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A Conditional Value-at-Risk Based Inexact Water Allocation Model

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Abstract A conditional value-at-risk (CVaR) based inexact two-stage stochastic programming (CITSP) model was developed in this study for supporting water resources allocation problems under uncertainty. A CITSP model was formulated through incorporating a CVaR constraint into an inexact two-stage stochastic programming (ITSP) framework, and could be used to deal with uncertainties expressed as not only probability distributions but also discrete intervals. The measure of risks about the second-stage penalty cost was incorporated into the model, such that the trade-off between system economy and extreme expected loss could be analyzed. The developed model was applied to a water resources allocation problem involving a reservoir and three competing water users. The results indicated that the CITSP model performed better than the ITSP model in its capability of reflecting the economic loss from extreme events. Also, it could generate interval solutions within which the decision alternatives could be selected from a flexible decision space. Overall, the CITSP model was useful for reflecting the decision maker's attitude toward risk aversion and could help seek cost-effective water resources management strategies under complex uncertainties.

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1 Introduction

Allocation planning is an important component of managing water uses. It involves deciding how much water is available from a particular resource and how much water can be taken. The process is quite intricate since balancing the water demands of competing users, such as public, industry and agriculture, is not an easy task. The problem is further complicated by the uncertainties that may be associated with almost every components of a water allocation system (Maqsood et al. 2005). Such uncertainties may arise from water quantity promised to various users, available amount of water resources, water demand of different users, benefit from water consumption, and penalty from water shortage. This leads to the difficulties for water managers or local authorities in generating cost-effective allocation plans.

Over the past decades, inexact optimization models have been widely used to tackle uncertainties and complexities in water allocation problems (Wagner et al. 1994; Huang 1996; Chang et al. 1996; Russell and Campbell 1996; Kira et al. 1997). Based on literature review (provided in the next section), it appears that the impact of uncertainties may lead to two major problems: infeasibility and instability. Infeasibility means that the system may fail. The parameters or related information involved in the model are usually affected by many uncertain factors in practical applications. Neglecting such uncertainties or quantifying them inaccurately may result in the model's deviation from the real world system. For example, the promised quantity of water is a key factor for various users, because they need to know how much water they can expect in order to make decisions to support their activities and investments. If the promised water fails to be delivered due to uncertainty impacts, they will have to suffer from the water shortage. Conversely, while practical available water amount is sufficient, users may miss the chance of development opportunities (Maqsood et al. 2005). The deterministic models are incapable of generating rational allocation alternatives for water resources management in light of uncertainty impacts (Huang et al. 1993; Wang et al. 2003). Instability means that the expected value of a random parameter is hardly representative for extreme conditions. Over the past years, the stochastic mathematical programming (SMP) models were widely used in tackling probabilistic uncertainties associated with water resources systems (e.g. river flow). Most of these models focused on applying the expected values of the random variables. However, this may somewhat overlook the extreme effects as the minimization of the expected value can hardly guarantee minima under all probability levels.

Therefore, the problem under consideration turns into how to effectively allocate the available water resources to various sectors in consideration of both uncertainty and risk control. To solve such a problem, this study aims to develop a CVaR-based inexact two-stage stochastic programming (CITSP) model for water resources allocation planning. It is a hybrid of CVaR and ITSP, and capable of handling multiple uncertainties and controlling the risk at extreme probability levels. The objective entails: (1) formulation of a CITSP model based on CVaR and ITSP models; (2)

application of the developed model to a water resources management case; (3) analysis of the results and discussion of the model's applicability.

2 Literature Review

In recent decades, a magnitude of inexact optimization techniques were developed to tackle uncertainties in water resources or environmental management problems, and a majority of them were based on fuzzy, stochastic, and interval methods, as well as their combinations (Wagner et al. 1994; Huang 1996; Chang et al. 1996; Russell and Campbell 1996; Kira et al. 1997; Qin et al. 2007; Li and Huang 2007; Li et al. 2008; Guo et al. 2008; Xu and Qin 2010). Among various alternatives, the inexact two-stage stochastic programming (ITSP) method, firstly proposed by Huang and Loucks (2000), has been widely used in many environment management problems. Maqsood et al. (2005) incorporated fuzzy programming into an ITSP model for handling water resources allocation problems. The integrated model was able to reflect the pre-defined water policies in optimization and describe multiple uncertainties presented as stochastic, interval and fuzzy information. Guo et al. (2008) developed an interval-parameter two-stage stochastic semi-infinite programming model to address uncertainties expressed as discrete intervals, functional intervals and probability distributions, and applied it to a solid waste management case. Li et al. (2008) advanced an inexact fuzzy-robust two-stage programming model by fusing fuzzy robust programming into an ITSP model to enhance the robustness of optimization model in air quality management. Generally, ITSP was found to be advantageous in its effectiveness of handling pre-defined policies and coupling parameter uncertainties presented in both stochastic and interval formats. However, ITSP takes the system benefit as the objective without considering risk control issues which need the decision maker's implicit knowledge. Consequently, the model would tend to reach high allocation target, which may lead to serious water shortage problems when the available amount of water resources is in a disadvantageous condition.

Conditional Value-at-Risk (CVaR) model is a new risk measurement method based on probability distributions of random variables. It is modified from the Value-at-Risk (VaR) model which is widely used for the portfolio selection (Rockafellar and Uryasev 2000). Previously, application of CVaR in water resources management field was relatively limited. For example, Yamout et al. (2007) investigated the applicability of the stochastic programming model with CVaR and other five types of models in dealing with water resources allocation problems in east central Florida. Piantadosi et al. (2008) developed a stochastic dynamic programming model with CVaR for supporting urban storm water management. These studies demonstrated that a CVaR-based model was advantageous in: (1) addressing the expected loss under extreme conditions, and (2) applying a linear model to tackle risk rather than using nonlinear ones, as were the cases in many other risk models. However, a CVaR model also has the following limitations: (1) the quality of uncertain data in many practical applications may not always be good enough to generate PDFs; (2) the selection of CVaR values is somewhat subjective, such that the obtained solution would be largely dependent on the decision makers' preferences towards risk aversion; (3) CVaR was incapable of handling multiple forms of uncertainties.

Based on the above-mentioned facts, it is found that ITSP is effective in helping analyze pre-defined policy scenarios and dealing with coupled uncertainties in both stochastic and interval formats; but it is incapable of reflecting the expected losses from extreme events. The CVaR model can be used to analyze the trade-off between system benefit and risk; however, it is relatively weak in handling multiple forms of uncertain information and analyzing implications of various policy scenarios. With varied pros and cons, there exists a potential for compensating each other when these two methods are integrated within a general optimization framework.

3 Model Formulation

3.1 Overview of the Problem

Consider a problem wherein a water manager is responsible for allocating water to multiple users, including municipal, industrial, and agricultural sectors (Huang and Loucks 2000; Maqsood et al. 2005; Li and Huang 2007). The objective is to maximize the economic benefit in the region over the planning horizon. Based on the local policies of water resources management, a target amount of water can be promised to each user sector in the region. If the promised water amount is delivered on time, it will bring net benefits to the system; otherwise, penalties of economic loss will be incurred as a result (Li and Huang 2007). For such a problem, the water allocation target needs to be designed at the beginning of the decision-making process; at a future time point when uncertain information of water flow is quantified, a recourse or corrective action should be taken (Maqsood et al. 2005).

3.2 Inexact Two-Stage Stochastic Programming

Depending on the quality of data, different uncertainty-analysis methods are available. Among various alternatives, SMP is mainly used to tackle uncertainties expressed as random variables described by probabilistic distribution functions (PDFs). They are normally generated from statistical analysis. In water resources management, the available water amount may exhibit random characteristic and the related distribution information could normally be obtained by analyzing the long-term time-series hydrological data (Eiger and Shamir 1991); however, in practical applications, such information may not always be available or sufficient. According to the available data information, assuming that water demands from different users are deterministic and the total water available is a random variable; other uncertain parameters are in interval formats. Then, an inexact two-stage stochastic programming model for water allocation problems can be formulated as (Kall 1979; Huang and Loucks 2000; Dai et al. 2000; Maqsood et al. 2005):

$$Maximize\ \ f^{\pm} = \sum_{j=1}^{J} NB_j^{\pm} T_j^{\pm} - \sum_{j=1}^{J} \sum_{s=1}^{S} p_s^{\pm} C_j^{\pm} D_{js}^{\pm} \tag{1a}$$

Subject to:

[Water availability constraints]

$$q_s^{\pm} \geq \sum_{j=1}^{J} \left(T_j^{\pm} - D_{js}^{\pm} \right), \quad \forall s \tag{1b}$$

[Non-negativity and technical constraint]

$$T_{j\max}^{\pm} \geq T_j^{\pm} \geq D_{js}^{\pm} \geq 0 \tag{1c}$$

where $T_j^{\pm}$, $D_{js}^{\pm}$ are the decision variables presented as discrete intervals; $NB_j^{\pm}$, $q_s^{\pm}$, $C_j^{\pm}$, $T_{j\max}^{\pm}$ are interval parameters; j is the index of water users, where $j = 1, 2, \ldots,$ J, and J is the total number of water users; p_s is the probability of scenario s; s is the index of scenarios where $s = 1, 2, \ldots, S$, and S is the total number of scenarios; $NB_j^{\pm}$ is the net benefit of user j per unit water allocated; $C_j^{\pm}$ is the reduction of net benefit to user j per unit water not delivered ; q_s is the available water resources under scenario s; $T_j^{\pm}$ is the water allocation target promised to user j; $D_{js}^{\pm}$ is the water deficit amount by which the water allocation target $T_j^{\pm}$ is not met under scenario s; $T_{j\max}^{\pm}$ is the maximum allowable allocation amount for user j.

From model (1), uncertainties associated with various water resources management components are presented as either discrete random variables or interval numbers. Optimal solutions leading to the maximum net benefit can be obtained due to the maximization of economic benefit in the objective function. Such a model normally has the tendency of promising a large amount of water resources to the user sector with a high net benefit; meanwhile, the penalty from the lack of water in such a sector is significant when there is a serious water shortage problem. In model (1), the possible loss is computed as the expected value of different probability conditions, such that the severity of extreme risks will be somewhat underestimated. The solutions from model (1) may lead to tremendous losses should an extremely adverse condition occur.

3.3 Conditional Value-at-Risk Model

The Conditional Value-at-Risk (CVaR) model, as a modified form of the Value-at-Risk (VaR) one, is used to examine the risk loss under specific probabilistic distributions (Andersson et al. 2001). According to Rockafellar and Uryasev (2002), VaR is defined as the maximum loss to be incurred over a certain time horizon or among different scenarios at a given level of cumulative distribution probability. Similarly, CVaR is defined as the mean loss given that the loss is greater than or equal to the VaR value. The relationship between VaR and CVaR can be found in the work of Webby et al. (2007) and Carneiro et al. (2010). Based on Huang and Loucks (2000), a SMP model with random constraints can be written as follows:

$$Maximize\ f(x) = \sum_{j=1}^{J} c_j x_j \tag{2a}$$

Subjective to

$$\sum_{j=1}^{J} a_{ij} x_j \leq b_i, \quad \forall i \tag{2b}$$

$$\sum_{j=1}^{J} u_j x_j \leq v(\omega) \tag{2c}$$

$$x_j \geq 0, \quad \forall j \tag{2d}$$

$$c_j, \ a_{ij}, \ u_j \neq 0, \quad \forall i, \ j \tag{2e}$$

where j is the index of decision variables, where $j = 1, 2, \ldots, J$, and J is the total number of decision variables; i is the index of deterministic constraints, where $i = 1, 2, \ldots, I$, and I is the total number of deterministic constraints; x_j are deterministic decision variables; $v(\omega)$ is a random number with probability distribution $p(\omega)$; c_j, a_{ij}, b_j and u_j are deterministic coefficients. Equation 2b represents the deterministic constraints with crisp parameters and Eq. 2c represents the stochastic constraints with random parameters in the right-hand side and crisp parameters in the left-hand side. Equations 2d and 2e are technical constraints, respectively.

To account for random risk aversion, it is preferable to maximize the economic returns subject to a certain level of risk loss. Risk aversion could be understood as the inverse of risk tolerance, which is the behavior of a manager to stay away from risky water allocation practices, even if these practices have high chances of profits. In this study, the possible risk loss will be estimated and reflected in the model constraints through incorporating CVaR concept. The transformed model can be written as follows (Carneiro et al. 2010)

$$Maximize \ f(x) = \sum_{j=1}^{J} c_j x_j \tag{3a}$$

subject to

$$CVaR[x, v(\omega)] \leq \beta \tag{3b}$$

$$\sum_{j=1}^{J} a_{ij} x_j \leq b_i, \quad \forall i \tag{3c}$$

$$\sum_{j=1}^{J} u_j x_j \leq v(\omega) \tag{3d}$$

$$x_j \geq 0, \quad \forall j \tag{3e}$$

$$c_j, \ a_{ij}, \ u_j \neq 0, \quad \forall i, \ j \tag{3f}$$

where Eq. 3b means that the acceptable loss should not exceed a threshold ; β is the maximum acceptable loss set by decision makers. The following equation can be used to compute the value of CVaR (Rockafellar and Uryasev 2002):

$$\begin{aligned} CVaR\left(x\right) &= E\left\{\zeta \in R | \Psi\left(x, \zeta\right) \geq \alpha\right\} \\ &= \left(1-\alpha\right)^{-1} \int_{L(x,\omega)>\zeta_\alpha(x)} L\left(x, \omega\right) P\left(\omega\right) d\omega \leq \beta \end{aligned} \tag{4}$$

where x is the vector of decision variables; $p(\omega)$ is the probability that the loss is not greater than ζ which represents a threshold; ω is a random parameter with a probability distribution $p(\omega)$; ζ is the maximum loss; α is the predefined confidence level; $\zeta_\alpha(x)$ means the maximum loss associated with the cumulative probability α and the decision variables; $L(x, \omega)$ is the loss function; $\Psi(x, \zeta)$ is the cumulative distribution function of $L(x, \omega)$. In order to solve the optimization model involved with CVaR, discrete scenarios of CVaR should be used. Thus, model (3) can be converted to (Rockafellar and Uryasev 2002):

$$Maximize\ \ f\left(x\right) = \sum_{j=1}^{J} c_j x_j \tag{5a}$$

subject to

$$\zeta_\alpha + \frac{1}{1-\alpha} \sum_{s=1}^{S} p_s \eta_s \leq \beta, \quad \forall s \tag{5b}$$

$$L_s\left(x, \omega\right) - \zeta_\alpha - \eta_s \leq 0, \quad \forall s \tag{5c}$$

$$\sum_{j=1}^{J} a_{ij} x_j \leq b_i, \quad \forall i \tag{5d}$$

$$\sum_{j=1}^{J} u_j x_j \leq v\left(\omega\right) \tag{5e}$$

$$\eta_s \geq 0 \tag{5f}$$

$$x_j \geq 0, \quad \forall j \tag{5g}$$

$$c_j,\ a_{ij},\ u_j \neq 0, \quad \forall i,\ j \tag{5h}$$

where p_s is the probability of scenario s; s is the index of scenarios where $s = 1, 2, \ldots,$ S, and S is the total number of scenarios; η_s is a positive auxiliary variable. Finally, the deterministic objective function values and decision variables (i.e. f_{opt} and x_{jopt}) at different confidence levels can be obtained.

3.4 CVaR-Based Inexact Two-Stage Programming Model

To reflect and calculate the extreme expected loss in ITSP model, the CVaR model can be incorporated. This leads to the formulation of a CVaR-based inexact two-stage programming model (CITSP). The lost function will be defined as the expected mean of penalty when the promised water is not delivered. In detail, a CITSP model can be written as:

$$Maximize\ \ f^{\pm} = \sum_{j=1}^{J} NB_j^{\pm} T_j^{\pm} - \sum_{j=1}^{J} \sum_{s=1}^{S} p_s^{\pm} C_j^{\pm} D_{js}^{\pm} \tag{6a}$$

Subject to:

[Water availability constraints]

$$q_s^{\pm} \geq \sum_{j=1}^{J} \left(T_j^{\pm} - D_{js}^{\pm} \right), \forall s \tag{6b}$$

[CVaR constraints]

$$\zeta_\alpha + \frac{1}{1-\alpha} \sum_{s=1}^{S} p_s \eta_s \leq \beta \tag{6c}$$

$$L_s \left(D_{js}, \omega \right) - \zeta_\alpha - \eta_s \leq 0 \tag{6d}$$

$$\eta_s \geq 0 \tag{6e}$$

[Non-negativity and technical constraint]

$$T_{j\max}^{\pm} \geq T_j^{\pm} \geq D_{js}^{\pm} \geq 0 \tag{6f}$$

where $D_{js}^{\pm}$ and $T_j^{\pm}$ are the decision variables; $D_{js}^{\pm}$ is the amount of water deficit by which the water allocation target $T_{js}^{\pm}$ is not met in scenario s; $T_j^{\pm}$ is the allocation target of water that is promised to user j; j is the index of water users, where $j = 1, 2, \ldots, J$, and J is the total number of water users; s is the index of scenarios where $s = 1, 2, \ldots, S$, and S is the total number of scenarios; $NB_j^{\pm}$ is the net benefit of user j per unit of water allocated; $C_j^{\pm}$ is the reduction of net benefit to user j per unit of water not delivered; q_s is the available water resources in scenario s; $L_s(\cdot)$ is the loss function at scenario s; $T_{j\max}^{\pm}$ is maximum allowable allocation amount for user j; p_s is the probability of scenario s; α is the confidence level, indicating that the cumulative probability of loss being lower than ζ_α is α; ζ_α is an auxiliary variable, which is the maximum loss at the cumulative probability α; β is the maximum acceptable loss set; η_s is a positive auxiliary variable; ω is a stochastic factor.

Generally, an ITSP model can be transformed into two deterministic submodels, corresponding to the upper and lower bounds of the desired objective-function values. According to Huang and Loucks (2000), if $T_j^{\pm}$ is considered as an uncertain input parameter, the general interval linear programming (ILP) approach cannot be used directly. An additional decision variable y_j $(0 \leq y_j \leq 1)$ can be introduced to mitigate the problem. Let $\Delta T_j = T_j^{+} - T_j^{-}$, $T_j^{\pm}$ would be equivalent to $T_j^{-} + \Delta T_j y_j$.

Such a setting will correspond to the highest system benefit given the uncertain water-allocation targets (Huang and Loucks 2000). Thus, the upper submodel of CITSP can be written as:

$$Maximize \quad f^{+}=\sum_{j=1}^{J} NB_j^{+}\left(T_j^{-}+\Delta T_j y_j\right)-\sum_{j=1}^{J}\sum_{s=1}^{S} p_s C_j^{-} D_{js}^{-} \tag{7a}$$

Subject to:

$$\sum_{j=1}^{J}\left(\Delta T_j-D_{js}^{-}\right)\leq q_s^{+}-\sum_{j=1}^{J} T_j^{-}, \quad \forall s \tag{7b}$$

$$\Delta T_j y_j \leq T_{j\max}^{+}-T_j^{-}, \quad \forall j \tag{7c}$$

$$D_{js}^{-}-\Delta T_j y_j \leq T_j^{-}, \quad \forall j, s \tag{7d}$$

$$\zeta_\alpha+\frac{1}{1-\alpha}\sum_{s=1}^{S} p_s \eta_s \leq \beta \tag{7e}$$

$$L\left(D_{js}, \omega\right)-\zeta_\alpha-\eta_s \leq 0 \tag{7f}$$

$$\eta_s \geq 0 \tag{7g}$$

$$D_{js}^{-} \geq 0, \ \forall j, s \tag{7h}$$

$$0 \leq y_j \leq 1, \ \forall j \tag{7i}$$

where D_{js}^{-} and y_j are decision variables. The solution of f^{+} provides the extreme upper bound of system benefit. Let $y_{j,opt}$ and $D_{js,opt}^{-}$ be the solution of model (7), the optimized water allocation targets can be generated as $T_{j,opt}=T_j^{-}+\Delta T_j y_{j,opt}$. Based on the optimized water allocation targets, the lower submodel of f^{-} can be formulated as:

$$Maximize \quad f^{-}=\sum_{j=1}^{J} NB_j^{-}\left(T_j^{-}+\Delta T_j y_{j,opt}\right)-\sum_{j=1}^{J}\sum_{s=1}^{S} p_s C_j^{+} D_{js}^{+} \tag{8a}$$

Subject to:

$$\sum_{j=1}^{J}\left(\Delta T_j y_{j,opt}-D_{js}^{+}\right)\leq q_s^{-}-\sum_{j=1}^{J} T_j^{-}, \quad \forall s \tag{8b}$$

$$D_{js}^{+}-\Delta T_j y_{j,opt} \leq T_j^{-}, \quad \forall j, s \tag{8c}$$

$$D_{js}^{+} \geq D_{js}^{-}, \quad \forall j, s \tag{8d}$$

where D^{+}_{js} are decision variables. Submodels (7) and (8) are deterministic linear programming problems and can be solved easily. The CVaR constraint is not included in the second submodel. This is because, the major function of CVaR is to identify the optimized water allocation targets (i.e. $T_{j,opt}$) under different scenarios. Since $T_{j,opt}$ has already be obtained from solving the upper submodel (i.e. f^{+}), it wouldn't be necessary to re-address the CVaR constraint. The final solutions for model (6) can be generated as follows: $f^{\pm}_{opt} = \left[f^{-}_{opt}, f^{+}_{opt}\right]$ and $D^{\pm}_{js,opt} = \left[D^{-}_{js,opt}, D^{+}_{js,opt}\right], \forall j, s$, where f^{+}_{opt} and $D^{-}_{js,opt}$ are solutions of submodels (7), and f^{-}_{opt} and $D^{+}_{js,opt}$ are those of submodel (8). The optimal water allocation amount will be $A^{\pm}_{js,opt} = T_{j,opt} - D^{\pm}_{js,opt}, \forall j, s$.

Figure 1 shows the general framework of the CITSP model. It is based on integration of ITSP and CVaR, and each technique has a unique contribution in enhancing the model's capability in dealing with system uncertainties. Based on models (7) and (8), a CITSP model could be transformed into its deterministic sub-models, and solved by standard linear optimization algorithm. The optimization model is coded and solved by using LINGO 12.0, which is a commercial optimization platform. Many applications have demonstrated that LINGO provides an easy-to-use language to describe the components and interactive relationships among complex systems, and is mostly suitable for tackling large-scale real-world problems. The hardware setting for running LINGO in this study is listed as follows: (1) Operation System: Microsoft windows XP home edition (32 bits/SP2/DirectX 9.0c); (2) CPU: Intel core duo T2450 at 2.00 GHz; (3) SIMM: 2 GB (HLX DDR2 667 MHZ). Generally, the computational time of solving a CITSP model under different parameter settings is just within a few seconds. Such a short computational requirement offers a chance to investigate the problem under a large number of scenarios, such that a variety of alternatives at different confidence levels could be obtained. This also reveals that the CITSP model could be easily used by decision makers to evaluate the trade-off between system economy and risk provided that a suitable computational platform is available. The

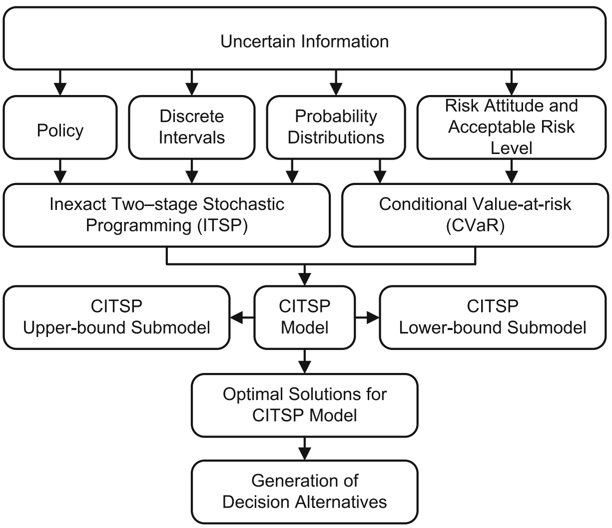

Fig. 1 General framework of the CITSP model

detailed procedures of formulating and solving a CITSP model are summarized as follows (Maqsood et al. 2005):

Step 1: Identify uncertain parameters and procure the related probabilistic distributions, discrete interval information, and acceptable risk levels;
Step 2: Formulate a CITSP model and transform the CITSP model into two sub-models;
Step 3: Solve the upper sub-model (f^+) with CVaR constraint, and obtain $D^-_{js,opt}$, f^+ and $y_{j,opt}$;
Step 4: Calculate $T_{j,opt} = T^-_j + \Delta T_j y_{j,opt}$;
Step 5: Solve the lower submodel (i.e. f^-) and obtain $D^+_{js,opt}$ and f^-;
Step 6: Generate the solutions of the ITSRP model as follows: $D^{\pm}_{js,\,opt} = \left[D^-_{js\,,opt}, D^+_{js\,,opt}\right], \quad \forall j, s.$, $f^{\pm}_{opt} = \left[f^-_{opt}, f^+_{opt}\right]$ and $A^{\pm}_{js,\,opt} = \left[T_{j,opt} - D^{\pm}_{js,opt}\right], \quad \forall j, s.$

4 Case Study

4.1 Overview of the Study System

A hypothetical water resources allocation problem is used to demonstrate the applicability of the proposed CITSP model (Huang and Loucks 2000). A water manager is responsible for allocating water from a reservoir to three users during a dry season: a municipality unit, an industrial unit, and an agricultural unit (Maqsood et al. 2005; Li and Huang 2007). Figure 2 shows a flow diagram of such a system. The three users want to know how much water they can expect to obtain in the future. If the promised water amount is delivered, the net benefit to the local economy will be generated. However, if the promised water amount is not delivered, either the water must be obtained from higher priced alternatives or the demand must be curtailed by reduced production, resulting in a reduced net system benefit (Huang and Loucks 2000; Maqsood et al. 2005).

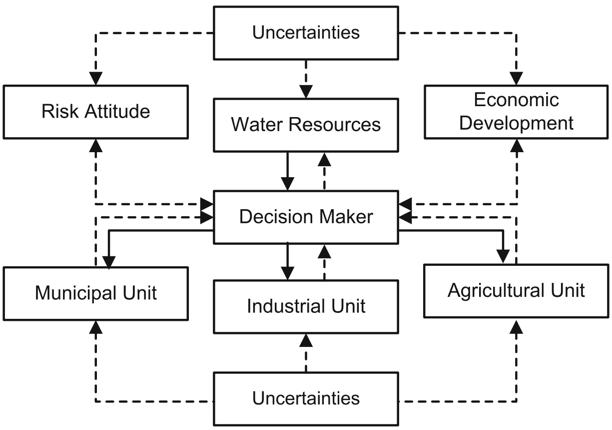

Fig. 2 Flow diagram of an integrated regional water resources allocation system

Table 1 Parameters related to water users and economic benefit

Parameters	Parameters meaning	Municipal ($j = 1$)	Industrial ($j = 2$)	Agricultural ($j = 3$)
T_{jmax} (million cubic meters)	Maximum allowable water allocation	15.0	20.0	35.0
T_j (million cubic meters)	Water allocation target	[6.0, 10.0]	[9.5, 15.0]	[16.0, 27.0]
NB_j (million \$/ million cubic meters)	Net benefit when water demand is satisfied	[0.8, 1.1]	[0.55, 0.7]	[0.25, 0.35]
C_j (million \$/ million cubic meters)	Reduction of net benefit when water is not delivered	[1.8, 2.1]	[0.95, 1.1]	[0.5, 0.6]

Part of the data is modified from Huang and Loucks (2000); j is the index of water users

However, the allocation problem becomes more complicated when the study region is vulnerable to droughts and the water resources availability has high variations. Since the local economic development heavily relies on the availability of water supply, the adaptive strategies to water shortage crisis are of high importance to local government. When water shortage is on a small scale, the major consequence may simply be a loss of benefit. However, the extent of the damage may change from economic loss to system impairment when the degree of water shortage increases. When the undelivered water amount reaches an extreme high level, the impairment may lead to collapse of the regional socio-economic and political system. Furthermore, the existence of multiple uncertainties associated with the water resources system will aggravate the risk of system impairment and failure. Therefore, it is also desired that the risk control be considered in the water allocation planning. The problem under consideration turns into how to effectively allocate water to various sectors in order to achieve a maximum benefit under uncertainties, subjected to the constraint of a certain level of risk aversion. To solve such a problem, the proposed CITSP model will be used.

4.2 Parameter Identification

Tables 1 and 2 present the water resources and economic data, which are based on literatures (Huang and Loucks 2000; Maqsood et al. 2005). After a number of test

Table 2 Streamflow data under various probabilistic scenarios

Streamflow level	Scenarios (s)	Probability (p_s)	Flow (q_s) (million cubic meters)
Low	1	0.05	[18, 20]
	2	0.05	[22, 24]
	3	0.05	[26, 28]
Medium	4	0.15	[31, 33]
	5	0.20	[37, 39]
	6	0.20	[43, 45]
	7	0.15	[48, 50]
High	8	0.05	[52, 54]
	9	0.05	[56, 58]
	10	0.05	[60, 62]

runs, it was found that, if β value is too small (e.g. lower than \$2 million), the optimal solutions would not be obtained as the risk constraint is too stringent; meanwhile, when the β value is very large (e.g. more than \$20 million), the risk constraint becomes insignificant as the decision makers are willing to accept a high economic loss. Consequently, we choose \$6, \$8 and \$10 million (denoted by β_1, β_2, β_3) as the representative β values in result analysis. Similarly, we choose 0.5, 0.7, 0.9, and 0.95 as the representative values of α. The effects of α and β on the model solutions will be discussed through sensitivity analysis.

4.3 Result Analysis

Tables 3 and 4 present the solutions obtained from CITSP model under different α and β levels, and Fig. 3 presents the corresponding optimized allocation targets of different users. Since CITSP is based on ITSP, the solution shows characteristics of an ITSP model. Firstly, the objective values and most of the decision variables are interval numbers, indicating the promulgation effect of interval inputs. For example, when β is β_2 and α is 0.7, the objective function value would be [16.41, 24.88] million \$. The solution corresponding to the upper bound of the net benefit is obtained under the most optimistic condition (e.g. high net benefit income) when the water deficit (i.e. $D_{js}^{\pm}$) are at their lower bounds; meanwhile, the objective function value corresponds to the most pessimistic condition when the water allocation deficit reaches their lower-bound levels. In practical applications, the decision makers could adjust the decision variables within their solution intervals by incorporating the stakeholder's implicit knowledge and preferences.

The obtained solutions from the CITSP model show the tendency to achieve high benefit under a specific CVaR constraint. Generally, water allocation would firstly be guaranteed to the municipal sector, followed by the industrial and the agricultural sectors. This is because the municipal water consumption brings the highest benefit when the water demand is satisfied and is subject to the highest penalty if the

Table 3 Objective function value and optimal targets of the CITSP model

β levels	$T_{j,opt}$ (million cubic meters) and f_{opt} (million \$)	α levels			
		$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.95$
β_1	$T_{1,opt}$	10.0	10.0	8.5	6.5
	$T_{2,opt}$	14.0	13.9	9.5	9.5
	$T_{3,opt}$	20.3	16.0	16.0	16.0
	Sum of $T_{j,opt}$	44.3	39.9	34.0	32.0
	f_{opt}	[16.3, 25.0]	[16.7, 24.5]	[14.7, 20.8]	[13.4, 18.8]
β_2	$T_{1,opt}$	10.0	10.0	10.0	10.0
	$T_{2,opt}$	14.0	14.0	11.4	10.0
	$T_{3,opt}$	21.0	19.9	16.0	16.0
	Sum of $T_{j,opt}$	45.0	43.9	37.4	36.0
	f_{opt}	[16.2, 25.0]	[16.4, 24.9]	[16.2, 23.2]	[15.7, 22.5]
β_3	$T_{1,opt}$	10.0	10.0	10.0	10.0
	$T_{2,opt}$	14.0	14.0	14.0	12.1
	$T_{3,opt}$	21.0	21.0	16.2	16.0
	Sum of $T_{j,opt}$	45.0	45.0	40.2	38.1
	f_{opt}	[16.2, 25.0]	[16.2, 25.0]	[16.7, 24.5]	[16.3, 23.6]

Table 4 Solutions of the CITSP model under different α and β levels

Solution of D_{js} from CITSP model (million cubic meters)

j	s	β_1				β_2				β_3			
		$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.95$
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	[4.0, 6.0]	[3.9, 5.9]	0	0	[4.0, 6.0]	[4.0, 6.0]	[1.4, 3.4]	[0, 2.0]	[4.0, 6.0]	[4.0, 6.0]	[4.0, 6.0]	[2.1, 4.1]
3	1	20	16	[14, 16]	[12, 14]	21	20	16	16	21	21	16	16
1	2	0	0	0	0	0	0	0	0	0	0	0	0
2	2	[0, 2.0]	[0, 1.9]	0	0	[0, 2.0]	[0, 2.0]	0	0	[0, 2.0]	[0, 2.0]	[0, 2.0]	0
3	2	20	16	[10, 12]	[8.0, 10]	21	20	[13, 15]	[12, 14]	21	21	16	[14, 16]
1	3	0	0	0	0	0	0	0	0	0	0	0	0
2	3	0	0	0	0	0	0	0	0	0	0	0	0
3	3	[16, 18]	[12, 14]	[6.0, 8.0]	[4.0, 6.0]	[17, 19]	[16, 18]	[9.4, 11]	[8.0, 10]	[17, 19]	[17, 19]	[12, 14]	[10, 12]
1	4	0	0	0	0	0	0	0	0	0	0	0	0
2	4	0	0	0	0	0	0	0	0	0	0	0	0
3	4	[11, 13]	[6.9, 8.9]	[1.0, 3.0]	[0, 1.0]	[12, 14]	[11, 13]	[4.4, 6.4]	[3.0, 5.0]	[12, 14]	[12, 14]	[7.2, 9.2]	[5.1, 7.1]
1	5	0	0	0	0	0	0	0	0	0	0	0	0
2	5	0	0	0	0	0	0	0	0	0	0	0	0
3	5	[5.3, 7.3]	0.9	0	0	[6.0, 8.0]	[4.9, 6.9]	0	0	[6.0, 8.0]	[6.0, 8.0]	[1.2, 3.2]	[0, 1.1]
1	6	0	0	0	0	0	0	0	0	0	0	0	0
2	6	0	0	0	0	0	0	0	0	0	0	0	0
3	6	[0, 1.3]	0	0	0	0	[0, 0.9]	0	0	[0, 2.0]	[0, 2.0]	0	0

When $s \geq 7$, the solutions under different α and β level are zero; s is the index of the streamflow scenario; j is the index of water users

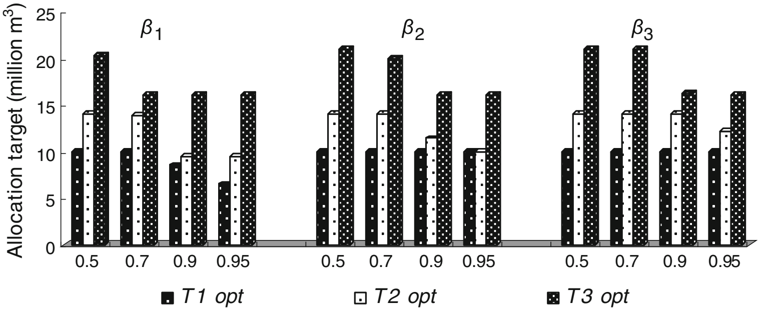

Fig. 3 Optimized allocation targets of different users under different α and β levels

promised water amount is not delivered; whereas, the industrial and agricultural sectors have lower benefits and penalties. For example, when α value is 0.7 and β is β_2, the solutions of $D_{12}^{\pm}$, $D_{22}^{\pm}$, and $D_{32}^{\pm}$ would be 0, [0, 2.0], and 20 million cubic meters. This indicates that, under a low streamflow level, there would be no shortage of water for municipal sector; but shortages at the levels of [0, 2.0] and 20 million cubic meters could occur for industrial and agricultural sector, respectively. The results of $D_{15}^{\pm}$, $D_{25}^{\pm}$ and $D_{35}^{\pm}$ would be 0, 0, [4.9, 6.9] million cubic meters. This indicates that, under a medium flow condition, there will be no shortages of water for municipal and industrial uses, and a shortage level of [4.9, 6.9] million cubic meters could occur for agricultural sector. The solutions of $D_{19}^{\pm}$, $D_{29}^{\pm}$, $D_{39}^{\pm}$ (i.e. all zero) denote that, under a high streamflow level, there would be no shortage of water for all sectors.

Tables 3 and 4 demonstrate that the CITSP model also possesses characteristics of a CVaR model where the trade-off between system economy and risk aversion could be analyzed. This could be reflected by assigning different α values to the model at the stage of target formulation, given a fixed β value. From Tables 3 and 4 as well as Fig. 3, a number of decision variables such as the target values ($T_{j,opt}$), and the upper and lower bounds of the shortage amount ($D_{js,opt}^{\pm}$) would vary with the change of α. For example, $T_{3,opt}$ would decrease from 20.3 to 16 million cubic meters when the α value increases from 0.5 to 0.95 and β is fixed at β_1. When the available water quantity is at the low streamflow level, the water shortage ($D_{32,opt}^{\pm}$) would be 20, 16, [10, 12], [8, 10] million cubic meters under α values of 0.5, 0.7, 0.8 and 0.95, respectively; at the medium streamflow level, the water shortage ($D_{35,opt}^{\pm}$) would decrease to [5.3, 7.3], [0.9, 2.9], 0 and 0 million cubic meters, respectively. This shows that the effect of the risk measure on the modeling outputs could be adjusted by changing α value. Generally, as α value increases, the allocation target values would decrease, leading to reduced amount of water shortage. In such a case, the extreme risk could be lowered and the system feasibility be enhanced. On the contrary, a lower α value would result in a higher possibility of system loss in extreme conditions.

Moreover, a tradeoff could be analyzed by assigning different β values in the model constraints when α is fixed. From Tables 3 and 4, a number of decision variables such as the target values ($T_{j,opt}$) and the upper and lower bounds of the shortage amount ($D_{js,opt}^{\pm}$) would vary with different β values. For example, $T_{2,opt}$ would increase from 9.5 to 14 million cubic meters when α is fixed at 0.9 and β changes from β_1 to β_3. When the available quantity of water is at the low level of stream flow, the amount of water shortage $D_{21opt}^{\pm}$ would turn to 0, [1.4, 3.4] and [4.0, 6.0] at β

values of β_1, β_2 and β_3, respectively. Generally, as β increases, the allocation target will increase, leading to an increased amount of water shortage. The reason is that when the acceptable risk level β increases, the risk constraint would become looser; consequently, the development scheme with a higher benefit is more attractive to decision makers. On the contrary, a lower β value would result in alternatives with higher risk aversions.

Figure 4 shows the varying trend of the total allocation target and the sum of water deficit intervals (from three users) under different α and β levels at the low streamflow condition (i.e. s= 1, 2, 3). It appears that the related outputs are affected by the risk constraint. For example, the total allocation target would increase from 34 to 40.2 million cubic meters when α is fixed at 0.9 and β changes from β_1 to β_3; the corresponding water shortages at β_3 for the municipal, industrial and agricultural users would be [6, 16], [9.4, 19.4] and [12, 22] million cubic meters, respectively. When β is fixed at β_3 and α is 0.5, 0.7,0.9 and 0.95, the total allocation target would become 45, 45, 40.2 and 38.1 million cubic meters, respectively, and the corresponding water shortages of the whole region will be [17, 27], [17, 27], [12, 22] and [10, 20.1] million cubic meters, respectively. This is due to the fact that, with the increase of strictness of the risk constraint (increase of α or decrease of β levels), the optimal allocation target will be compromised; this leads to an alleviated water deficit problem.

Figure 5 shows the varying trend of the system net benefit. Generally, the intervals of the total benefits would narrow down as α value increases. For example, when β is fixed at β_2, the total expected system benefits would become 8.9, 8.5, 7.0 and 6.7 million $ under α values of 0.5, 0.7, 0.9 and 0.95, respectively. This implies that a lower system benefit would guarantee a lower system risk; conversely, if the planner aims towards a greater return, a higher expected loss may have to be confronted. In real-world applications, the decision makers may need to choose between a conservative solution with a lower benefit and a more risky solution with a higher benefit. Thus, a trade-off analysis will have to be made.

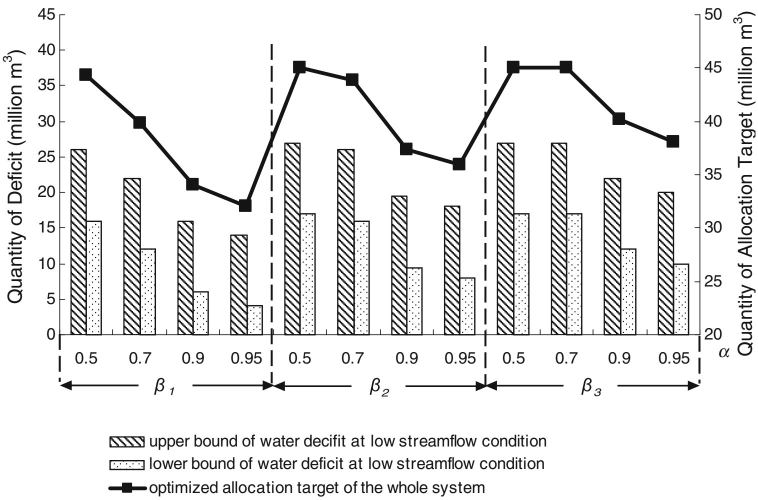

Fig. 4 Optimized total allocation target and water deficit intervals at the low streamflow condition

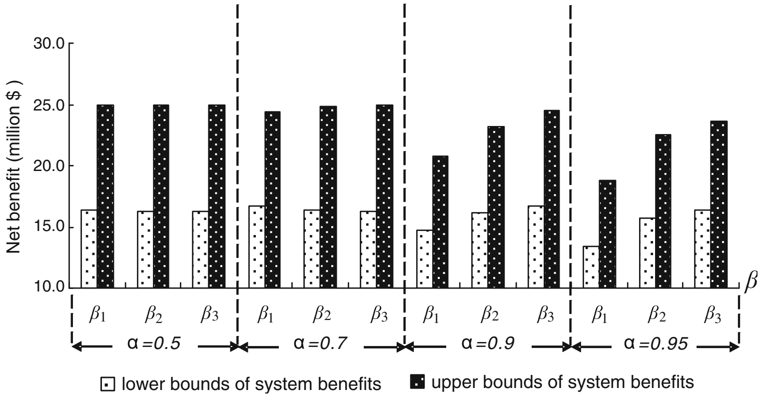

Fig. 5 Net benefits obtained from the CITSP model under different α levels

Generally, both the ITSP model and the CVaR model have specific merits and limitations, and their integration performs better. Firstly, CITSP could address uncertainties in water resources allocation systems in different formats due to features of ITSP. Secondly, CITSP could help establish an effective linkage between the pre-defined water policies, the associated economic implications and the possible expected loss due to incorporation of CVaR concept. The linkage forms the base for selecting the desired water-allocation strategies in light of multiple uncertainties and risk aversions. Finally, the interval-format solutions offer a flexible decision space for the water managers to choose suitable management schemes.

4.4 Comparison between ITSP and CITSP

ITSP can also be used to solve the same problem. Table 5 shows the corresponding solutions. Different from CITSP model, an ITSP model searches for the optimal solution to obtain the maximum benefit without regard to risk aversion; consequently,

Table 5 Solutions of the ITSP model

	Water users		
	Municipal ($j=1$)	Industrial ($j=2$)	Agricultural ($j=3$)
$T_{i,opt}$ (million cubic meters)	10.0	14.0	21.0
$D_{js,opt}$ (million cubic meters)			
$s=1$	0	[4.0, 6.0]	21.0
$s=2$	0	[0, 2.0]	21.0
$s=3$	0	0	[17.0, 19.0]
$s=4$	0	0	[12.0, 14.0]
$s=5$	0	0	[6.0, 8.0]
$s=6$	0	0	[0, 2.0]
$s=7$	0	0	0
f_{opt} (million $)	[16.2, 25.0]		

When $s \geq 7$, the solutions of $D_{js,opt}$ are zero it is incapable of analyzing the trade-off between the system benefit and risk. For example, a higher target value ($T_{j,opt}$) would be generated by ITSP to bring higher net benefit to the system, compared with that by CITSP. Moreover, the benefit interval of ITSP is wider than that of CITSP. This implies that the system income largely relies on the water resources condition, and tends to fluctuate more intensively with the change of available water resources when CVaR constraint is not incorporated. Finally, the solutions from ITSP seem to be exactly the same as those from CITSP when the α is zero or β is sufficiently large. This shows that the ITSP model is a special or simplified case of the CITSP one.

4.5 Further Discussions

Figure 6 shows the sensitivity analysis results for the effects of α and β on the system benefit, where the upper and the lower parts of the figure correspond to the upper and lower bounds of net benefit, respectively. It appears that different combination of α and β levels will notably influence the optimal objective value. A lower acceptable risk level (β) and a higher confidence level (α) could give rise to a lower system benefit; conversely, a higher acceptable risk level (β) and a lower confidence level (α) would create a higher system benefit. The water managers could assign different α and β values to adjust risk-control levels based on their preferences.

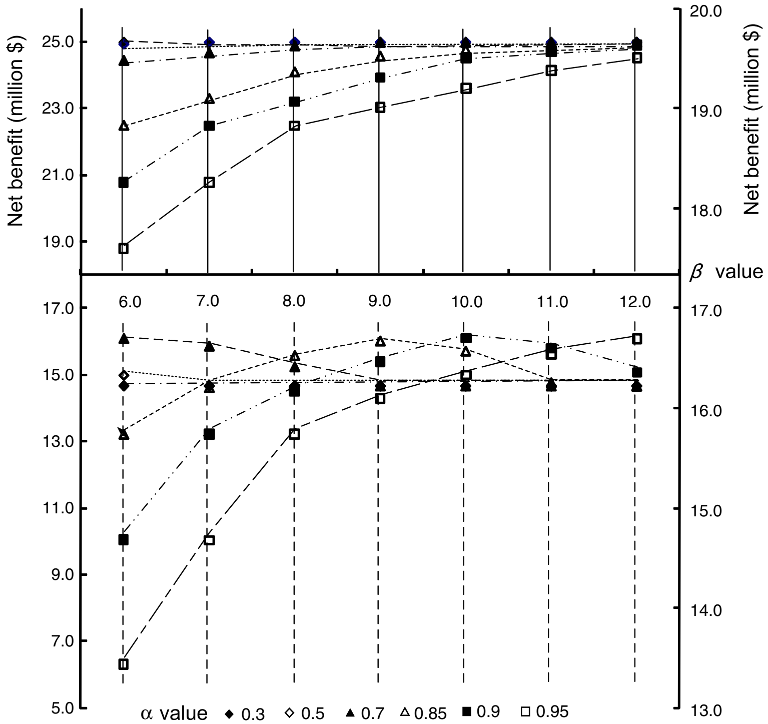

Fig. 6 Distribution of the lower and upper bounds of the net benefits under different α and β levels

Another fact from Fig. 6 is that α and β have different sensitivity levels. When β value is 6 million \$, different objective values can be obtained at different α levels. However, with an increase of β value, the variation degree of the objective value would decrease gradually. This indicates that the risk constraint in effect is weakening. When β value increases to 12 million \$, the objective value under different α levels would be nearly constant, indicating that the risk constraint is of insignificant effect. Thus, the α and β values in the CVaR constraint should be properly chosen in order to avoid the invalidation of risk control. Generally, in choosing appropriate solutions for real-world applications, it is suggested that: (1) the multi-criteria decision analysis (MCDA) technologies be used for ranking potential alternatives; (2) the decision makers incorporate their implicit knowledge (such as socio-economical conditions) and preferences about the risk and benefits into the problems and generate more practical decisions (Xu et al. 2009).

The CITSP model also shows a number of limitations and needs improvement in further studies. Firstly, a CITSP model may encounter difficulties when the model's right side parameters are highly uncertain (i.e. parameters with wide intervals). Such a limitation could be mitigated through incorporation of other uncertainty-analysis algorithms such as chance-constrained programming (Qin et al. 2010) or fuzzy flexible programming (Xu et al. 2009). Secondly, the obtained solutions from CITSP are represented as a spectrum of alternatives under different risk control levels. To make the final decision, not only the attitude of decision makers toward risk, but also their abilities of comprehensive consideration and integrated assessment to different development schemes must be considered. This may become an extra burden to decision makers and affect the applicability of the model. A potential solution would be the use of multiple attribute decision making (MADM) methods (Roy 1991).

5 Conclusions

A conditional value-at-risk (CVaR) based inexact two-stage stochastic programming (CITSP) model was developed in this study for supporting water resources allocation planning. In CITSP, uncertainties could be described as both discrete intervals and probabilistic distribution functions (PDFs).A risk measure, as described by CVaR, was incorporated within the CITSP model to represent the expected losses on extreme resources conditions. A hypothetical case of water resources allocation was used to demonstrate the applicability of the proposed model. The obtained first-stage allocation-target values could be used to reflect decision makers' opinions on the long-term development plan, and the percentile α and risk acceptance β could be used to reflect decision maker's preference towards system benefit and risk control. The results demonstrated that the CITSP model was able to help the decision makers generate cost-effective allocation schemes, gain in-depth insights into the effects of uncertainties, and analyze the trade-offs between system economy and risk aversion.

Compared with other methods, CITSP showed a number of advantages and limitations. It could address uncertainties in water resources allocation systems in different formats, and help establish an effective linkage between the pre-defined water policies, the associated economic implications and the possible expected loss. However, CITSP would encounter difficulties when the model's right-hand-side coefficients are highly uncertain; the selection of a suitable alternative among the obtained interval solutions under different α and β values is of significant complexity and becomes an extra burden for water resources managers. Further investigations are desired to tackle these issues. The proposed integrated optimization technique appears to be not only suitable for water allocation planning, but also applicable to many other water resources and environmental management problems.

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