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A Cost-Effective Method to Control Seawater Intrusion in Coastal Aquifers

Hany F. Abd-Elhamid · Akbar A. Javadi

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Abstract Intrusion of seawater into coastal aquifers is considered one of the most important processes that degrade water-quality by raising the salinity to levels exceeding acceptable drinking standards. Therefore saltwater intrusion should be prevented or at least controlled to protect groundwater resources. This paper presents a cost-effective method to control seawater intrusion in coastal aquifers. This methodology ADR (Abstraction, Desalination and Recharge) includes; abstraction of saline water and recharge to the aquifer after desalination. A coupled transient density-dependent finite element model is developed for simulation of fluid flow and solute transport and used to simulate seawater intrusion. The simulation model has been integrated with an optimization model to examine three scenarios to control seawater intrusion including; abstraction, recharge and a combination system, ADR. The main objectives of the models are to determine the optimal depths, locations and abstraction/recharge rates for the wells to minimize the total costs for construction and operation as well as salt concentrations in the aquifer. A comparison between the combined system (ADR) and the individual abstraction or recharge system is made in terms of total cost and total salt concentration in the aquifer and the amount of repulsion of seawater achieved. The results show that the proposed ADR system performs significantly better than using abstraction or recharge wells alone as it gives the least cost and least salt concentration in the aquifer. ADR is considered an effective tool to control seawater intrusion and can be applied in areas where there is a risk of seawater intrusion.

Keywords Seawater intrusion · Control · Finite element · Genetic algorithm · Simulation–optimization · ADR

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1 Introduction

Saltwater Intrusion (SWI) is a major problem in coastal regions all over the world. The intrusion of saline water in groundwater is considered a special category of pollution, making groundwater unsuitable for human, industry and irrigation uses. SWI reduces the freshwater storage in coastal aquifers and in extreme cases can result in abandonment of freshwater supply wells if concentration of dissolved salts exceeds drinking water standard. Therefore, groundwater resources should be protected from saltwater intrusion, using suitable measures. Salinization presents a constraint on use of groundwater as saline groundwater should be treated before use. Remediation of groundwater could be very costly and could take a long time depending on the source and level of salinization. Different measures have been presented to control SWI in coastal aquifers. Todd (1974) presented various methods of preventing saltwater from contaminating groundwater sources including; reduction of pumping rates, relocation of pumping wells, use of subsurface barriers, natural recharge, artificial recharge, abstraction of saline water and combination techniques.

Extensive research has been carried out to investigate SWI in coastal aquifers. However, a limited amount of researches has been directed to study the control of SWI. The existing control methods are based on one or more of the abovementioned measures to control SWI. Reduction of abstraction rate aims to reduce the pumping rates and use other water resources (Scholze et al. 2002). Relocation of abstraction wells aims to move the wells further inland (Sherif and Al-Rashed 2001). Subsurface barriers prevent the inflow of seawater into the basin (Harne et al. 2006). Natural recharge aims to recharge aquifers with additional surface water (Ru et al. 2001). Artificial recharge can increase the groundwater levels, using surface spread for unconfined aquifers and recharge wells for confined aquifers. The source of water for injection may be surface water, groundwater, treated wastewater or desalinated water (Papadopoulou et al. 2005). Abstraction of saline water aims to reduce the volume of saltwater by extracting brackish water from the aquifer and disposing it to the sea (Sherif and Hamza 2001). The combination of injection of freshwater and extraction of saline water can reduce the volume of saltwater and increase the volume of freshwater (Rastogi et al. 2004; Mahesha 1996c).

Some previous studies have concentrated on the control of seawater intrusion using simulation models to study the effect of abstraction, recharge and combination systems on the intrusion of seawater by trial-and-error. Some research has been done to study the control of SWI using individual recharge or abstraction system. Kashef (1976) studied the effect of recharge wells of different patterns on the degree of saltwater retardation in confined coastal aquifers. The batteries (recharge wells) were located at various distances parallel to the shoreline; each battery consisting of equally spaced recharge wells of a certain number. It was found that, the optimum location of the batteries should lie between 0.7 L and L, where L is initial length of the intruded saltwater wedge. Mahesha (1996a) studied the effect of battery of injection wells on seawater intrusion in confined coastal aquifers. He used a quasi three-dimensional areal finite element model considering a sharp interface. He studied various conditions by changing the well spacing and intensity and duration of fresh water injection. He concluded that, spacing between wells, injection rate and duration of injection control the repulsion of the saline wedge. Mahesha (1996b) presented steady state solutions for the motion of the fresh water/seawater interface due to a series of injection wells in confined aquifer using a sharp interface finite element model. The model was used to perform parametric studies on the effect of location of the series of injection wells, spacing of the wells and fresh water injection rate on the sea water intrusion. It was found that reduction of seawater intrusion (of up to 60–90%) could be achieved through proper selection of injection rate and spacing between the wells. Also, he studied the effect of double series of injection wells and compared with the single series. He found that a double series performs slightly better than single series. Also, it was found that the staggered system of wells in the series is slightly better than the straight well system for long spacing. Sherif and Hamza (2001) used finite element model to examine the effect of abstraction of brackish water and its disposal to the sea on SWI. Sherif and Kacimov (2008) suggested pumping of brackish water, encountered between the freshwater and saline water bodies, as a method to reduce the extension of seawater intrusion. They used SUTRA to examine different pumping scenarios in the vertical view and identified the equiconcentration lines and velocity vectors for the different cases. They concluded that seawater intrusion problems could be controlled through proper pumping of saline groundwater from the coastal zone.

On the other hand, a limited amount of research has been directed on using a combination system to control SWI. Mahesha (1996c) studied the control of seawater intrusion by a series of abstraction wells for saline water alone and combination with the freshwater injection wells in confined aquifer using a vertically integrated two dimensional sharp interface model under steady state condition. It was found that the combination of injection wells with extraction wells produced excellent results to the individual cases for larger well spacing and small rates of injection. Rastogi et al. (2004) developed a simulation model to study the effect of abstraction and recharge on the intrusion of seawater. They studied the effect of recharge wells and combination of abstraction and recharge wells on saltwater intrusion. The study approved that using a combination of abstraction and recharge wells is more efficient in retarding seawater intrusion.

The setback of the combination system presented by Mahesha (1996c) and Rastogi et al. (2004) is the prohibitive cost factor involved in the construction and maintenance of the wells. Also the other methods of control have a number of limitations that can be summarized as follows. Most of these methods are costly and some of them may not be applicable in certain cases. Furthermore, they are generally temporary solutions and with the population growth and increasing demand the intrusion will be increased. The source and the cost of fresh water for injection, especially in areas that suffer from scarcity of water and the fact that the disposal of the brine into the sea can cause many environmental problems, have usually been ignored (Abd-Elhamid and Javadi 2008). In general, the principal of protection aims to increase the volume of fresh groundwater and reduce the volume of saltwater to retard the intrusion of saltwater into coastal aquifers. Not all the solutions are economically feasible because they are generally long term solutions, and the time to reach the state of dynamic equilibrium between freshwater and brackish water may take tens of years (Bear et al. 1999).

This study presents a cost-effective methodology, ADR (Abstraction, Desalination and Recharge), to control SWI in coastal aquifers. This methodology aims to overcome all or at least most of the limitations of the previous models. ADR consists of three steps; abstraction of brackish water from the saline zone, desalination of the abstracted brackish water using reverse osmosis (RO) treatment process and recharge of the treated water into the aquifer. Figure 1 shows a diagram of the ADR methodology. This method combines the advantages of these three components; abstraction of brackish water helps to reduce the volume of saline water in the aquifer and reduce the intrusion of saltwater; recharge of treated water helps to increase the fresh groundwater volume to prevent the intrusion of saltwater; and desalination of abstracted brackish water helps to produce fresh water for recharge. Desalination of brackish water using RO helps to overcome the scarcity of water in these areas. It is generally less expensive than other sources of freshwater for injection. For example, desalination of seawater has a lot of problems such as; high cost, high pollution (mainly carbon emission), and disposal of the brine. Desalinating brackish water is an efficient alternative to seawater desalination, because the salinity of brackish water is less than one-third of that of seawater. Therefore, brackish water can be desalinated at a significantly lower cost than seawater (Jaber and Ahmed 2004).

In this study, a coupled transient density-dependent finite element model has been developed for simulation of fluid flow and solute transport in soils and applied to simulate seawater intrusion in coastal aquifers. Numerical simulation models can be used to examine a limited number of design options by trial and error. However, optimization methods can be combined with simulation models to search for the optimal solution in a wide search space of design variables. A number of researchers have applied simulation models for the control of seawater intrusion seeking for optimal management strategy by trial-and-error (e.g., Mahesha (1996c) and Rastogi et al. (2004)). This has proved to be time consuming and laborious. In addition, the results obtained may not be optimal. The main reason for this is

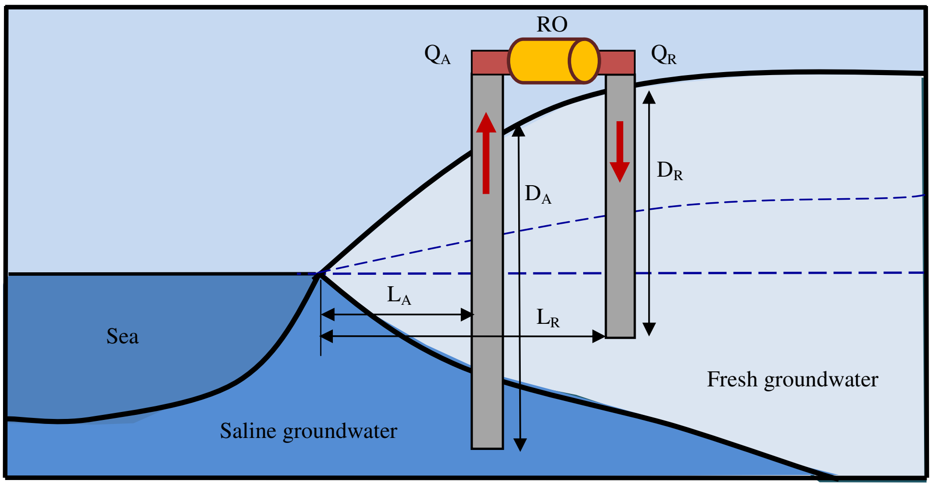

Fig. 1 Diagram of the ADR methodology

the inability of this approach to consider important physical and operational constraints. To accommodate these constraints, coupling of the simulation model with a management model is the generally adopted procedure to solve the optimization problem because of its effectiveness in finding the global optimum solution. The motivation for the development of the simulation–optimization (S/O) approach for the control of seawater intrusion is the enormous cost of groundwater remediation. The S/O approach can be used efficiently in groundwater remediation system design and also in other groundwater quality management problems. Development and application of optimization techniques in association with seawater intrusion models was presented by a number of researchers (e.g. Cheng et al. 2000; Das and Datta 2000; Bhattacharjya and Datta 2005; Qahman et al. 2005). Park et al. (2008) investigated the effects of pumping of saltwater as a measure to protect an over-exploited freshwater pumping well. They used a S/O model to identify the minimum saltwater pumping rate required to protect the over-exploited pumping well. Arvanitidou et al. (2010) applied a generic operational tool for the optimal management of coastal aquifers to a real unconfined coastal aquifer in the Greek island of Kalymnos. They combined a numerical model for prediction of the seawater intrusion due to flow perturbations and a GA. The presented management scenarios were based on the hydrodynamic control of seawater intrusion due to the operation of a network of production wells and the seasonal use of a recharge canal as an alternative method for increasing aquifer's sustainable productivity.

Rejani et al. (2009) developed two optimization models for the efficient utilization of water resources in Balasore coastal groundwater basin of Orissa in eastern India during non-monsoon periods. These included a non-linear hydraulic management model for optimal pumping, and a linear optimization model for optimal cropping pattern, both in integration with a groundwater flow simulation model. The study concluded that in order to ensure sustainable groundwater utilization in the basin, the optimal cropping pattern and pumping schedule should be adopted by the farmers. Ataie-Ashtiani and Ketabchi (2011) presented an evolutionary approach for optimal management of coastal aquifers to control saltwater intrusion. An improved Elitist Continuous Ant Colony Optimization (ECACO) algorithm was employed in this study. The objectives of the optimal management were to maximizing the total water-pumping rate, while controlling the drawdown limits and protecting the wells from saltwater intrusion. Kourakos and Mantoglou (2011) proposed a combined management plan for the island of Santorini. The plan involved: (1) desalinization of pumped water (if needed) to a potable level using reverse osmosis and (2) injection into the aquifer of biologically-treated waste water. The management plan was formulated in a multi-objective optimization framework, where simultaneous minimization of economic and environmental costs was desired, subject to a constraint so that cleaned water satisfies demand. The decision variables were the well locations and the corresponding pumping and recharge rates. They used the SEAWAT code for the simulation.

According to the literature, only a limited number of studies have concentrated on reduction of costs of construction and operation of control systems. This paper presents the development and application of S/O model to control seawater intrusion in coastal aquifers. A simulation model is integrated with a genetic algorithm (GA) optimization model to study the control of seawater intrusion. Three management scenarios have been considered; abstraction of brackish water, recharge of fresh water and combination of abstraction, recharge and desalination (ADR). The objectives of these management scenarios include minimizing the total construction and operation cost, minimizing salt concentrations in the aquifer and determining the optimal depths, locations and abstraction/recharge rates. The developed model is applied to study the control of seawater intrusion in a hypothetical case (Henry's problem) and real world problem (Biscayne aquifer at Coulter area, Florida, USA). Comparison is made between the proposed ADR system and the individual abstraction or recharge system in terms of total cost and total salt concentration in the aquifer and the amount of repulsion of seawater achieved.

2 Development of Simulation Model

The governing equations for water flow, air flow and solute transport are obtained, based on mass balance equations and law of conversation of mass for solute transport, in terms of three primary variables; pore-water pressure (u_w), pore-air pressure (u_a) and salt concentration (c). The governing differential equations for water flow, air flow and solute transport and the numerical solution of coupled fluid flow and solute transport are presented in the following sections.

2.1 Governing Equations of Fluid Flow

The theoretical formulation of hydraulic behavior in unsaturated soil describes coupled water and air flow in unsaturated soil. The governing differential equations for water flow and air flow in terms of two primary variables; pore-water pressure (u_w) and pore air pressure (u_a) can be expressed as (Javadi et al. 2008):

Water flow equation

$$C_{ww}\frac{\partial u_w}{\partial t} + C_{wa}\frac{\partial u_a}{\partial t} = \nabla\left[K_{ww}\nabla u_w\right] + \nabla\left[K_{wa}\nabla u_a\right] + \rho_w\nabla\left(K_w\nabla z\right) \tag{1}$$

Air flow equation

$$C_{aw}\frac{\partial u_w}{\partial t} + C_{aa}\frac{\partial u_a}{\partial t} = \nabla\left(K_{aw}\nabla u_w\right) + \nabla\left(K_{aa}\nabla u_a\right) + H_a\rho_{da}\nabla\left(K_w\nabla z\right) \tag{2}$$

2.2 Governing Equation of Solute Transport

The law of conversation of mass for solute transport requires that the rate of change of solute mass within a control volume is equal to the net rate at which the solute enters or leaves the control volume through the control surfaces plus the net rate at which solute is produced within the control volume by various chemical and physical processes (Istok 1989). The governing differential equation of solute transport in unsaturated soil can be written based on conservation of mass as (Javadi et al. 2008):

$$
\begin{aligned}
\frac{\partial\left(\left(\theta_w+H\theta_a\right)c\right)}{\partial t}=&-\left[\left(\frac{\partial}{\partial x}\left(v_{w_x}c\right)+\frac{\partial}{\partial y}\left(v_{w_y}c\right)\right)+\left(\frac{\partial}{\partial x}\left(v_{a_x}c\right)+\frac{\partial}{\partial y}\left(v_{a_y}c\right)\right)\right]\\
&+\left[\left\{\frac{\partial}{\partial x}\left(D_{w_{xx}}\frac{\partial}{\partial x}\left(\theta_w c\right)+D_{w_{xy}}\frac{\partial}{\partial y}\left(\theta_w c\right)\right)\right.\right.\\
&\left.+\frac{\partial}{\partial y}\left(D_{w_{yy}}\frac{\partial}{\partial y}\left(\theta_w c\right)+D_{w_{yx}}\frac{\partial}{\partial x}\left(\theta_w c\right)\right)\right\}\\
&+H\left\{\frac{\partial}{\partial x}\left(D_{a_{xx}}\frac{\partial}{\partial x}\left(\theta_a c\right)+D_{a_{xy}}\frac{\partial c}{\partial y}\left(\theta_a c\right)\right)\right.\\
&\left.\left.+\frac{\partial}{\partial y}\left(D_{a_{yy}}\frac{\partial}{\partial y}\left(\theta_a c\right)+D_{a_{yx}}\frac{\partial}{\partial x}\left(\theta_a c\right)\right)\right\}\right]\\
&-\frac{\partial}{\partial t}\left(\rho_b K_d c\right)-\left(\lambda_w\theta_w+H\lambda_a\theta_a\right)\left(1+\rho_b K_d/\theta_w\right)c=0
\end{aligned}
\tag{3}
$$

The definition of the coefficients in Eqs. 1, 2 and 3 is presented in Appendix 1.

2.3 Numerical Solution of the Governing Equations

The coupling of fluid flow and solute transport in unsaturated soil is modeled using two sets of equations. The first set of equations describes water flow and air flow and the second set describes solute transport. The nonlinear governing differential equations of fluid flow and solute transport are solved using the finite element method in the space domain and a finite difference scheme in the time domain. The solution is divided into two stages. In the first stage the flow equations for water and air are solved simultaneously for the two primary variables; pore-water pressure (u_w) and pore air pressure (u_a) and then in the second stage the solute transport equation is solved for solute concentration (C). The current work employs two dimensional quadrilateral isoparametric elements. After spatial discretization, the temporal integration is employed to obtain the transient solution of the spatially discretized equations for both fluid flow and solute transport. The numerical solution of fluid flow and solute transport governing equations is presented in Appendix 2.

2.3.1 Coupling of Fluid Flow and Solute Transport

The nonlinear governing differential equations of fluid flow and solute transport, considering the mixing between seawater and freshwater and density-dependent flow in the transition zone, are solved using the finite element method in the space domain and a finite difference scheme in the time domain. In the examples presented in this paper the pore air pressure has been considered constant and equal to atmospheric pressure. The output of the simulation model is values of pore water pressures and salt concentrations at various points within the domain. The predicted values of pore water pressure are used to calculate pressure head, ψ, and total hydraulic head, h, through Bernoulli's equations. The coupling between the flow and transport equations makes the problem of seawater intrusion highly nonlinear. Through Darcy's law and a constitutive equation relating fluid density to salt concentration, the fluid flow and solute transport equations are coupled and solved. The constitutive equation relating fluid density to salt concentration can be expressed as (Rastogi et al. 2004):

$$\rho = \rho_f(1 + \varepsilon C) \tag{4}$$

In this equation, C is the relative salt concentration, normalized with respect to the maximum salt concentration of seawater, $\varepsilon = (\rho_s - \rho_f)/\rho_f$, ρ_s is density of seawater and ρ_f is density of fresh water. The numerical solution of coupled fluid flow and solute transport is based on solving the governing equations with the boundary and initial conditions by the iterative solution scheme described above. The flow equations are solved to compute pore water pressure at all nodes. The pressure head, fluid velocity and fluid density are then calculated. The calculated velocities are used to define the dispersion coefficient for the solute transport equation. The solute transport equation is then solved for salt concentrations at every node in the domain and this process is repeated at every time step.

3 Development of Simulation–Optimization Model

Coupling of simulation model with a management model has been recently used in management of groundwater resources to predict the behavior of the system and provide optimal solutions for problems such as planning of long term water supply or preventing seawater intrusion. Only a limited number of studies have concentrated on the problems of seawater intrusion control. The proposed simulation–optimization model consists of two main components. The first is a simulation model for flow and solute transport that is used to compute heads and salt concentrations in the aquifer. The second component is a GA-based evolutionary optimization model that calls the simulation model iteratively to evaluate fitness of different alternative solutions. Genetic algorithms (GAs) have received much attention regarding their potential as global optimization techniques for problems with large and complex search spaces. Many variations of the GA have been developed, but the algorithm implemented here can be summarized as being a simple binary encoded GA with roulette wheel selection, uniform crossover and an elitist replacement strategy. The main objective of this study is to determine a cost-effective method to control the intrusion of seawater in coastal aquifers. Three main factors contributing to the construction and operation costs are considered as the decision variables; the depths of abstraction and/or recharge wells, the locations of wells (distance from the seashore) and abstraction/recharge rates.

The problem is formulated as an optimization problem of finding the optimum management options in terms of depth, location and abstraction/recharge rates for wells to reduce the construction and operation costs and minimize salt concentrations in the aquifer. Different sets of values for the design parameters are generated and evolved using the principles of genetic algorithm and the behavior of each of the generated management systems (corresponding to a set of design parameters) is analyzed using the finite element model to evaluate its fitness in competition with other generated systems. During successive generations the genetic algorithm operators gradually improve the fitness of the solution and direct the search towards the optimal or near optimal solution.

The simulation–optimization model developed in this work is based on the integration of a GA with a coupled transient density-dependent FE simulation model for flow and solute transport. In the developed framework, the simulation model is repeatedly called by the GA to calculate the response of the system to each set of design variables generated by the GA. The simulation model is used to compute pressure heads and salt concentrations for every node in the domain. These values for pressure heads and salt concentrations are returned to the GA routine and used to evaluate the objective function values and determine the fitness of each solution in competition with other generated solutions. The steps followed for finding the optimal solution can be summarized as:

1. An initial population of decision variables is generated randomly in the form of binary strings. The binary strings are then decoded to real values representing the design parameters of the system.
2. The values of the design variables are used by the FE simulation model to compute the hydraulic heads and salt concentrations at different points within the domain.
3. The calculated values of heads and concentrations are used by the GA process to evaluate the objective function (fitness) values for all members of the population.
4. The processes of selection, crossover and mutation are performed in the genetic algorithm procedure and a new population of individuals (new generation) is created.
5. The new generation replaces the current generation and the entire process (steps 2–4) is repeated until convergence is achieved or the maximum number of generations is exceeded.

3.1 Formulation of the Management Models

In this work, a GA-based optimization approach together with a FE simulation model for seawater intrusion are used within a management model to evolve an optimal management strategy. The main objective of the simulation–optimization models is to minimize the construction and operation costs by identifying the optimal depths, locations and abstraction and/or recharge rates for abstraction/recharge wells to control the intrusion of seawater. Three management models are developed in this work to control seawater intrusion in coastal aquifers; abstraction of brackish water, recharge of fresh water and combination of abstraction, recharge and desalination (ADR). The objective functions of the three management models can be represented as:

Management model 1 (Abstraction of brackish water)

$$\text{Min } f = P_1 \sum_{i=1}^{N} c_i + P_2 \sum_{i=1}^{N} Q_{A_i} (C_A + C_T) + P_3 \sum_{i=1}^{N} D_{A_i} (C_{DW}) \tag{5}$$

Management model 2 (Recharge of fresh water)

$$\text{Min } f = P_1 \sum_{i=1}^{N} c_i + P_4 \sum_{i=1}^{N} Q_{R_i} (C_{PW} + C_R) + P_5 \sum_{i=1}^{N} D_{R_i} (C_{DW}) \tag{6}$$

Management model 3 (Abstraction, Desalination and Recharge ADR)

$$\text{Min } f = P_1 \sum_{i=1}^{N} c_i + P_2 \sum_{i=1}^{N} Q_{A_i} (C_A + C_T) + P_3 \sum_{i=1}^{N} D_{A_i} (C_{DW}) + P_4 \sum_{i=1}^{N} Q_{R_i} (C_R) + P_5 \sum_{i=1}^{N} D_{R_i} (C_{DW}) \tag{7}$$

where

f	is the objective function in terms of the total cost
N	is the total number of nodes in the domain
D_A	is the depth of abstraction well (m)
D_R	is the depth of recharge well (m)
Q_R	is the recharge rate (m^3/s)
Q_A	is the abstraction rate (m^3/s)
c_i	is total amount of solute mass in the aquifer (mg/l)
C_A	is the cost of abstraction (US\$/m^3)
C_T	is the cost of treatment (US\$/m^3)
C_R	is the cost of recharge (US\$/m^3)
C_{PW}	is the price of water (US\$/m^3)
C_{DW}	is the cost of installation/drilling of well (US\$/m)
P_1, P_2, P_3, P_4 and P_5	are the weighting parameters

The management objectives are achieved within a set of constraints includeing side constraints for well depths, well locations and abstraction/recharge rates as: $Q_{min} < Q < Q_{max}$, $L_{min} < L < L_{max}$ and $D_{min} < D < D_{max}$. In these management models the costs are introduced based on the available data from literature. According to the literature these costs are considered as; cost of installation/drilling of well per unit depth: US\$1000, cost of abstraction per cubic meter: US\$0.42, cost of recharge per cubic meter: US\$0.48, cost of treatment (i.e. desalination) per cubic meter: US\$0.6 and price of water per cubic meter: US\$1.5 (Qahman et al. 2005).

4 Validation and Application of the Simulation Model

The developed simulation model is validated by application to a number of examples including Henry's problem and then applied to a real world case study (Biscayne aquifer, Florida, USA) to simulate saltwater intrusion in coastal aquifers.

4.1 A Hypothetical Case Study (Henry's Problem)

The developed finite element model is validated by application to Henry's problem to simulate seawater intrusion in a confined coastal aquifer. This case involves seawater intrusion into a confined aquifer in a vertical cross section. Henry's problem

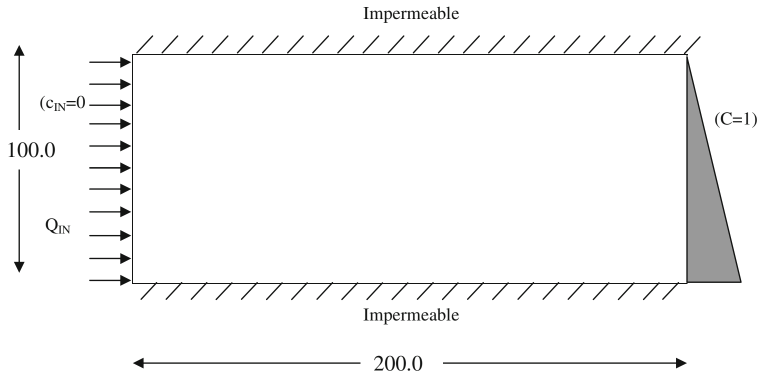

Fig. 2 Boundary conditions of Henry's problem

was studied by various researchers including Pinder and Cooper (1970), Lee and Cheng (1974), Segol et al. (1975), Huyakorn et al. (1987), Frind (1982), Cheng et al. (1998) and Rastogi et al. (2004). The aquifer is homogenous and isotropic and is bounded above and below by impermeable strata. The aquifer is exposed to seawater on the right hand side and a constant freshwater flux on the left hand side. No flow occurs across the top and bottom boundaries. The source of freshwater is set along the left vertical boundary of the domain and is implemented by employing source nodes, which inject freshwater at a rate of Q_{IN} (or the equivalent hydrostatic head) and concentration of c_{IN}. The prescribed pressure along the right vertical boundary is set at hydrostatic seawater pressure with water density of $\rho_s = 1025$ kg/m^3. The boundary conditions are defined in Fig. 2. The soil parameters used in Henry's problem are summarized in Table 1. Freshwater concentrations ($c = 0$) and natural steady-state pressures are set as the initial conditions everywhere in the aquifer. The domain considered is 100.0 m high and 200.0 m long. The finite element mesh consists of 661 nodes and 200 elements, each of size 10×10 m. The seawater wedge is chosen to be represented by 0.5 isochlor, which is an approach adopted by many researchers.

Henry (1964) and some other researches assumed that dispersion is represented by a constant coefficient of diffusion, rather than being velocity-dependent. However, Rastogi et al. (2004) considered the dispersion coefficients to be velocity-dependent. They presented an analysis using values for longitudinal and transverse dispersivities

Table 1 The parameters used in Henry's problem

D_m: coefficient of water molecular diffusion (m^2/s)	6.6×10^{-6}
α_T: transverse dispersivity (m)	0.0
α_L: longitudinal dispersivity (m)	0.0
N: porosity	0.35
M: fluid viscosity (kg/m)	0.001
Q_{in}: inland fresh water flux (m^3/s)	6.6×10^{-5}
g: gravitational acceleration (m/s^2)	9.81
ρ_w: density of fresh water (kg m^{-3})	1000
ρ_s: density of sea water (kg m^{-3})	1025
K: hydraulic conductivity (m/s)	1×10^{-2}
K: permeability (m^2)	1×10^{-9}

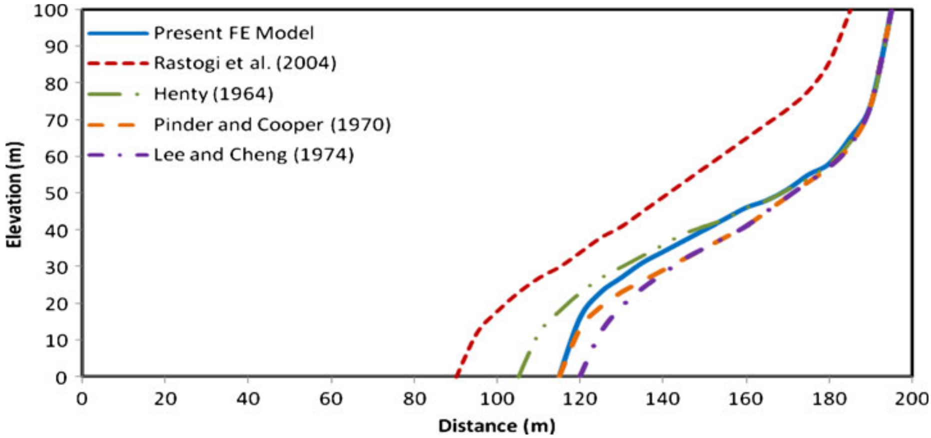

Fig. 3 Isochlor distribution for Henry's problem

α_L and α_T as 0.5 and 0.1 m respectively. This case is also modeled using the developed model and the results are compared with a number of known solutions from the literature. Figure 3 shows the results in terms of position of 0.5 isochlors obtained by the present model compared with the results of Henry (1964), Pinder and Cooper (1970), Lee and Cheng (1974) and Rastogi et al. (2004). The present model results are consistent with the existing solutions in terms of the shape of the interface and are very close to those reported by Pinder and Cooper (1970) and Lee and Cheng (1974).

4.2 Real World Case Study (Biscayne Aquifer, Florida, USA)

Lee and Cheng (1974) presented a steady-state 2-D finite element model using triangular elements to study saltwater intrusion problem. They applied their model to simulate saltwater intrusion to Biscayne aquifer at Coulter area, Florida, where a field investigation has been made by Kohout (1960). The developed model is applied to simulate saltwater intrusion into Biscayne aquifer and the results are compared with Lee and Cheng (1974) and the observations of Kohout (1960).

The geological and hydraulic aspects of Biscayne aquifer have been investigated and reported by Kohout (1960). The study region considered in this study is 300 × 30 m similar to the one considered by Lee and Cheng (1974) and Kohout (1960). Coefficient of permeability of 3×10^{-10} m/s and porosity of 0.39 are considered. The longitudinal and transverse dispersivities α_L and α_T are taken as 5.0 and 0.5 m, respectively. The densities of fresh water ρ_f and seawater ρ_{fs} are 1000 and 1025 kg/m^3 respectively. The dispersion coefficients in x-direction D_x and y-direction D_y are velocity-dependant. The same hydraulic parameters and boundary conditions are considered to compare the results with Lee and Cheng (1974) and Kohout (1960). The schematic sketch of the Biscayne aquifer at Coulter area and the corresponding boundary conditions applied in this case are shown in Fig. 4. The domain is represented by quadrilateral isoparametric element, the mesh consisting of 250 elements and 861 nodes.

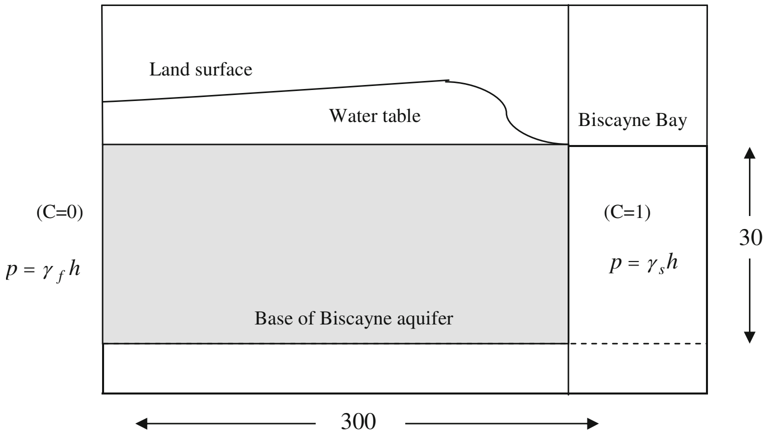

Fig. 4 Schematic sketch and boundary conditions of Biscayne aquifer, Florida

The flow and transport in Biscayne aquifer is studied to test the applicability of the present model to simulate saltwater intrusion in real world coastal aquifers. The results are compared with those of Lee and Cheng (1974) and the observations of Kohout (1960). Figure 5 shows 1, 0.75, 0.5 and 0.25 isochlor contours obtained by the current model and those of Lee and Cheng (1974), who compared their results with the observations of Kohout (1960). Good qualitative agreement is observed between current predictions, those of Lee and Cheng (1974) and observations of Kohout (1960).

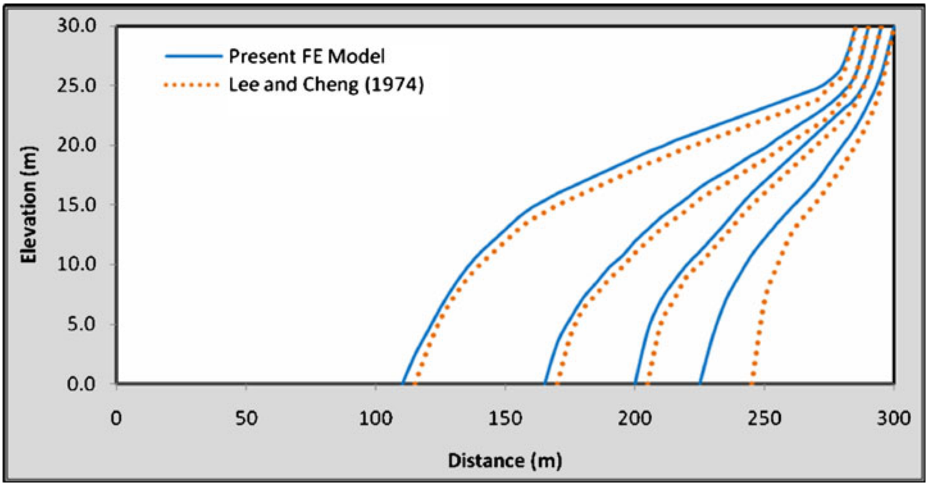

Fig. 5 1, 0.75, 0.5 and 0.25 isochlor counters in Biscayne aquifer, Florida

5 Application of the Simulation–Optimization Model to Control Seawater Intrusion

The developed simulation–optimization model is applied to Henry's problem and a real case study (Biscayne aquifer, Florida, USA) to study the control of seawater intrusion using ADR method and compare the results with the other two scenarios. The results are presented and a comparison is presented between the three scenarios in terms of total cost, total amount of water abstracted or recharged and total salt concentration remained in the aquifer.

5.1 A Hypothetical Case Study (Henry's Problem)

The developed simulation–optimization model is applied to Henry's problem to study the control of seawater intrusion. The GA is applied using the following parameters: population size = 100, probability of crossover = 0.7 and probability of mutation = 0.03 (Qahman et al. 2005). The upper and lower limits for the side constraints are taken as: $Q_{\min} = 0$; $Q_{\max} = 0.1\ \mathrm{m}^3/\mathrm{s}$;$L_{\min} = 0.$; $L_{\max} = 200$ m; $D_{\min} = 0.$ and $D_{\max} = 100$ m. The simulation–optimization model is used to determine the optimal values of decision variables; depth and location of wells and abstraction/recharge rates subject to constraints which have been defined for the three management models presented above. The objective function for the three models is to minimize the total construction and operation costs and the total salt concentration in the aquifer associated with management options.

Results and Discussion The results in terms of the optimal depths, locations and abstraction rates for the wells, together with the corresponding total cost are summarized in Table 2. The final concentration distribution for this model is shown in Fig. 6 in the form of 0.5 isochlors. In management model 1 the objective is to determine the optimum depth, location, abstraction rate for the abstraction well to minimize the cost of construction and operation of the system and the salt concentrations in the aquifer. The construction costs include installation and drilling costs, while the operation costs include abstraction and treatment costs. Using management model 1 the value of the total cost is determined as US\$2.62 million per year. The optimal

Table 2 Summary of the results obtained from the simulation–optimization models for the hypothetical case study

Model	Norm. L	Norm. D	Q (m^3/ s)	Norm. C	Q(Mm^3/year)	F(cost US$/year)
No management model	No abstraction or recharge wells have been used			167		
Abstraction well						
A	50	90	−0.083	149	2.6	2.62E+6
Recharge well						
R	90	60	0.095	151	3.0	5.72E+6
Combination system						
ADR						
A	50	90	−0.048	142	1.5	1.32E+6
R	110	80	0.018		0.5	

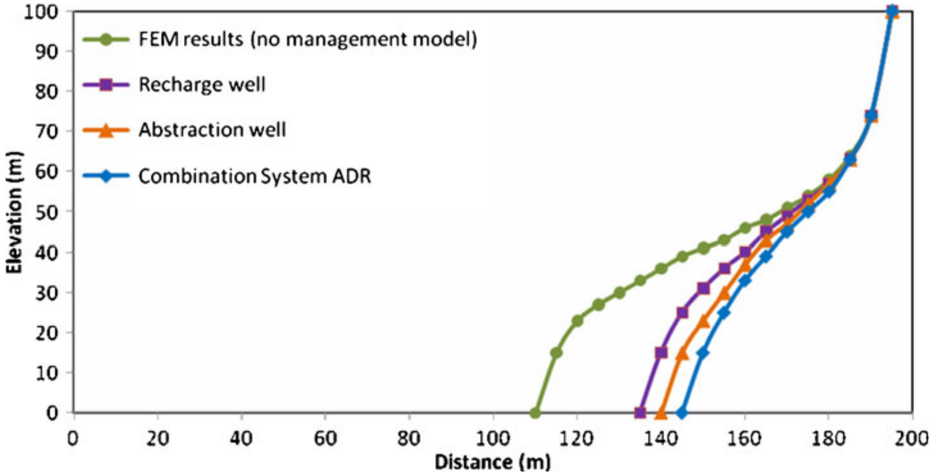

Fig. 6 0.5 isochlors from simulation–optimization models for the hypothetical case

depth is 90 m, the optimal location is 50 m from the seashore and the optimal abstraction rate is 0.083 m^3/s while the total concentration in the aquifer is reduced from 167 to 149 mg/l.

In the second management model the objective is to determine the optimal well location, depth and recharge rate for the recharge well to minimize total construction and operation costs of using recharge well to control seawater intrusion. Using management model 2 the total cost is US$5.72 million per year, the optimal depth is 60 m, the optimal location is 90 m from the seashore and the optimal abstraction rate is 0.095 m^3/s while the total concentration has reduced from 167 to 151 mg/l. The third management model is based on combination of management models 1 and 2. This model seeks to minimize the total construction and operation costs of the combined ADR system. Using management model 3 the value of the minimized objective function is US$1.32 million per year. The optimal depths for abstraction and recharge wells are 90 and 80 m respectively; the optimal locations for abstraction and recharge wells are 50 and 110 m from the seashore and the optimal rates for abstraction and recharge wells are 0.018 and 0.048 m^3/s respectively. The total concentration in the aquifer has reduced from 167 to 142 mg/l.

A comparison between the combined ADR system and the individual abstraction or recharge system is made in terms of total cost and total salt concentration in the aquifer and abstraction/recharge rates are presented in Fig. 7. From Figs. 6 and 7 it can be concluded that: using recharge wells alone has reduced the total salt concentration in the aquifer from 167 mg/l where no management model is used (N) to 151 mg/l through recharging 3 Mm^3/year of fresh water at a cost of US$5.72 million/year. Using abstraction wells alone reduced the salt concentration to 149 mg/l through abstracting 2.6 Mm^3/year of saline water at cost of US$2.62 million per year. However, using a combination of abstraction, desalination and recharge (ADR) reduced salt concentration to 142 mg/l through abstraction of 1.5 Mm^3/year of saline water and recharge of 0.5 Mm^3/year with cost of US$1.32 million/year.

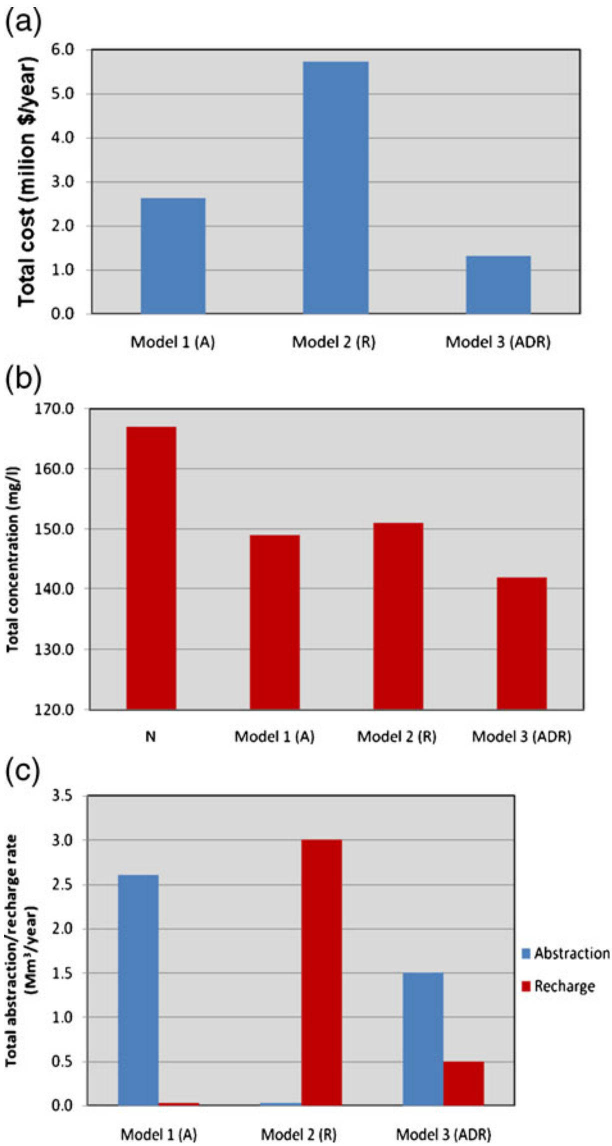

Fig. 7 Comparison between the results of model 1, 2 and 3 for: **a** total costs, **b** total salt concentrations in the aquifer and **c** total abstraction/recharge rates for the hypothetical case study

5.2 Real World Case Study (Biscayne Aquifer, Florida, USA)

In this case the developed simulation–optimization model is applied to a real case study to control saltwater intrusion in a coastal aquifer in Biscayne aquifer. The GA is applied using the same as in the previous case. The simulation–optimization model is applied to minimize the costs and salt concentration in the aquifer by determining the optimal depth, location and abstraction/recharge rates for the wells.

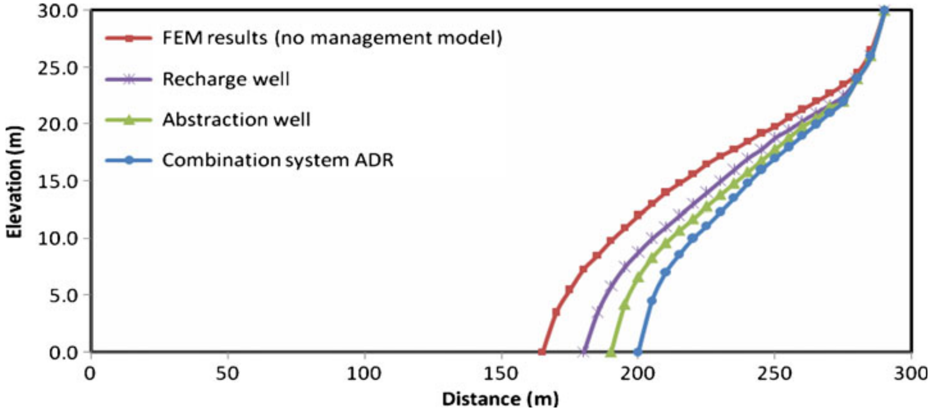

Fig. 8 0.5 isochlor distribution from S/O models for the hypothetical case

6 Results and Discussion

The final optimized concentration distribution for this example is summarized in Fig. 8. The results obtained from the simulation–optimization model for the three management models are presented in Table 3. Using management model 1 the total cost is determined as US$0.25 million per year. The optimal depth is 24 m, the optimal location is 66 m from the sea shore and the optimal abstraction rate is 0.008 m^3/s while the total concentration in the aquifer is reduced from 305 to 248 mg/l. Using management model 2 the total cost is US$0.5 million per year. The optimal depth is 24 m, the optimal location is 156 m from the sea shore and the optimal abstraction rate is 0.009 m^3/s while the total concentration in the aquifer is reduced from 305 to 255 mg/l. Using ADR in management model 3 the total cost is US$0.13 million per year. The optimal depths for abstraction and recharge wells are 24 and 18 m respectively, the optimal locations for abstraction and recharge wells are 72 and 186 m respectively from the sea shore and the optimal abstraction rates for abstraction and recharge wells are 0.005 and 0.002 m^3/s respectively while the total concentration in the aquifer is reduced from 305 to 227 mg/l.

Table 3 Summary of the results obtained from the simulation–optimization models for the real case study

Model	Norm. L	Norm. D	Q (m^3/s)	Norm. C	Q (Mm^3/year)	F (cost US$/year)
No management model	No abstraction or recharge wells have been used			305		
Abstraction well						
A	66	24	−0.008	248	0.252	0.25E+6
Recharge well						
R	156	24	0.009	255	0.283	0.5E+6
Combination system ADR						
A	72	24	−0.005	227	0.158	0.13E+6
R	186	18	0.002		0.063	

A comparison between the combined ADR system and the individual abstraction or recharge system is made in terms of total cost and total salt concentration in the aquifer and abstraction/recharge rates and the results are shown in Fig. 9. From Figs. 8 and 9 it can be seen that using the proposed ADR system in model 3 reduced salt concentration from 305 to 227 mg/l which is less than the amount of salt concentration achieved by model 1 (248 mg/l) or 2 (255 mg/l). Using ADR requires abstraction of 0.158 Mm^3/year of saline water and recharge of 0.063 Mm^3/year of fresh water which is less than the amount of water abstracted using model 1

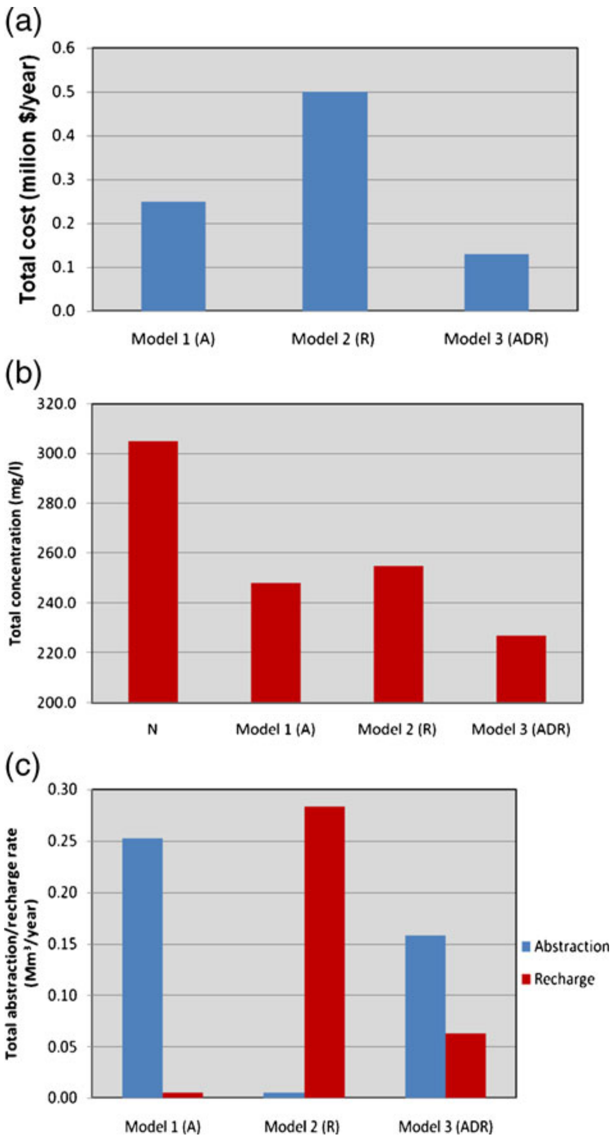

Fig. 9 Comparison between the results of model 1, 2 and 3 for: **a** total costs, **b** total salt concentrations in the aquifer and **c** total abstraction/recharge rates for the real case study

(0.252 Mm^3/year) or recharged using model (0.283 Mm^3/year). It is also gives the least cost (US$0.13 million per year) compared with the cost for recharge (US$0.5 million per year) and abstraction (US$0.25 million per year) which represents 50% of the abstraction costs and 25% of the recharge costs.

Although in the cases presented in this paper, ADR has shown to be the most effective method, in practice, there might be exceptional cases in which other methods could prove more efficient. For example, in cases where the transition zone is close to the sea, the ADR methodology may not be effective. This is mainly because the desalination may involve water with very high salinity which could increase desalination costs. In this case, abstraction of saline water and disposal to the sea could be more cost effective. Also, if sufficient source of freshwater is available in the area, recharge of fresh water could be more efficient.

7 Conclusion

Seawater intrusion is a major problem in the coastal regions all over the world that should be controlled to protect groundwater resources from depletion. This study presented a cost-effective method (ADR) to control seawater intrusion in coastal aquifers. ADR method involves; abstraction of saline water, desalination of the abstracted water and recharge of desalinated water. The development and application of S/O model was presented to study the control of seawater intrusion in coastal aquifers using three management scenarios: abstraction of brackish water, recharge of fresh water and the combined system ADR. A coupled fluid flow and solute transport model was developed, validated and applied to simulate seawater intrusion in coastal aquifers. GA optimization technique was used to search for optimal solution of the problem using the three scenarios. The developed simulation–optimization model was applied to evaluate the three management scenarios to control seawater intrusion in coastal aquifers and to determine the optimal locations, depths and rates of abstraction and/or recharge wells to minimize the construction and operation costs. The efficiencies of the three management scenarios were examined and compared. The results show that all the three scenarios could be effective in controlling seawater intrusion but using ADR (model 3) is the most efficient method that resulted in the least cost and salt concentration. ADR produced excellent results as compared to individual abstraction or recharge wells. For the case studies considered in this paper, the total costs of ADR was about 50% less compared with the case of abstraction wells only and about 75% less in comparison with the case of using recharge wells only. The reduction in costs was due to smaller abstraction and recharge rates and the use of treated brackish water for recharge to overcome the problem of scarcity of water which is common in coastal areas. The amount of abstracted and treated water was larger than the amount required for recharge; therefore the remaining treated water could be directly used for different proposes. ADR is considered an efficient method to control saltwater intrusion and helps to repulse seawater intrusion. Finally, ADR appears to be more efficient and more practical, since it is a cost-effective method to control seawater intrusion in coastal aquifers. It can be used for sustainable development of water resources in coastal areas where it provides a new source of water coming from treated water for recharge or human uses.

Appendix 1: Governing Equations

Governing Equations of Fluid Flow

The governing differential equations for water flow and air flow were presented in Eqs. 1 and 2. The coefficients C and K in these equations are defined as:

$$C_{ww} = -n\left(\rho_w - \rho_v\right)\frac{\partial S_w}{\partial s} - nS_a\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s} C_{wa} = n\left(\rho_w - \rho_v\right)\frac{\partial S_w}{\partial s} + nS_a\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s}$$

$$C_{aw} = -n\rho_{da}\left(H-1\right)\frac{\partial S_w}{\partial s} + n\left(S_a + HS_w\right)\left(\frac{R_v}{R_{da}}\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s}\right)$$

$$C_{aa} = n\rho_{da}\left(H-1\right)\frac{\partial s_w}{\partial s} + n\left(S_a + HS_w\right)\left(\frac{R_v}{R_{da}}\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s}\right)$$

$$K_{ww} = \frac{\rho_w K_w}{\gamma_w} + \rho_w\frac{D_{atms}V_v n}{\rho_w}\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s} K_{wa} = \rho_v K_a - \rho_w\frac{D_{atms}V_v n}{\rho_w}\rho_0\frac{\partial h}{\partial \psi}\frac{\partial \psi}{\partial s}$$

$$K_{aw} = \frac{H\rho_{da}}{\gamma_w}K_w K_{aa} = K_a\nabla u_w$$

In these equations, K_w is the conductivity of water [L][T]$^{-1}$, K_a is the conductivity of air [L][T]$^{-1}$, n is the porosity of the soil, S_w is the degree of saturation of water, S_a is the degree of saturation of air, ρ_w, ρ_v, ρ_0 and ρ_{da} are densities of water, water vapor, saturated water vapor and dry air respectively [M][L]$^{-3}$, s is the soil suction [M][L]$^{-1}$[T]$^{-2}$, the suction is the difference between pore air pressure and pore water pressure, V_v is the mass flow factor, D_{atms} is the molecular diffusivity of vapor through air, γ_w is the unit weight of water [M] [L]$^{-2}$[T]$^{-2}$, ψ is the capillary potential [L], h is the relative humidity, ∇z is the unit normal oriented downwards in the direction of the force of gravity, H_a is Henry's volumetric coefficient of solubility, R_{da} and R_v are the specific gas constants for dry air and water vapor respectively. In the above equations, the velocities of pore water and pore air are based on the generalized Darcy's law. The pore air is considered to exist in the forms of bulk air and dissolved air. The portion of dry air in the pore water is described using Henry's law. Two major components affecting the flow of water in unsaturated soils include the liquid transfer and vapor transfer. The vapor transfer in unsaturated soils is a result of the diffusive and pressure flows. The part of the vapor that is transferred by the pressure flow is governed by the movement of the air mixture and is described by Darcy's law while the part transferred by diffusion is governed by the theory of Phillip and de Vries (1957).

Governing Equation of Solute Transport

The governing differential equation of solute transport in unsaturated soil was presented in Eq. 3. In Eq. 3 the term in the left hand side is the rate of change of solute mass in the control volume. In the right hand side of the equation, the first two terms represent the effect of advection, the following four terms represent the effect of mechanical dispersion in pore water and pore air, the seventh term shows the effect of adsorption/desorption of solute on the solid skeleton of the soil and the last term represents the effect of radioactive decay or biodegradation (Javadi et al.

2008). In this equation v_{w_x} and v_{w_y} are the components of apparent groundwater velocity in the x and y directions respectively $[L][T]^{-1}$; v_{a_x} and v_{a_y} are the components of apparent air velocity in the x and y directions $[L][T]^{-1}$, θ_w and θ_a are the volumetric water content and volumetric air content respectively, ρ_b is the bulk density of the porous medium $[M][L]^{-3}$, K_d is the equilibrium distribution coefficient $[L]^3[M]^{-1}$, λ_w and λ_a are the chemical reaction rates for water and air respectively $[T]^{-1}$, $D_{w_{xx}}$, $D_{w_{xy}}$, $D_{w_{yx}}$, $D_{w_{yy}}$ are the coefficients of dispersivity tensor for water $[L][T]^{-1}$, H is the Henry's constant ($H = \frac{c_a}{c_w}$), c_a and c_w are the concentrations in air and water phases respectively and $D_{a_{xx}}$, $D_{a_{xy}}$, $D_{a_{yx}}$, $D_{a_{yy}}$ are the coefficients of dispersivity tensor for air $[L][T]^{-1}$. The coefficients of mechanical dispersion can be computed from the following expressions:

$$D_{w_{xx}} = \alpha_{L_w} \frac{\bar{v}^2_{w_x}}{|v_w|} + \alpha_{T_w} \frac{\bar{v}^2_{w_y}}{|v_w|} + D_{m_w} \tag{8}$$

$$D_{w_{yy}} = \alpha_{T_w} \frac{\bar{v}^2_{w_x}}{|v_w|} + \alpha_{L_w} \frac{\bar{v}^2_{w_y}}{|v_w|} + D_{m_w} \tag{9}$$

$$D_{w_{xy}} = D_{w_{yx}} = \left(\alpha_{L_w} - \alpha_{T_w}\right) \frac{\bar{v}_{w_x}\bar{v}_{w_y}}{|v_w|} + D_{m_w} \tag{10}$$

$$D_{a_{xx}} = \alpha_{L_a} \frac{\bar{v}^2_{a_x}}{|v_a|} + \alpha_{Ta} \frac{\bar{v}^2_{a_y}}{|v_a|} + D_{m_a} \tag{11}$$

$$D_{a_{yy}} = \alpha_{T_a} \frac{\bar{v}^2_{a_x}}{|v_a|} + \alpha_{L_a} \frac{\bar{v}^2_{a_y}}{|v_a|} + D_{m_a} \tag{12}$$

$$D_{a_{xy}} = D_{a_{yx}} = \left(\alpha_{L_a} - \alpha_{Ta}\right) \frac{\bar{v}_{a_x}\bar{v}_{a_y}}{|v_a|} + D_{m_a} \tag{13}$$

where, α_{Lw}, α_{T_w}, α_{L_a} and α_{T_a} are the longitudinal and transverse dispersivity coefficients for water and air phases [L], D_{m_w} and D_{m_a} are the coefficients of molecular diffusion for water and air $[L]^2[T]^{-1}$, $\bar{v}_{w_x}$ and $\bar{v}_{w_y}$ are the components of the pore water velocity in x and y directions ($\bar{v}_{w_x} = \frac{v_x}{\theta_w}, \bar{v}_{w_y} = \frac{v_y}{\theta_w}$), $\bar{v}_{a_x}$ and $\bar{v}_{a_y}$ are the components of the pore air velocity in x and y directions ($\bar{v}_{a_x} = \frac{v_x}{\theta_a}, \bar{v}_{a_y} = \frac{v_y}{\theta_a}$), and $|v_w|$ and $|v_a|$ are the magnitudes of the water and air velocities respectively ($|v_w| = \sqrt{\bar{v}^2_{w_x} + \bar{v}^2_{w_y}}, |v_a| = \sqrt{\bar{v}^2_{w_x} + \bar{v}^2_{w_y}}$).

Equation 3 can be written using simpler notations as following:

$$\begin{aligned} &\frac{\partial((\theta_w + H\theta_a)c)}{\partial t} + \frac{\partial}{\partial t}(\rho_b K_d C) + \nabla(v_w c) + \nabla(v_a c) - \nabla(\theta_w D_w \nabla_c) - \nabla(\theta_a D_a \nabla_c) \\ &\quad + (\lambda_w \theta_w + H\lambda_a \theta_a)(1 + \rho_b K_d/\theta_w)c = 0 \end{aligned} \tag{14}$$

or in a more compact form

$$\frac{\partial\left(\theta c\right)}{\partial t} + \nabla\left(vc\right) - \nabla\left(D\nabla c\right) + \lambda c = 0 \tag{15}$$

where,

$v = \left(v_{w_x} + v_{w_y}\right) + H\left(v_{a_x} + v_{a_y}\right);$

$\lambda = (\lambda_w\theta_w + H\lambda_a\theta_a)\,R$

$D = \theta_w\left[\left(D_{wxx} + D_{wxy}\right) + \left(D_{wyy} + D_{wyx}\right)\right] + H\theta_a\left[\left(D_{axx} + D_{wxy}\right) + \left(D_{ayy} + D_{ayx}\right)\right]$

$\theta = (R\theta_w + H\theta_a)$

$R =$ retardation factor.

The governing equation for saturated soil can be obtained by setting $\theta_a = 0$ and H = 0 in the above equations.

Appendix 2: Numerical Solution of the Governing Equations

Numerical Solution of Fluid Flow Equations

The governing differential equations for water flow and air flow were defined in Section 2, in terms of two primary unknowns; u_w and u_a. The spatial discretization of the governing differential equation for water flow and air flow can be written as (Javadi et al. 2008)

$$C_{ww}\frac{\partial u_{ws}}{\partial t} + C_{wa}\frac{\partial u_{as}}{\partial t} + K_{ww}u_{ws} + K_{wa}u_{as} = f_w \tag{16}$$

$$C_{aw}\frac{\partial u_{ws}}{\partial t} + C_{aa}\frac{\partial u_{as}}{\partial t} + K_{aw}u_{ws} + K_{aa}u_{as} = f_a \tag{17}$$

where,

$$C_{ww} = \sum_{e=1}^{m}\int_{\Omega^e}\left[N^T C_{ww} N\right]d\Omega^e; \quad ; \quad C_{wa} = \sum_{e=1}^{m}\int_{\Omega^e}\left[N^T C_{wa} N\right]d\Omega^e;$$

$$C_{aw} = \sum_{e=1}^{m}\int_{\Omega^e}\left[N^T C_{aw} N\right]d\Omega^e; \quad ; \quad C_{aa} = \sum_{e=1}^{m}\int_{\Omega^e}\left[N^T C_{aa} N\right]d\Omega^e;$$

$$K_{ww} = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T K_{ww}\nabla N\right]d\Omega^e \quad ; \quad K_{wa} = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T K_{wa}\nabla N\right]d\Omega^e$$

$$K_{aw} = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T K_{aw}\nabla N\right]d\Omega^e \quad ; \quad K_{aa} = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T K_{aa}\nabla N\right]d\Omega^e;$$

$$f_w = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T\left(K_w\rho_w\nabla z\right)\right]d\Omega^e - \sum_{e=1}^{m}\int_{\Gamma^e}N^T\left(\rho_w\hat{v}_{wn} + \rho_w\hat{v}_{vd} + \rho_w\hat{v}_{va}\right).d\Gamma^e$$

$$f_a = \sum_{e=1}^{m}\int_{\Omega^e}\left[\nabla N^T(K_w\rho_{da}H_a\nabla z)\right]d\Omega^e - \sum_{e=1}^{m}\int_{\Gamma^e}N^T\rho_{da}\left(\hat{v}_{fn} + \hat{v}_{an}\right).d\Gamma^e$$

In the above equations, u_{ws} and u_{as} are the approximate nodal values of pore-water and pore-air pressures, $\hat{v}_{wn}$, $\hat{v}_{vd}$, $\hat{v}_{va}$ the approximated water velocity normal to the boundary surface, the approximated diffusive vapor velocity normal to the boundary surface and the approximated pressure vapor velocity normal to the boundary surface respectively; Γ^e is the element boundary surface; $\hat{v}_{fn}$ is the approximated velocity of free dry air; $\hat{v}_{an}$ is the approximated velocity of dissolved dry air normal to the boundary and N is the shape function.

The spatially discretized equations for coupled flow of water and air (Eqs. 16 and 17) can be combined in a matrix form as:

$$\begin{bmatrix} K_{ww} & K_{wa} \\ K_{aw} & K_{aa} \end{bmatrix} \cdot \begin{bmatrix} u_{ws} \\ u_{as} \end{bmatrix} + \begin{bmatrix} C_{ww} & C_{wa} \\ C_{aw} & C_{aa} \end{bmatrix} \cdot \begin{bmatrix} \dot{u}_{ws} \\ \dot{u}_{as} \end{bmatrix} = \begin{bmatrix} f_w \\ f_a \end{bmatrix} \tag{18}$$

where,

$$\dot{u}_{ws} = \frac{\partial u_{ws}}{\partial t} \qquad \dot{u}_{as} = \frac{\partial u_{as}}{\partial t}$$

Equation 18 can be rewritten in the following form (Istok 1989):

$$K\left[\varphi\right] + C\left[\frac{\partial \varphi}{\partial t}\right] = [F] \tag{19}$$

where, K is the global conductance matrix, C is the global capacitance matrix, F is the vector of source or sink and ϕ is the vector of unknowns. These can be written as following:

$$K = \begin{bmatrix} K_{ww} & K_{wa} \\ K_{aw} & K_{aa} \end{bmatrix}; \; C = \begin{bmatrix} C_{ww} & C_{wa} \\ C_{aw} & C_{aa} \end{bmatrix}; \; F = \begin{bmatrix} f_w \\ f_a \end{bmatrix}; \; \varphi = \begin{bmatrix} \dot{u}_{ws} \\ \dot{u}_{as} \end{bmatrix}$$

Equation 19 is a system of ordinary differential equations, the solution of which provides values of unknowns at each node in the finite element mesh. This equation is highly non linear and can only be solved using an iteration procedure. The finite difference approach is used to discretize the governing equations for water flow and air flow as (Istok 1989):

$$\left[C + \theta K \Delta t\right] \varphi_{i+1} = \left[C - (1-\theta)\, K \Delta t\right] \varphi_i + \left[(1-\theta)\, F_i + \theta F_{i+1}\right] \Delta t \tag{20}$$

where, i is the iteration number, θ takes the value of 0, 1/2 and 1 for the forward, central, and backward difference schemes respectively. A value of θ equal to 1 (for the backward difference scheme) has been used in the current work as it has proven its efficiency in solving highly non-linear equations (Istok 1989).

Equation 20 can be rewritten as:

$$K_{eff} \phi_{i+1} = F_{eff} \tag{21}$$

where,

$$K_{eff} = \left[C + \theta K \Delta t\right] \tag{22}$$

$$F_{eff} = \left[C - (1-\theta)\, K \Delta t\right] \varphi_i + \left[(1-\theta)\, F_i + \theta F_{i+1}\right] \Delta t \tag{23}$$

The iteration procedure commences with assuming an initial value for ϕ_i as the initial conditions. Therefore, the right hand side terms F_{eff} of Eq. 21 can be obtained at the beginning of the time step. The first approximation for ϕ_{i+1} is obtained based on the result of the previous time step as:

$$\varphi_{i+1} = K_{eff}^{-1} F_{eff} \tag{24}$$

The iterative solution continues until the problem converges. It is assumed that convergence occurs when the following norm criterion is satisfied;

$$\left| \frac{\varphi_{i+1}^{n+1} - \varphi_{'i}^{n+1}}{\varphi_i^{n+1}} \right| \leq tolerance \tag{25}$$

where, n is the time step level.

A solution of the fully coupled water flow and air flow formulation provides values of u_w and u_a at various points within the soil and at different times, taking into account the interaction between the flow of water and air.

Numerical Solution of the Solute Transport Governing Equation

Employing the Galerkin weighted residual approach to Eq. 15, the discretized global finite element equation for a single component of contaminant takes the form:

$$M\frac{dc}{dt} + Hc = F^c \tag{26}$$

where,

$$M = \sum_1^n \int_{\Omega^e} \frac{\theta c}{\Delta t} A_{ij}; \quad H = \sum_1^n \int_{\Omega^e} vcB_{ij} + DcE_{ij} + \lambda c A_{ij};$$

$$F^c = \sum_1^n N_r \left(vc - D\frac{\partial c}{\partial x}\frac{\partial c}{\partial y} \right) |_{\Omega^e}$$

$$A_{ij} = \int NNd\Omega^e; \quad B_{ij} = \int N\frac{\partial N}{\partial x}N\frac{\partial N}{\partial y}d\Omega^e; \quad E_{ij} = \int \frac{\partial N}{\partial x}\frac{\partial N}{\partial y}\frac{\partial N}{\partial x}\frac{\partial N}{\partial y}d\Omega^e$$

The finite difference approach can be used to discretize the governing equations for solute transport as (Istok 1989):

$$H[\phi] + M\left[\frac{\partial \phi}{\partial t}\right] = \left[F^c\right] \tag{27}$$

$$[M + \theta H\Delta t]\,\phi_{i+1} = [M - (1-\theta)\,H\Delta t]\,\phi_i + \left[(1-\theta)\,F_i^c + \theta F_{i+1}^c\right].\Delta t \tag{28}$$

where, φ is the vector of unknowns, M, H and F^c are matrices that have been defined above.

Equation 27 can be rewritten as:

$$H_{eff}\phi_{i+1} = F_{eff}^c \tag{29}$$

where

$$H_{eff} = [M + \theta H\Delta t] \tag{30}$$

$$F_{eff}^c = [M - (1-\theta)\,H\Delta t]\,\phi_i + \left[(1-\theta)\,F_i^c + \theta F_{i+1}^c\right]\Delta t \tag{31}$$

Similarly, the iteration procedure commences with assuming initial values for φ_i as the initial conditions, therefore, the right hand side terms F^c_{eff} of Eq. 29 can be obtained at the beginning of the time step. The first estimation for φ_{i+1} is obtained based on the result of the previous time step φ_i Therefore the value of φ_{i+1} for each iteration can be obtained by:

$$\phi_{i+1} = H^{-1}_{eff} F^c_{eff} \tag{32}$$

The iterative solution proceeds until the problem converges. A solution of the solute transport Eq. 32, following the solution of Eq. 24, at different time steps will give the distribution of the solute concentrations c at various points within the soil and at different times, taking into account the interaction between the flow of water, air and various mechanisms of solute transport.

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