141117	155228
T1	5042368	5546604	5042367	5546604	5042367	5546603	5042367	5546604
P1	152798033	168077836	15279008	168077809	152798004	168077805	152798010	168077811

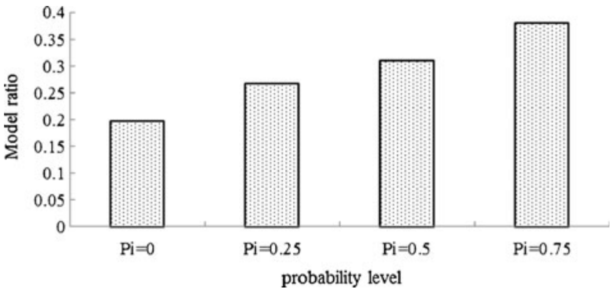

Fig. 1 Model ratio under different p_i

higher system benefits with relatively less water resources investment, but the system reliability will be reduced accordingly. While, the lower the p_i level, the smaller the system benefits, but the system reliability will be increased accordingly. The decision maker can get the ratio of output/input (value added per unit water resources) and system benefits clearly from Figs. 1 and 2. A according to the system benefits and the ratio of output/input, the decision makers will get the water use efficiency directly. As a whole, how to determine the level of p_i depends on the discussion of interested parties. In other words, decision makers should have higher corresponding knowledge background. All the results indicate that the SLFP model can be applied to allocate water resources optimally under uncertainty, and different decision schemes would be obtained under different violating probability by solving the SLFP model. Generally, in comparison SLFP model with the other optimization methods, the former has the following advantages. i), it can be used to analyze two objectives without modifying the original magnitudes; ii), it can handle the problem about ratio optimization and reflect the efficiency of system; iii), it can account for multi-uncertainties characteristics of the modeling constraints; iv), it can give the constraints a relaxation limit, so it can deal with more situation rather than extreme situation only (p_i=0), that is, it can provide the different optimization schemes under different risk probabilities; v), it can provide in-depth analysis of the interrelationships among system efficiency, the investment of water resources and system-failure risk. Therefore, the SLFP method can also be applied to other resources, such as air quality management and energy systems planning. SLFP can also be further enhanced by integrating other methods, such as fuzzy theory and interval analysis.

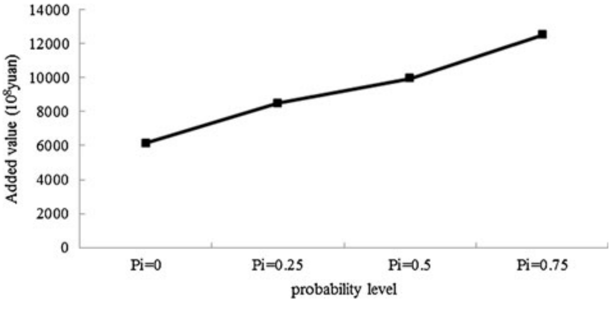

Fig. 2 Added value under different probability levels

5 Conclusion

Stochastic linear fraction programming (SLFP) model was developed to deal with water resources optimal allocation under uncertainty. Moreover, the SLFP model can handle the optimal ratio problems with random information by introducing Chance Constraint Programming (CCP). The developed model has the following advantages :(1) The model reflected the relationship between local economic development planning and water resources carrying capacity and paid more attention to the efficiency of the water resources according to actual situation. (2) Different optimization schemes were given under different risk probabilities. (3) Water quality and quantity issues were integrated in the developed model.

SLFP model was used to deal with economic planning problem in Jinchang city, Gansu Province. In this application, the local water carrying capacity and Environmental Chemical Oxygen Demand (ECCOD) were expressed by random parameters. The results of SLFP under different p_i can help decision makers make the following judgment: (1) Whether the "Twelfth Five-Year" economic planning in Jinchang can be realized; (2) How to optimize the local industrial structure; (3) How to optimize the local industrial structure (primary industry, secondary industry and tertiary industry); (4)Analysis the relationship between local economic development scale and water resources carrying capacity.

This study attempts to provide a new modeling framework for solving ratio optimization problems associated with random inputs. The results suggest that it is also applicable to other water resources management and environmental management problems, such as waste disposal management, water pollution management etc. The SLFP could be further enhanced through incorporating methods of interval analysis, fuzzy set theory and integer programming into its framework.

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