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Multivariate Bayesian Regression Approach to Forecast Releases from a System of Multiple Reservoirs

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Abstract This research presents a model that simultaneously forecasts required water releases 1 and 2 days ahead from two reservoirs that are in series. In practice, multiple reservoir system operation is a difficult process that involves many decisions for real-time water resources management. The operator of the reservoirs has to release water from more than one reservoir taking into consideration different water requirements (irrigation, environmental issues, hydropower, recreation, etc.) in a timely manner. A model that forecasts the required real-time releases in advance from a multiple reservoir system could be an important tool to allow the operator of the reservoir system to make better-informed decisions for releases needed downstream. The model is developed in the form of a multivariate relevance vector machine (MVRVM) that is based on a sparse Bayesian regression model approach. With this Bayesian approach, a predictive confidence interval is obtained from the model that captures the uncertainty of both the model and the data. The model is applied to the multiple reservoir system located in the Lower Sevier River Basin near Delta, Utah. The results show that the model learns the input–output patterns with high accuracy. Computing multiple-time-ahead predictions in real-time would require a model which guarantees not only good prediction accuracy but also robustness with respect to future changes in the nature of the inputs data. A bootstrap analysis is used to guarantee good generalization ability and robustness of the MVRVM. Test results demonstrate good performance of predictions and statistics that indicate robust model generalization abilities. The MVRVM is compared in terms of performance and robustness with another multiple output model such as Artificial Neural Network (ANN).

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1 Introduction

The per capita availability of water resource is decreasing worldwide due to population growth, climate change, rapidly increasing demands (for irrigation, domestic supply, recreation, etc.), and pollution. In order to achieve greater operational efficiency in meeting these increasing demands and decreasing relative supplies, water managers must have better information about future conditions of their water systems. The purpose of this paper is to use a real-time model that can provide valuable information to the operator of a multiple reservoir system in the form of multi-time-ahead release predictions with predictive confidence intervals.

Techniques based on physical modeling have been developed to characterize current and future states of water resources systems. However, the lack of required data and the expense of data acquisition can limit the practical applications of physical based models (Khalil et al. 2005a). To overcome these limitations, researchers have used data-driven modeling as an alternative, or a complement, to physically based models (Lobbrecht and Solomatine 2002; Khalil et al. 2005a). Examples of such models include artificial neural network (ANNs), support vector machines (SVMs), and relevance vector machines (RVMs). These types of models are derived from the emerging area of machine-learning theory. They are characterized by their ability to capture the underlying physics of the system simply by examination of the inputs and outputs of the system. They can be used to provide predictions of the system behavior using only historical data; they "let the data speak" (Khalil et al. 2005d).

Machine learning models have often been applied in water resources management. Sivapragasam and Muttil (2005) used SVMs in the extrapolation of stage-discharge rating curves. Their results showed that a SVM model was better suited for extrapolation than a comparable ANN. El-Shafie et al. (2007) proposed an adaptive neuro-fuzzy inference system (ANFIS) to forecast the monthly inflow from the Nile River to a dam. Their results demonstrated that the ANFIS model was more accurate than an ANN model. Nourani et al. (2009) presented a multivariate ANN-wavelet model to predict short- and long-term runoff discharges. Their ANN-wavelet conjunction model performed better when compared with an Auto Regressive Integrated Moving Average (ARIMA) model. Guldal and Tongal (2010) compared four models to predict lake-level changes in Turkey: Recurrent neural network (RNN), ANFIS, auto regressive (AR) and auto-regressive moving average (ARMA). Their results indicated that RNN and ANFIS models were more accurate than AR and ARMA models.

Dams and reservoir systems have been built to regulate storage and manage water distribution. However, many of theses systems are not producing benefits that would economically justify their development (World Commission on Dams 2000; Labadie 2004). As a result, we must focus on improving the operational effectiveness of existing reservoir systems to maximize their value (Labadie 2004). Providing a model that forecasts releases from a system of multiple reservoirs could be an important tool in integrated reservoir operation and management. Information on predicted releases can allow the operator of the multiple reservoir system to make better-informed decisions for releases needed downstream.

The multiple reservoir system operation depends on physical behavior of the watershed (hydrologic, climatic, environmental, etc.) and human behavior. Human behavior takes the form of the reservoir operator who has to release water from more than one reservoir to fulfill different water requirements. The combination of all these behaviors may cause unexpected future changes in the reservoir system operation which could be extremely difficult to predict. Therefore, it is necessary to develop predictive models which have the ability to guarantee robustness towards futures changes in the system behavior.

Khalil et al. (2005a) applied RVM, which is based on Bayesian learning theory, to predict the real-time operation of a single reservoir. The target output of their model is the hourly prediction of the quantity of water to be released from a single reservoir in order to meet downstream diversion requirements. Their performance results showed that the RVM model was able to predict future system states (generalization ability) and had the capability to estimate the uncertainty of the predictions (predictive confidence intervals). The research reported here extends this capability to a multiple-day-ahead for multi reservoir setting.

In order to obtain multiple-time-ahead predictions with (predictive) confidence intervals, the model exploits the capability of the Multivariate Relevance Vector Machine (MVRVM) (Thayananthan 2005). The model forecasts the water releases of two reservoirs simultaneously having as inputs recent historical data on reservoir releases, diversions into canals, weather, and streamflows. The target outputs are the predictions of the required water releases from two reservoirs. These predictions are made 1 and 2 days ahead, simultaneously for each reservoir. Therefore, the model recognizes the patterns between future reservoir releases and historical data collected from the system.

The MVRVM is a Bayesian regression tool extension of the RVM algorithm developed by Tipping and Faul (2003) to produce multivariate outputs when given a set of inputs. In addition to its ability to predict multiple outputs, the MVRVM has the same properties of the conventional RVM: high prediction accuracy, robustness and characterization of uncertainty in the predictions. Therefore, developing a model with all these properties can work as a practical decision support tool in real-time water resources management by providing multiple predictions that are difficult (or not practical) to obtain from traditional modeling approaches.

The remainder of the paper describes the MVRVM learning model, the area of study where the model has been applied, how the model has been developed for a multiple reservoir system, the results of the MVRVM application, the comparison with the performance of an ANN model, and conclusions that can be drawn.

2 Model Description

Thayananthan (2005) proposed the Multivariate Relevance Vector Machine (MVRVM) to provide a regression tool capable of generating multivariate outputs. This model is an extension of the sparse Bayesian model developed by Tipping and Faul (2003). It is developed as follows.

Given a training data set of input-target vector pairs $\{\mathbf{x}^{(n)}, \mathbf{t}^{(n)}\}_{n=1}^{N}$, where N is the number of observations, $\mathbf{x} \in R^D$ is a D-dimensional input vector, $\mathbf{t} \in R^M$ is a M-dimensional output target vector; the model has to "learn" the dependency between input and output target with the purpose of making accurate predictions of $\mathbf{t}$ for previously unseen values of $\mathbf{x}$:

$$\mathbf{t} = \mathbf{W}\boldsymbol{\Phi}(\mathbf{x}) + \varepsilon \tag{1}$$

where $\mathbf{W}$ is a $M \times Q$ weight matrix and $Q = N + 1$. A fixed kernel function $K(\mathbf{x}, \mathbf{x}_j)$ is used to create a vector of basis functions of the form $\boldsymbol{\Phi}(\mathbf{x}) = \left[1, K\left(x, x^{(1)}, \ldots K\left(x, x^{(N)}\right)\right)\right)$. The error $\boldsymbol{\varepsilon}$ is conventionally assumed to be zero-mean Gaussian with diagonal covariance matrix $\mathbf{S} = \mathrm{diag}\left(\sigma_1^2, ..., \sigma_M^2\right)$.

Let $\mathbf{t} = [\boldsymbol{\tau}_1, ..., \boldsymbol{\tau}_r, ..., \boldsymbol{\tau}_M]^T$ and $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_r, ..., \mathbf{w}_M]^T$. A likelihood distribution of the weight matrix can be written as a product of Gaussians of the weight vectors ($\mathbf{w}_r$) corresponding to each target output ($\boldsymbol{\tau}_r$) (Thayananthan et al. 2008):

$$p\left(\left\{\mathbf{t}^{(n)}\right\}_{n=1}^{N} \left|\mathbf{W}, \mathbf{S}\right.\right) = \prod_{n=1}^{N} N\left(\mathbf{t}^{(n)} \left|\mathbf{W}\boldsymbol{\Phi}\left(\mathbf{x}^{(n)}\right), \mathbf{S}\right.\right) = \prod_{r=1}^{M} N\left(\boldsymbol{\tau}_r \left|\mathbf{w}_r \boldsymbol{\Phi}, \sigma_r^2\right.\right) \tag{2}$$

where $\boldsymbol{\Phi} = [1, \boldsymbol{\Phi}(\mathbf{x}_1), \boldsymbol{\Phi}(\mathbf{x}_2), ..., \boldsymbol{\Phi}(\mathbf{x}_N)]$. Equation 2 contains several parameters. As a result, there is a danger that the maximum likelihood estimation of $\mathbf{w}_r$ and σ_r^2 will suffer from severe over-fitting. To avoid this, Tipping (2001) proposed constraining the selection of parameters by applying a Bayesian perspective and defining an explicit zero-mean Gaussian prior probability distribution over the weights (Thayananthan et al. 2008):

$$p(\mathbf{W}|\mathbf{A}) = \prod_{r=1}^{M}\prod_{j=1}^{Q} N\left(w_{rj}|0, \alpha_j^{-2}\right) = \prod_{r=1}^{M} N(\mathbf{w}_r|0, \mathbf{A}) \tag{3}$$

where $\mathbf{A} = \mathrm{diag}(\alpha_1^{-2}, ..., \alpha_Q^{-2})^T$ is a hyperparameter matrix and w_{rj} is the (r,j)th element of the weight matrix $\mathbf{W}$. Each α_j controls the strength of the prior over its associated weight (Tipping and Faul 2003).

Bayesian inference considers the posterior distribution of the model parameters, which is proportional to the product of the likelihood and prior distributions:

$$p\left(\mathbf{W}\right| \{\mathbf{t}\}_{n=1}^{N}, \mathbf{S}, \mathbf{A}\Big) \propto p\left(\{\mathbf{t}\}_{n=1}^{N} \left|\mathbf{W}, \mathbf{S}\right.\right) p\left(\mathbf{W}|\mathbf{A}\right) \tag{4}$$

The posterior parameter distribution conditioned on the data can be written as the product of Gaussians for the weight vectors of each target output dimension (Thayananthan et al. 2008):

$$p\left(\mathbf{W}\right| \{\mathbf{t}\}_{n=1}^{N}, \mathbf{S}, \mathbf{A}\Big) \propto p\left(\{\mathbf{t}\}_{n=1}^{N} \left|\mathbf{W}, \mathbf{S}\right.\right) p\left(\mathbf{W}|\mathbf{A}\right) \propto \prod_{r=1}^{M} N\left(\mathbf{w}_r|\boldsymbol{\mu}_r, \boldsymbol{\Sigma}_r\right) \tag{5}$$

The posterior distribution of the weights is Gaussian $N(\mathbf{u}_r, \boldsymbol{\Sigma}_r)$ with the covariance and mean, $\boldsymbol{\Sigma}_r = (\mathbf{A} + \boldsymbol{\sigma}_r^{-2}\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}$ and $\boldsymbol{\mu}_r = \boldsymbol{\sigma}_r^{-2}\boldsymbol{\Sigma}_r\boldsymbol{\Phi}^T\boldsymbol{\tau}_r$, respectively. Given this posterior, we can obtain an optimal weight matrix by getting a set of hyperparameters that maximizes the data likelihood over the weights in Eq. 5. The marginal likelihood is then:

$$p\left(\{\mathbf{t}\}_{n=1}^{N}\left|\mathbf{A},\mathbf{S}\right.\right)=\int p\left(\{\mathbf{t}\}_{n=1}^{N}\left|\mathbf{W},\mathbf{S}\right.\right)p\left(\mathbf{W}|\mathbf{A}\right)d\mathbf{W},$$
$$=\prod_{r=1}^{M}\int N\left(\boldsymbol{\tau}_r\left|\mathbf{w}_r\boldsymbol{\Phi},\boldsymbol{\sigma}_r^2\right.\right)N\left(\mathbf{w}|\mathbf{0},\mathbf{A}\right)=\prod_{r=1}^{M}\left|\mathbf{H}_r\right|^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\boldsymbol{\tau}_r^T\mathbf{H}_r^{-1}\boldsymbol{\tau}_r\right)\quad(6)$$

where $\mathbf{H}_r=\sigma_r^2\mathbf{I}+\boldsymbol{\Phi}\mathbf{A}^{-1}\boldsymbol{\Phi}^T$. Then, we can obtain an optimal set of hyperparameters $\boldsymbol{\alpha}^{opt}=\left\{\boldsymbol{\alpha}_j^{opt}\right\}_{j=1}^{Q}$ and noise parameters $(\boldsymbol{\sigma}^{opt}.)^2=\left\{\boldsymbol{\sigma}_r^{opt}\right\}_{r=1}^{M}$ by maximizing the marginal likelihood using the fast marginal likelihood maximization algorithm proposed by Tipping and Faul (2003). During the optimization process, many elements of $\boldsymbol{\alpha}$ go to infinity, for which the posterior probability of the weight becomes zero. The few nonzero weights are the relevance vectors (RVs) which generate a sparse representation. Inducing sparsity can be an effective method to control model complexity, avoid over-fitting and control computational characteristics of model performance (Tipping and Faul 2003). Tipping (2001) and Tipping and Faul (2003) used synthetic and benchmark datasets as model examples to prove that the Bayesian learning procedure is capable of producing highly sparse models. Also, Thayananthan et al. (2008) showed the sparse properties of the MVRVM when modeling the motion of the hand and the whole human body from a single camera. Moreover, Khalil et al. (2005a, b, c) and Ghosh and Mujumdar (2007) used RVMs to model reservoir releases, lake volumes, groundwater contaminant levels, and streamflow, respectively. Their papers demonstrated that the RVM approach results in sparse models.

The optimal parameters are used to obtain the optimal weight matrix with optimal covariance $\boldsymbol{\Sigma}^{opt}=\left\{\boldsymbol{\Sigma}_r^{opt}\right\}_{r=1}^{M}$ and mean $\boldsymbol{\mu}^{opt}=\left\{\boldsymbol{\mu}_r^{opt}\right\}_{r=1}^{M}$.

In order to make predictions we can compute the predictive distribution for a new input $\mathbf{x}^*$:

$$p\left(\mathbf{t}^*\left|\mathbf{t},\boldsymbol{\alpha}^{opt},\left(\boldsymbol{\sigma}^{opt}\right)^2\right.\right)=\int p\left(\mathbf{t}^*\left|\mathbf{W},\left(\boldsymbol{\sigma}^{opt}\right)^2\right.\right).p\left(\mathbf{W}\left|\mathbf{t},\boldsymbol{\alpha}^{opt},\left(\boldsymbol{\sigma}^{opt}\right)^2\right.\right)d\mathbf{W}\quad(7)$$

Taking into consideration that both terms in the integrand are Gaussian, Eq. 7 is computed as:

$$p\left(\mathbf{t}^*\left|\mathbf{t},\boldsymbol{\alpha}^{opt},\left(\boldsymbol{\sigma}^{opt}\right)^2\right.\right)=N\left(\mathbf{t}^*\left|\mathbf{y}^*,\left(\boldsymbol{\sigma}^*\right)^2\right.\right)\quad(8)$$

where $\mathbf{y}^*=\left[\mathbf{y}_1^*,...,\mathbf{y}_r^*,...\mathbf{y}_M^*\right]^T$ is the predictive mean with $\mathbf{y}_r^*=\left(\boldsymbol{\mu}_r^{opt}\right)^T\boldsymbol{\Phi}(\mathbf{x}^*)$; and $(\boldsymbol{\sigma}^*)^2=\left[(\sigma_1^*)^2,...\ (\sigma_r^*)^2,...,\ (\sigma_M^*)^2\right]^T$ is the predictive variance with $\left(\sigma_r^*\right)^2=\left(\sigma_r^{opt}\right)^2+\boldsymbol{\Phi}(\mathbf{x}^*)^T\boldsymbol{\Sigma}_r^{opt}\boldsymbol{\Phi}(\mathbf{x}^*)$ which contains the sum of two variance terms: the noise on the data and the uncertainty in the prediction of the weight parameters (Tipping 2001).

Readers interested in greater detail regarding multivariate sparse Bayesian regression, its mathematical formulation and the optimization procedures of the model are referred to Thayananthan (2005), Thayananthan et al. (2008), Tipping (2001) and Tipping and Faul (2003). A MATLAB code developed by Thayananthan (2005) is available from http://mi.eng.cam.ac.uk/~at315/MVRVM.

3 Study Area

The Sevier River Basin is used here to demonstrate the MVRVM modeling approach as applied to the operation of multiple reservoirs. The basin located in south-central Utah and is the largest area drainage in the state (approximately 12.5 percent of the state's area). Average annual precipitation ranges from 6.4 to 13.0 in. in the valleys to more than 40 in. in the high mountains. Elevation, precipitation, and temperatures are highly variable over the basin, and as a result there are several vegetative types that grow in the area. The population of the basin was more than 56,700 people in 1997, with most of it residing in small farming communities. The population of the basin is expected to reach over 86,000 people by 2020 (based on the current annual growth rate of 1.82%). The economy of the Basin is based primarily on agriculture and also there are other important economic activities such as tourism, few mining and manufacturing enterprises (Berger et al. 2003).

The Sevier River Basin is a highly instrumented and controlled basin. Automated data collection equipment (sensors on reservoirs, canals, diversions, and the river itself) was installed beginning in 1999 (Berger et al. 2002), and is currently collecting data at stations throughout the entire basin. The data collected are displayed in the Sevier River Water Users website, www.sevierriver.org. This website provides the user several data retrieval and display options, such as hourly flow data for the previous 7 days or the current river and canal flow information displayed in spatial diagrams (Berger et al. 2002).

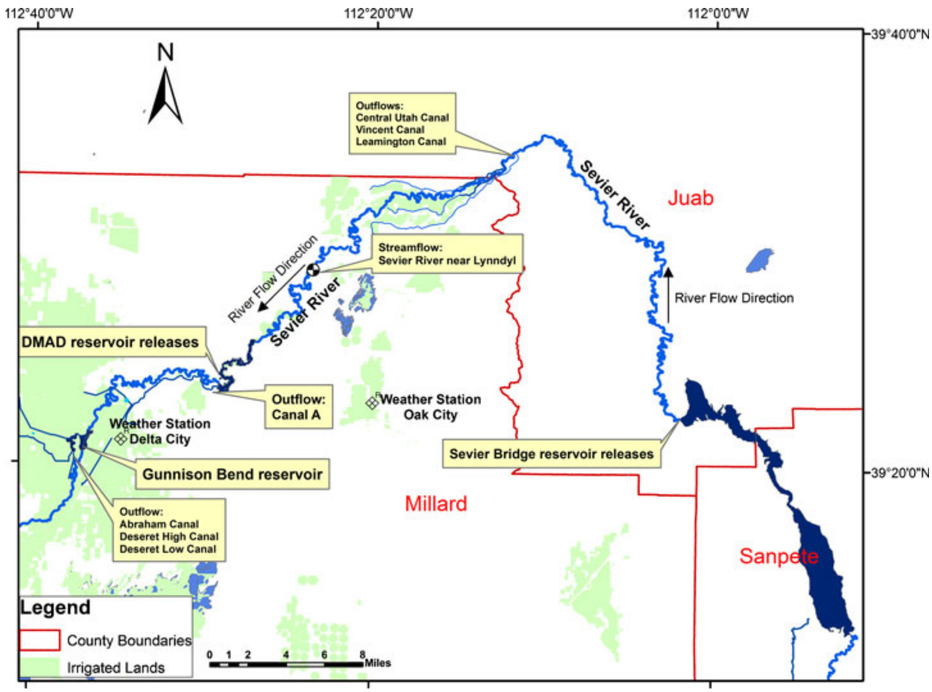

Fig. 1 Lower Sevier River Basin and the spatial distribution of the sensor station locations (reservoir releases, streamflows, inflows and outflows)

In this paper we focus on the Lower Sevier River Basin, which is regulated by three reservoirs: Sevier Bridge (or Yuba) Reservoir, the Delta-Millard Association Dam (DMAD) Reservoir, and Gunnison Bend Reservoir (Fig. 1).

4 Model Application to Multiple Reservoir System

The MVRVM previously described is applied to the multiple reservoir system located in the Lower Sevier River Basin. Monitoring data are posted hourly to the Sevier River website. This information enables real-time operations of reservoir releases and canal diversions. Daily averages taken from the Sevier River database and the United States Geological Survey (USGS) website (http://waterdata.usgs.gov/nwis) from 2001 to 2007 were used to build the MVRVM reservoir model. Daily data from the irrigation seasons of 2001 through 2006 were used to train the MVRVM and find the model parameters. Daily data from the 2007 irrigation season were used to test the model.

The inputs are available past daily data collected by sensors on the reservoirs, canals, diversions, weather and the river itself. The multiple output target vectors are the predictions of water releases 1 and 2 days ahead from Sevier Bridge and DMAD reservoirs.

The USGS gauging station located on the Sevier River near Juab measures the releases from Sevier Bridge Reservoir into the Sevier River, and the station located on the Sevier River bellow DMAD Reservoir measures the releases from DMAD reservoir into the Sevier River (Fig. 1).

Four sensor stations are located between Sevier Bridge and DMAD reservoirs: three measure diversions to irrigation canals, and one USGS gauging station measures streamflow on the Sevier River near Lynndyl. One measure diversion is made from DMAD reservoir to an irrigation canal called canal A. Three diversions from Gunnison Bend reservoir are measured, corresponding to irrigation releases to serve the Abraham canal, the Deseret High canal, and the Deseret Low canal.

Daily maximum and minimum temperature from Delta city wais obtained from The Community Environmental Monitoring Program (CEMP) website, and daily maximum and minimum temperature from Oak City was obtained from The National Oceanic and Atmospheric Administration (NOAA).

Inputs to the MVRVM model also include past data on daily Sevier Bridge reservoir releases and data for DMAD Reservoir releases.

The inputs used in the model to predict reservoir releases are expressed as:

$$x = \left[X1_{d-nd}, X2_{d-nd}, X3_{d-nd}, X4_{d-nd}, X5_{d-nd}\right]^T \tag{9}$$

where,

d	day of prediction
nd	number of days previous to the prediction time
$\mathbf{X1}_{d-nd}$	diversions to the Central Utah canal, Vincent canal, Leamington canal, canal A, Abraham canal, Deseret High canal, and Deseret Low canal.
$\mathbf{X2}_{d-nd}$	streamflow on the Sevier River near Lynndyl.
$\mathbf{X3}_{d-nd}$	Sevier Bridge reservoir releases.
$\mathbf{X4}_{d-nd}$	DMAD reservoir releases.
$\mathbf{X5}_{d-nd}$	maximum and minimum daily Temperature from Oak City and Delta City.

The multiple output target vector of the model is expressed as:

$$t = \left[R1_d, R1_{d+1}, R2_d, R2_{d+1}\right]^T \tag{10}$$

where,

$\mathbf{R1}_d$	prediction of Sevier Bridge reservoir release 1 day ahead
$\mathbf{R1}_{d+1}$	prediction of Sevier Bridge reservoir release 2 days ahead
$\mathbf{R2}_d$	prediction of DMAD reservoir releases 1 day ahead
$\mathbf{R2}_{d+1}$	prediction of DMAD reservoir releases 2 day ahead

Finally, the model can be defined as in Eq. 1 with a data set of input–output pairs $\left\{x^{(n)}, t^{(n)}\right\}_{n=1}^{N}$, where N is the number of observations.

5 Results and Discussion

5.1 Model Selection

In Eq. 1, the basis function ($\boldsymbol{\Phi}$) is defined in terms of a fixed kernel function. It is necessary to choose the type of kernel function and also to determine the values for its associated parameter, the kernel width (Tipping 2001). As mentioned previously, the MVRVM model automatically sets the majority of its parameters (i.e., hyperparameters $\boldsymbol{\alpha}$ and noise parameters $\boldsymbol{\sigma}^2$). The kernel width is the only model parameter that must be set by the user. This is an important advantage if we want to compare the proposed MVRVM model with a SVM model where three parameters (the kernel width, cost parameter and the error-insensitive parameter) must be determined by using additional procedures. For example, Hong and Pai (2007) used a simulated annealing algorithm (SA) to optimize the SVM parameters for rainfall forecasting. Also, Pal and Goel (2007) carried out a large number of trials to find suitable values of SVM parameters to estimate discharge and end depth in a trapezoidal channel.

The statistics used for the selection of the model are the coefficient of efficiency (E) and the correlation coefficient (R). E is equal to 1 minus the ratio of the mean square error to the variance in the observed data. This statistic ranges from minus infinity (poor model) to 1.0 (a perfect model) (Legates and McCabe 1999). The R value measures the correlation between observed and predicted reservoir release.

Several MVRVM models were built with variation in the type of kernel, kernel width and the number of days previous to the prediction time (from 1 to 5 days). The selected model was the one with the maximum E of the average outputs corresponding to the testing phase. Table 1 shows the model selected for each type of kernel. The average results from the four kernels are equally good. The model with the Gaussian kernel shows slightly higher accuracy of the average results than the others with different kernel types. Also, the Gaussian kernel has been used by Thayananthan et al. (2008) and several authors in water resources and hydrology applications (Khalil et al. 2005b; Tripathi and Govindaraju 2006). Therefore this type of kernel was selected for the MVRVM model.

Table 1 Selected MVRVM for each type of kernel function

Testing phase								
Type of kernel function	Kernel width	Number of days	Statistics	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$	Average
Gauss	2,900	2	E	0.947	0.868	0.930	0.805	0.888
			R	0.973	0.932	0.968	0.909	0.946
Laplace	3,100	1	E	0.925	0.841	0.900	0.780	0.861
			R	0.964	0.917	0.952	0.892	0.931
Cauchy	2,900	3	E	0.943	0.866	0.935	0.800	0.886
			R	0.972	0.931	0.969	0.903	0.944
Cubic	1,700	2	E	0.936	0.842	0.927	0.813	0.879
			R	0.968	0.918	0.966	0.914	0.942

5.2 Performance Evaluation

Figure 2 illustrates the training phase of Sevier Bridge reservoir release prediction 1 day ahead. The training phase of release predictions for Sevier Bridge reservoir 2 days ahead and DMAD reservoir 1 and 2 days ahead are shown in Appendix 1.

In the RVM approach, the relevance vectors (RVs) are subsets of the training data set that are used for prediction (Khalil et al. 2005a); as a consequence, the complexity of the model is proportional to the number of RVs. The model only utilizes 39 RVs from the full data set (1248 observations) that was used for training (2001 through 2006 irrigation seasons). This low number of vectors illustrates that the Bayesian learning procedure embodied in the MVRVM is capable of producing very sparse models.

The RVs are the summary of the most essential features (observations) of the training data set to build the MVRVM (Khalil et al. 2005a). The MVRVM identifies the greatest number of RVs (10 RVs) from the 2006 irrigation season (Fig. 2f), and the lowest number (1 RVs) from the 2004 season (Fig. 2d). The 2006 irrigation season has the largest number of relevance observations, while the majority of the observations from the 2004 irrigation season have been ignored to build the model.

The predicted outputs of the MVRVM for the testing phase (2007 irrigation season) are shown as the full lines in Fig. 3. The figure shows good performance of the machine. The model explains well the observed releases (dots) for the releases 1 day ahead for both reservoirs. The releases 2 days ahead from Sevier Bridge Reservoir also illustrated good performance (Fig. 3a, b, and c). The performance accuracy decreases for DMAD Reservoir releases 2 days ahead (Fig. 3d). This decrease in accuracy is found in most of the multiple-time-ahead prediction models, where the further ahead we predict into the future, the less accurate the prediction becomes.

Figure 3 also shows the 0.90 confidence interval (shaded region) associated with the predictive variance of the MVRVM in Eq. 8. The confidence intervals for the 2-day-ahead prediction (Fig. 3b and d) become wider than the confidence interval for their corresponding 1-day-ahead predictions for both reservoirs (Fig. 3a and c). We can see how the uncertainty in the predictions increases when predicting further into the future.

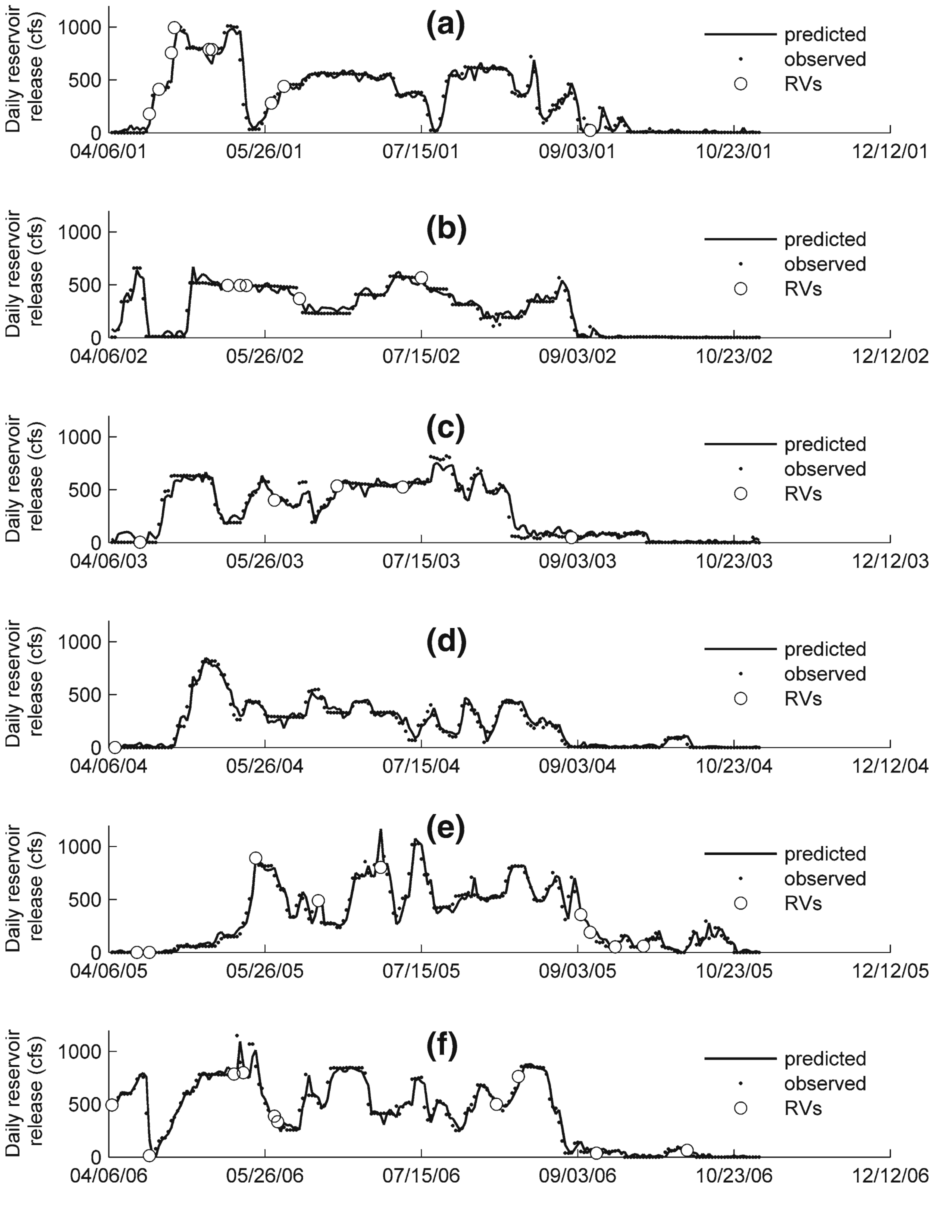

Fig. 2 Plot of observed versus predicted releases of Sevier Bridge reservoir 1 day ahead, and RVs of the MVRVM. Training phase (2001 (**a**)–2006 (**f**) irrigation seasons)

Table 2 shows some statistics to measure MVRVM performance for both the training and testing phases. Again, we can see good performance of the machine in the testing phase for the 1-day-ahead prediction for both reservoir releases ($R1_d$ and $R2_d$) with a high E, 0.95 and 0.93, respectively, for Sevier Bridge and DMAD reservoirs; and the 2-day-ahead prediction for Sevier Bridge Reservoir

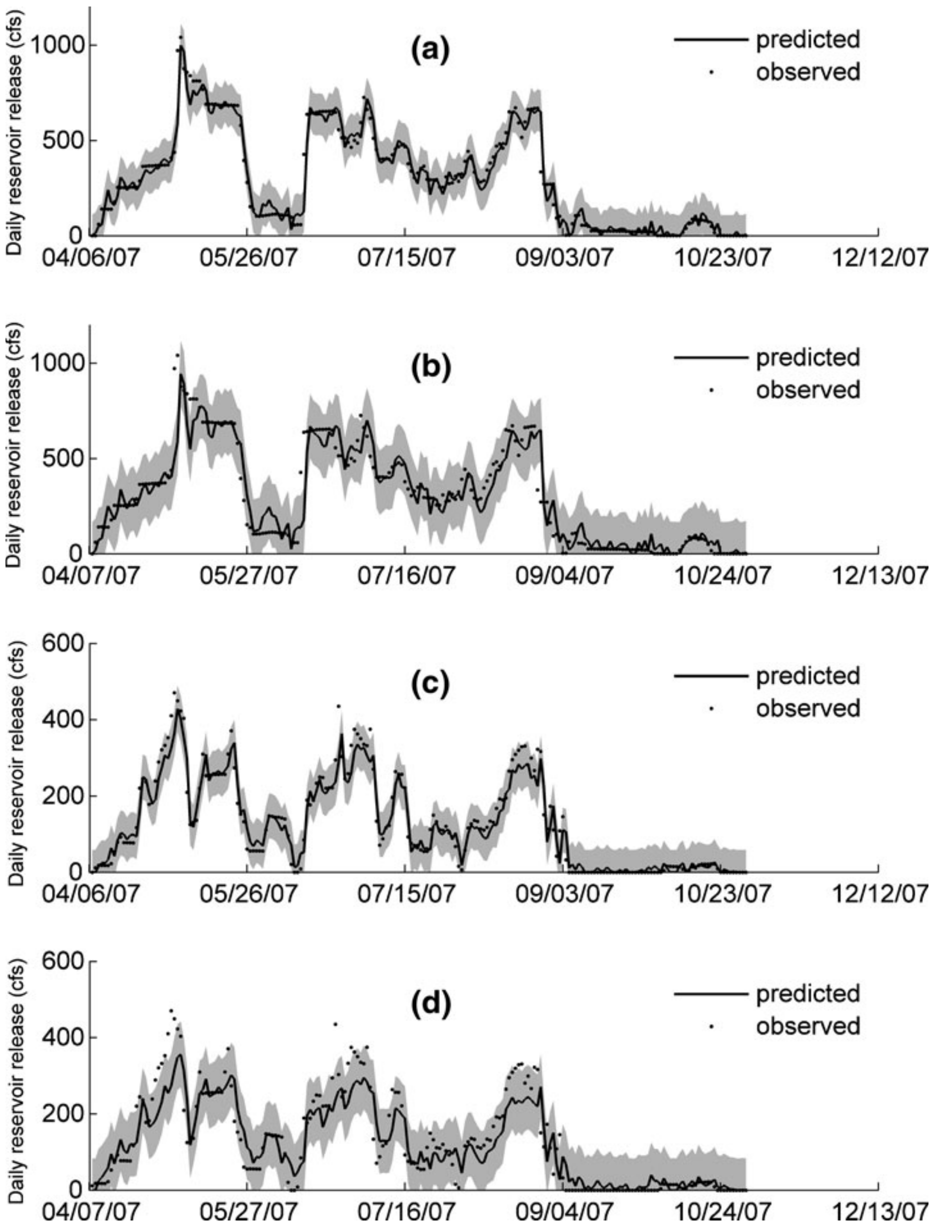

Fig. 3 Observed versus predicted releases of the MVRVM with 0.90 confidence intervals (*shaded region*) for the testing phase (2007 irrigation season): **a** prediction of Sevier Bridge reservoir releases 1 day ahead, **b** prediction of Sevier Bridge reservoir releases 2 days ahead, **c** prediction of DMAD reservoir releases 1 day ahead, **d** prediction of DMAD reservoir releases 2 days ahead

releases ($R1_{d+1}$) also has a high E, 0.87. The performance accuracy is reduced for the 2-day-ahead prediction of DMAD Reservoir releases ($R2_{d+1}$) with a lower E of 0.80.

Table 2 MVRVM performance using different statistics

Statistics	Multivariate relevance vector machine							
	Training				Testing			
	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$
Coefficient of efficiency E	0.95	0.87	0.89	0.77	0.95	0.87	0.93	0.80
Correlation coefficient R	0.97	0.93	0.94	0.88	0.97	0.93	0.97	0.91
Root mean square error RMSE, cfs	64.42	99.24	34.84	49.66	59.45	93.82	32.55	54.49

5.3 Bootstrap Analysis

In order to avoid overfitting and evaluate the performance of machine learning models, several authors in hydrology and water resources modeling research (El-Shafie et al. 2009; Rezaeian Zadeh et al. 2010; Shirsath and Singh 2010; Trichakis et al. 2010) calibrated their models with one training data set and evaluated the performance of their models with a different unseen test data set. However, it is necessary to develop models to guarantee not only high accuracy during the test period but also good generalization and robustness of model parameter estimation with respect to future changes on the nature of the input data. Changes in the training data used to build a model may give different test results. Different sets of training data may produce models with very different generalization accuracies. The bootstrap method (Efron and Tibshirani 1998) was used to explore the implications of the change in the nature of input data and to guarantee good generalization ability and robustness of the MVRVM (Khalil et al. 2005b).

The bootstrap data set was generated by randomly selecting (with replacement) from the whole training data set. Because the selection is from the whole training data set, there is nearly always duplication of individual points in a bootstrap data set. In this paper, this selection process was independently repeated 1,000 times to yield 1,000 bootstrap training data sets, which are treated as independent sets (Duda et al. 2001). For each of the bootstrap training data sets, a model was trained and evaluated over the original test data set. Figure 4 shows the bootstrap histograms based on 1,000 bootstrap training data sets of the E and RMSE test.

Efron and Tibshirani (1998) emphasized that it is always wise to look at the bootstrap data graphically, rather than relying entirely on a single summary statistic estimator. The bootstrap method provides information on the uncertainty in the statistics estimator evaluated in the model. The width of the bootstrapping confidence intervals provides information on the uncertainty in the model parameters. A narrow confidence interval implies low variability of the statistics with respect to possible future changes in the nature of the input data, which indicates that the model is robust (Khalil et al. 2005b). According to Khalil et al. (2005b) a robust model is one that shows narrow confidence bounds in the bootstrap histogram, such as those illustrated in Fig. 4.

5.4 Comparison Between MVRVM and ANN

ANNs have been widely applied in hydrology and water resources modeling (ASCE Task Committee on the Application of ANNs in Hydrology 2000a, b; Khalil et al.

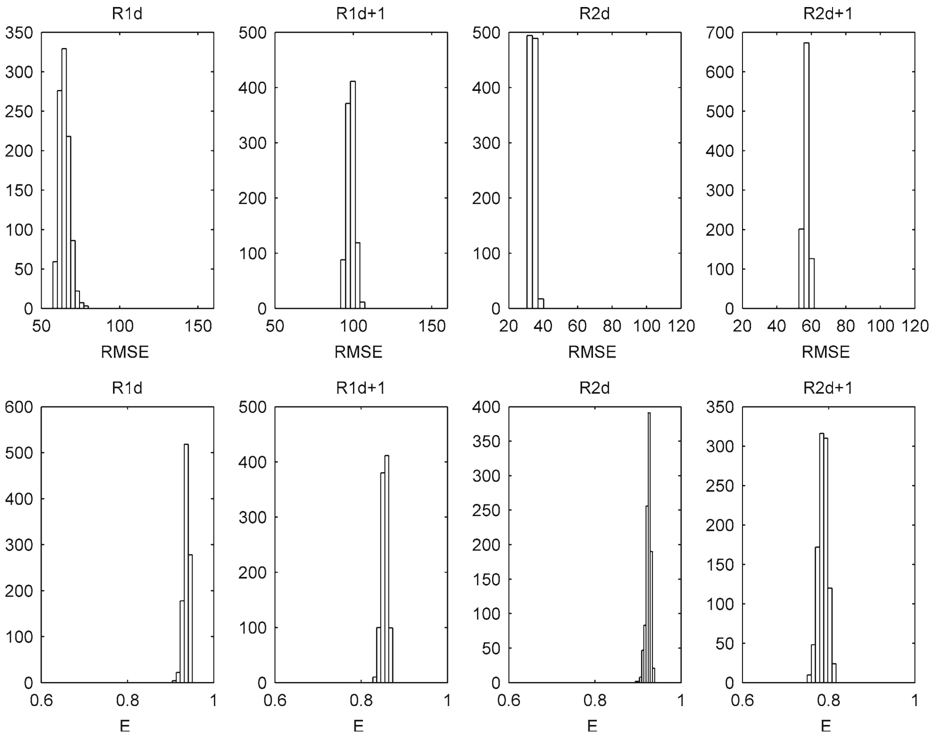

Fig. 4 Bootstrap histogram of the MVRVM model for the RMSE and E test

2005d; Adeloye 2009). A comparative analysis between the developed MVRVM and ANNs is performed in terms of performance and robustness. Readers interested in greater detail regarding ANNs and their training functions are referred to Demuth et al. (2009).

The ANN model selection is similar to the MVRVM model selection described in Section 5.1. Several feed-forward ANN models were built using different types

Table 3 Selected ANN model for each type of training function

Testing phase								
Type of training function	Size of layer	Number of days	Statistics	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$	Average
Quasi-Newton	3	2	E	0.942	0.867	0.898	0.777	0.871
			R	0.971	0.932	0.952	0.891	0.936
Conjugate gradient with Powell-Beale restarts	3	4	E	0.928	0.846	0.922	0.821	0.879
			R	0.964	0.922	0.961	0.908	0.939
Levenberg-Marquardt	3	5	E	0.917	0.860	0.923	0.811	0.878
			R	0.959	0.929	0.963	0.908	0.940
Scaled conjugate gradient	4	2	E	0.943	0.856	0.928	0.834	0.890
			R	0.972	0.929	0.966	0.917	0.946

of training function, sizes of layer, and numbers of days previous to the prediction time (from 1 to 5 days). The selected model was the one with the maximum E of the average outputs corresponding to the testing phase. Table 3 shows the selected model

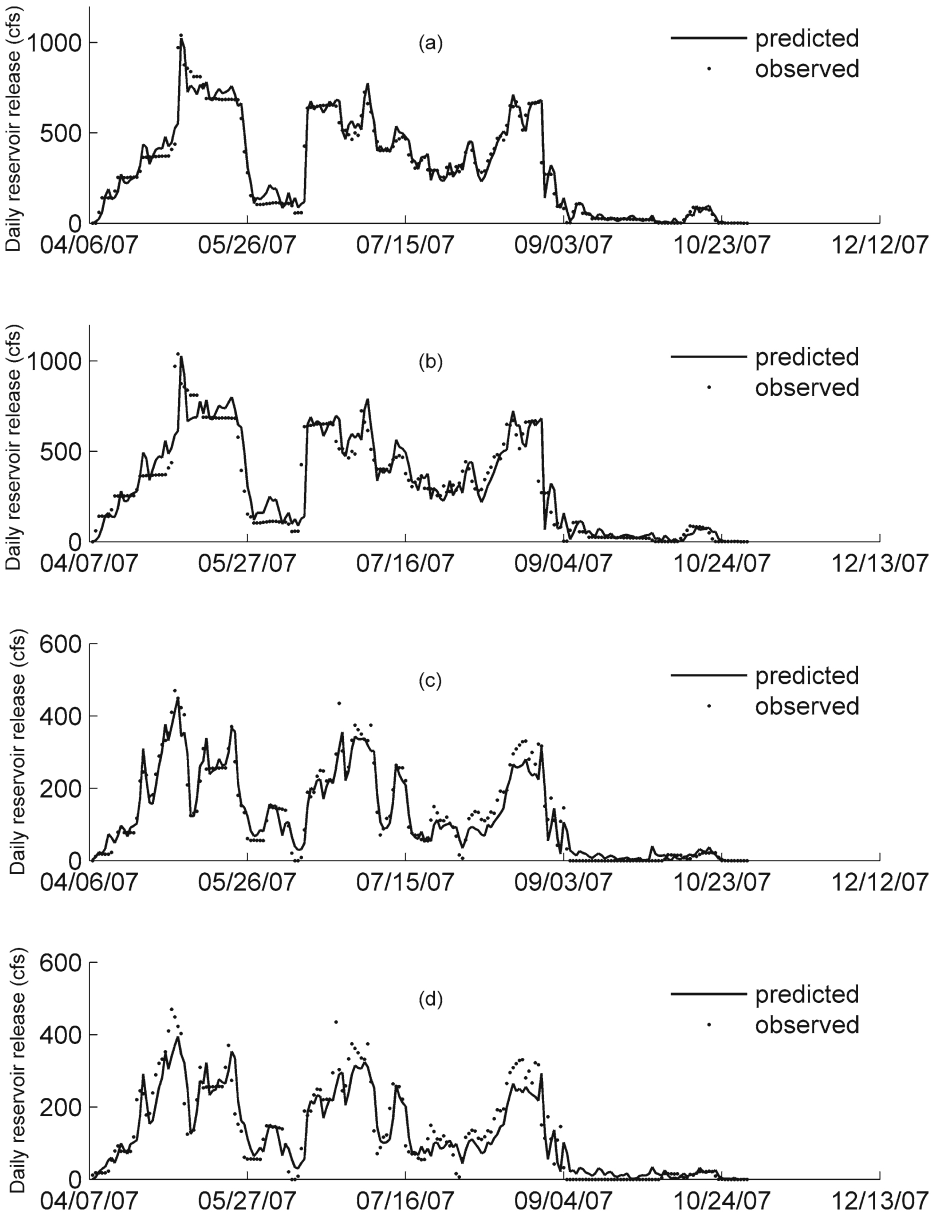

Fig. 5 Observed versus predicted releases of the ANN for the testing phase (2007 irrigation season): **a** prediction of Sevier Bridge reservoir releases 1 day ahead, **b** prediction of Sevier Bridge reservoir releases 2 days ahead, **c** prediction of DMAD reservoir releases 1 day ahead, **d** prediction of DMAD reservoir releases 2 days ahead

Table 4 ANN Performance using different Statistics

Statistics	Artificial neural network							
	Training				Testing			
	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$	$R1_d$	$R1_{d+1}$	$R2_d$	$R2_{d+1}$
Coefficient of efficiency E	0.96	0.90	0.89	0.79	0.94	0.86	0.93	0.83
Correlation coefficient R	0.98	0.95	0.94	0.89	0.97	0.93	0.97	0.92
Root mean square error RMSE, cfs	55.47	88.21	34.50	48.29	61.77	98.11	33.02	50.29

for each type of training function. The ANN model with the scaled-conjugated-gradient-training function shows slightly higher E and R of the average results than do the other ANN models with different training functions. Therefore this training function was chosen for the ANN model.

The observed (dots) and predicted (full lines) outputs of the ANN for the testing phase (2007 irrigation season) are shown in Fig. 5. Table 4 shows the ANN performance for both the training and testing phases. From Table 4 we can see that the performance results are fairly similar to the MVRVM performance (Table 2).

Figure 6 shows the bootstrap histograms based on 1,000 bootstrap training data sets of the ANN model for the RMSE and E test. The bootstrapped histograms of

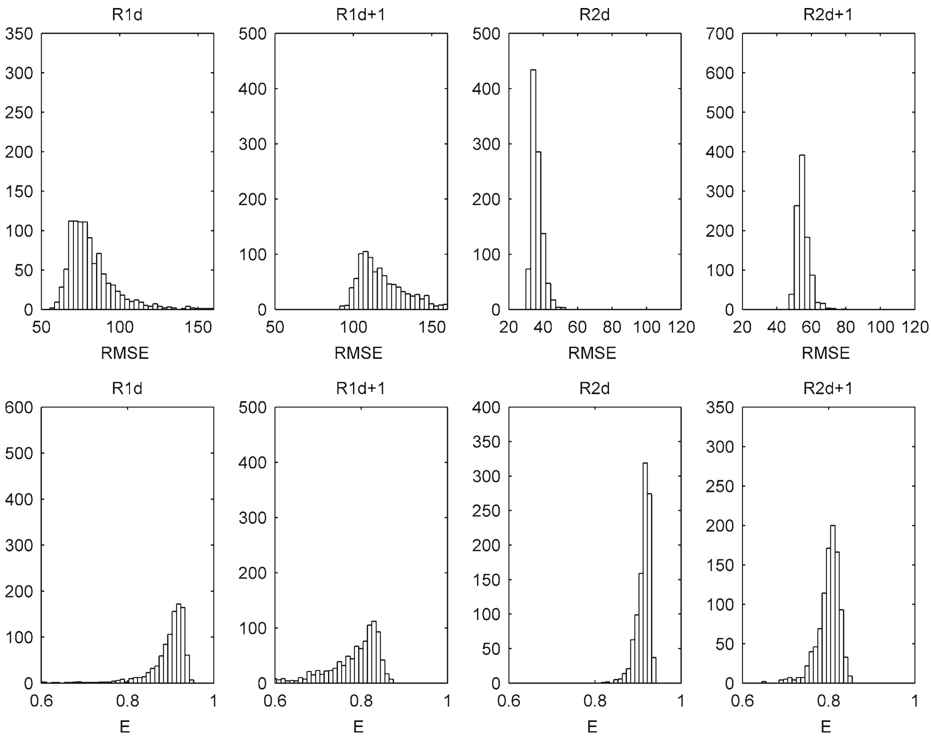

Fig. 6 Bootstrap histogram of the ANN model for the RMSE and E test

the MVRVM model (Fig. 4) show very narrow confidence bounds in comparison to the histograms of the ANN model (Fig. 6). Therefore, the MVRVM appears to be more robust.

6 Summary and Conclusions

This paper presents a first attempt to use a MVRVM model to develop multiple-time-ahead predictions of daily releases from a multiple reservoir system. The MVRVM is a regression tool extension of the RVM algorithm to produce multivariate outputs (with predictive confidence intervals) when given a set of inputs. The model is illustrated by application to the Lower Sevier River near Delta, Utah. The predictions are water releases 1 and 2 days ahead from Sevier Bridge and DMAD reservoirs.

The results show that the model learns the input–output patterns with high accuracy consistent with the statistics for the test results. The statistical results indicate good performance of the model for the 1-day prediction for the releases of Sevier Bridge and DMAD reservoirs. The performance decreased slightly for the 2-day prediction of DMAD reservoir release.

The MVRVM model has the property of sparse formulation. The model only utilizes 39 RVs from the full data set (out of a possible 1248 observations) that was used for training. The parsimonious structure of this empirical model is sufficient to explain the data and to avoid data over-fitting. Therefore, we can see an important advantage of the Bayesian learning procedure, which is the capability of the MVRVM to produce very sparse models.

Another important advantage of utilizing MVRVM is its generalization capabilities while achieving sparse representation. Generalization ability is associated with the capability of the model to predict future system states when presented with a range of input vectors. Multiple reservoir system operation could become a difficult process to predict since this involves many decisions for real-time water resources management. The model presented here ensures good generalization providing robustness with new oncoming data.

The performance results are fairly similar by both the MVRVM and ANN. Bootstrap analysis is used to explore the robustness of the models. Narrow confidence bounds in the bootstrap histograms imply low variability of the test statistics when presented with a range of input vectors, which indicates that the model is robust. The bootstrap histograms show that the MVRVM model is more robust than the ANN model.

In summary, the results presented in this paper have demonstrated the successful performance and robustness of MVRVM for multiple reservoir release forecasts. Simultaneous multiple-time-ahead release predictions from a multiple reservoir system have potential value to assist the reservoir operator in efficiently selecting the real-time operation and management decisions for available water resources.

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Appendix 1

The Appendix 1 gives the training phase plots of release predictions for Sevier Bridge reservoir 1 day ahead (Fig. 7), and DMAD reservoir 1 and 2 days ahead (Figs. 8 and 9, respectively).

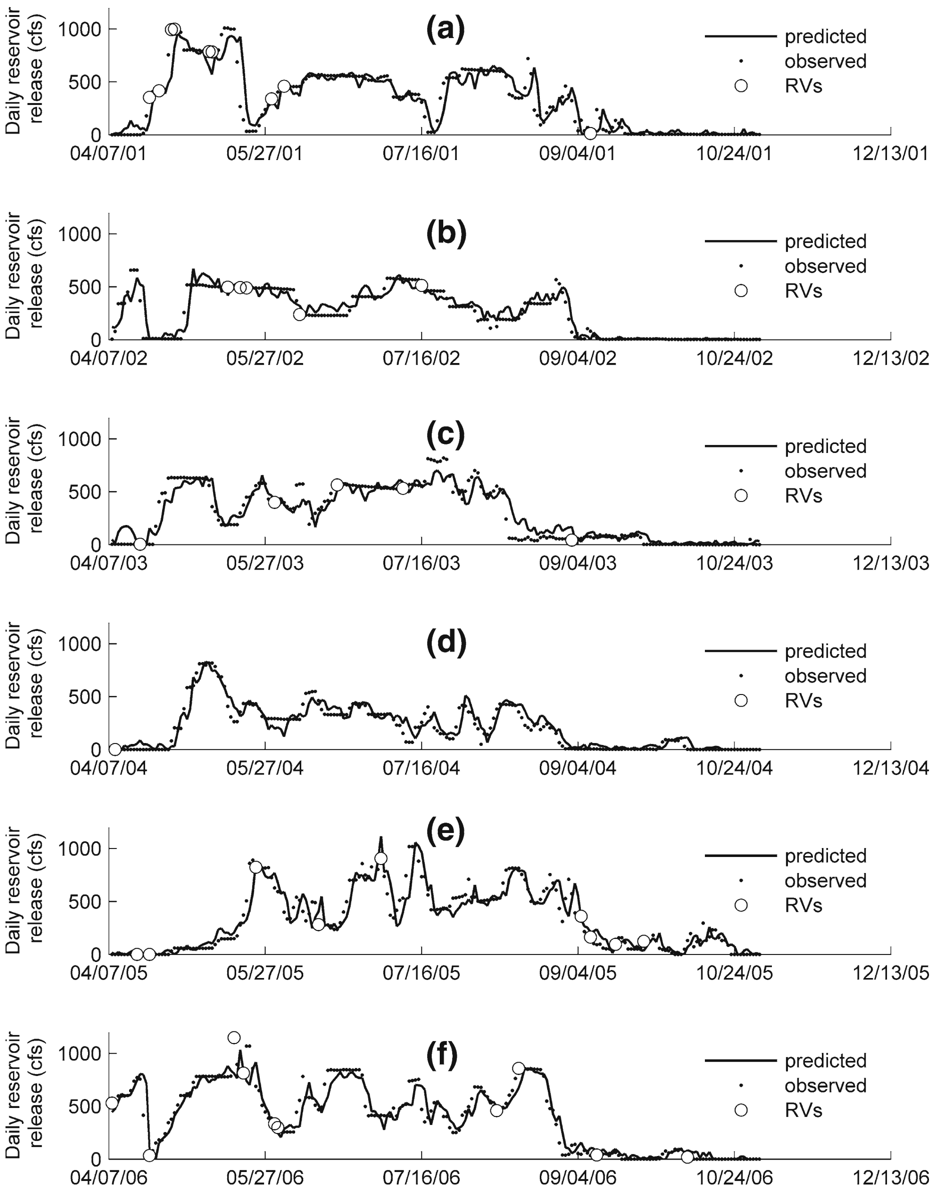

Fig. 7 Plot of observed versus predicted releases of Sevier Bridge reservoir 2 days ahead, and RVs of the MVRVM. Training phase (2001 (**a**)–2006 (**f**) irrigation seasons)

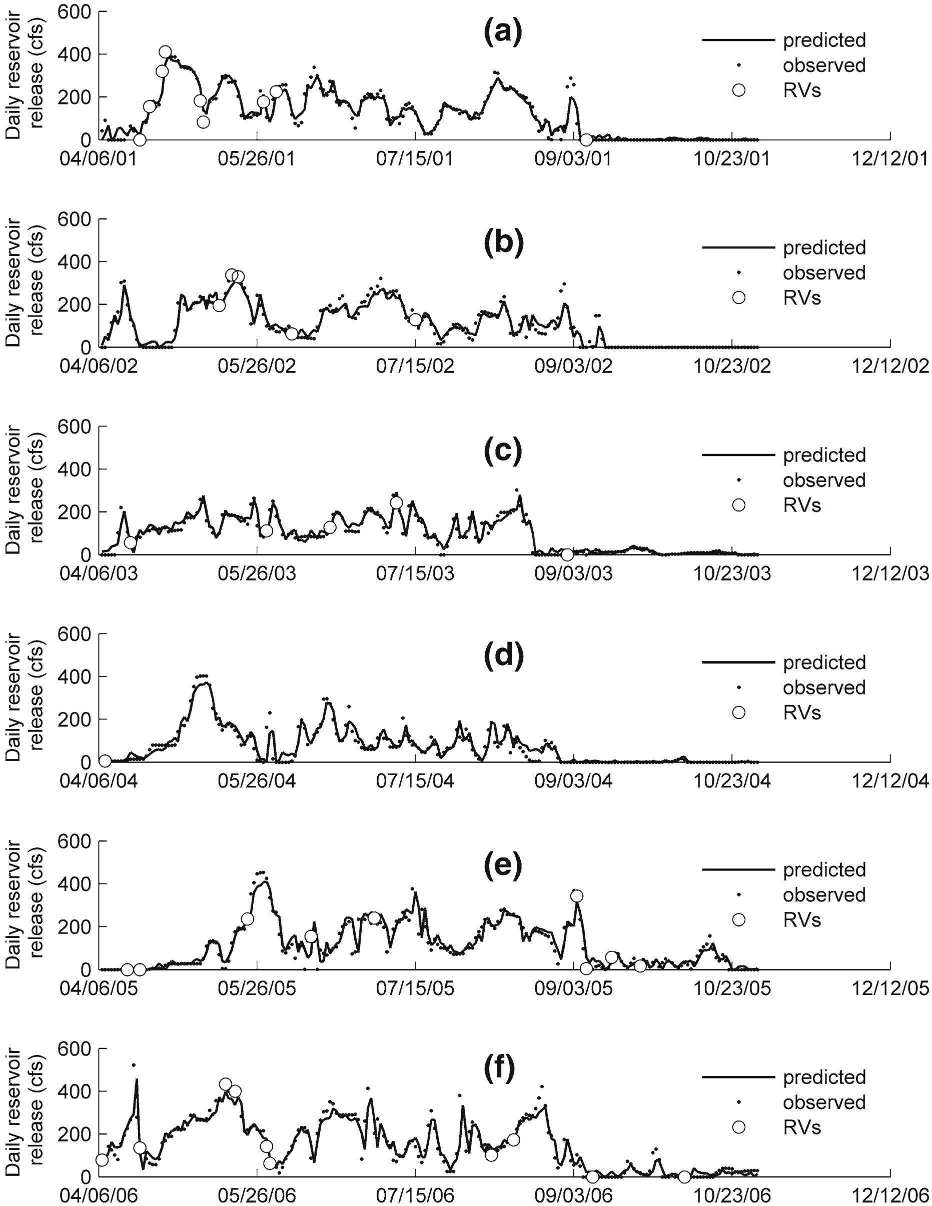

Fig. 8 Plot of observed versus predicted releases of DMAD reservoir 1 day ahead, and RVs of the MVRVM. Training phase (2001 (**a**)–2006 (**f**) irrigation seasons)

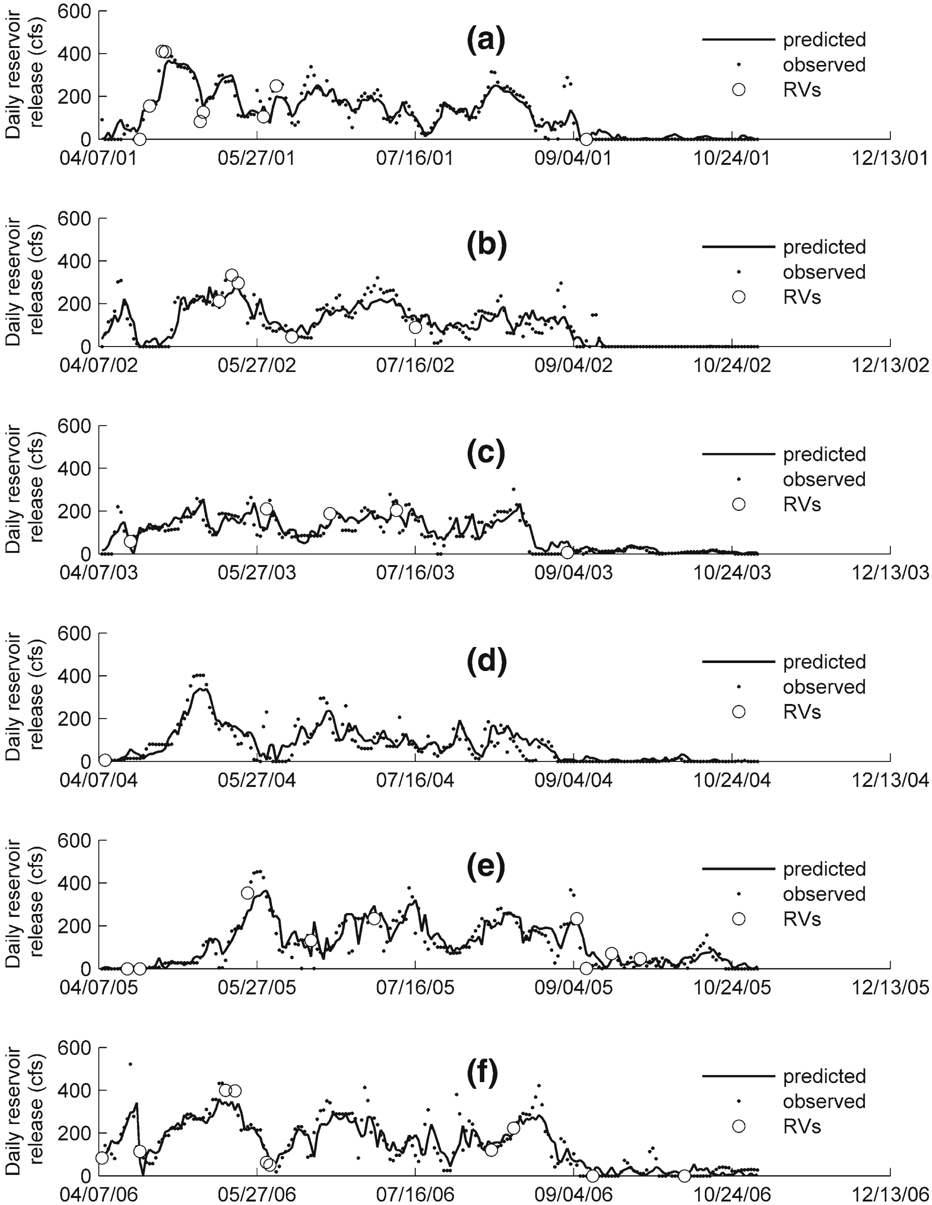

Fig. 9 Plot of observed versus predicted releases of DMAD reservoir 2 days ahead, and RVs of the MVRVM. Training phase (2001 (**a**)–2006 (**f**) irrigation seasons)

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