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# Resilience Indexes for Water Distribution Network Design: A Performance Analysis Under Demand Uncertainty

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**Abstract** Traditionally, the optimal design of water distrubution networks has been dealt with using single-objective constrained approaches, where the aim is to minimize the network investment cost while maintaining minimum pressure head constraints at all nodes. However, in the last decade some authors have proposed multi-objective approaches which optimize other objectives than network investment cost. In most cases, these objectives have been formulated using the concept of resilience index, which mimics the design aim of providing excess head above the minimum allowable head at the nodes and of designing reliable loops with practicable pipe diameters. Although several authors have proposed different resilience indexes for this pupose, to date there is no empirical study that analyzes the advantages and disadvantages of these proposals. This paper evaluates the performance of a well-known multi-objective evolutionary algorithm, the Strength Pareto Evolutionary Algorithm 2, using three different resilience indexes. The results obtained in two

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water supply networks under a large number of simulated over-demand scenarios show the advantages and disadvantages of these measures.

**Keywords** Water distribution systems · Multi-objective optimization · Cost · Resilience indexes · Demand uncertainty

## 1 Introduction

The optimal design of water distribution networks is a combinatorial optimization problem included in the class of complex combinatorial problems known as NP-hard (Gupta et al. 1993), which consists of finding the best way of conveying water from the sources (reservoirs) to the users (demand nodes), satisfying certain requirements. The typical single-objective constrained formulation of this problem aims to obtain a combination of pipe diameters (decision variables) that minimize the network investment cost, while pipe layout, connectivity and demands are input data (Montesinos et al. 1999; Reca et al. 2008a; Cisty 2010). The main drawback of the single-objective constrained formulation is that it does not adequately establish a trade-off between the cost and reliability or robustness of a design (Todini 2000). Reliability can be understood as the capability of providing adequate supply under both normal and abnormal conditions (Farmani et al. 2005a), including demand uncertainty, pipe failure, etc. One difficulty that must be faced is finding an accurate way to express reliability analytically, which is why in recent years interest has increased in the application of so-called reliability surrogate measures, such as flow entropy, resilience indexes, etc. (Raad et al. 2010). One of the most widely used reliability measures is the concept of resilience index proposed by Todini (2000), which is a measure of the capability of the network to cope with failures and is related indirectly to system reliability. In recent years, some authors have extended this resilience index in order to overcome certain drawbacks (Prasad and Park 2004; Jayaram and Srinivasan 2008). The authors of these extensions have enumerated some theoretical advantages of their approaches, but none of them have compared the performance of these resilience indexes in the same conditions.

Therefore, it would be novel and interesting to evaluate the performance of these resilience indexes in addition to the network investment cost. Taking into account that classical formulations of the problem aim to maintain the minimum pressure constraints and that the resilience indexes measure the ability of the network to cope with uncertainties (Todini 2000), the methodology of the study consists of simulating a large number of scenarios where the flow in demand nodes is increased in order to determine which resilience index obtains the best performance. For this purpose, the Strength Pareto Evolutionary Algorithm 2 (SPEA2) (Zitzler et al. 2001) is used to generate the trade-off between the cost and Todini's, Prasad's and Jayaram's resilience indexes. The solutions are evaluated in all the over-demand scenarios in order to determine whether or not they still satisfy the pressure constraints. SPEA2 was chosen due to the conclusions of Farmani et al. (2005b), who demonstrated the good performance of SPEA2 when solving a multi-objective formulation of this problem using Todini's resilience index.

## 2 Multi-Objective Formulations of Water Distribution Network Design Using Different Resilience Indexes

Typical single-objective formulation (Montesinos et al. 1999; Reca et al. 2008a) of the optimal design of water distribution networks tries to minimize the cost with pipe diameters as decision variables, while pipe layout, connectivity and demands are input data. Equation 1 shows the cost function that is typically used to solve the single-objective formulation of this problem, where $C$ is the total cost of the network, $c_i$ is the cost of the pipe with diameter $i$ per unit length, $L_i$ is the total length of pipe $i$ in the network, *npipes* is the number of available pipe diameters and $ha_j$ and $hr_j$ are the pressure available and required at node $j$, respectively.

$$\begin{aligned} &C = \textstyle\sum_{i=1}^{npipes} c_i L_i \\ &\text{subject to: } ha_j \geq hr_j, \quad \forall j \end{aligned} \tag{1}$$

However, some authors have extended the traditional single-objective formulation of the problem in order to consider not only the network investment cost, but also the reliability of the system. The reliability of a network can be understood as the capability of providing adequate supply under both normal and abnormal conditions (Farmani et al. 2005a). However, there are many different alternatives to consider reliability. In particular, two main research lines have been analyzed in the past to include the concept of reliability (Farmani et al. 2005a): surrogate-based measures (deterministic modelling) and stochastic analysis of uncertainty (probabilistic modelling).

One of the most widely used surrogate-based measures is the *resilience index* (RI) introduced by Todini (2000), which is strongly related to the intrinsic capability of the system to overcome failures while still satisfying demands and pressures in nodes. This index, which has recently been used by other authors (Farmani et al. 2005a; Reca et al. 2008b), is described in Eq. 2, where: $RI$ is the resilience index, $N$ the number of demand nodes, $q_j$ the demand at node $j$, $R$ the number of reservoirs, $Q_r$ the flow from reservoir $r$ when it is feeding the system, $H_r$ the elevation plus water level in the reservoir $r$, $B$ the number of pumps, $P_b$ the power introduced by pump $b$ into the system, and $\gamma$ the specific weight of water.

$$RI = \frac{\sum_{j=1}^{N} q_j (ha_j - hr_j)}{\left(\sum_{r=1}^{R} Q_r H_r + \sum_{b=1}^{B} \frac{P_b}{\gamma}\right) - \sum_{j=1}^{N} q_j hr_j} \tag{2}$$

Although most previous studies considering reliability in this problem have used Todini's resilience index, some authors have tried to improve it in recent years. Prasad and Park (2004) introduced another reliability measure called *network resilience index* (NRI), which incorporates the effects of both surplus power and reliable loops. Reliable loops can be ensured if the pipes connected to a node do not vary greatly in diameter. If $D1_j$, $D2_j$, ..., $Dnp_j$ (where $D1_j \geq D2_j, \geq .. \geq Dnp_j$) are the diameters of the $np$ pipes connected to node $j$, then uniformity of that node is given by Eq. 3,

$$c_j = \frac{\sum_{i=1}^{np} Di_j}{np * \max(Di_j)} \tag{3}$$

where *npj* is the number of pipes connected to node *j*. The value of $c_j = 1$ if pipes connected to a node have the same diameter; and $c_j < 1$ if pipes connected to a node have different diameters. For nodes connected with only one pipe, the value of $c_j$ is taken to be one.

$$NRI = \frac{\sum_{j=1}^{N} c_j q_j \left(ha_j - hr_j\right)}{\left(\sum_{r=1}^{R} Q_r H_r + \sum_{b=1}^{B} \frac{P_b}{\gamma}\right) - \sum_{j=1}^{N} q_j hr_j} \tag{4}$$

Theoretically, the value of network resilience may vary between 0 and 1. However, for real systems it never attains a value of 1, since imposing the same diameter for all pipes in a network need not always provide a Pareto-optimal solution in Cost-*NRI* space, as *NRI* is a measure of the combined effect of surplus power and nodal uniformity.

Recently, Jayaram and Srinivasan proposed the *modified resilience index* (*MRI*), which theoretically overcomes the drawback of Todini's resilience index when evaluating networks with multiple sources (Jayaram and Srinivasan 2008). In contrast to Todini's resilience index, the value of the modified resilience index is directly proportional to the total surplus power at the demand nodes. Equation 5 describes *MRI*, which only considers those solutions with pressures equal to or higher than that required in all nodes. While Todini's *RI* and Prasad's *NRI* take values up to a maximum of 1, Jayaram's *MRI* can be greater than 1.

$$MRI = \frac{\sum_{j=1}^{N} q_j \left(ha_j - hr_j\right)}{\sum_{j=1}^{N} q_j hr_j} \times 100 \tag{5}$$

The multi-objective formulation of water distribution network design consists of finding the best diameter in each pipe so that the network investment cost (*C*) is minimized (Eq. 1), while the selected resilience index used (*RI*, *NRI* or *MRI*) is simultaneously maximized. It is important to determine how unfeasible solutions are managed. Some authors have indicated the possibility of using a penalty multiplier for handling the pressure constraints. Todini (2000) suggested a failure index, which is used to identify unfeasibilities during the opimization process. Another alternative, used in the present paper, consists of considering the pressure in demand nodes as hard constraints, i.e. all the solutions must satisfy the pressure constraints in all the demand nodes ($ha_j \geq hr_j$, $\forall j$).

## 3 Methodology of the Study

Most multi-objective computational approaches apply Pareto-optimization concepts (Goldberg 1989), which, instead of giving a scalar value to each solution, establish relationships between solutions according to Pareto-dominance relations. The aim of Pareto-based approaches is to find the Pareto-optimal set, or a representative sample of it. The practical advantage of using Pareto-based multi-objective algorithms is that the set (front) of non-dominated solutions returned can be used in a subsequent phase to select one of the solutions according to several criteria.

The methodology of this research is based on executing SPEA2 three times, using the three resilience indexes here evaluated. Three fronts of non-dominated solutions are obtained, which are then analyzed in terms of their capability to maintain the required pressures in the demand nodes when the flow in these nodes is randomly increased. A set of 10,000 over-demand scenarios is generated, each one consisting of a random increase in the flow demand (*OverDemand*) in a random percentage of demand nodes (*NodesOverDemand*), i.e. each simulation randomly selects %*NodesOverDemand* nodes, whose base demand is randomly increased up to %*OverDemand*. Thus, total overdemand of each simulation (*TotalOverDemand*) is within the interval: $0\% \leq \mathit{TotalOverDemand} \leq \mathit{OverDemand}{*}\mathit{NodesOverDemand}\%$. Once these 10,000 stochastic scenarios are generated, each one of the solutions of the three non-dominated fronts (SPEA2_RI, SPEA2_NRI, SPEA2_MRI) is tested in each scenario in order to determine whether they are still feasible solutions, i.e. if they are able to maintain the pressure in the demand nodes. These runs will provide information about the following aspects:

- The percentage of over-demand simulations that renders unfeasible the non-dominated solutions found by SPEA2 using different resilience indexes.
- The minimum over-demand that makes the non-dominated solutions unfeasible.
- The percentage of non-dominated solutions that become unfeasible in each over-demand simulation.

SPEA2 (Zitzler et al. 2001) is an improved version of SPEA (Zitzler and Thiele 1999) that incorporates a fine-grained fitness assignment strategy, which takes into account, for each individual, the number of individuals that dominate it, and the number of individuals it dominates. It uses a nearest neighbor density estimation technique which guides the search more efficiently by avoiding crowding on the front. SPEA2 uses an external archive to store promising solutions found in the search. When the number of non-dominated individuals exceeds the size of this archive, SPEA2 applies an enhanced archive truncation method that guarantees the preservation of boundary solutions.

## 4 Empirical Analysis

### 4.1 Test Problems

The empirical study of the quality of the solutions generated by SPEA2 using the three resilience indexes described above uses two gravity-fed looped water distribution networks that are often used in the context of water distribution systems. The main characteristics of each network are summarized below and are described in further detail in Reca and Martinez (2006).

- Alperovits and Shamir network (Alperovits and Shamir 1977) is a simple two-loop network, with seven nodes and eight pipes arranged in two loops. A total of 14 commercial pipe diameters can be selected, i.e. there exist $14^8 = 1.4758 * 10^9$ possible configurations.
- Hanoi network (Fujiwara and Khang 1990) consists of 32 nodes, 34 pipes, and 3 loops. A set of 6 available commercial-diameter pipes is used, which implies a total of $6^{34} = 2.8651 * 10^{26}$ possible configurations.

The minimum pressure limitation is 30 m above ground level for each node ($hr_{rj} = 30$) in both networks. The interface of the program and the memetic algorithm have been programmed in the Visual-Basic programming language. A database management system has been implemented using a relational database and the ActiveX Data Objects (ADO) model. EPANET network solver (Version 2.00.07) (Rossman 2000) has been used considering its default values.

## 4.2 Parameter Settings

The guide algorithm used in the study is SPEA2, which has been run using a main population (P) and an external one (ND), each with 50 individuals ($P_{\text{size}} = ND_{\text{size}} = 50$ individuals). Since SPEA2 is run three times (one per each resilience index), the stop criterion is that all the methods perform the same number of evaluations of the fitness function. As discussed in a previous work (Baños et al. 2009), all the runs perform the same number of evaluations, $n_e$, which should depend on the complexity of the network. The size of the search space is a function of the number of links $n_l$ and the number of possible pipe diameters $n_d$. Equation 6 has been adopted to establish a ratio between the number of evaluations in two different networks ($\alpha$ and $\beta$). The resulting fitness function evaluations are 91,690 and 264,570 for the Alperovits-Shamir and Hanoi networks, respectively.

$$\frac{n_e^{\beta}}{n_e^{\alpha}} = \frac{n_l^{\beta} * ln\left(n_d^{\beta}\right)}{n_l^{\alpha} * ln\left(n_d^{\alpha}\right)} \tag{6}$$

The following over-demand parameters are used: $0\% \leq OverDemand \leq 50\%$, $0\% \leq NodesOverDemand \leq 33.3\%$ in the Alperovits-Shamir network, and $0\% \leq OverDemand \leq 20\%$, $0\% \leq NodesOverDemand \leq 10\%$ in the Hanoi network. The over-demand scenarios used in Alperovits-Shamir are greater than those used in the Hanoi network, since preliminary results showed that the solutions obtained in Alperovits-Shamir become unfesasible with a greater overdemand than those obtained in the Hanoi network. Figure 1 shows the distribution of the over-demand scenarios for 10,000 simulations in both networks.

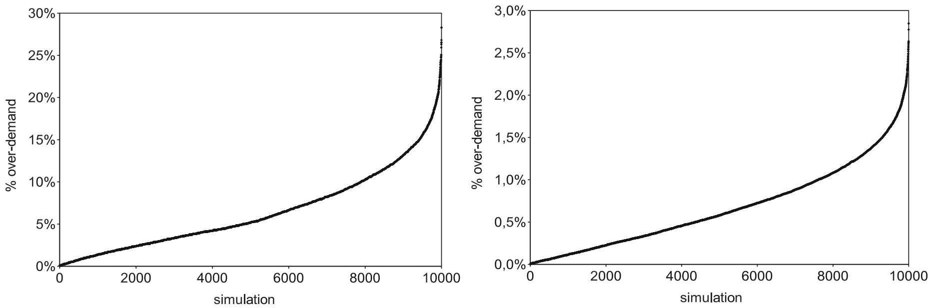


**Fig. 1** Distrubution of the global over-demand scenarios in Alperovits-Shamir (*left*) and Hanoi (*right*)

### 4.3 Performance Measures

The quality of the sets of non-dominated solutions is evaluated using two specific metrics: Average maximum over-demand (AverageMaxOverDemand(S)) and average percentage of unfeasible scenarios (AveragePercentageUnfeasibleScenarios(S)).

*Average Percentage of Unfeasible Scenarios (APUS)* Given a front of non-dominated solutions $S$ ($S = \{s_1, s_2, .., s_n\}$),the function *PercentageUnfeasibleScenarios*($s_i$) returns the percentage of the 10,000 scenarios in which solution $s_i$ becomes unfeasible. Function APUS(S) returns the average of all *PercentageUnfeasibleScenarios*($s_i$) values. The lower the value of APUS(S), the better the resistence of the solutions included in $S$.

*Average Minimum Over-Demand (AMOD)* Given a front of non-dominated solutions $S$ ($S = \{s_1, s_2, .., s_n\}$ ), the function *MinOverDemandUnfeasible*($s_i$) returns the lowest over-demand of the 10,000 scenarios in which solution $s_i$ becomes unfeasible. Function AMOD(S) returns the average of all *MinOverDemandUnfeasible*($s_i$) values. The higher the value of AMOD(S), the better the resistence of the solutions included in $S$.

### 4.4 Results and Discussion

The quality of the solutions obtained by SPEA2 using the three resilience indexes described above is analyzed from the point of view of the two metrics previously proposed. A total of 15 independent runs of SPEA2 have been performed for each network and resilience index. Of the 15 non-dominated fronts obtained for each configuration, the front which is best distributed has been selected as the representative front. The representative non-dominated fronts obtained in the Alperovits-Shamir network using the three resilience indexes (*RI*, *NRI*, *MRI*) are shown in Fig. 2. It is noteworthy that, since the best known solution for this network in the single-objective case is 419,000 monetary units, these fronts are relatively close to the true (unknown) Pareto-optimal front. The quality of these solutions in terms of the metrics described above is shown in Table 1, where non-dominated individuals (solutions) are sorted by increasing cost. These results indicate that, in general, the higher the cost of the solutions, the lower the percentage of the 10,000 scenarios which make solutions unfeasible, and the higher the minimum over-demand that makes solutions unfeasible. According to the two metrics described above, Table 1 shows that the values of APUS are 21.30%, 59.59% and 29.94%, in *RI*, *NRI* and *MRI*, respectively. On the other hand, the results in terms of AMOD metric are 8.486%, 2.407% and 6.155%, respectively. These results denote that solutions obtained by Todini's *RI*, in general, resist better the over-demand scenarios, while Prasad's *NRI* obtains the worst results in this network.

Figure 3 shows the results obtained by using the three indexes in terms of the APUS metric. Taking into account that the solutions obtained are numbered according to increasing cost, it can be seen that unfeasibility tends to decrease in the more expensive solutions, which reinforces the numerical conclusions shown in Table 1.

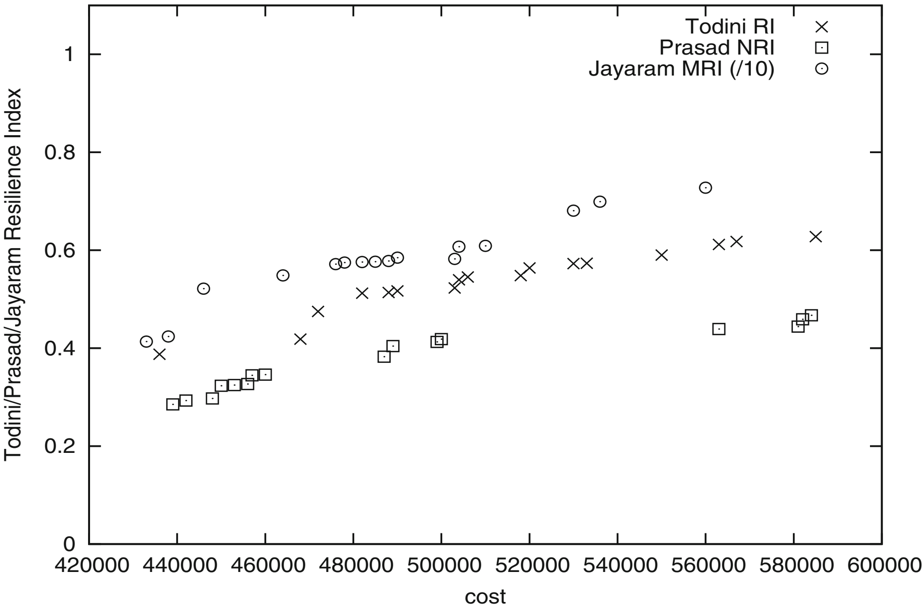


**Fig. 2** Front of non-dominated solutions obtained by SPEA2 using the three resilience indexes (*RI*, *NRI*, *MRI*) in Alperovits-Shamir network

Figure 4 shows that solutions of over 550,000 monetary units are still feasible in most over-demand scenarios, while the percentage of over-demand scenarios that make solutions unfeasible increases when the cost is closer to 450,000 monetary

**Table 1** Results obtained in Alperovits-Shamir network

| IND | TODINI | | PRASAD | | JAYARAM | |
|---|---|---|---|---|---|---|
| | APUS | AMOD | APUS | AMOD | APUS | AMOD |
| 1 | 0.7971 | 1.00536 | 0.9399 | 1.00179 | 0.6798 | 1.01607 |
| 2 | 0.5108 | 1.02330 | 0.7604 | 1.00357 | 0.8540 | 1.00804 |
| 3 | 0.4242 | 1.02357 | 0.9351 | 1.00179 | 0.3570 | 1.03929 |
| 4 | 0.2889 | 1.04286 | 0.7356 | 1.00893 | 0.3612 | 1.04714 |
| 5 | 0.2775 | 1.04464 | 0.6319 | 1.01179 | 0.2348 | 1.06964 |
| 6 | 0.2567 | 1.04643 | 0.6645 | 1.00884 | 0.3546 | 1.03214 |
| 7 | 0.2120 | 1.06790 | 0.8922 | 1.00295 | 0.3668 | 1.02857 |
| 8 | 0.1076 | 1.09429 | 0.9387 | 1.00295 | 0.3525 | 1.03214 |
| 9 | 0.2510 | 1.03393 | 0.8922 | 1.00295 | 0.3162 | 1.03750 |
| 10 | 0.1179 | 1.08545 | 0.4032 | 1.03536 | 0.2807 | 1.06187 |
| 11 | 0.1385 | 1.05000 | 0.2723 | 1.04304 | 0.1138 | 1.09429 |
| 12 | 0.0942 | 1.09830 | 0.4635 | 1.02357 | 0.1110 | 1.09607 |
| 13 | 0.0996 | 1.09723 | 0.2583 | 1.05893 | 0.0135 | 1.10438 |
| 14 | 0.0427 | 1.13848 | 0.2898 | 1.05304 | 0.0936 | 1.06071 |
| 15 | 0.0011 | 1.21339 | 0.1478 | 1.07857 | 0.0012 | 1.19536 |
| 16 | 0.0002 | 1.18830 | 0.3088 | 1.04714 | | |
| 17 | 0.0004 | 1.19536 | | | | |
| AVG | 0.2130 | 1.08486 | 0.5959 | 1.02407 | 0.2994 | 1.06155 |

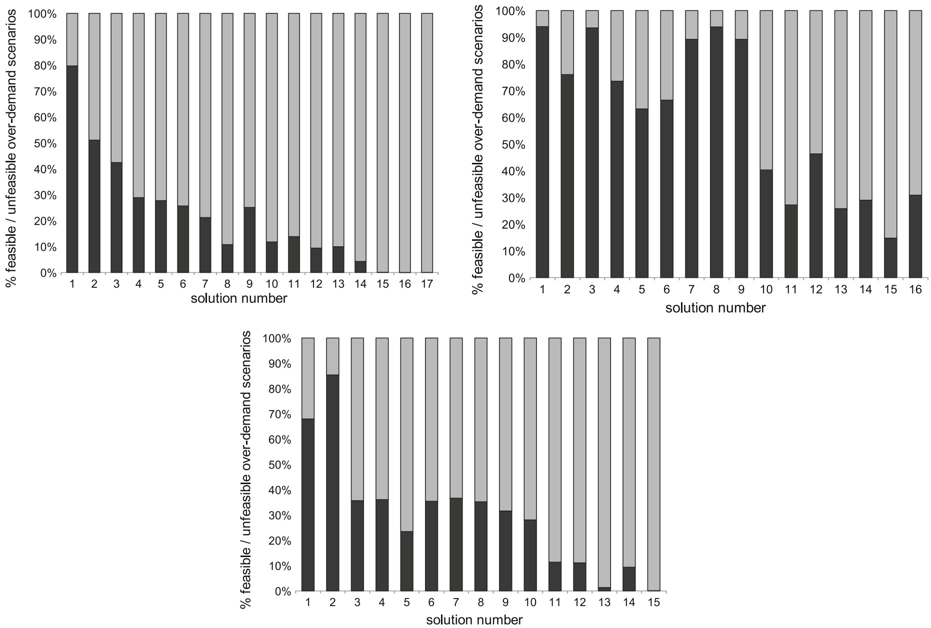


**Fig. 3** Percentage of feasible (*grey*) and unfeasible (*black*) solutions obtained by SPEA2 using Todini (*upper-left*), Prasad-Park (*upper-right*) and Jayaram (*lower*) indexes after 10,000 simulations

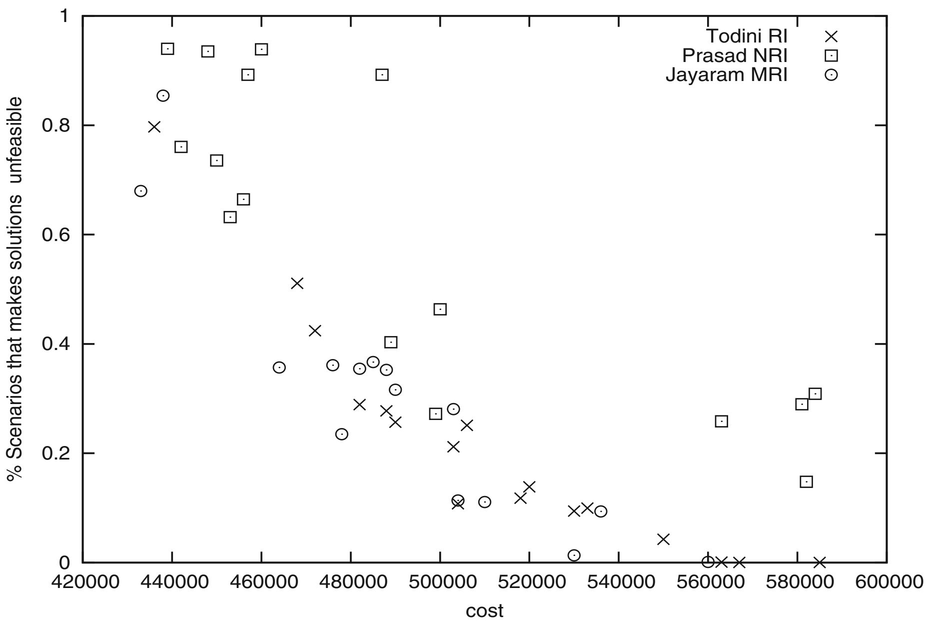


**Fig. 4** Comparison of the three non-dominated fronts in terms of cost and percentage of unfeasible scenarios in Alperovits-Shamir network

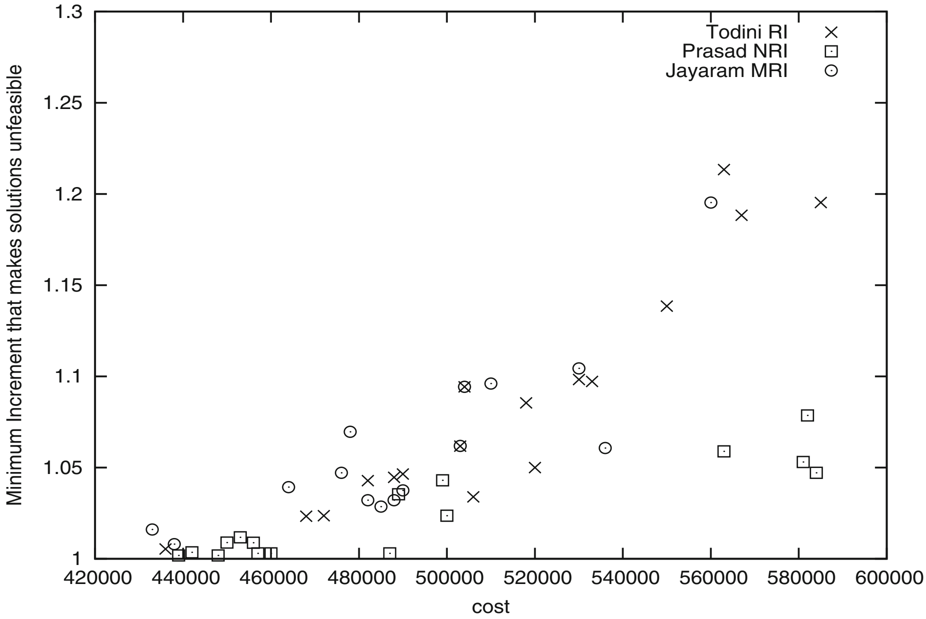


**Fig. 5** Comparison of the three non-dominated fronts in terms of cost and minimum over-demand scenario which makes the solutions unfeasible in Alperovits-Shamir network

units. In reference to the minimum over-demand that makes solutions unfeasible, Fig. 5 shows that this value increases with network investment cost. However, even expensive non-dominated solutions found by Prasad's *NRI* become unfesasible with low over-demand scenarios, while expensive non-dominated solutions obtained by SPEA2 using Todini's *RI* and Jayaram's *MRI* only become unfeasible with significant over-demands. Figure 6 shows the percentage of solutions that become unfeasible in the 10,000 different over-demand scenarios using the three resilience indexes, and the linear extrapolation of these points. Those simulations in which

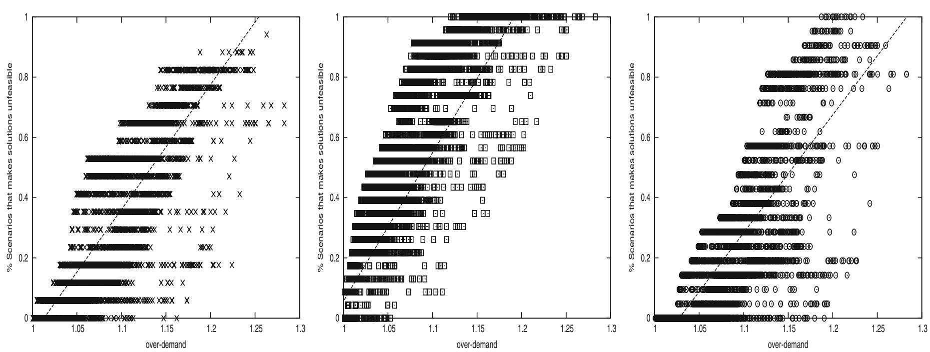


**Fig. 6** Percentage of unfeasible individuals in the 10,000 over-demand scenarios analyzed in Alperovits-Shamir network using Todini's *RI* (*left*), Prasad's *NRI* (*middle*) and Jayaram's *MRI* (*right*)

over-demand is over 12% lead to 100% of the non-dominated solutions obtained by SPEA2 using Prasad's *NRI* becoming unfeasible, while this situation occurs when over-demand is greater than 18% using Jayaram's MRI. Finally, none of the 10,000 over-demand scenarios leads to 100% of non-dominated solutions found by SPEA2 using Todini's *RI* becoming unfeasible.

To analyze the quality of solutions in the larger network (Hanoi), Fig. 7 shows the representative fronts obtained by applying SPEA2 using the three resilience indexes. These fronts are also good approximations to the true (unknown) Pareto-optimal fronts, since the best known solution for this network in the single-objective case is 6,081.127 monetary units. Table 2 again shows that, in general, the higher the cost of the solutions, the lower the over-demand and the lower the percentage of the 10,000 scenarios which make solutions unfeasible. According to the two metrics previously described, Table 2 shows that the values of APUS are 40.36%, 32.35% and 57.85%, in *RI*, *NRI* and *MRI*, respectively. On the other hand, the results in terms of AMOD metric are 0.162%, 0.220% and 0.124%, respectively. These results also denote that, on the whole, solutions obtained by Prasad's *NRI* resist better the over-demand scenarios, while Jayaram's *NRI* obtains the worst results in this network.

However, Fig. 8 shows that, irrespective of the resilience index used, there are some non-dominated solutions that become unfeasible in a higher percentage of over-demand scenarios than other cheaper solutions. In reference to the minimum over-demand scenario that makes solutions unfeasible, Fig. 9 also shows that some expensive solutions become unfeasible with a lower over-demand than other cheaper solutions of the same non-dominated front. Figure 10 shows the percentage of

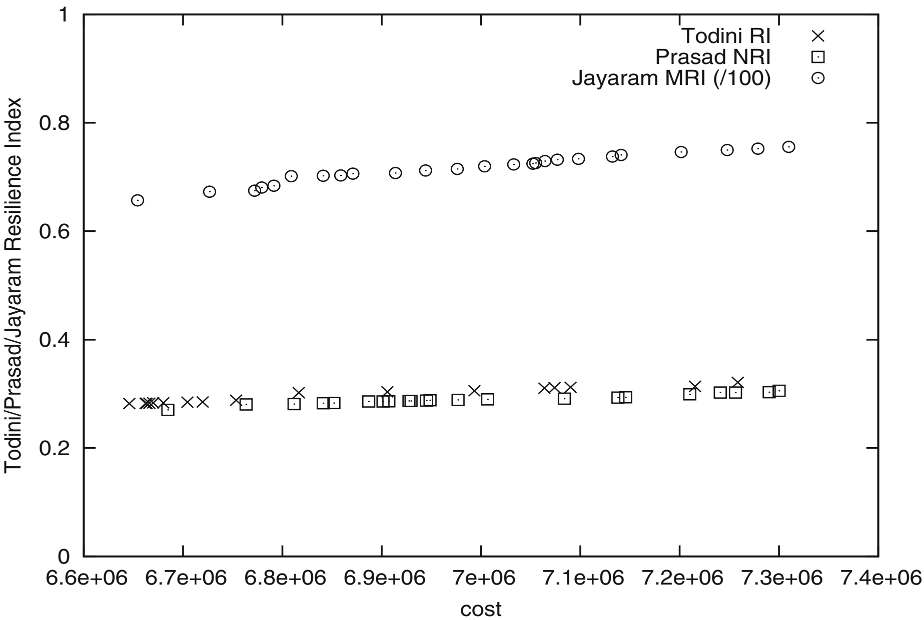


**Fig. 7** Front of non-dominated solutions obtained by SPEA2 using the three resilience indexes (*RI*, *NRI*, *MRI*) in the Hanoi network

**Table 2** Results obtained in the Hanoi network

| IND | TODINI | | PRASAD | | JAYARAM | |
|---|---|---|---|---|---|---|
| | APUS | AMOD | APUS | AMOD | APUS | AMOD |
| 1 | 0.2976 | 1.00189 | 0.9408 | 1.00015 | 0.8949 | 1.00047 |
| 2 | 0.2981 | 1.00189 | 0.7865 | 1.00047 | 0.6244 | 1.00094 |
| 3 | 0.4436 | 1.00141 | 0.4375 | 1.00141 | 0.6421 | 1.00094 |
| 4 | 0.3004 | 1.00189 | 0.4669 | 1.00141 | 0.7713 | 1.00047 |
| 5 | 0.3056 | 1.00154 | 0.7453 | 1.00047 | 0.7713 | 1.00047 |
| 6 | 0.6628 | 1.00094 | 0.6168 | 1.00094 | 0.8069 | 1.00047 |
| 7 | 0.6431 | 1.00094 | 0.6237 | 1.00094 | 0.7487 | 1.00047 |
| 8 | 0.2796 | 1.00141 | 0.4178 | 1.00141 | 0.8515 | 1.00047 |
| 9 | 0.8245 | 1.00047 | 0.2992 | 1.00141 | 0.8515 | 1.00047 |
| 10 | 0.2143 | 1.00189 | 0.3891 | 1.00141 | 0.0436 | 1.00424 |
| 11 | 0.6202 | 1.00094 | 0.1626 | 1.00236 | 0.9380 | 1.00028 |
| 12 | 0.1896 | 1.00189 | 0.0391 | 1.00446 | 0.6937 | 1.00094 |
| 13 | 0.0369 | 1.00471 | 0.2590 | 1.00189 | 0.8930 | 1.00047 |
| 14 | 0.2143 | 1.00189 | 0.0306 | 1.00566 | 0.9205 | 1.00028 |
| 15 | 0.6431 | 1.00094 | 0.2303 | 1.00189 | 0.2956 | 1.00141 |
| 16 | 0.2447 | 1.00189 | 0.0664 | 1.00330 | 0.5973 | 1.00094 |
| 17 | 0.6431 | 1.00094 | 0.0323 | 1.00566 | 0.6710 | 1.00094 |
| 18 | | | 0.1215 | 1.00236 | 0.3533 | 1.00141 |
| 19 | | | 0.1341 | 1.00236 | 0.3729 | 1.00141 |
| 20 | | | 0.1444 | 1.00236 | 0.5160 | 1.00094 |
| 21 | | | 0.1281 | 1.00236 | 0.1033 | 1.00236 |
| 22 | | | 0.0450 | 1.00377 | 0.0554 | 1.00330 |
| 23 | | | | | 0.0606 | 1.00330 |
| 24 | | | | | 0.0554 | 1.00330 |
| 25 | | | | | 0.9295 | 1.00028 |
| AVG | 0.4036 | 1.00162 | 0.3235 | 1.00220 | 0.5785 | 1.00124 |

solutions that become unfeasible in the 10,000 over-demand scenarios using each index.

It is interesting to analyze why some cheaper non-dominated solutions resist better the over-demand scenarios than other more expensive ones. For instance, Table 2 shows that solution number 11 of Todini's *RI*, solution 13 of Prasad's *NRI*, and solution 20 of Jayaram's *MRI* become unfeasible in a higher percentage of over-demand scenarios, while their immediately cheaper solutions (10, 12 and 19, respectively) become unfeasible in a lower number of over-demand scenarios. Similar situations are shown graphically in Figs. 8 and 9. These strange results pose two questions: 1) Why do cheaper solutions maintain their feasibility in more demand scenarios than more expensive ones? 2) Why is this strange situation not usual in the smaller test network (Aperovits-Shamir)?

In order to answer to both questions a second analysis was carried out to analyze the real differences between solutions, i.e. which pipe diameters differ among solutions. For instance, solutions 5 and 6 obtained by SPEA2 using Todini's *RI* in Hanoi are displayed in Table 3. The only difference between both solutions is that while solution 5 has pipes of diameter 2 in link number 24 and diameter 3 in link number 33, solution 6 has pipes of diameter 3 and 2 in those links. However, while the cost of solution 5 is 6,669,852.158 monetary units and the percentage of unfeasible

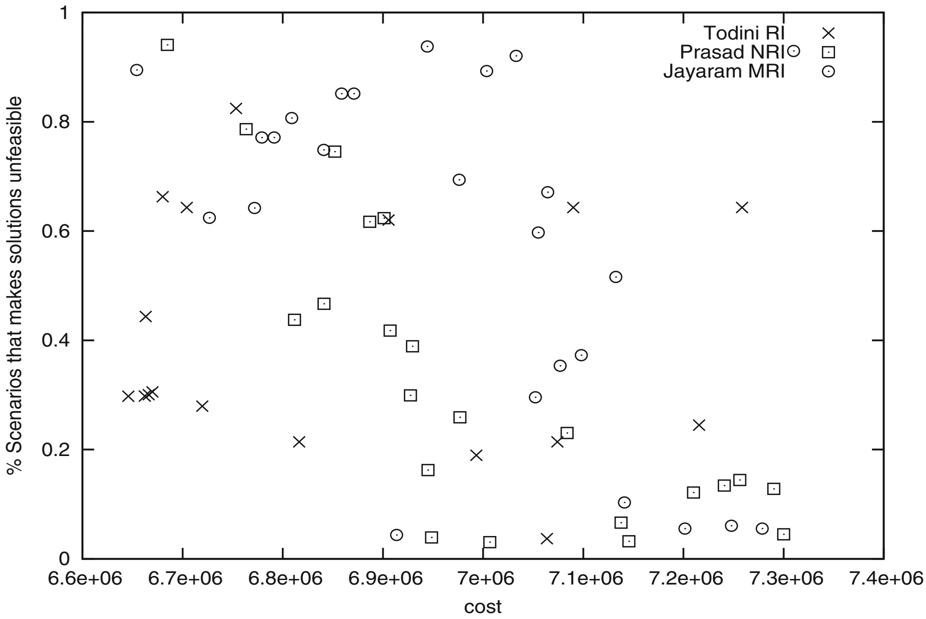


**Fig. 8** Comparison in the Hanoi network of the three non-dominated fronts in terms of cost and percentage of unfeasible scenarios

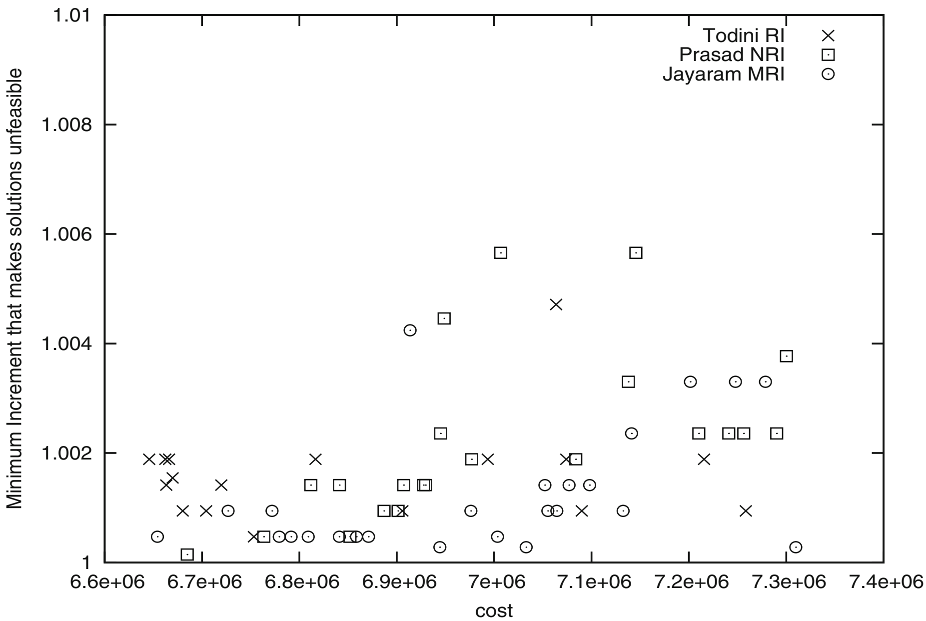


**Fig. 9** Comparison in the Hanoi network of the three non-dominated fronts in terms of cost and minimum over-demand scenario which make solutions unfeasible

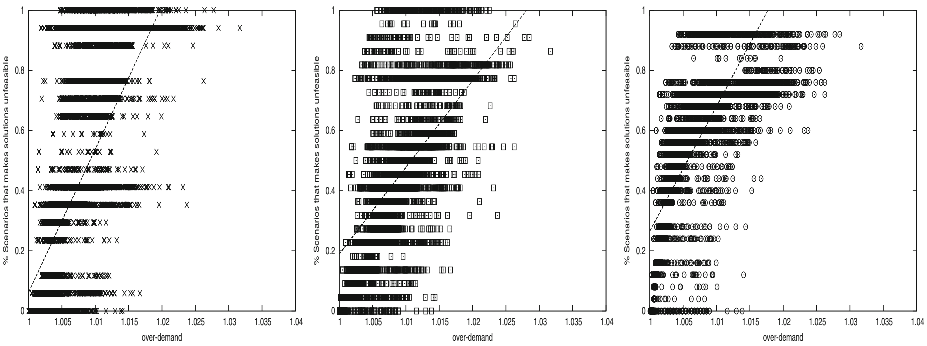


**Fig. 10** Percentage of unfeasible individuals in the 10,000 over-demand scenarios analyzed in the Hanoi network using Todini's *RI* (*left*), Prasad's *NRI* (*middle*) and Jayaram's *MRI* (*right*)

scenarios is 30.56%, solution 6 becomes unfeasible in 66.28% of the 10,000 over-demand scenarios despite being more expensive (6,680,207.345 monetary units). Therefore, this example shows that a very small difference between two solutions constitutes a large difference in the ability to provide adequate supply under demand uncertainty.

The same analysis has been performed in the Alperovits-Shamir network. Although this situation is not so usual as in the Hanoi network, it is also observed. For instance, solutions 21 and 22 obtained by SPEA2 using Prasad's *NRI* in Alperovits-Shamir are displayed in Table 4. In this case, although solution 22 (cost 1,133,000 monetary units) has larger pipe diameters in links 5 and 6 than solution 21 (cost 1,044,000 monetary units), while only pipe number 8 of solution 22 has a pipe diameter one degree smaller than the same link in solution 21, the percentage of unfeasible scenarios is 47.54% in the former, compared to 7.85% in the latter.

From this analysis, which can be generalised to other non-dominated solutions obtained by SPEA2 using any of the three resilience indexes here evaluated, it can be concluded that, although the three resilience indexes have been widely used to measure the quality of the network design, in practice none of them correctly measures the ability to provide adequate supply under abnormal demand conditions (demand uncertainty). In other words, an improved resilience index would consider not only the global excess of pressure in the network, but also the distribution of pressure in demand nodes, i.e. the network topology.

**Table 3** Special case in the Hanoi network

| | i1 | i2 | i3 | i4 | i5 | i6 | i7 | i8 | i9 | i10 | i11 | i12 | i13 | i14 | i15 | i16 | i17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 3 | 1 | 3 | 4 | 6 | 6 |
| s6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 3 | 1 | 3 | 4 | 6 | 6 |

| | i18 | i19 | i20 | i21 | i22 | i23 | i24 | i25 | i26 | i27 | i28 | i29 | i30 | i31 | i32 | i33 | i34 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s5 | 6 | 6 | 6 | 4 | 2 | 4 | 2 | 1 | 5 | 5 | 5 | 2 | 1 | 1 | 2 | 3 | 4 |
| s6 | 6 | 6 | 6 | 4 | 2 | 4 | 3 | 1 | 5 | 5 | 5 | 2 | 1 | 1 | 2 | 2 | 4 |

**Table 4** Special case in the Alperovits-Shamir network

| | i1 | i2 | i3 | i4 | i5 | i6 | i7 | i8 |
|---|---|---|---|---|---|---|---|---|
| s21 | 14 | 11 | 11 | 9 | 8 | 7 | 9 | 7 |
| s22 | 14 | 11 | 11 | 9 | 10 | 10 | 9 | 6 |

In reference to the second question, it is observed that all the demand nodes of the Alperovits-Shamir network are connected with at least two other demand nodes and all them are relatively close to the reservoir, while some nodes in the Hanoi network are not in looped areas and/or they are relatively far from the reservoir. Therefore, the reason why it is more usual for cheaper solutions to maintain their feasibility than more expensive ones in the Hanoi network than in Alperovits-Shamir could be because an over-demand of $\Delta D\%$ has a higher impact in some demand nodes. In particular, this seems to occur more frequently in nodes which are closer to the reservoirs and those which are in branched parts of the network than in those which are further from the reservoirs and/or in looped parts of the network.

## 5 Conclusions

This paper analyzes the quality of several resilience indexes of frequent use in multi-objective formulations of the Water Distribution Network Design problem. In particular, it proposes a simulation-based analysis where the non-dominated solutions generated by a well-known multi-objective evolutionary algorithm are tested in a large number of over-demand scenarios with the aim of determining whether these solutions become unfeasible using two different metrics. The results obtained show that Todini's *RI* is slightly better than Prasad's *NRI* and Jayaram's *MRI* in the Alperovits-Shamir network, while Prasad's *NRI* is slightly better in the Hanoi network. Furthermore, and even more importantly, the results obtained demonstrate that given a random over-demand scenario the demand nodes where over-demand is applied become more important than the global over-demand of the network. The reason is that a given over-demand has a higher impact in some demand nodes, especially those which are closer to the reservoirs and those which are in branched parts of the network. Based on these results, it can be concluded that, as none of the resilience indexes consider where over-demand is applied, but rather the global excess of pressure in the network, they do not accurately determine the capability of the network to provide adequate supply under demand uncertainty. Therefore, it is suggested that resilience indexes consider the topology of the network in order to determine its critical points, where over-demand could make solutions unfeasible, i.e. where the head pressure is lower than the required pressure. These conclusions can offer very useful guidelines for designing new measures to determine the quality of water distribution networks, not only from the point of view of static pressures, but also from the point of view of dynamic over-demand scenarios, which are more realistic.

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